

1 Calcolo di limiti

Calcolare i seguenti limiti:

$$1 \quad \lim_{n \rightarrow +\infty} \left(\frac{3n+5}{n+3} \right)^5$$

$$2 \quad \lim_{x \rightarrow +\infty} \frac{\sin x + x}{x^3 + 3}$$

$$3 \quad \lim_{x \rightarrow +\infty} \left(\frac{x^3}{x^2 + 1} - x \right)$$

$$4 \quad \lim_{x \rightarrow +\infty} x(\sin x - 2)$$

$$5 \quad \lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1}$$

$$6 \quad \lim_{n \rightarrow +\infty} \frac{3n^2 + 5n + 1}{5n^2 + 2n + 7}$$

$$7 \quad \lim_{n \rightarrow +\infty} \frac{2^n + (-1)^n}{2^n}$$

$$8 \quad \lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x^2 - 6x + 9}$$

$$9 \quad \lim_{x \rightarrow +\infty} \left(\sqrt{2+x^3} - \sqrt{1+2x^2+x^3} \right)$$

$$10 \quad \lim_{x \rightarrow +\infty} \left(\sqrt[3]{2+x^3} - \sqrt[3]{1+2x^2+x^3} \right)$$

$$11 \quad \lim_{x \rightarrow +\infty} \frac{\log_2(e^x + 1)}{x + \sin x}$$

$$12 \quad \lim_{x \rightarrow 0} \frac{\cos x^3 - 1}{\sin x^5}$$

$$13 \quad \lim_{t \rightarrow +\infty} \frac{\sqrt{t^6 + 1} \left(\cos \frac{2}{t} - 1 \right)}{t}$$

$$14 \quad \lim_{x \rightarrow +\infty} \frac{\ln(1 + 2e^{x^3})}{x^3}$$

$$15 \quad \lim_{x \rightarrow 0} \frac{2 - 2 \cos x}{\sin x^2 - 3 \sin x - x^2}$$

$$16 \quad \lim_{x \rightarrow 0} \frac{\log_a(x+2) - \log_a 2}{x}$$

$$17 \quad \lim_{x \rightarrow 0} \frac{e^{\operatorname{tg}^3 x} - 1}{x(\cos x - e^{x^2})}$$

$$18 \quad \lim_{t \rightarrow +\infty} \frac{t^3 \log \left(1 + \frac{3}{t^2} \right)}{\sqrt{t^2 + 1}}$$

$$19 \quad \lim_{t \rightarrow +\infty} \frac{\sqrt{t^6 + 1} \left(\cos \frac{2}{t} - 1 \right)}{t}$$

$$20 \quad \lim_{x \rightarrow 0} \frac{\log(1 + \arctg x)^x}{e - e^{\cos^4 x}}$$

$$21 \quad \lim_{x \rightarrow 0^+} (1 + \sin(x^\alpha))^{1/x} \text{ al variare di } \alpha > 0$$

$$22 \quad \lim_{x \rightarrow 0} \frac{(\cos x)^{\tan x} - 1}{x^3},$$

$$23 \quad \lim_{x \rightarrow +\infty} x^2(e^{\frac{1}{x}} - 1)^{\frac{3}{2}}$$

$$24 \quad \lim_{x \rightarrow +\infty} x^{\frac{1}{3}}(e^{\frac{1}{x}} - 1)^{\frac{1}{2}}$$

$$25 \quad \lim_{x \rightarrow 0} \sin(x^{\frac{2}{5}}) \frac{\log(1 + x^{\frac{1}{5}})}{4x^{\frac{3}{5}}}$$

$$26 \quad \lim_{x \rightarrow +\infty} \sin\left(\frac{1}{x^{\frac{7}{2}}}\right) \left(\log\left(1 + \frac{e}{x^{\frac{2}{7}}}\right) \right)^{-2}$$

$$27 \quad \lim_{x \rightarrow +\infty} \sqrt{x} \sqrt{\left(1 + \frac{1}{x}\right)^{\frac{1}{4}} - 1}$$

$$28 \quad \lim_{x \rightarrow 0} \frac{(1 + x^{\frac{3}{2}})^{\frac{2}{3}} - 1}{e^{x^{\frac{3}{2}}} - 1}$$

$$29 \quad \lim_{x \rightarrow 0} \frac{|4^x - 1 - (\log 2) \sin x|}{x^3 - |x|^2}$$

$$30 \quad \text{Determinare } c \text{ in } \mathbb{R} \text{ tale che } \lim_{x \rightarrow +\infty} \left(\frac{x+c}{x-c} \right)^x = 4.$$

$$31 \quad \lim_{n \rightarrow +\infty} \operatorname{tg} \frac{n\sqrt{n} + 1}{2n\sqrt{n} - \sqrt{n}}$$

$$32 \quad \lim_{n \rightarrow +\infty} \frac{n \sin n + 5}{n^2 + \sqrt{n}}$$

$$33 \quad \lim_{n \rightarrow +\infty} (2^n - n)$$

$$34 \quad \lim_{n \rightarrow +\infty} \frac{2^n + 4^n}{3^{n+1} + 5^n}$$

$$35 \quad \lim_{n \rightarrow +\infty} \frac{n2^n}{3^n}$$

$$36 \quad \lim_{n \rightarrow +\infty} \frac{n^2}{n^2 + (\log n)^{1000} - n \sin(n^{500})}$$

$$37 \quad \lim_{n \rightarrow +\infty} \frac{n \log n}{2^n}$$

$$38 \quad \lim_{n \rightarrow +\infty} e^n \sin \frac{1}{n^{100}}$$

$$39 \quad \lim_{n \rightarrow +\infty} \frac{n(1 - \cos \frac{1}{n})}{\sin \frac{1}{n}}$$

$$40 \quad \lim_{n \rightarrow +\infty} \frac{n+1}{\sqrt{n}+2} \operatorname{tg} \frac{1}{\sqrt{n}}$$

$$41 \quad \lim_{n \rightarrow +\infty} \operatorname{tg} \frac{\pi n^2}{4n^2 - 5n}$$

$$42 \quad \lim_{n \rightarrow +\infty} \operatorname{tg} \frac{\pi n^2}{2n^2 - 5n}$$

$$43 \quad \lim_{n \rightarrow +\infty} (3n+2) \operatorname{sen} \frac{\pi n - 5}{n+7}$$

$$44 \quad \lim_{n \rightarrow +\infty} (2n+5) \cos \frac{\pi n^2 + 7}{2n^2 + n}$$

$$45 \quad \lim_{n \rightarrow +\infty} \frac{n^4}{n+1} \operatorname{sen} \frac{\log n}{n^4}$$

$$46 \quad \lim_{n \rightarrow +\infty} \frac{(n+1)^{n+1}}{(n+2)^n} \operatorname{sen} \frac{1}{n}$$

$$47 \quad \lim_{n \rightarrow +\infty} \left(1 - \frac{1}{n^2}\right)^n$$

$$48 \quad \lim_{n \rightarrow +\infty} \left(1 - \frac{1}{n}\right)^{n^2}$$

$$49 \quad \lim_{n \rightarrow +\infty} \left(1 + \sin^2 \frac{1}{n}\right)^n$$

$$50 \quad \lim_{n \rightarrow +\infty} \cos^n \frac{1}{n}$$

$$51 \quad \lim_{n \rightarrow +\infty} \cos^{n^2} \frac{1}{n}$$

$$52 \quad \lim_{n \rightarrow +\infty} \left(1 + \sin \frac{1}{n}\right)^n$$

$$53 \quad \lim_{n \rightarrow +\infty} (n^{\sqrt{n}} - 2^n)$$

$$54 \quad \lim_{n \rightarrow +\infty} \sqrt{n} \left(\operatorname{sen} \frac{2}{\sqrt{n}} - \frac{1}{\sqrt{4n+3}} \right)$$

$$55 \quad \lim_{n \rightarrow +\infty} n \left(1 - \cos \frac{1}{\sqrt{n}} + e^{-n} \right)$$

$$56 \quad \lim_{n \rightarrow +\infty} \left(\frac{\log(2n) + 1}{\log(2n) - 5} \right)^{e^n}$$

$$57 \quad \lim_{n \rightarrow +\infty} \left(\frac{4 - 3 \log n}{2 - 3 \log n} \right)^{e^{n+2}}$$

$$58 \quad \lim_{n \rightarrow +\infty} \left(\frac{1}{3} \log(n^2 + 1) - \log \sqrt[3]{n^2 + 2} \right) \operatorname{sen} n$$

$$59 \quad \lim_{n \rightarrow +\infty} \frac{n+1}{\sqrt{n}(n+2)} (\operatorname{sen} n - \cos n)$$

$$60 \quad \lim_{n \rightarrow +\infty} \left(\cos \frac{1}{n^2} \right)^{\sqrt{4n^8 + 7n + 3}}$$

$$61 \quad \lim_{n \rightarrow +\infty} \frac{n}{\log n} (n^{1/n} - 1)$$

$$62 \quad \lim_{n \rightarrow +\infty} \frac{1}{n} (\sqrt{n^2 + 2} - \sqrt{2n^2 + 5})$$

$$63 \quad \lim_{n \rightarrow +\infty} (\sqrt[4]{n+1} - \sqrt[4]{n+5})$$

$$64 \quad \lim_{n \rightarrow +\infty} \frac{n^{\frac{5}{2}} + n^2 (\log n)^3 - n (\log n)^5}{\log \log 2n + n^{\frac{1}{2}} (n^4 + \log n)^{\frac{1}{2}}}$$

$$65 \quad \lim_{n \rightarrow +\infty} \frac{n^{(\log n)^3} + e^{(\log n)^2}}{1 + (\log n)^n}$$

$$66 \quad \lim_{n \rightarrow +\infty} \frac{(1 + e^{\frac{2}{3}n})^{\frac{3}{2}} + 3^n}{(1 + 2^n)^3 + e^n}$$

$$67 \quad \lim_{n \rightarrow +\infty} \left(\frac{n-5}{n+1} \right)^n$$

$$68 \quad \lim_{n \rightarrow +\infty} \frac{n}{\log(n^6 + \sqrt{n})}$$

$$69 \quad \lim_{n \rightarrow +\infty} \frac{n}{\log(1 + e^n)}$$

$$70 \quad \lim_{n \rightarrow +\infty} (\sqrt{n^2 + 5n} - n)$$

$$71 \quad \lim_{n \rightarrow +\infty} \frac{\sqrt{2n + (-1)^n}}{\sqrt{2n - (-1)^n}}$$

$$72 \quad \lim_{n \rightarrow +\infty} n^2 \operatorname{sen} \frac{2\pi}{(n+5)^2}$$

$$73 \quad \lim_{n \rightarrow +\infty} \sqrt[n]{e^n + \log n}$$

$$74 \quad \lim_{n \rightarrow +\infty} (n^{\sqrt{n}} - (\sqrt{n})^n)$$

$$75 \quad \lim_{n \rightarrow +\infty} \frac{3n^2 + (\cos n)n \log n}{(3+n)\sqrt{n^2 + 1}}$$

$$76 \quad \lim_{n \rightarrow +\infty} \left(2 - \frac{3}{n} \right)^{\log n}$$

$$77 \quad \lim_{n \rightarrow +\infty} \frac{\sqrt{n^4 + 3n^3} - \sqrt{n^4 - 2n^3}}{n}$$

$$78 \quad \lim_{x \rightarrow +\infty} x^{10} \ln(1 + 2e^{-x^3})$$

$$79 \quad \lim_{n \rightarrow +\infty} \frac{\sqrt[p]{p} - 1}{n-1}, \quad p > 0,$$

$$80 \quad \lim_{x \rightarrow +\infty} [x\sqrt{1+x^2} - \sqrt{x^2-1}],$$

$$81 \quad \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x} \right)^{\frac{1+x}{x}},$$

$$82 \quad \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^2},$$

$$83 \quad \lim_{x \rightarrow 0} \left(\frac{x^2 - 2x + 3}{x^2 - 3x + 2} \right)^{\frac{\sin x}{x}},$$

$$84 \quad \lim_{x \rightarrow \infty} (\sqrt{x^2 + 8x + 3} - \sqrt{x^2 + 4x + 3}),$$

$$85 \quad \lim_{x \rightarrow 1} \frac{x^3 - 6x^2 + 11x - 6}{x^2 - 3x + 2},$$

$$86 \quad \lim_{x \rightarrow -\infty} x^{-10} 3^{-(x+1)} 2^x,$$

$$87 \quad \lim_{x \rightarrow 0} \frac{\sin 2x}{\log(x+1)}.$$

2 Risposte ad alcuni esercizi

- | | |
|--|--|
| 1: 243; | 23: $+\infty$; |
| 2: 0; | 24: 0; |
| 3: 0; | 25: $\frac{1}{4}$; |
| 4: $-\infty$; | 26: $\frac{1}{e^2}$; |
| 5: 2; | 27: $\frac{1}{2}$; |
| 6: $\frac{3}{5}$; | 28: $\frac{2}{3}$; |
| 7: 1; | 29: $-\infty$; |
| 8: non esiste, il limite per $x \rightarrow 3^+$ vale $+\infty$, mentre il limite per $x \rightarrow 3^-$ vale $-\infty$; | 30: $c = \ln 2$; |
| 9: $-\infty$; | 31: $\operatorname{tg} \frac{1}{2}$; |
| 10: $-\frac{2}{3}$; | 32: 0; |
| 11: $\frac{1}{\ln 2}$; | 33: $+\infty$; |
| 12: 0; | 34: 0; |
| 13: -2; | 35: 0; |
| 14: 1; | 36: 1; |
| 15: 0; | 37: 0; |
| 16: $\frac{1}{2 \ln a}$; | 38: $+\infty$; |
| 17: $-\frac{2}{3}$; | 39: $\frac{1}{2}$; |
| 18: 3; | 40: 1; |
| 19: -2; | 41: 1; |
| 20: $\frac{1}{2e}$; | 42: $-\infty$; |
| 21: 1 se $\alpha > 1$, e se $\alpha = 1$, $+\infty$ se $0 < \alpha < 1$; | 43: $3(7\pi + 5)$; |
| 22: $-\frac{1}{2}$; | 44: $\frac{\pi}{2}$; |
| | 45: 0; |

46: $\frac{1}{e};$

70: $\frac{5}{2};$

47: $1;$

71: $1;$

48: $0;$

72: $2\pi;$

49: $1;$

73: $e;$

50: $1;$

74: $-\infty;$

51: $e^{-1/2};$

75: $3;$

52: $e;$

76: $+\infty;$

53: $-\infty;$

77: $\frac{5}{2};$

54: $\frac{3}{2};$

78: $0;$

55: $\frac{1}{2};$

86: $+\infty.$

56: $+\infty;$

57: $0;$

58: $0;$

59: $0;$

60: $e^{-1};$

62: $1 - \sqrt{2};$

63: $0;$

64: $1;$

65: $0;$

66: $0;$

67: $e^{-6};$

68: $+\infty;$

69: $1;$