Handout: The Cagan's model estimation

Cagan's original model consists of two equations, one which describes money market equilibrium and another which describes the evolution of inflation expectations over time (which are assumed to be adaptive):

$$m_t - p_t = -\eta\pi^e_{t+1}$$

$$\pi^e_{t+1} = \lambda\pi^e_t + (1-\lambda)(p_t - p_{t-1})$$

where $\lambda \in (0,1)$. If λ is close to one, then individuals' expectations are slow to update, they place a lot of weight on past expectations and little weight on current expectations. But if λ is close to zero, individuals place a lot of weight on current experience.

It is worth noting that expectations are not observed. Therefore, the model in the above form cannot be estimated. A form that can be estimated would the following:

$$p_t = b_1 p_{t-1} + b_2 m_t + b_3 m_{t-1}$$

By inverting the money equilibrium, we get

$$\pi^e_{t+1} = -\frac{1}{\eta}(m_t - p_t)$$

then

$$\pi^e_{t-1} = -\frac{1}{\eta}(m_{t-1} - p_{t-1})$$

Substituting back in the expectation equation:

$$-\frac{1}{\eta}(m_t - p_t) = -\frac{\lambda}{\eta}(m_{t-1} - p_{t-1}) + (1-\lambda)(p_t - p_{t-1})$$

By rearranging, we obtain:

$$p_t = \underbrace{\frac{\lambda - \eta(1-\lambda)}{1-\eta(1-\lambda)}}_{b_1} p_{t-1} + \underbrace{\frac{1}{1-\eta(1-\lambda)}}_{b_2} m_t - \underbrace{\frac{\lambda}{1-\eta(1-\lambda)}}_{b_3} m_{t-1}$$

The above equation is a linear one and we could estimate its parameters with regression methods so that we could study inflation and expected inflation over time

The most important properties of the solution are governed by the coefficient b_1. If $b_1 \in (0,1)$, (i.e., $|b_1| < 1$), then the inflation dynamics of the system are *dynamically stable*, i.e., if the government stabilizes the money supply process, then the price dynamics will stabilize too. In the case of $|b_1| < 1$, once a government gets control of the money supply process, inflation will eventually come under control too.

Conversely, if b_1 is too large, then even a stable monetary process may lead to hyperinflations driven purely by momentum –by individuals extrapolating from past inflation behavior. Cagan estimated the equation coefficients for monthly data on Germany (1922-1923), he found $b_1 = 3.17$, i.e., inflation had a significant momentum component.