‘‘On recognizing ‘law without law,’ ’’ Oersted Medal Response at the joint APS–AAPT Meeting, New York, 25 January 1983

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"On recognizing 'law without law,'" Oersted Medal Response at the joint APS–AAPT Meeting, New York, 25 January 1983

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The belief is expressed that particles, fields of force, spacetime, and "initial conditions" are only intermediate entities in the building of physics, that at bottom there is no "law," that everything is built higgledy-piggledy on the unpredictable outcomes of billions upon billions of elementary quantum phenomena, and that the laws and initial conditions of physics arise out of this chaos by the action of a regulating principle, the discovery and proper formulation of which is the number one task of the coming third era of physics. What a regulating principle means and how it works is illustrated in the far more modest content of (1) Boltzmann's law for the distribution of energy among molecules, (2) universality of exponents near thermodynamic critical points, (3) Wigner's "semicircle law" for the distribution of characteristic frequencies of a randomly coupled system, and (4) a new "physicist's version" of the problem of the traveling salesman. The regulating principles to be seen in these simple examples fall far short—in scope and simplicity—of the sought-for regulating principle. The search for it lies in the new domain of "recognition physics," being explored today on four fronts and at least half a dozen centers of investigation.

OERSTED, MILLIKAN, CREW

Thank you, William Kelly; and thanks to the great organization that you represent, the American Association of Physics Teachers, that has done so much to advance physics and the teaching of physics in this country for so many years.

No one could be more touched than I at the great honor that the Oersted Medal represents. Many a story I know from Copenhagen days of the unique man that Hans Christian Oersted was, his modesty, his admiration of his friend the writer, H. C. Anderson, whom Oersted always called the *great* Hans Christian, his encouragement to students, his part in making the education of engineers in Denmark what it is today, his great discovery linking electricity and magnetism. To me it is also a very great honor indeed to have this link with past Oersted Medal recipients whom I have known and admired; among them two who are no longer with us I remember with special vividness and affection. Robert Millikan, one of the great talent scouts of all time, used to pull that little black notebook out of his pocket to make notes about one or another young person who was giving a talk at a Physical Society meeting, or whom he had just met. Henry Crew, I recall in his nineties, thin, erect, white-haired, attending a Princeton Alumni Day celebration, the oldest living alumnus, one of the founding members of the American Physical Society, who could tell me what it was to attend the lectures of Helmholtz in Berlin.

THE SEARCH FOR THE GREAT SIMPLICITY

In making your decision, how much or how little weight to give to anything I may say today, it may help if I say a word or two about my background and goal. Already in high school days, I had fallen in love with the classic writings of J. Arthur Thomson, Charles Steinmetz, and H. A. Lorentz, and yet also with the workings of radio circuits, mechanical calculators, and automatic machinery. By the time I was a graduate student, my life goal had become the same as it is for so many of us, to understand the inner machinery of this strange and beautiful world. If, thanks, to the guidance of many a thoughtful student, I have had a little hand in opening up one and another new area of physics, it is perhaps because I have always been in search of something deeper, the wider perspective, the great unifying simplicity behind all we see and know. I am willing to go anywhere, talk to anybody, raise any question, make a fool of myself one hundred times over to make some small advance toward this great goal.

But how?

Some of our most distinguished friends believe that we will best see the larger unity of physics by exploring the interface between general relativity and quantum theory, by developing "quantum gravity." I sympathize with their endeavors and join when I can, because I have no doubt that important insights are to be won in this way.[1]

Other wonderful colleagues believe that supersymmetry and gauge theory[2] offer our greatest hope of recognizing great new unity. I sympathize wholeheartedly with these endeavors, too.

As we look at the achievements in these and other great frontier areas of physics, both experimental and theoretical, we can only say, "magnificent." Magnificent, too, are the regularities and laws that have been uncovered, from electrodynamics to the structure of matter and from Einstein's geometrodynamics to modern chromodynamics.

Are we then to believe that all of physics will one day be expressed in one or more beautiful equations, chiseled as it were on a tablet of granite and standing there from everlasting to everlasting?

However, much thought, many discussions and long study lead me to the directly opposite vision. All of physics, in my view, will be seen someday to follow the pattern of thermodynamics and statistical mechanics,[3] of regularity based on choas, of "law without law." Specifically, I believe that everything is built higgledy-piggledy on the unpredictable outcomes of billions upon billions of elementary quantum phenomena, and that the laws and initial conditions of physics arise out of this chaos by the action of a regulating principle, the discovery and proper formulation of which is the number one task of the coming third era of physics. In era number one Copernicus, Galileo, and

Kepler taught us the simplicity of motion. Era number two began with Newton teaching us the laws of motion and can be called the era of physical law. The coming third era, in my view, will show us the choas behind the law. It is my main purpose today to illustrate by four examples what a regulating principle means and how it acts to bring order out of chaos. The examples I shall give are taken out of modest contexts. Not one of them deals with the single quantum phenomenon, much less billions upon billions of elementary quantum phenomena. Therefore you will be left as much in the dark as I am, what the grand regulating principle is that produces the laws—and the initial conditions—of physics out of so many billions of individual phenomena, each by itself unpredictable. But I can at least, before I come to my examples of regulating principles meant for more modest context, recall what the individual quantum phenomenon is.

"QUANTUM PHENOMENON"

That single word, "phenomenon" is, we know, the distillation of the great twenty-eight year dialog between Bohr and Einstein about the meaning of the quantum.[4] It is an animal new to the thought of the older physics. It is illustrated nowhere better than in the "split-beam experiment" (Fig. 1). Operating in the mode shown at the lower left (no second half-silvered mirror in place), we find equal numbers of photon counts in the two photodectors. When the one counter clicks, it is nevertheless wrong to say that "the photon has traveled the high road," and again wrong to say "the photon has traveled the low road" when the other counter clicks. When the second half-silvered mirror is in place, destructive interference kills all radiation going to the one counter. The other counter goes off every time. It is nevertheless a mistaken form of expression to say then that "each photon travels both routes." It is the lesson of the great Bohr–Einstein dialog that is it wrong to speak at all about "the route of the photon" or "what it is doing" between point of entry and point of reception. *No elementary quantum phenomenon is a phenomenon until it is registered, recorded, "brought to a close" by an "irreversible act of amplification,"* such as the blackening of a grain of photographic emulsion or the triggering of a counter.[5]

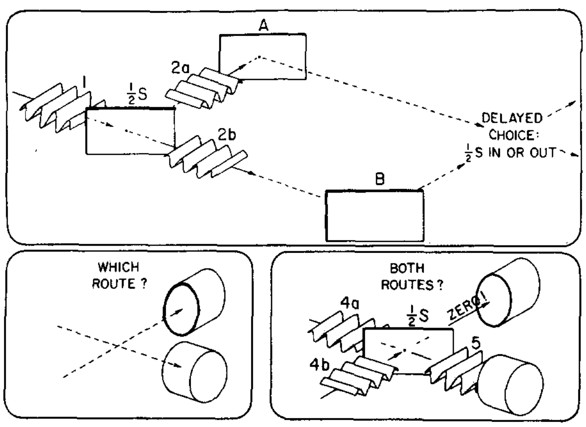

Fig. 1. Beam splitter (above) and its use in a delayed-choice experiment (below). We can put the second half-silvered mirror in place (lower right) or remove it (lower left) at the very last instant, after the photon has *already* accomplished most of its travel. Nevertheless, it is *wrong* to say that we thereby "decide whether the photon shall have come by one route, or by both routes" after it has "*already done* its travel." No elementary quantum phenomenon *is* a phenomenon until it is a registered phenomenon.

The elementary quantum phenomenon is a great smoky dragon-shaped cloud. The mouth of the dragon is sharp, where it bites the counter. The tail of the dragon is sharp where we inject the electron or the photon into the apparatus. About what it does in between we have no right to speak, neither in the double-slit experiment, nor in the split-beam experiment, nor in the famous Einstein–Podolsky–Rosen experiment.[6] We get the yes or no message but we neither know nor have the right to speak about how it came. It is the strangest thing in this strange world!

Quantum phenomena, recorded by counters or by blackened grains of emulsion, may seem to obey law when registered in large numbers; but the individual event, the individual yes or no decision, is as lawless as anything on the face of the Earth. Moreover, unlocalized and unlocalizable as the phenomenon is in space and time, it is the only thing we know in all of physics which has the smell of an element logically prior to spacetime on which spacetime—and all the rest of the structure of physics—might be considered to be built. [It is AIP policy to hyphenate space-time, the author chooses to spell it as one word as in all his previous publications.]

Built—but built how? Built with the help of what regulating principle? It is an inspiring task for the future to discover and formulate the regulating principle. Surely it is deep, otherwise we would have recognized it long ago. Surely, however, it is also simple, so utterly simple that when we finally see it we will exclaim in sudden illumination, "That is it." If today we do not know what the principle is, perhaps we may get some guidance in searching for it, and some pleasure, by looking at a few elementary examples where order comes out of disorder through the action of a regulating principle.

THE SHARING OF ENERGY

The well-known order that we see in molecular chaos provides the first of our four examples of law without law. We would all be lost today if we did not have Boltzmann's law to help us in understanding what we see. He discovered it,[7] we know, in Vienna in 1868. It tells us—to use modern language—that the probability for a molecule to be in a state of energy E is given by some normalizing constant or proportionality factor, depending on the number of molecules, multiplied by a final factor, the number $e = 2.718\cdots$ to a negative power. The higher this power is, the lower the

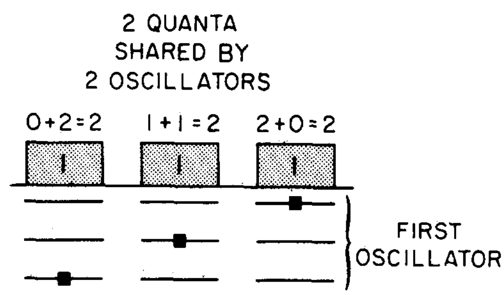

Fig. 2. Two oscillators sharing two quanta. The energy levels of only the first oscillator are shown, with a black square to designate the level of excitation. There is only one way (cross-hatched bar) to achieve each pattern of sharing.

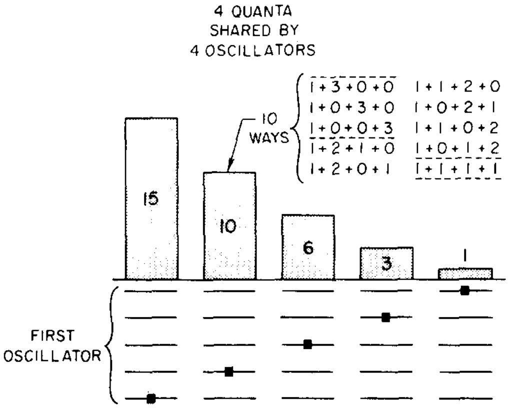

Fig. 3. Four quanta shared by four oscillators.

probability is. This power is given by an utterly simple formula, the quotient of the energy of the molecular state in question divided by a quantity, the temperature, common to all of the molecules of a system in equilibrium. Here we follow the convention that temperature is measured in the same units as energy, a convention that might have been adopted long ago if we had understood the nature of temperature better at the time the units were settled on for measuring it. If we want to give a simple meaning to temperature in the light of this formula we can use, we know, these words: "Temperature is that amount of energy difference between two states which makes the occupation probability of one 2.718··· times as great as the occupation probability of the other."

How can stupid molecules ever be conceived to obey a law so simple and so general? It is one of the joys of statistical mechanics to see how naturally the Boltzmann law follows from the elementary sharing of energy between oscillators. Figure 2 illustrates two oscillators sharing two quanta of energy. The first oscillator can have all the energy, both quanta. Or each oscillator can have one quantum. Or the first oscillator can have no energy at all. There are thus three ways of sharing the energy. Molecules being as stupid as they are, we assume that in the course of time all three ways of sharing energy occur with equal probability, a probability of one-third.

When we have four oscillators sharing four quanta of energy (Fig. 3) the average energy per oscillator is the same as before, one quantum. However, there are now more ways to share the energy. For example, there are, as illustrated, ten ways for the first oscillator to have a single quantum. However, there are still more ways for that first oscillator to have no energy at all; fifteen ways, to be precise. The reason, we know, is simple. There is more energy for the other oscillators, and therefore more ways for them to share it. That is why the state of lowest energy is always the most probable, no matter how much energy the whole system has available for sharing.

How does the number of ways open for the first oscillator to have any given energy fall off with that energy? Almost exponentially. Moreover, the distribution in energy comes still closer to the ideal exponential when we turn from four oscillators sharing four quanta of energy to six oscillators sharing six quanta of energy (Fig. 4).

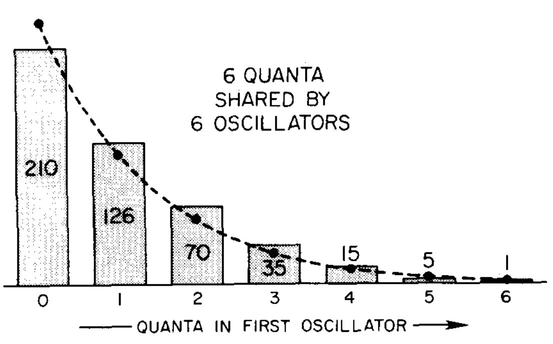

Fig. 4. Six quanta shared by six oscillators. The dashed curve is the prediction of the Boltzmann law.

(HIGGLEDY-PIGGLEDY) + (REGULATING PRINCIPLE) → (LAW OF PHYSICS)

Fig. 5. Role of the regulating principle.

Where more transparently than in these simple examples does one see how stupid molecules, sharing energy higgledy-piggledy, nevertheless end up on the average obeying Boltzmann's law? If this is the first of our examples of law without law, it is also an occasion to remind ourselves once again that no law springs unguided out of absolute chaos. It demands the guidance of a regulating principle (Fig. 5). We would get nowhere if we did not know that the sum of the energies of all the oscillators is fixed, regardless of how they share this energy. No such regulating principle? No Boltzmann law!

UNIVERSALITY IN CRITICAL POINT PHENOMENA

A second example of law without law, and of the regulating principle behind it, is seen in the "principle of universality" established in statistical mechanics by the work of many hands,[8] both in experiment and in theory, and not least through the investigations of our distinguished colleague Kenneth Wilson, happily recognized in the award to him this past December of the Nobel Prize in physics. Substances as different as iron and CO_2, as helium and a binary alloy of cobalt and zinc, as gadolinium and xenon, studied near the critical point, show order parameters—such as magnetization or departure from the critical density, such as magnetic susceptibility or compressibility, such as specific heat, such as cross section per unit volume for the forward scattering of slow neutrons—that vary near the critical point as one and another universal, and simply related, power of the temperature difference.[9] This is the principle of universality in action.

Down underneath this law of the universality of critical points exponents lies wild disorder. Figure 6 shows a computer simulation of a ferromagnet worked out by Wilson.[10] In the upper diagram each square symbolizes the magnetic moment associated with a single atom in the solid. Black squares designate atoms with an "up" moment: white squares, a "down" moment. A first look at such a diagram leaves one with the dismaying impression that there is not the slightest regularity in all this disorder. A closer examination reveals correlations between the magnetization of any chosen atom and its nearest neighbors. We recognize, after all, that we are dealing with a cooperative phemon-

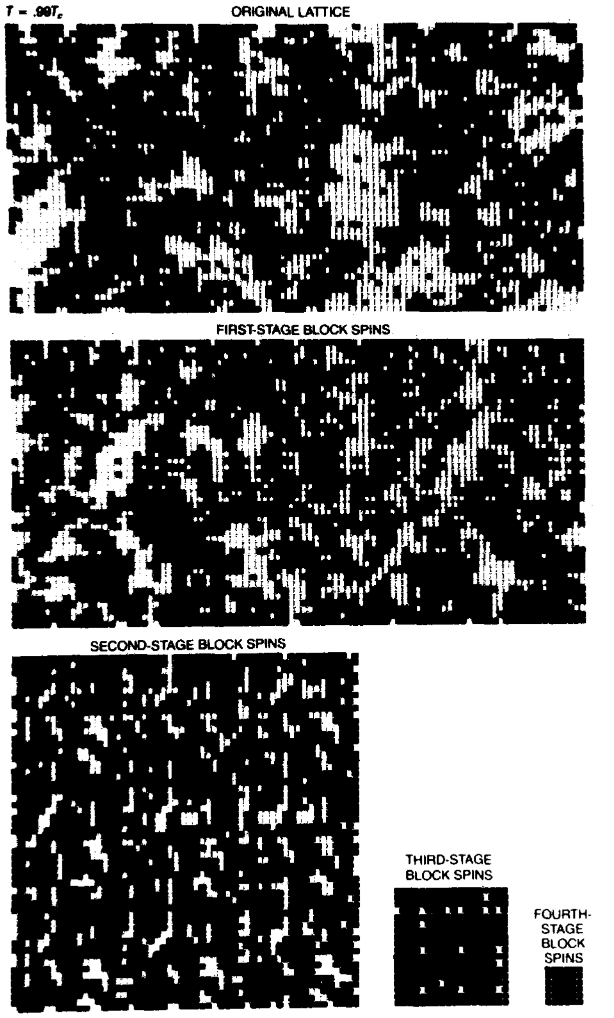

Fig. 6. Magnetization examined for order by a computer model at a temperature of 0.99 times the Curie temperature. Reproduced from K. G. Wilson.[10]

enon when we study the magnetization of iron or the melting of ice or the onset of superfluidity. Moreover, it is impossible to have a coupling between atoms A and B, and between B and C, without having indirectly a coupling between A and C. The same reasoning tells us that such linkages can in principle extend over indefinitely great distances. These linkages bring about an order. That order comes to light on closer examination of the pattern of magnetization in the upper part of Fig. 6.

Divide the original lattice in the diagram at the top of Fig. 6 into three by three blocks. Conduct a poll of the nine atoms. If the majority vote black, we make a new little square (second diagram in Fig. 6) and color it all black; conversely if the vote goes to the whites. Continue in this way until the original lattice has all been reworked into a lattice of one-ninth the original size. At first sight the new pattern appears to be as disorderly as the original one. But once again combine the blocks of nine into similar squares, and again, and again until we arrive at the pattern of "fourth-stage block spins" illustrated at the lower right of Fig. 6.

How can we describe what we see? Evidently we are studying the pattern of magnetization at larger and larger scales of distance. This pattern, moreover, is dominated to an ever greater extent by a single direction of magnetization. This is the sense in which a long-range order shows itself, a consequence, of course, of the linkage between atom and atom. A closer study shows that the degree of order scales with dimension in a simple way as we go to larger and larger blocks. The order increases. As Wilson puts, "Merely looking at a configuration of ... spins just below the Curie temperature will seldom reveal that the model is slightly magnetized. At this temperature there is only a small excess of one spin direction over the other, and the many small-scale fluctuations obscure the overall bias. After several applications of the block-spin transformation, however, the smaller fluctuations disappear and the long-range magnetization becomes obvious."

A more detailed examination shows more. There is an intimate connection between the scaling of magnetization with distance, on the one hand, and, on the other hand, the scaling of magnetization with difference of temperature from the critical point. This is where the principle of universality arises. In brief, the identity of "critical exponents" in the power law dependence of magnetization, specific heat, neutron scattering, and other properties of very different substances has its origin in the scaling of the physics over many orders of magnitude of distance. Universality comes out of geometry of similarity and the physics of coupling between neighbor and neighbor. Together physics and geometry provide the regulating principle that enforces universality. We have here a second example of a regulating principle generating law without law, order out of chaos.

RANDOM COUPLINGS

Our third example lies in the realm of "random matrices," a mathematical idealization employed by Wigner,[11] Mehta,[12] the late Charles Porter,[13] and many other distinguished colleagues in the analysis of the spacing and

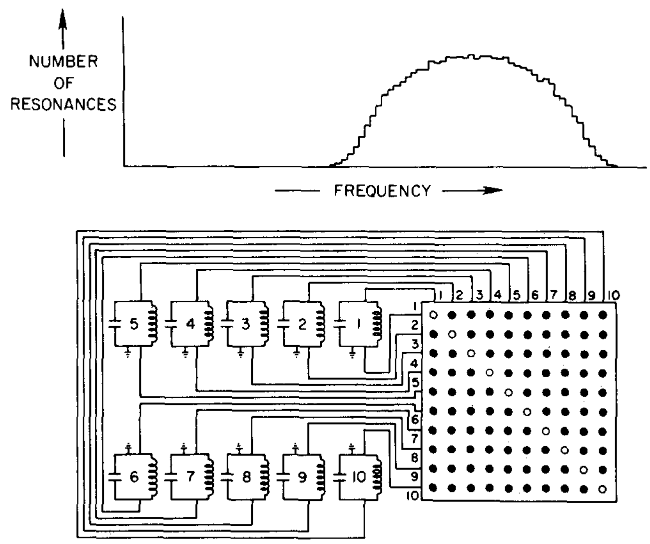

Fig. 7. Below, oscillators that can be coupled by way of the circuit board at the lower right. Above, result of plugging in capacitances selected at random from a Gaussian distribution. Reworked by the author from C. E. Porter,[15] who generated 10 000 random real symmetric matrices and diagonalized them and made a histogram of their eigenvalues. Already at the level of a ten by ten matrix the curve for distribution of eigenvalues is approximating Wigner's semicircle.

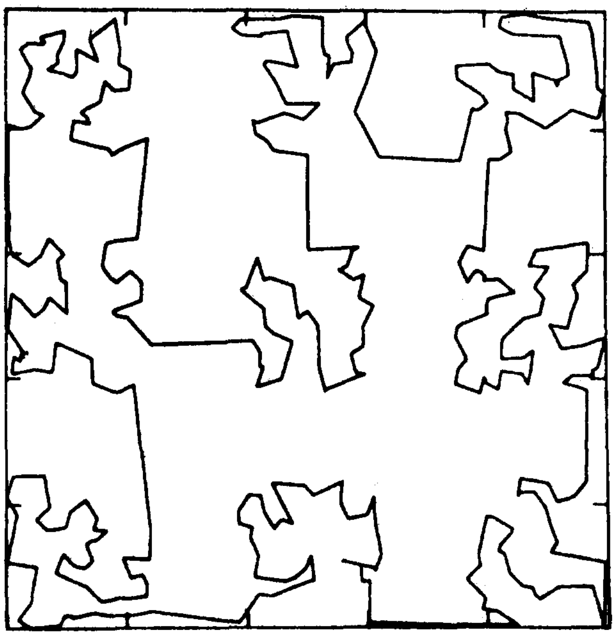

Fig. 8. Traveling salesman's tour of 400 cities optimized—or close to being optimized—for minimum length. Reproduced from Bertram M. Schwarzschild, *Physics Today*.[16]

"strength" of nuclear energy levels and other physical phenomena. I know no simpler example than a set of ten identical oscillators (Fig. 7) that can be coupled to each other with the help of the "plug-in circuit board" shown at the lower right in Fig. 7. With nothing plugged in, all ten oscillators, disturbed, vibrate at the same frequency. Someone comes in the room with a box of little capacitors of random rating. Paying no attention to the label specifying this, that, or the other number of microfarads, operating on the principle of the grab bag, we reach blindfolded into the box, pick out one, and plug it into a hole in the circuit board. We reach into the box, pick out another capacitor and plug it into another hole. Proceeding in this way over and over again we fill up the circuit board with an array of capacitors of random capacity. They couple the ten electrical circuits. No longer do we have ten identical frequenices. Instead, the frequencies are spread. Moreover, the pattern of resonances—provided that the number of oscillators is large and the random distribution of coupling capacitances is Gaussian—follows Wigner's famous "semicircle law."[14] We can see the approach to the semicircle law in Fig. 7. In constructing it I have reworked calculations made by Charles Porter in another connection.[15] He diagonalized 10 000 matrices, each consisting of ten rows and ten columns. The figure is a histogram of his findings. Theory tells us that the larger the number of identical oscillators, the closer we approach Wigner's ideal semicircle law of distribution of characteristic frequencies. Here indeed is law without law, order out of disorder. And what is the regulating principle operating behind the scenes to generate this regularity? The *Gaussian distribution* of the couplings. Without this—or something equivalent—there would be no semicircle law.

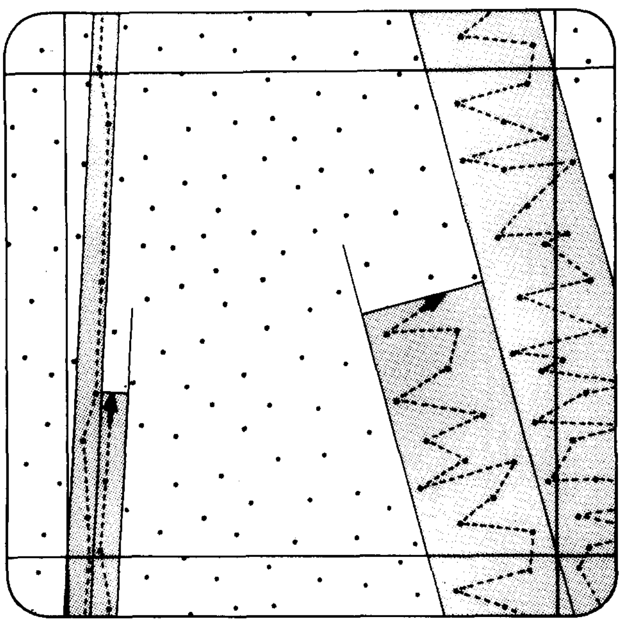

Fig. 9. Physicist's version of the problem of the traveling salesman. The cities to be visited are "uniformly random" in location, and the landscape has the topology of a two-torus (symbolized here by phantom additional landscapes that reproduce the pattern of the actual landscape.)

THE TRAVELING SALESMAN AND THE FOLIATIONS OF HIS FURLONGS

Our fourth and last example of regularity out of chaos is furnished by a new look at the famous problem of the traveling salesman. Figure 8, taken from an article of Bertram Schwarzschild[16] in a recent issue of *Physics Today*, shows the route taken by the traveling salesman who has to visit his customers, scattered over many circles, one after another, and then at the completion of his task start the same circuit all over again. He wants to cut his cost and to minimize the average distance traveled per customer visited. The number of possible routings to be compared grows exponentially with the number of cities visited. When that number is more than a few hundred, it is beyond the power of any computer to compare all these possiblities, one with another, and find the shortest route, even were it granted a century in which to do the calculations. Neither has the most diligent search by our friends in the world of mathematics disclosed any algorithm whatsoever that will single out the circuit of absolutely shortest length.[17] However, the path illustrated in Fig. 8, if not absolutely the best for visiting the four hundred indicated cities, is close to it. And what a tortured route it is! Surely no one seeking order would look for it here! But so we shall.

What happens to the problem of the traveling salesman when we approach it in the spirit of the physicist? First we kill the boundaries. In field theory, it is an old procedure to replace the box with its enclosing copper walls by a region of the same volume with periodic boundary conditions. Then we free ourselves form any restrictions and reflections. We do the same here. Second, we free ouselves from the idea that the cities to be visited are clustered in favored regions. Instead, we conceive them as spread about in the "uniform randomness" that we customarily attribute to the molecules in a dilute gas. The problem in this "physicist's version" takes the form illustrated in Fig. 9.

Having a job to do, we get on with it after the fashion of the busy farmer. He cuts the hay in swaths with a mowing machine. Its blade looks neither to the left nor to the right, but straightforwardly cuts every stalk in the order of encounter. The farmer has some problem with choice of di-

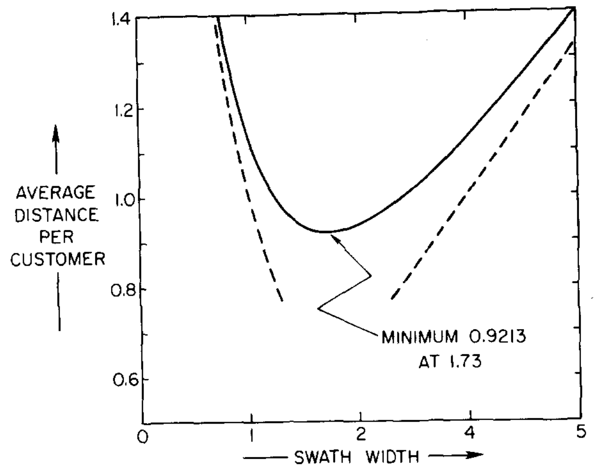

Fig. 10. Average distance traveled, per customer visited, in its dependence on the swath width of Fig. 9. Analysis done by Rollin Armour and the author.[18]

rection because of the boundaries of his field. We don't. For us one direction is as good as another. Only about the width of the swaths do we have to worry. What happens when we make the swath width too narrow? Then the mowing machine has to advance a great distance from one stalk of hay to the next. What does this mean for the traveling salesman? His travel takes place almost exclusively in the direction of the advance of the mythical mowing machine. Any lateral motion contributes negligibly to his mileage and gasoline costs. The average distance traveled per customer visited is given by one of the most elementary of statistical quantities, the mean free path,

$$\begin{pmatrix}\text{Average distance}\\ \text{per customer}\\ \text{visited}\end{pmatrix} = \begin{pmatrix}\text{mean}\\ \text{free}\\ \text{path}\end{pmatrix} = \frac{1}{\begin{pmatrix}\text{swath}\\ \text{width}\end{pmatrix}\begin{pmatrix}\text{number of cities}\\ \text{per unit area}\end{pmatrix}}.$$

This result is argument enough for not making the swaths too narrow.

It is also expensive to make the swaths too wide. Why? Because then even a little advance of the blade encounters many stalks. In other words, the salesman has to go zigzagging laterally back and forth a great deal for even a small advance along his swath of cities to be visited. In this mathematical limit, it is easy to show that the average distance per customer visited is

$$\begin{pmatrix}\text{Average distance}\\ \text{per customer}\\ \text{visited}\end{pmatrix} = \frac{1}{3}\begin{pmatrix}\text{the swath}\\ \text{width}\end{pmatrix}.$$

The lesson is clear: reduce the swath width.

Rollin Armour, an undergraduate student at the University of Texas, and I have used elementary statistical arguments[18] to calculate the dependence of average "mileage per customer" on swath width for cases between these two idealized extremes (Fig. 10). The optimum economy is achieved, we find, when the swath width is chosen to have the value

$$\begin{pmatrix}\text{swath}\\ \text{width}\end{pmatrix} = 1.73\begin{pmatrix}\text{number of}\\ \text{cities}\\ \text{per unit}\\ \text{area}\end{pmatrix}^{-1/2}.$$

It would be a complete misrepresentation of what we are talking about to say that we have here the long-sought perfect path for the traveling salesman, even in this new "physicist's version" of the old problem. It will take considerable mathematical work to establish an upper limit—5%? 10%?—to the typical departure of our optimum from mathematical perfection. We don't have this ideal. We have a practical person's approach to this ideal. More important, we have a lesson, a lesson summarized in the single word, "foliation." The salesman should foliate the cities into layers, sheets, ribbons, swaths of the width calculated in Fig. 10, if he would minimize his cost.

With the lesson of foliation in mind, we can turn back to the tortured path of Fig. 8 for a new look. Suddenly we recognize that foliation is there staring us in the face. In any region where there is a cluster of enough cities and they are located in something like a pattern of "uniform randomness," the optimum, or near-to-optimum, path is indeed foliated. Only in the regions where cities are few and far between does the pattern change character.

This irregularity is no surprise to the physicist. Metallurgy never shows us perfect iron, never a perfect crystal lattice. Usually we find lumps, occlusions, cracks and imperfections separating domains of nearly ideal—but never absolutely ideal—crystal structure. It is no reason for astonishment that we find a similar result here.

We do not abandon the concept of the ideal crystal when we learn that nature never offers one. There is no reason to abandon the concept of foliation in the problem of the traveling salesman for any similar reason.

In this foliation, we have the "order" that we seek. It emerges from the disorder of the arrangement of the cities. Insofar as we can speak of this foliation as a "law," as law without law, we have it once again enforced by a regulating principle, in this case, the principle of "minimum distance per customer visited."

THE COMING THIRD ERA OF PHYSICS

If I have been able to offer you four examples of order brought out of disorder by a reglating principle, I have still given only a faint and inadequate indication of the prize that lies ahead for physics in its coming third era.

What is the order that we seek to understand? The structure of spacetime and the particles and fields that so many distinguished colleagues do so much today to lay out in simplest form.

What is the disorder out of which, in my view, we must hope to see this structure build? The higgledy-piggledy, the randomness, the unpredictability of billions upon billions of elementary quantum phenomena, each unlocalized in space and time.

What regulating principle can we conceive that is general enough, deep enough, and independent enough in character from all this physics to be nevertheless the ultimate source of it all? Aye, there's the rub; that's where the mystery lies.

The question we raise it not in character a new one, although in our time it has new tonalities, above all a fundamentally quantum flavor. The great philosopher Immanuel Kant argued that space and time do not exist "out there," they are concepts developed in the mind to make sense of our experience,[19] the principle without which "reality" would not be recognizable. His far-reaching program, which has had so much influence on so many great thinkers, was in the end unsuccessful. Why did it fail? Because it aimed to high? No, in my opinion; it did not aim high enough. In my view, the task ahead is no small task and it demands no small courage. We have to account not merely for space and time. We have to account for all the structure that makes physics what it is. We have at the same time to account for that strange feature that lies outside the scope of law, the particularities that do more than anything to give our world the richness that we see around us: the *initial conditions* for all our dynamic laws.

What regulating principle accomplishes this miracle? How does it make room for domains of law, as well as the domain of initial conditons where there is no law? No small answer can ever hope to live up to a question so great.

Is the regulating principle behind the structure of physics the demand for meaning? Should we be asking, "What makes meaning?" Do we have to invade the world of philosphy to make headway with that problem? If so, perhaps we can modify Talleyrand's words and declare that philosphy is too important to be left to the philosophers. In older times, we had to take over from them the problem of motion. In our own century, Einstein established dominion over the subject of space and time. Tomorrow, will it not be existence itself that comes under the purview of physics?

THE FOUR FRONTS OF "RECOGNITION PHYSICS"

Physics today is making its way into this fascinating new domain of *recognition physics* on four fronts and at least half a dozen centers of investigation: (1) New aspects of what we mean by "meaning" are beginning to emerge from recent studies in computer science and in information theory that put the quantum at center stage.[20] (2) Clues have been discovered suggesting that some of the main features of quantum theory, rather than having descended by direct revelation from heaven, can be derived from deeper considerations closely related to the demand for distinguishability.[21] (3) Thanks to greater understanding of quantum gravity, and quantum field theory generally, we are developing a deeper insight than ever before into what quantum theory has to say in the context of relativity.[1] (4) New interest is developing in the great links which—we have always known—connect quantum theory and measurement theory.[4]

The coming third era of physics confronts us with a challenge greater than we have ever faced before. It is a time for hope. How can physics live up to its true greatness except by a new revolution in outlook which dwarfs all its past revolutions? And when it comes, will we not say to each other, "Oh, how beautiful and simple it all is! How could we ever have missed it so long!"

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[2]W. Marciano and H. Pagels, Phys. Rep. **36C**, 137 (1978); see also D. Bleecker, *Gauge Theory and Variational Principles* (Addison-Wesley, Reading, MA, 1981).

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[4]Documented in *Quantum Theory and Measurement*, edited by J. A. Wheeler and W. H. Zurek (Princeton University, Princeton, NJ, 1983).

[5]For the delayed-choice version of the split-beam and other experiments, see J. A. Wheeler, *Mathematical Foundations of Quantum Theory*, edited by A. R. Marlow (Academic, New York, 1978), pp. 9–48.

[6]The paper of A. Einstein, B. Podolsky and N. Rosen is reprinted with related papers in Ref. 4.

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[11]E. P. Wigner, Ann. Math. **53**, 36 (1951); reprinted in Ref. 13.

[12]M. L. Mehta, *Random Matrices* (Academic, New York, 1967).

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[14]E. P. Wigner, Ann. Math. **65**, 203–207 (1957), reproduced in Ref. 13.

[15]C. E. Porter, J. Math. Phys. **4**, 1039–1044 (1963). Two steps are required to translate from Porter's problem of the eigenvalues of the Hamiltonian to our problem of the resonances of an electric circuit. The first is to replace energy eigenvalue by square of the frequency. Before doing so we have put in a natural frequency high in comparison with the spread of the couplings. As a consequence it makes little difference whether the horizontal scale in Fig. 7 is frequency itself (as shown) or the square of the frequency, as by rights ought to be shown on the label. Second, in Porter's symmetric matrices the squared spread of the diagonal elements is twice as great as the squared spread of the typical off-diagonal element. It requires no great complication to arrange the same here. We have no cause for concern in the asymmetry of the pattern for plug-in capacitances, for in the circuits all that counts is sums of the type $(1/C_{5,9}) + (1/C_{9,5})$.

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