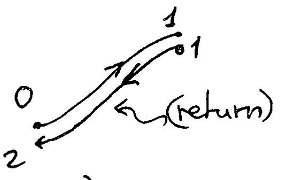
TIME REVERSAL

By construction the Verlet update is time reversible. Let us verify it



$$(\bar{r}_0, \bar{v}_0) \xrightarrow{\Delta t} (\bar{r}_1, \bar{v}_1)$$

if $(r_1, -v_1) \xrightarrow[\Delta t]{} (r_0, -v_0)$ [to be proved]

Start at (r_0, v_0)

$$\begin{cases} r_1 = r(\Delta t) = r_0 + v_0 \Delta t + \frac{1}{2m} F(r_0) \Delta t^2 \\ v_1 = v(\Delta t) = v_0 + \frac{1}{2m} \left(F(r_0) + F(r_1) \right) \Delta t \end{cases}$$

Now we start in $r'_0 = r_1$, $v'_0 = -v_1$ (reverse the velocity)

$$r_2 = r(\Delta t) = r'_0 + v'_0 \Delta t + \frac{1}{2m} F(r'_0) \Delta t^2$$

$$= r_1 - v_1 \Delta t + \frac{1}{2m} F(r_1) \Delta t^2$$

$$= r_0 + v_0 \Delta t + \frac{1}{2m} F(r_0) - v_0 \Delta t - \frac{1}{2m} \left(F(r_0) + F(r_1) \right) \Delta t^2$$

$$+ \frac{1}{2m} F(r_1) \Delta t^2 = r_0$$

[we go back to the starting point]

$$v_2 = v(\Delta t) = v'_0 + \frac{1}{2m} \left(F(r'_0) + F(r_2) \right) \Delta t$$

$$= -v_1 + \frac{1}{2m} \left(F(r_1) + F(r_0) \right) \Delta t$$

$$= -v_0 - \frac{1}{2m} \left(F(r_0) + F(r_1) \right) \Delta t + \frac{1}{2m} \left(F(r_1) + F(r_0) \right) \Delta t$$

$$= -v_0$$

[Same velocity, but in the opposite direction]