ENERGY FLUCTUATIONS AND SPECIFIC HEAT

Defining
\[ E = \langle H \rangle \]
\[ C_V = \frac{dE}{dT} \]

Specific heat at constant volume \( V \)

\[ \langle H \rangle = \frac{1}{Z} \int [d\rho d\sigma] \: H e^{-\beta H} = - \frac{1}{Z} \frac{\partial}{\partial \beta} \int [d\rho d\sigma] e^{-\beta H} \]
\[ = - \frac{1}{Z} \frac{\partial Z}{\partial \beta} = - \frac{\partial \ln Z}{\partial \beta} = \frac{\partial \langle \beta F \rangle}{\partial \beta} \]

\[ C_V = \frac{\partial}{\partial T} \langle H \rangle = - \frac{1}{kT^2} \frac{\partial}{\partial \beta} \left[ \frac{\int [d\rho d\sigma] H e^{-\beta H}}{\int [d\rho d\sigma] e^{-\beta H}} \right] \]
\[ = - \frac{1}{kT^2} \left[ - \frac{\int [d\rho d\sigma] H e^{-\beta H}}{\int [d\rho d\sigma] e^{-\beta H}} + \left( \frac{\int [d\rho d\sigma] H e^{-\beta H})^2}{\int [d\rho d\sigma] e^{-\beta H}} \right) \right] \]
\[ = \frac{1}{kT^2} \left[ \langle H^2 \rangle - \langle H \rangle^2 \right] \]

The specific heat

\[ \langle H^2 \rangle - \langle H \rangle^2 = kT^2 C_V(T) \]

The energy is extensive \( \Rightarrow \) \( C_V \) is extensive

\[ E \sim V, N \quad C_V \sim V, N \]

\[ \langle H^2 \rangle - \langle H \rangle^2 \sim V, N \]

\[ \frac{E}{N} = \frac{E}{V} \cdot \frac{1}{N^*} \]

Energy per particle

\[ \langle H/V \rangle \quad E/V \equiv \text{intensive} \]