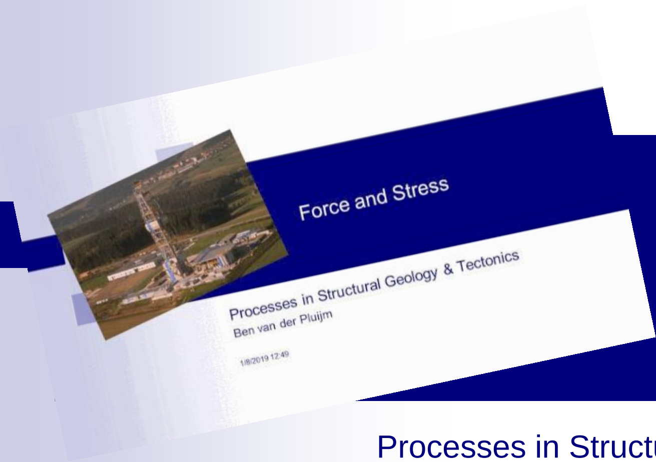


Force and Stress

Processes in Structural Geology & Tectonics

Ben van der Pluijm

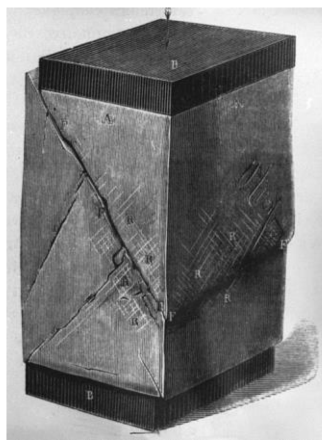
1/14/2019 9:32 AM

Rock Stories: Fracture

Fractures from compression of a material are typically oriented at angle of ~30º from the maximum compressive force (=maximum principal stress x area) in a confined environment.

Fractures are formed when the difference between maximum and minimum principal stresses (called, differential stress) reaches a certain value (failure criterion). Using trigonometry we can derive values of the normal (perpendicular) stress (σ_n) and the shear (parallel) stresses (σ_s or τ) on fracture surfaces.

The predictions from these relationships hold for fracture formation, but not necessarily for fracture reactivation.

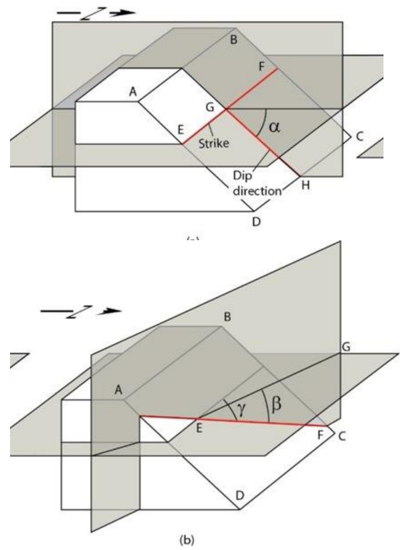


Daubrée's late 19th Century fracture experiment with wax.

This Week’s Lab: Orientation, Map Geometry

Attitude of a plane: dip, dip direction, and strike.
Attitude of a line: plunge, plunge direction, and pitch.
Load free Fieldmove Clino smartphone app.

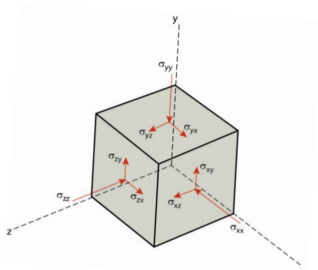
Structure contours (or strike lines), construct (not sketch) cross section, bed thickness calculation, “three-point” problem.

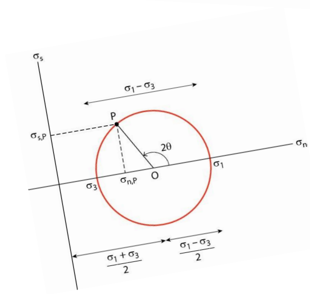


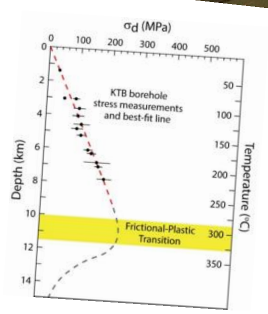
We Discuss …

Force and Stress

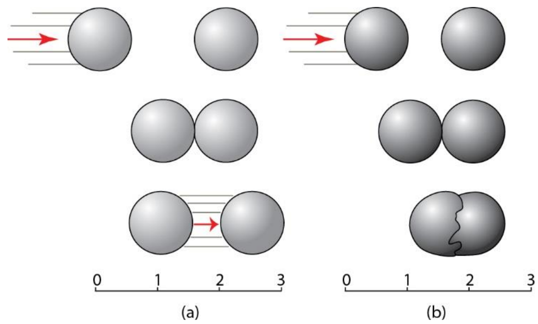
- Force and Units
- (Trigonometry)
- Newtonian and Continuum Mechanics
- 2D and 3D Stress
- Normal Stress and Shear Stress
- Mohr Construction of Stress
- Stress States
- Measurement of Stress
  - At surface
  - At depth
- Lithospheric Stress
  - Modern stress regimes
  - Geologic Provinces







Mechanics: Force and Displacements



(a) Newtonian mechanics: displacements **between** bodies

1st Law: No force on object means constant velocity (inertia law)

2nd Law: F = m .a

3rd Law: F = -F

(b) Continuum mechanics: displacement **between** and **within** bodies

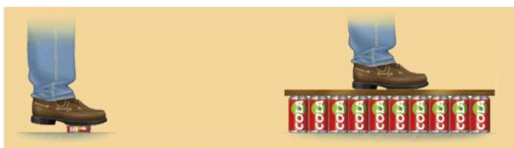
Stress and Tractions

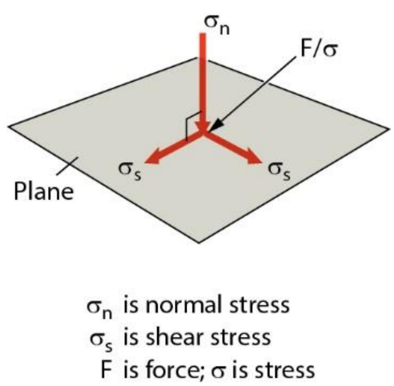
Stress = Force/Area
(or, stress is "intensity of force")

Stress on a plane in space is defined by

- stress acting perpendicular to plane (normal stress, σ_n),
- stress acting along plane (shear stress, σ_s).

These resolved stresses are also called *tractions*.





Units and Conversions

Stress (or Pressure) = Force/Area

= (m.a)/Area

= kg.m.s^{-2}.m^{-2}

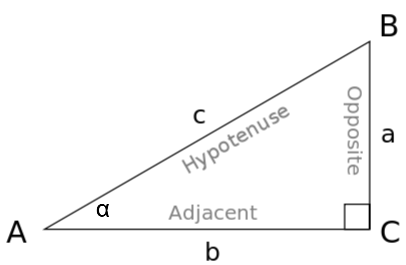
= N.m^{-2} = Pa (Pascal)

1 bar (atmospheric conditions) = 1.10^{5} Pa (1E5 Pa) = 0.1 MPa

1000 bar = 1 kbar (geologic conditions) = 100 MPa

	bar	dynes/cm^2	atmosphere	kg/cm^2	pascal (Pa)	pounds/in^2 (psi)
bar		10^6	0.987	1.0197	10^5	14.503
dynes/cm^2	10^{-6}		0.987×10^{-6}	1.919×19^{-6}	0.1	14.503×10^{-6}
atmosphere	1.013	1.013×10^6		1.033	1.013×10^5	14.695
kg/cm^2	0.981	0.981×10^6	0.968		0.981×10^5	14.223
pascal (Pa)	10^{-5}	10	0.987×10^{-5}	1.0197×10^{-5}		14.503×10^{-5}
pounds/in^2 (psi)	6.895×10^{-2}	6.895×10^4	6.81×10^{-2}	7.03×10^{-2}	6.895×10^3	

Trigonometry

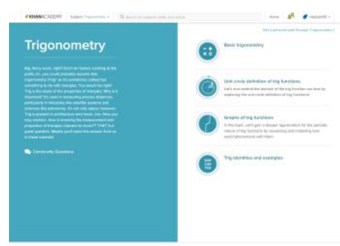


In a right triangle:

sin α = a/c

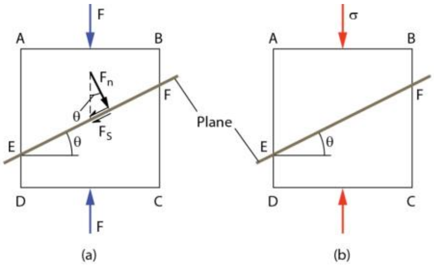
cos α = b/c

tan α = a/b



https://www.khanacademy.org/math/trigonometry

Relationship between Force and Stress (2D analysis)



$F_n = F.\cos\theta$
$F_s = F.\sin\theta$

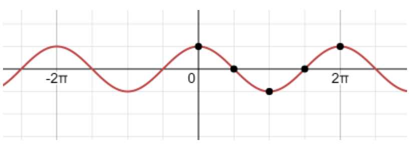
σ = F/Area; Here (2D): σ = F/AB
(AB = EF. $\cos\theta$)
Thus, corresponding stresses are:

$\sigma_n = F_n/EF = F.\cos\theta/EF = \sigma.AB.\cos\theta/EF$
$= \sigma.EF.\cos\theta.\cos\theta/EF = \sigma \cos^2 \theta$

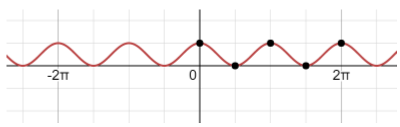
$\sigma_s = F_s/EF = F.\sin\theta/EF = \sigma.AB.\sin\theta/EF$
$= \sigma.EF.\sin\theta.\cos\theta/EF = \sigma.\sin\theta.\cos\theta = \sigma\ ½(\sin2\theta)$

Effect of length/area

$\sigma_n = \sigma \cos^2 \theta$

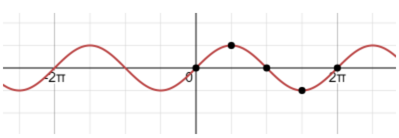


y=cosx

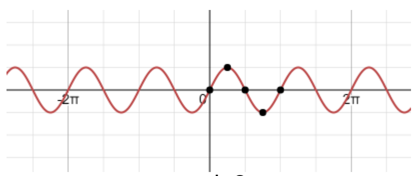


y=$\cos^2$x

$\sigma_s = \sigma\ ½(\sin2\theta)$

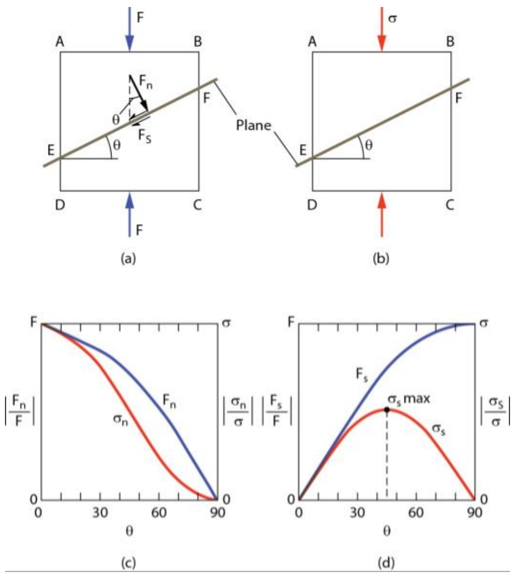


y=sinx



y=sin2x

Relationship between Force and Stress (2D analysis)



$F_n = F.\cos\theta$
$F_s = F.\sin\theta$

σ = F/Area; Here (2D): σ = F/AB
(AB = EF. $\cos\theta$)
Thus, corresponding stresses are:

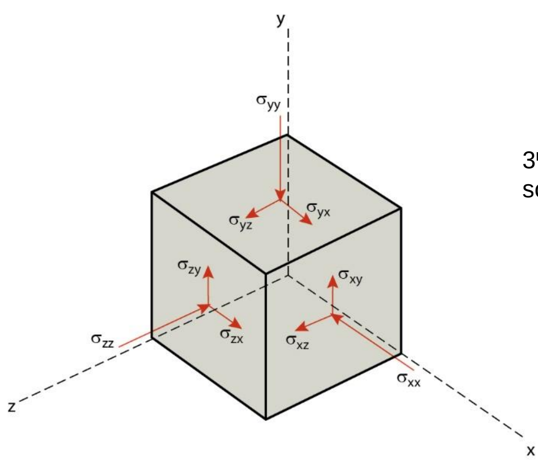
$\sigma_n = \sigma \cos^2 \theta$

$\sigma_s = \sigma\ ½(\sin 2\theta)$

c) normalized values of F_n and σ_n on plane with angle θ;
d) normalized values of F_s and σ_s on a plane with angle θ.

3D stress

Cube is 3D object with least number of unique planes (2)

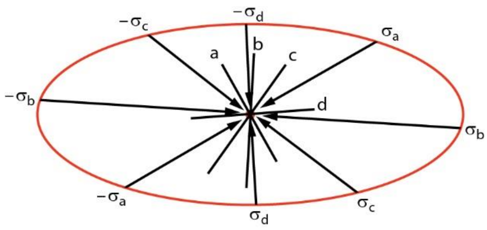


	In the direction of x:	y:	z:
stress on the face normal to *x*:	σ_{xx}	σ_{xy}	σ_{xz}
stress on the face normal to *y*:	σ_{yx}	σ_{yy}	σ_{yz}
stress on the face normal to *z*:	σ_{zx}	σ_{zy}	σ_{zz}

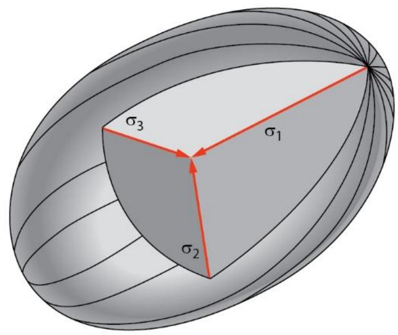
3rd Law: non-moving object,
so, balancing tractions define six independent components:

	In the direction of x:	y:	z:
stress on the face normal to *x*:	σ_{xx}	σ_{xy}	σ_{xz}
stress on the face normal to *y*:	σ_{xy}	σ_{yy}	σ_{yz}
stress on the face normal to *z*:	σ_{xz}	σ_{yz}	σ_{zz}

Infinitessimal Stress and Stress Ellipsoid



(a)

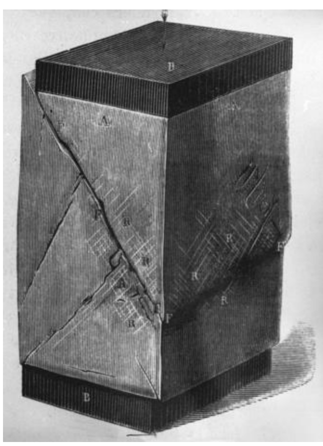


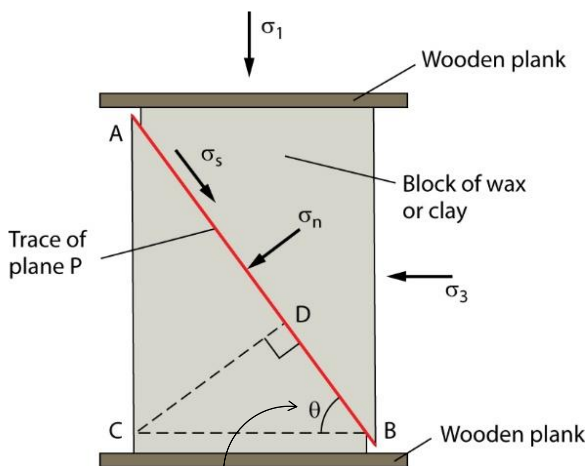
(b)

Point is intersection of infinite number of planes
Shrink cube to a point
(a) two-dimensions: stress ellipse
(b) three dimensions: stress ellipsoid

Ellipsoid axes are called **principal stresses**, with $\sigma_1 \geq \sigma_2 \geq \sigma_3$

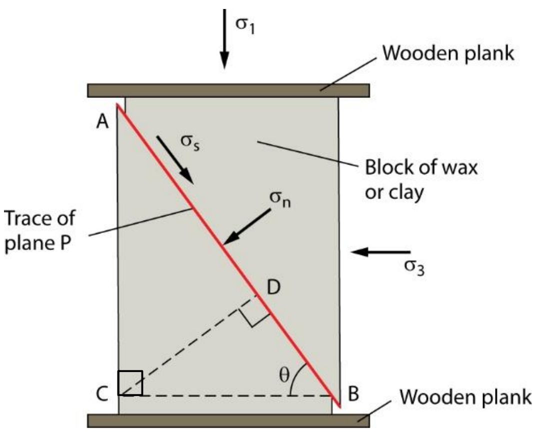
Normal and Shear Stress Relationships





Ɵ (~60º) is angle with σ_3
is 90-Ɵ (~30º) with σ_1

Deriving σ_{normal} and σ_{shear} from Principal Stresses



F = σ . Area
Here (2D): F = σ . Length

σ_{normal}

force ⊥ AB = force ⊥ BC resolved on CD +
force ⊥ AC resolved on CD

force on side $BC = \sigma_1 \cdot \cos\theta$
force on side $AC = \sigma_3 \cdot \sin\theta$

$$1 \cdot \sigma_n = \sigma_1 \cos\theta \cdot \cos\theta + \sigma_3 \sin\theta \cdot \sin\theta$$
$$\sigma_n = \sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta$$

$$\cos^2\theta = \tfrac{1}{2}(1 + \cos 2\theta)$$
$$\sin^2\theta = \tfrac{1}{2}(1 - \cos 2\theta)$$

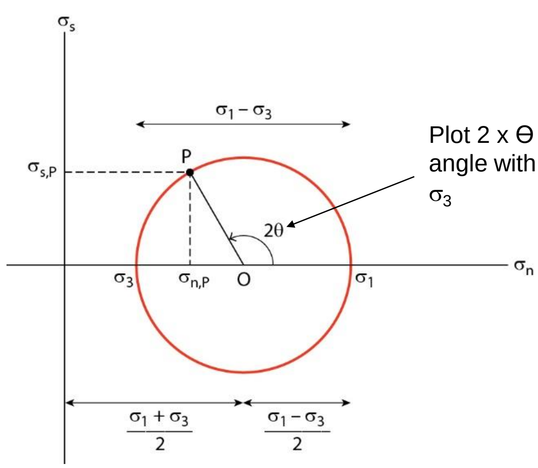
$$\sigma_n = \tfrac{1}{2}(\sigma_1 + \sigma_3) + \tfrac{1}{2}(\sigma_1 - \sigma_3)\cos 2\theta$$

σ_{shear}

force parallel AB = force ⊥ BC resolved on AB +
force ⊥ AC resolved on AB

$$\sigma_s = \tfrac{1}{2}(\sigma_1 - \sigma_3)\sin 2\theta$$

Graphical Solution: Mohr Diagram for Stress



Plot 2 x Θ angle with σ_3

$$\sigma_{n,P} = \tfrac{1}{2}(\sigma_1 + \sigma_3) + \tfrac{1}{2}(\sigma_1 - \sigma_3)\cos 2\theta$$
and
$$\sigma_{s,P} = \tfrac{1}{2}(\sigma_1 - \sigma_3)\sin 2\theta$$

Rearrange for σ_d, and square:

$$[\sigma_n - \tfrac{1}{2}(\sigma_1 + \sigma_3)]^2 + \sigma_s^2 = [\tfrac{1}{2}(\sigma_1 - \sigma_3)]^2$$

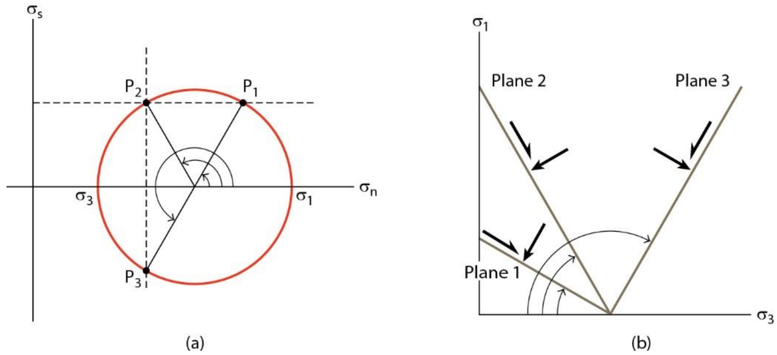
$$(x - a)^2 + y^2 = r^2$$

Circle with radius r, centered on x-axis at distance a from origin

Radius is $1/2(\sigma_1 - \sigma_3) = \sigma_s$,
Diameter is $(\sigma_1 - \sigma_3) = \sigma_d$

So, shear stress is half of differential stress

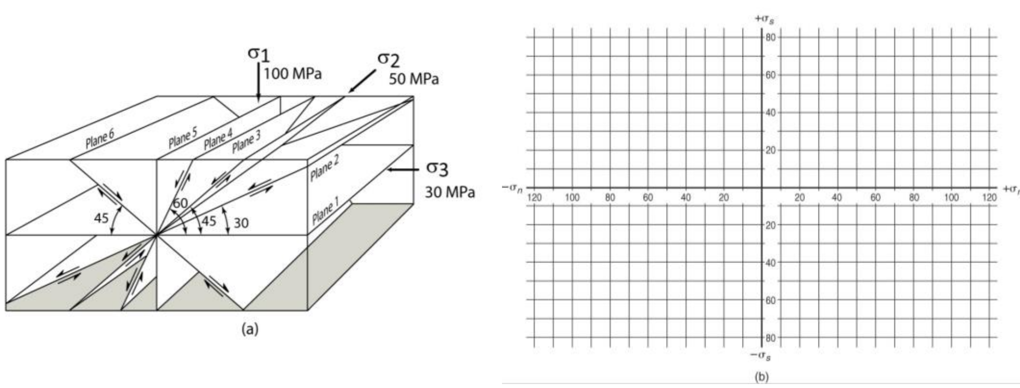
Planes in Stress and Real Space



(a) For each value of shear stress and normal stress there are two corresponding planes, shown in Mohr diagram. Note that this is stress solution space, *not* real space.

(b) Corresponding planes in ($\sigma_1 - \sigma_3$) space, which is real space.

Homework: Stress



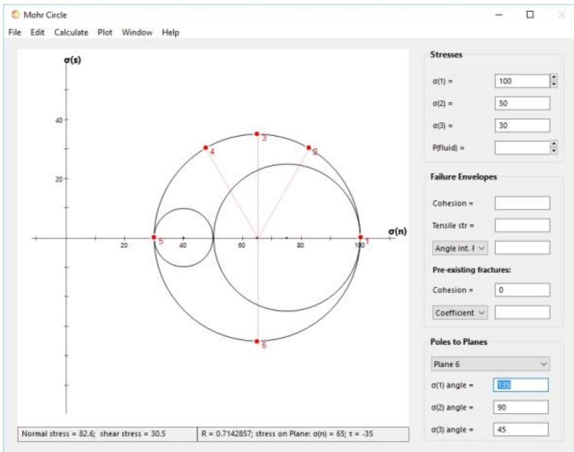
To estimate normal and shear stresses on six planes shown in (a) apply the Mohr construction in the graph (b). The principal stresses and angles θ are given. You should check your estimates from the construction by using the derived Equations for σ_n and σ_s:

$$\sigma_n = ½(\sigma_1 + \sigma_3) + ½(\sigma_1 - \sigma_3) \cos 2\theta$$

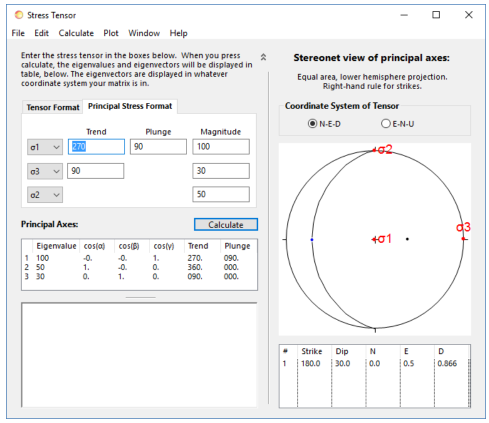
$$\sigma_s = ½(\sigma_1 - \sigma_3) \sin 2\theta$$

Name:
Date:

MohrPlotter



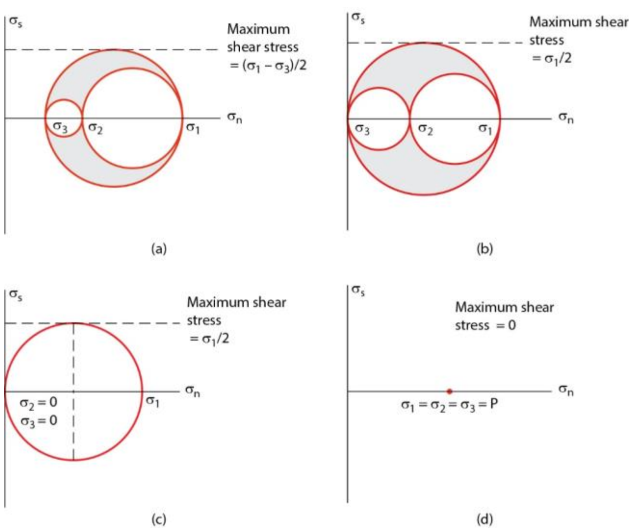
Note: MohrPlotter uses poles to planes, meaning angles are complements. So, 30º angle with σ_3 become 60º and 60º angle with σ_3 becomes 30º. Confusing, I know ….



Windows:
http://www.geo.cornell.edu/geology/faculty/RWA/programs/mohrplotterwin.zip
Mac: http://www.geo.cornell.edu/geology/faculty/RWA/programs/mohrplottermac.zip

© R Allmendinger

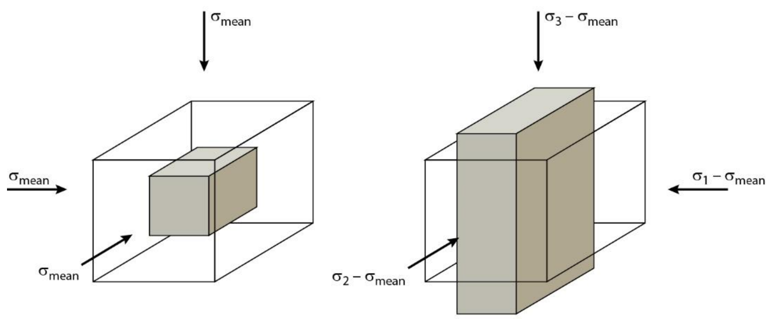
3D Stress states



Mohr circle construction allows 3D stress ellipsoid solution in 2D plot space

a) General triaxial stress: $\sigma_1 > \sigma_2 > \sigma_3 \neq 0$
b) Biaxial (or plane) stress: one axis = 0 (e.g., $\sigma_1 > \sigma_2 > 0$)
c) Uniaxial compression: $\sigma_1 > 0$; $\sigma_2 = \sigma_3 = 0$ (and Uniaxial tension: $\sigma_1 = \sigma_2 = 0$; $\sigma_3 < 0$)
d) Pressure (or Hydrostatic/Lithostatic stress): $\sigma_1 = \sigma_2 = \sigma_3$

Isotropic and Non-isotropic Stress



(a) (b)

σ_m + σ_{dev}

$$[\sigma_{ij}] = \begin{bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{bmatrix} + \begin{bmatrix} \sigma_{11}-\sigma_m & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}-\sigma_m & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}-\sigma_m \end{bmatrix}$$
where $\sigma_m = (\sigma_{11} + \sigma_{22} + \sigma_{33})/3$

(a) volume change, reflects isotropic mean stress, σ_m

(b) shape change, reflects non-isotropic stresses

Note: differential stress (=scalar) ≠ deviatoric stress (=tensor)

…. Matrix algebra

Extra: Stress Tensor and Matrix Algebra

The transformation of point *P* defined by coordinates *P*(*x, y, z*) to point *P*′(*x*′, *y*′, *z*′). We describe the transformation of the three coordinates of *P* as a function of *P*′ by

$$\begin{bmatrix} x' = ax + by + cz \\ y' = dx + ey + fz \\ z' = gx + hy + iz \end{bmatrix}$$

The tensor that describes the transformation from *P* to *P*′ is the matrix: $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

In matrix notation, the nine components of a stress tensor are:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = [\sigma_{ij}]$$

with σ_{11} oriented parallel to the 1-axis and acting on a plane perpendicular to the 1-axis, σ_{12} oriented parallel to the 1-axis and acting on a plane perpendicular to the 2-axis, and so on.

Mean stress and deviatoric stresses:

$$[\sigma_{ij}] = \begin{bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{bmatrix} + \begin{bmatrix} \sigma_{11}-\sigma_m & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}-\sigma_m & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}-\sigma_m \end{bmatrix}$$
where $\sigma_m = (\sigma_{11} + \sigma_{22} + \sigma_{33})/3$

Isotropic Stress: Lithostatic Pressure (P_l)

Mean stress is hydrostatic (water) pressure, P_h, or lithostatic (rock) pressure, P_l.

Consider rock at depth of 3 km, lithostatic pressure is :

$P_l = \rho \cdot g \cdot h$

$$[\sigma_{ij}] = \boxed{\begin{bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{bmatrix}} + \begin{bmatrix} \sigma_{11}-\sigma_m & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}-\sigma_m & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}-\sigma_m \end{bmatrix}$$

where $\sigma_m = (\sigma_{11} + \sigma_{22} + \sigma_{33})/3$

If ρ (density) is 2700 kg/m^3, g (gravity) is 9.8 m/s^2, h (depth) is 3000 m, we get:

P_l = 2700 · 9.8 · 3000 = 79.4 · 106 Pa ≈ 80 MPa (or 800 bars)

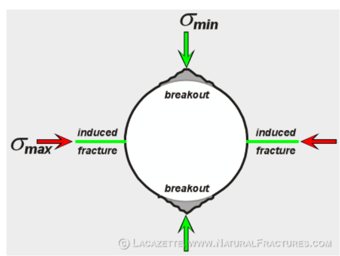
For every kilometer in Earth's crust, lithostatic pressure increases by approximately 27 MPa (and more with higher density rocks) -- or, ~1kbar per 3.3km.

Corresponding $P_h = \rho \cdot g \cdot h$ = 1000 · 9.8 · 3000 ≈ 29 Mpa (or 290 bars)

Note that stresses arising from plate movement are much smaller (see later)!

Modern Stress Measurements 1

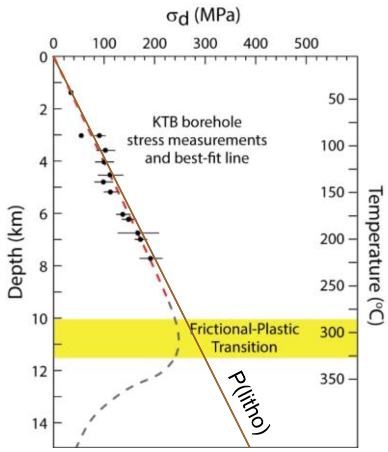
- **Borehole breakouts** - Shape of borehole changes after drilling in response to stresses in host rock. Hole becomes elliptical with long axis of ellipse parallel to minimum horizontal principal stress ($\sigma_{s\ hor}$).



- **Hydrofracture** - If water is pumped under sufficient pressure into sealed well, host rock will fracture. Fractures are parallel to maximum principal stress (σ_1), because water pressure necessary to open fractures is equal to minimum principal stress.

- **Strain release** - A strain gauge, consisting of tiny electrical resistors in a thin plastic sheet, is glued to bottom of borehole. The hole is drilled deeper with a hollow drill bit (called overcoring), separating core with strain gauge from wall of hole. Inner core expands (by elastic relaxation), which is measured by strain gauge. The direction of maximum elongation (**e**) is parallel to direction of maximum compressive stress and its magnitude is proportional to stress according to elasticity law (σ = E . **e**).

Differential Stress with Depth



KTB, Germany (9100m)

P(litho) = ρ · *g* · *h*

ρ (density) is 2700 kg/m³, *g* (gravity) is 9.8 m/s², and *h* (depth) is 1000 m, *P(litho)* ≈ 27 MPa (or 800 bars)

For every kilometer in Earth's crust, isotropic **lithostatic pressure** increases by approximately 27MPa (or 100Mpa=1kbar per ~3.3 km).

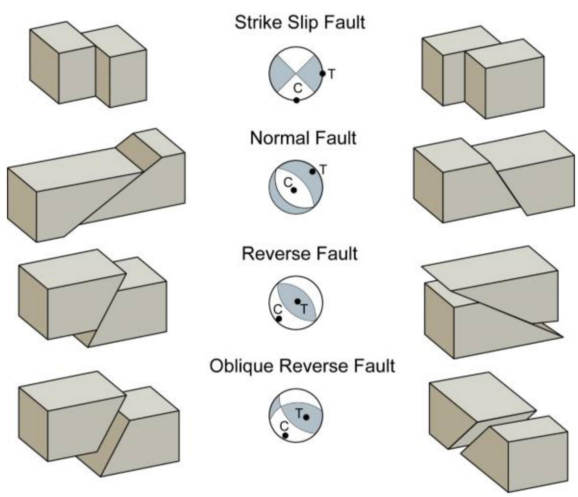
Instead, non-isotropic **differential stress** ($\sigma_d = \sigma_1 - \sigma_3$) is smaller, increasing to a few hundred MPa until dropping at Frictional-Plastic Transition (see later).

Modern Stress Measurements 2

Fault-plane solutions - When earthquake occurs, global seismometer records of first motion divide area into two sectors of compression (white) and two sectors of tension (gray), separated by two perpendicular planes.

One of these planes is fault plane on which earthquake occurred, and from distribution of compressive and tensile sectors, sense of slip on fault is determined.

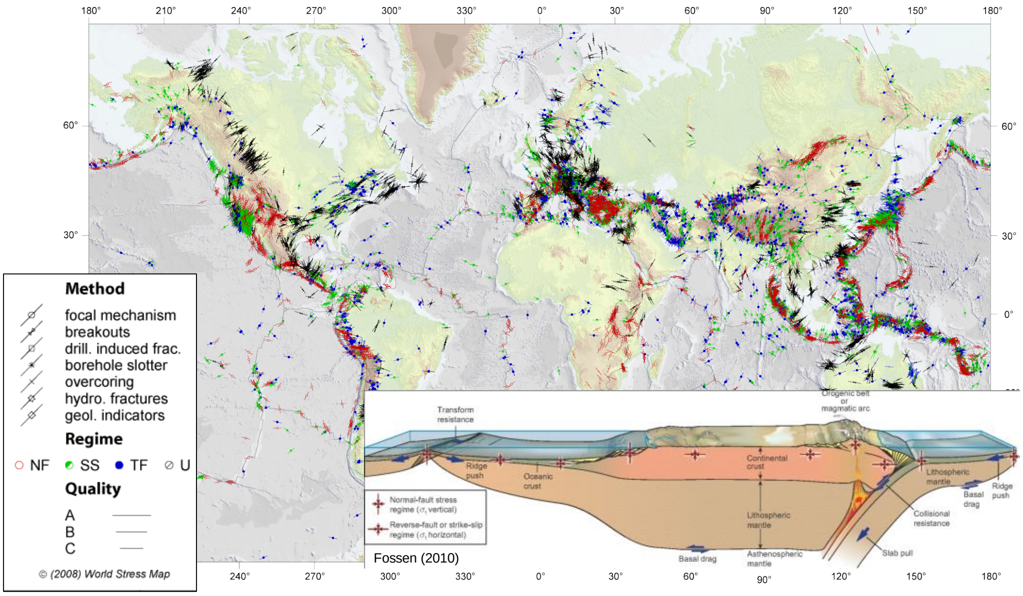
C and T define regions for σ_1 and σ_3, but *not* exact orientation (not a fracturing solution)!



USGS

World Stress Map (2008)

http://www.world-stress-map.org/

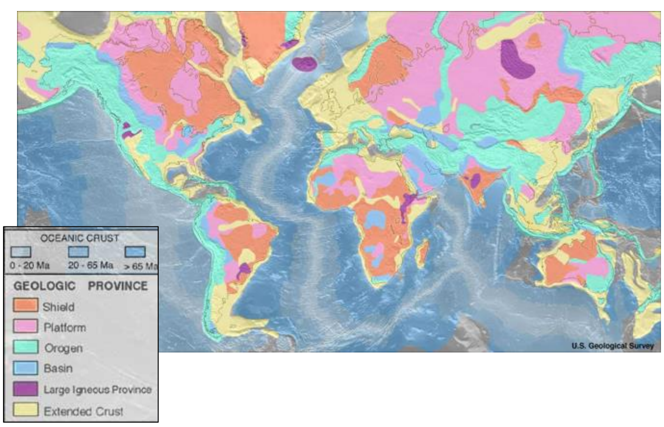


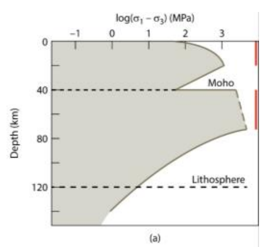
Heidbach, O., Tingay, M., Barth, A., Reinecker, J., Kurfeß, D., and Müller, B., The World Stress Map database release 2008 doi:10.1594/GFZ.WSM.Rel2008, 2008: www.world-stress-map.org

Differential Stress and Geologic Provinces

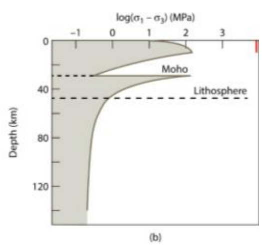
"Peanut butter sandwich" model

(σ_1-σ_3) is differential stress





Cold lithosphere (cratons; Precambrian rocks).



Hot lithosphere (orogens, ocean floor; Cenozoic rocks).