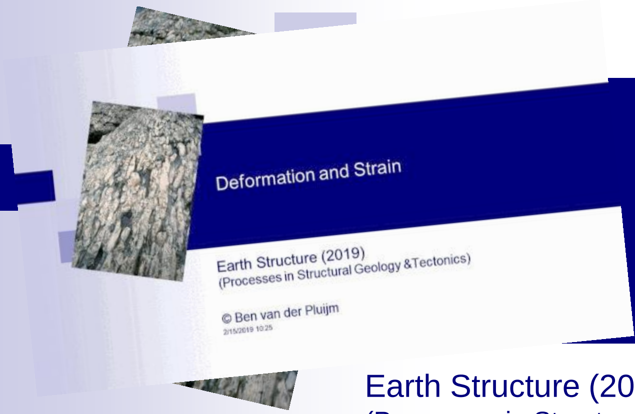


Deformation and Strain

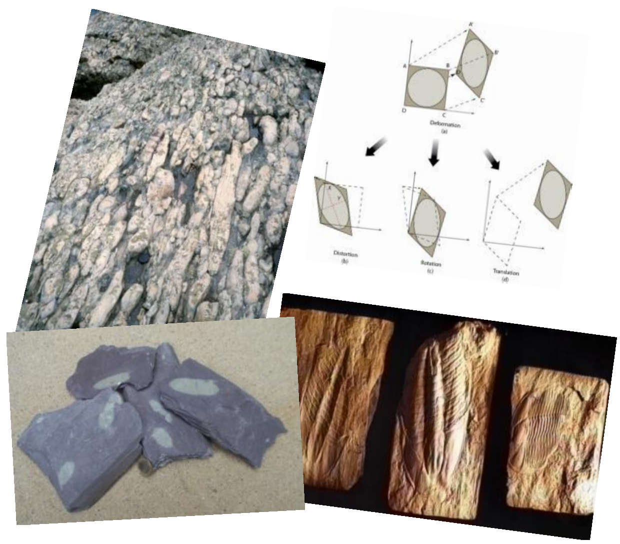
Earth Structure (2019)
(Processes in Structural Geology &Tectonics)

2/21/2019 16:32

We Discuss …

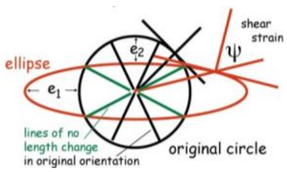
Deformation and Strain

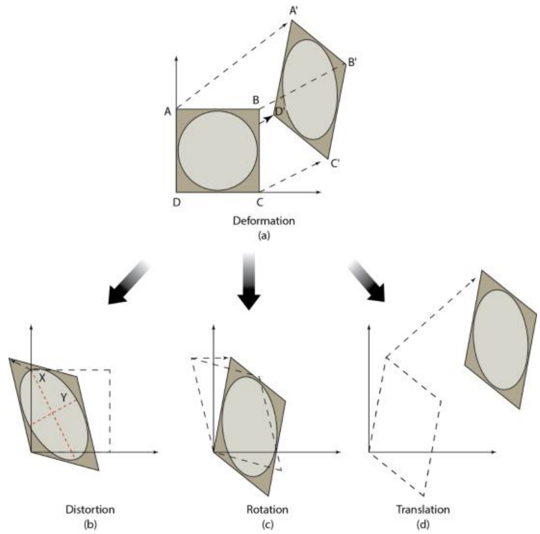
- Deformation Components
- Homogeneous vs. Heterogeneous Strain
- Strain Ellipse (and Ellipsoid)
- Coaxial and Non-coaxial Strain
- Superposed Strain
- Strain Quantities
- Representations of Strain
- (Finite) Strain Analysis
  - Spherical Objects
  - Angular Changes
  - Length Changes
- Understanding Strain Values



The Components of Deformation

1. Strain (distortion)
   a) Extension (or stretch)
      length changes
   b) Internal rotation (vorticity)
      finite strain axes rotate relative to instantaneous strain axes
   c) Volume change
2. Rigid-body rotation (or spin)
   instantaneous strain axes and finite strain axes rotate together
3. Rigid-body translation





Homogeneous vs. Heterogeneous Strain

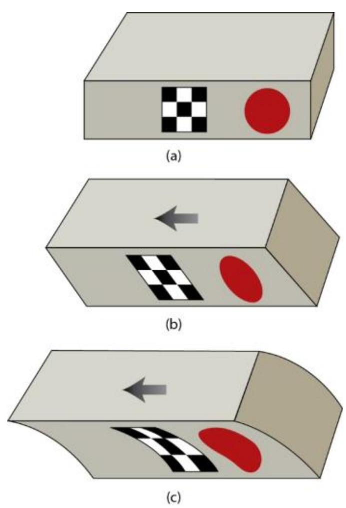
Homogeneous strain:

- Straight lines remain straight
- Parallel lines remain parallel
- Circles become ellipses (or spheres become ellipsoids)

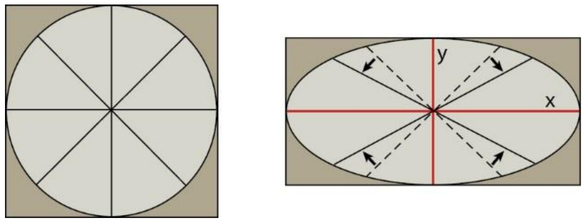
vs.

Heterogeneous strain

(deck of cards)

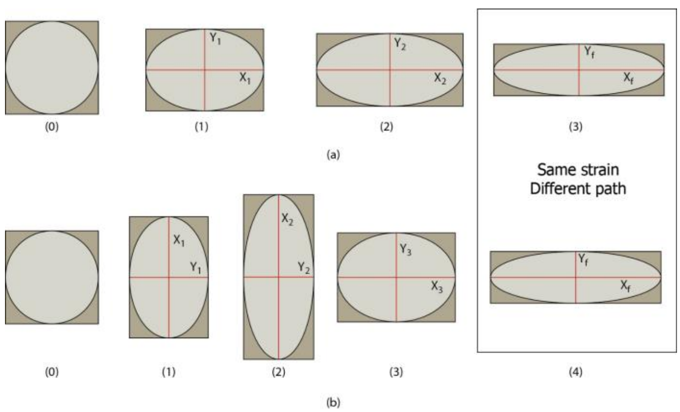


Homogeneous Strain: Principal Strain Axes



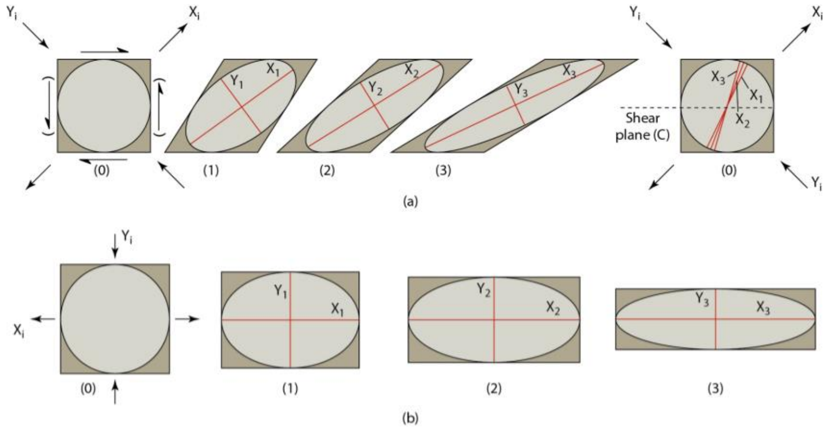
- Homogeneous strain: transformation of square to rectangle, circle to ellipse ("strain ellipse"). Note: lines in circle all equal length; line lengths in square vary by orientation.
- Two material lines perpendicular before and after strain are **principal axes** of strain ellipse (red lines): X≥Y≥Z.
- Dashed lines are **material lines** that do not remain perpendicular after strain; they rotate toward long axis of strain ellipse (X).

Strain path



- Incremental strain (steps)
- Finite strain (difference between unstrained and final shape)
- Infinitely small strain increments (mathematical): instantaneous strain

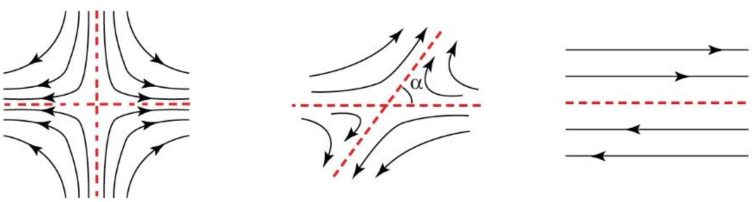
Strain Accumulation: coaxial and non-coaxial strain



a. (progressive) simple shear or non-coaxial strain
b. (progressive) pure shear, or coaxial strain

Compare incremental strain (X_i, Y_i) and finite strain ($X_{1,2,3}$, $Y_{1,2,3}$): **vorticity**

Vorticity (internal rotation) and Particle Paths



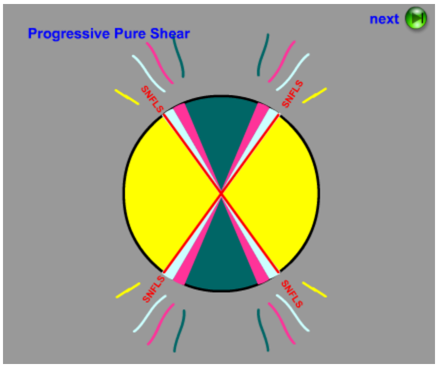
(a) coaxial (b) general shear (c) non-coaxial

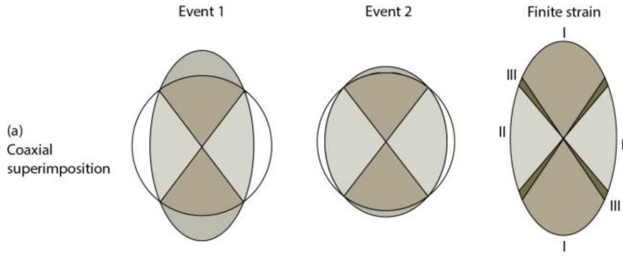
Rotation of material lines wrt to finite strain is kinematic vorticity.

Kinematic vorticity number: $W_k = \cos \alpha$

a. $W_k = 0$ (pure shear, coaxial strain end-member)
b. $0 < W_k < 1$ (general shear, non-coaxial strain)
c. $W_k = 1$ (simple shear, non-coaxial strain end-member)

Progressive Coaxial Strain – Pure Shear

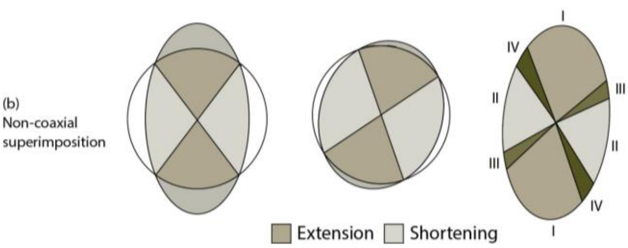




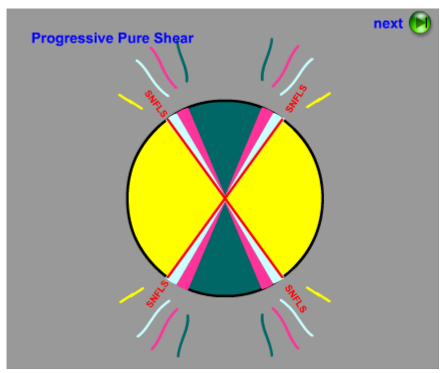
I. lines continue to be extended
II. lines continue to be shortened
III. shortening during event 1 is followed by extension during event 2.

DePaor, 2002

Progressive Non-coaxial Strain – Simple Shear



I. lines continue to be extended
II. lines continue to be shortened
III. shortening during event 1 is followed by extension during event 2.
IV. extension is followed by shortening during event 2

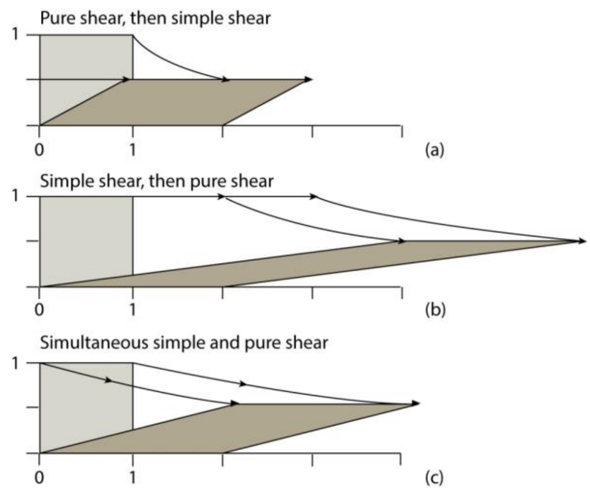


DePaor, 2002

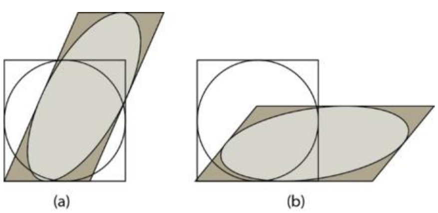
Strain Addition

"Adding" non-coaxial strain and coaxial strain is non-commutative:

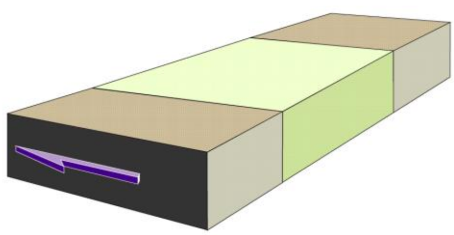
strainA + strainB ≠ strainB+ strainA



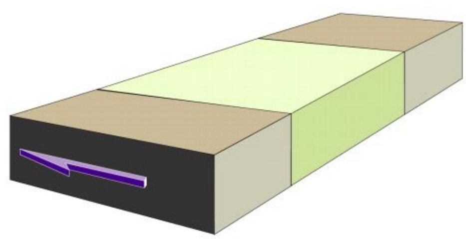
Combinations: Transtension and Transpression



a. Transtension
b. Transpression



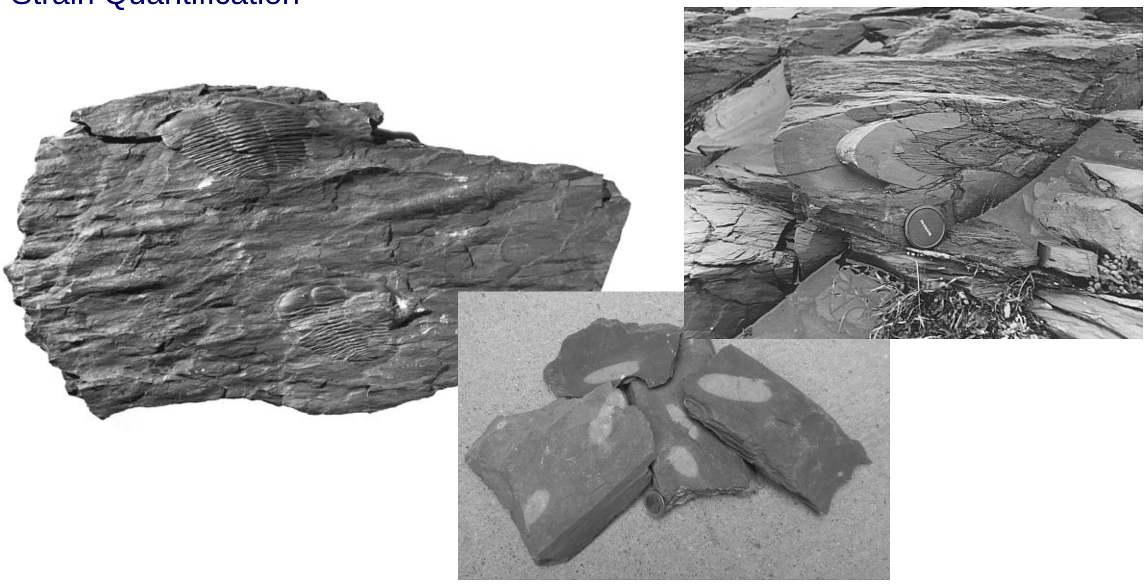
Transtension



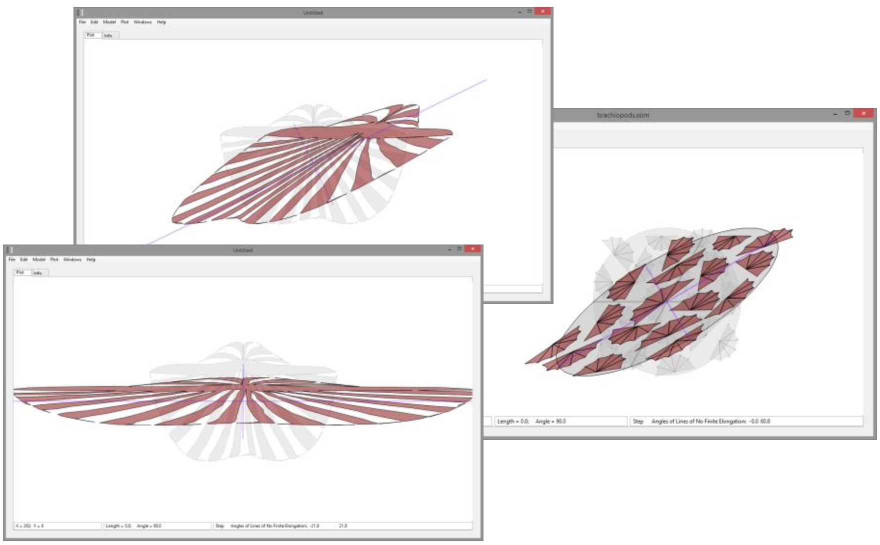
Transpression

Fossen, 2016

Strain Quantification

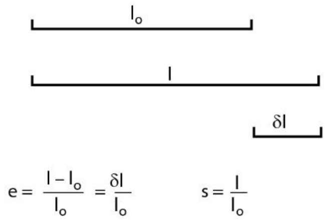


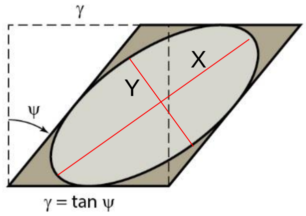
StrainSim (R. Allmendinger)



http://www.geo.cornell.edu/geology/faculty/RWA/programs/strainsim-v-3.html

Strain Quantities





e = elongation = $(l-l_o)/l_o = \delta l/l_o$
($e_1 \geq e_2 \geq e_3$; e_3 usually negative)

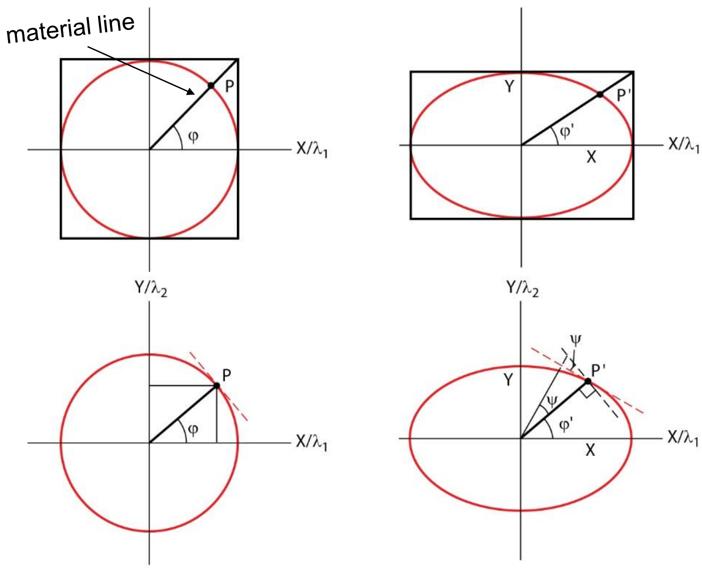
s = stretch = l/l_o (X ≥ Y ≥ Z; strain ellipsoid axes)
= $[(l-l_o)/l_o + l_o/l_o] =$ **e**+1

λ = quadratic elongation (lambda) = $s^2 = (e+1)^2$
$\lambda_1 \geq \lambda_2 \geq \lambda_3$

Misnomer: should be quadratic stretch

γ = shear strain (gamma) = tan ψ
ψ = angular shear (psi)

Relationship between strain ratio, ψ and line angle and length



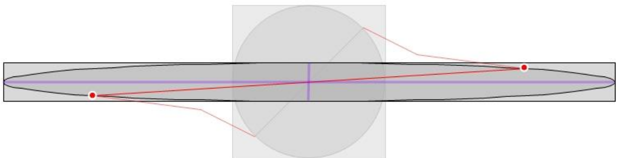
Angle:
φ to φ' is f^ion of strain ratio

tanφ' / tanφ = Y/X
(X.Y = 1, constant area)
So,
tanφ' = Y/X . tanφ
tanφ = X/Y . tanφ'

Length:
l_o to l is f^ion of strain ratio

$l^2 = X^2 + Y^2$
So ,
P' (x', y') =
X/Y.$\cos^2\varphi$, Y/X.$\sin^2\varphi$

Strain Calculator



StrainSim
http://www.geo.cornell.edu/geology/faculty/RWA/programs/strainsim-v-3.html

Strain Calculator		
strain ratio: X/Y	1	
initital angle, ϕ (deg)	45	45.0
deformed angle, ϕ' (deg)		45.0
initital length, lo	1	1.0
deformed length, l		1.0

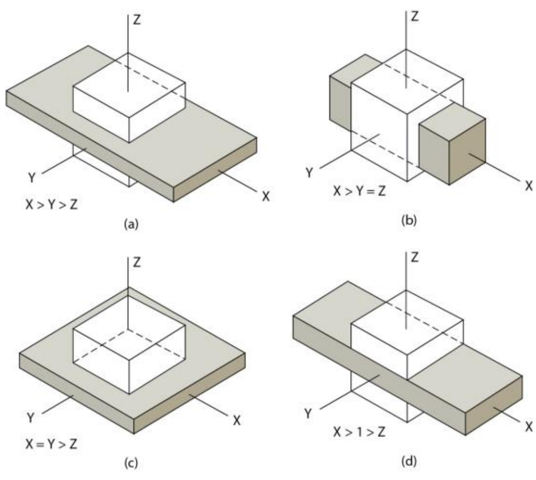
X=2, Y=1/2

Strain Calculator		
strain ratio: X/Y	4	
initital angle, φ (deg)	45	45.0
deformed angle, φ' (deg)		14.0
initital length, lo	1	1.0
deformed length, l		1.5

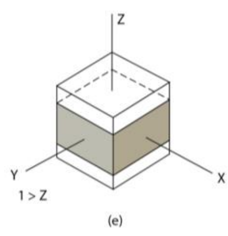
X=4, Y=1/4

Strain Calculator		
strain ratio: X/Y	16	
initital angle, φ (deg)	45	45.0
deformed angle, φ' (deg)		3.6
initital length, lo	1	1.0
deformed length, l		2.8

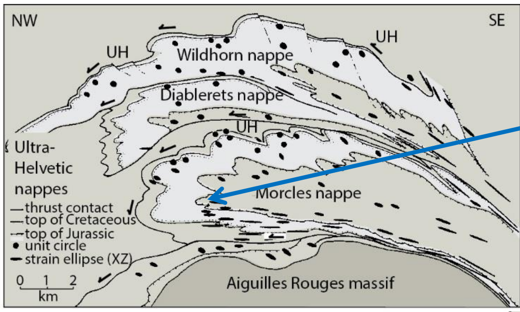
3D Strain States

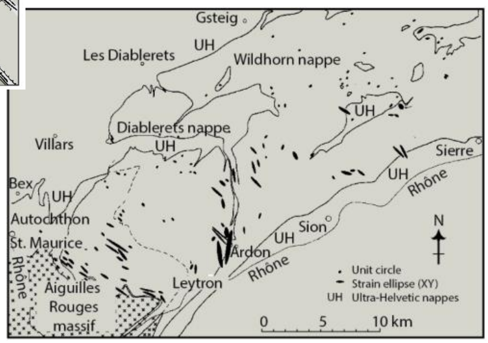


(a) General strain ($X > Y > Z$)
(b) Axially symmetric extension ($X > Y = Z$)
(c) axially symmetric shortening ($X = Y > Z$)
(d) plane strain ($X > 1 > Z$)
(e) simple shortening ($1 > Z$)



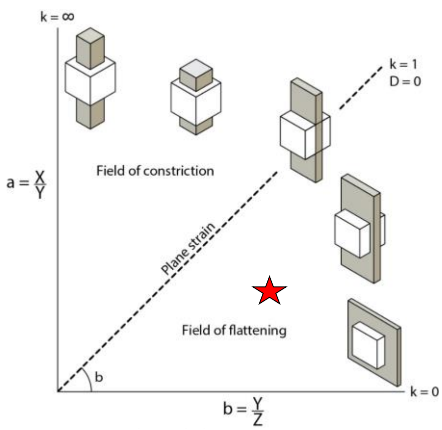
Representation of Strain – Section, Map

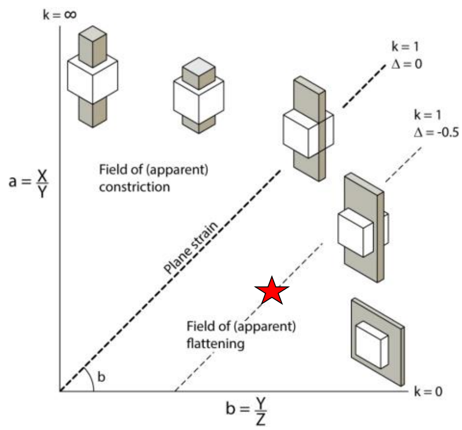




Strain in Helvetic "nappes" (large recumbent folds), Switzerland.

Representation of Strain – 3D in 2D-Space



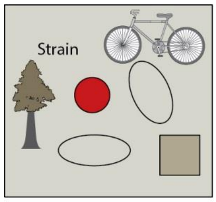


$\mathbf{i} = \{[(X/Y) - 1]^2 + [(Y/Z) - 1]^2\}^{1/2}$

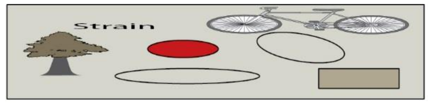
$\Delta + 1 = X \cdot Y \cdot Z$

3D strain geometry in 2D plot: axial ratios and volume change

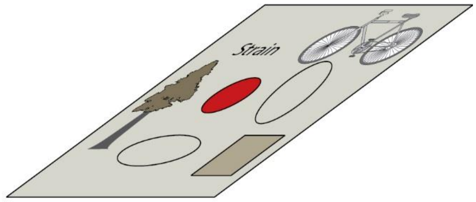
Strain Methods: Variably-shaped Objects



(a)

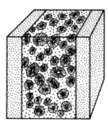


(b)

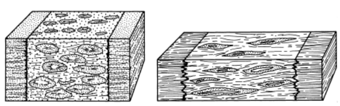


(c)

Strain Methods: Initially Spherical Objects



(a)

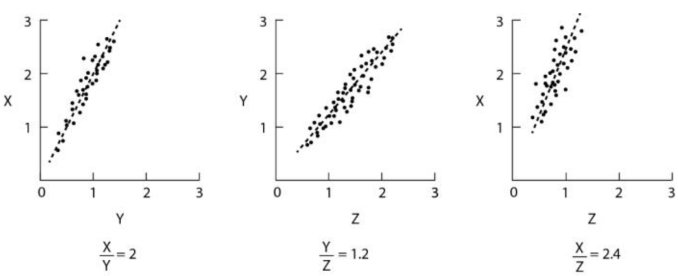


(b) (c)

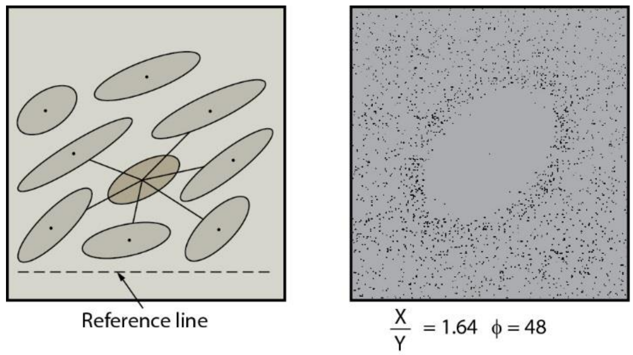
Deformed ooids after:

(b) 25% (X/Z = 1.8) shortening

(c) 50% (X/Z = 4.0) shortening



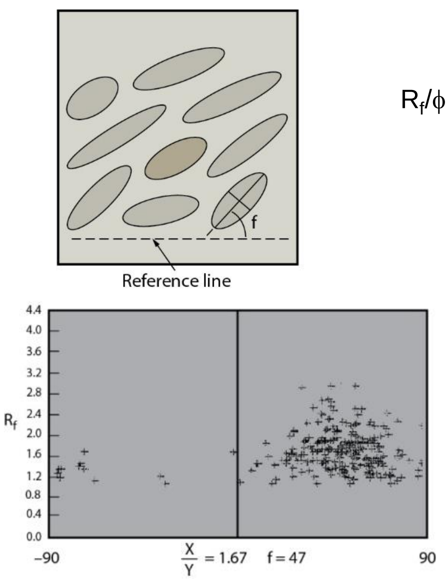
Strain Methods: Non-spherical Objects, c-t-c



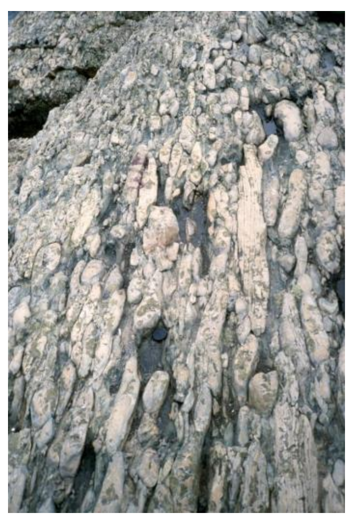
(a) (b)

Center-to-center method (or Fry method):
spacing varies as function of finite strain

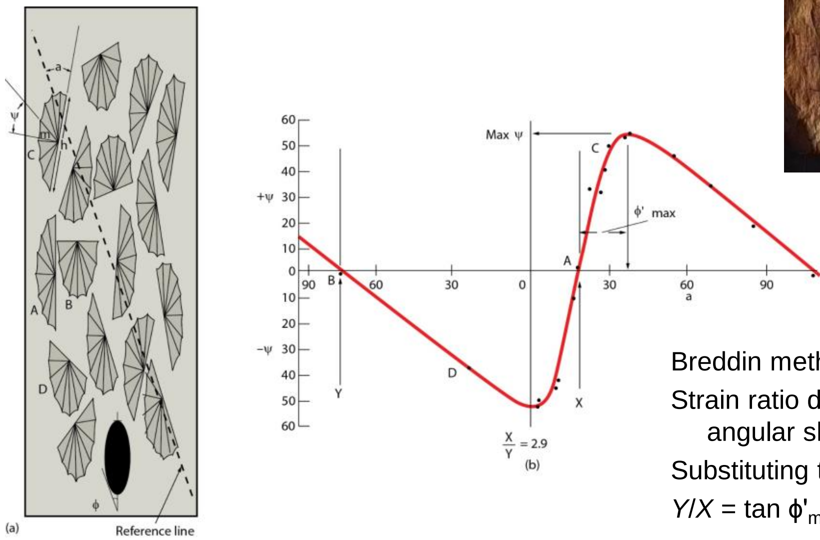
Strain Methods: Non-spherical Objects, R_f/ϕ



R_f/ϕ method



Strain Methods: Angular Changes



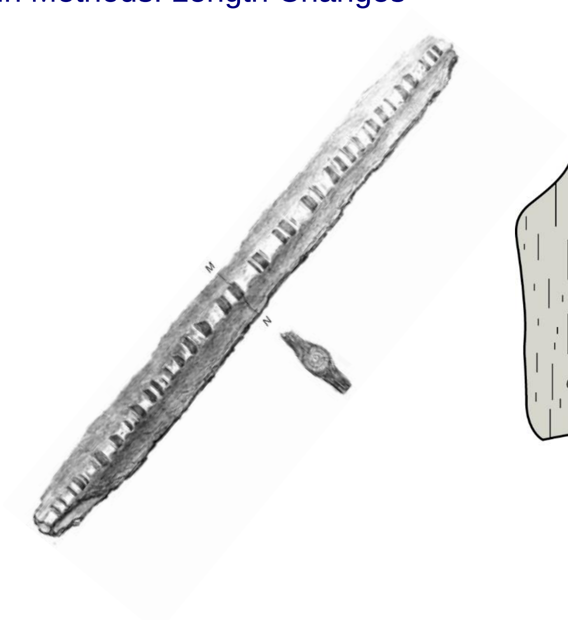
Breddin method:

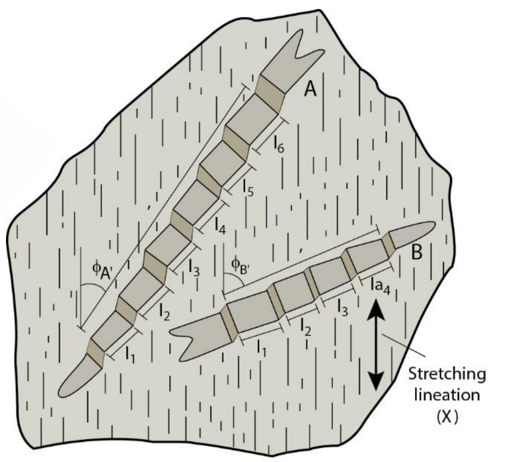
Strain ratio determined at maximum angular shear, angle ϕ = 45°

Substituting tan ϕ = 1

$Y/X = \tan \phi'_{max}$

Strain Methods: Length Changes





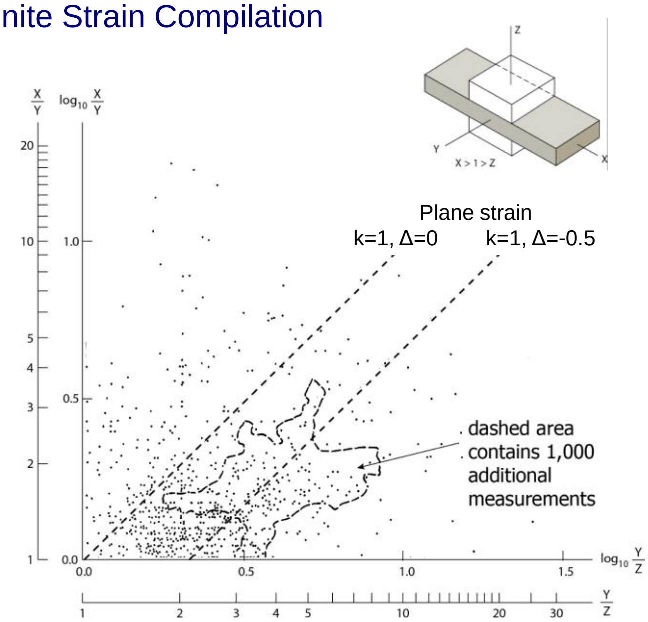
$\lambda'_A = [l/(l_1 + l_2 .. l_6)]^2$

$\lambda'_B = [l/(l_1 + l_2 .. l_4)]^2$

X = 1.36
Y = 1.11
Δ = 0.51

X/Y=1.23

Finite Strain Compilation

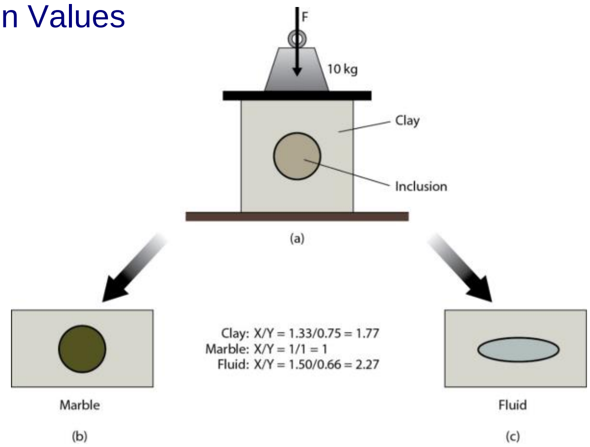


Typical strain values:
- 1 < X/Z < 20
- 1 < X < 3
- 0.13 < Z < 1

Plane strain: X>1>Z (k=1)
Δ is volume change

Where does lost rock volume go? Veins.

Understanding Strain Values



Strain and Mechanical Contrast:
- Passive markers have no mechanical contrast: bulk rock strain (clay and clay inclusion; oolite)
- Active markers have mechanical contrast: marker strain (clay and marble or fluid; conglomerate)