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Research report

A cognitive model for multidigit number reading: Inferences from individuals with selective impairments

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ABSTRACT

We propose a detailed cognitive model of multi-digit number reading. The model postulates separate processes for visual analysis of the digit string and for oral production of the verbal number. Within visual analysis, separate sub-processes encode the digit identities and the digit order, and additional sub-processes encode the number's decimal structure: its length, the positions of 0, and the way it is parsed into triplets (e.g., 314987 → 314,987). Verbal production consists of a process that generates the verbal structure of the number, and another process that retrieves the phonological forms of each number word. The verbal number structure is first encoded in a tree-like structure, similarly to syntactic trees of sentences, and then linearized to a sequence of number-word specifiers. This model is based on an investigation of the number processing abilities of seven individuals with different selective deficits in number reading. We report participants with impairment in specific sub-processes of the visual analysis of digit strings – in encoding the digit order, in encoding the number length, or in parsing the digit string to triplets. Other participants were impaired in verbal production, making errors in the number structure (shifts of digits to another decimal position, e.g., 3,040 → 30,004). Their selective deficits yielded several dissociations: first, we found a double dissociation between visual analysis deficits and verbal production deficits. Second, several dissociations were found within visual analysis: a double dissociation between errors in digit order and errors in the number length; a dissociation between order/length errors and errors in parsing the digit string into triplets; and a dissociation between the processing of different digits – impaired order encoding of the digits 2–9, without errors in the 0 position. Third, within verbal production, a dissociation was found between digit shifts and substitutions of number words. A selective deficit in any of the processes described by the model would cause difficulties in number reading, which we propose to term "dysnumeria".

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1. Introduction

Number reading is a complex cognitive operation involving several different sub-processes, each of which can be impaired and cause a different type of reading errors (Basso & Beschin, 2000; Cappelletti, Kopelman, Morton, & Butterworth, 2005; Cipolotti & Butterworth, 1995; Cipolotti, Warrington, & Butterworth, 1995; Cohen, Verstichel, & Dehaene, 1997; Dehaene, Piazza, Pinel, & Cohen, 2003; Delazer & Bartha, 2001; Deloche & Willmes, 2000; Dotan & Friedmann, 2015; Friedmann, Dotan, & Rahamim, 2010; McCloskey, Caramazza, & Basili, 1985; McCloskey, Sokol, & Goodman, 1986; McCloskey, Sokol, Caramazza, & Goodman-Schulman, 1990; Moura et al., 2013; Noël & Seron, 1993; Starrfelt & Behrmann, 2011; Starrfelt, Habekost, & Gerlach, 2010; Temple, 1989). In the present study, we propose a detailed model of how these cognitive mechanisms of number reading operate.

In doing so, we draw inspiration from models of word reading, another complex and potentially-similar cognitive function. Like number reading, word reading also involves a variety of processes: visually analyzing the sequence of letters, accessing the appropriate entries in orthographic, phonological, and semantic mental lexicons, generating the phonological output, and articulation. After several decades of research, we now have a cognitive model with detailed specification of the processes involved in word reading and of the flow of information among these processes (Coltheart, Rastle, Perry, Langdon, & Ziegler, 2001; Ellis & Young, 1996; Ellis, 1993; Friedmann & Coltheart, in press; Friedmann & Gvion, 2001; Humphreys, Evett, & Quinlan, 1990; Marshall & Newcombe, 1973; Patterson & Shewell, 1987; Shallice, 1988). This model turned out to be invaluable in several ways. From a theoretical point of view, an accurate model of word reading allows for better understanding of the reading mechanisms, and enables detailed investigation of other language processes such as morphology and lexical retrieval (Biran & Friedmann, 2012; Dotan & Friedmann, 2015; Friedmann, Biran, & Dotan, 2013; Funnell, 1983; Gvion & Friedmann, 2016; Job & Sartori, 1984; Reznick & Friedmann, 2009, 2015). From a clinical point of view, such a detailed model improves our ability to identify specific impairments in word reading, to learn about their characteristics, and consequently to diagnose and treat individuals with such impairments (Castles & Friedmann, 2014; Colenbrander, Nickels, & Kohnen, 2011; Coltheart & Kohnen, 2012; Friedmann & Coltheart, 2017; Friedmann & Gvion, 2001; Friedmann et al., 2013; Marshall & Newcombe, 1973; Nickels, 1997; Nickels, Rapp, & Kohnen, 2015; Rapp, 2005; Temple, 2006).

The cognitive model of word reading could not have been as useful had it not been very explicit in terms of information processing: the model accurately describes the function of each cognitive sub-process involved in reading, and the kind of information transferred between these processes, in a manner detailed enough to allow for computational simulation (Coltheart et al., 2001). This high level of granularity is what allows characterizing the interaction between reading and other language processes, and makes it possible to identify specific cognitive disorders in specific processing stages.

The reading of numbers (such as “256”) is implemented, at least in part, by separate mechanisms than the word reading mechanisms (Abboud, Maidenbaum, Dehaene, & Amedi, 2015; Friedmann, Dotan, & Rahamim, 2010; Hannagan, Amedi, Cohen, Dehaene-Lambertz, & Dehaene, 2015; Shum et al., 2013; for a review, see Dotan & Friedmann, 2018). However, number reading has not been investigated as much as word reading, and less is known about it. The present study aims to fill this gap.

1.1. *Existing models of number reading*

1.1.1. *The triple code model*

During the 1990's, there was much debate about the representation of symbolic numbers and the transcoding processes that convert between these representations (Cipolotti & Butterworth, 1995; Cipolotti et al., 1995; Cohen & Dehaene, 2000; Dehaene & Cohen, 1997; McCloskey, 1992; McCloskey et al., 1990, 1986; Sokol, McCloskey, Cohen, & Aliminosa, 1991). At present, a widely accepted model is the triple-code model of number processing (Dehaene & Cohen, 1995; Dehaene, 1992; Dehaene et al., 2003), which holds that separate cognitive and neural circuits represent numbers as sequences of digits, as verbal number words, and as quantities. With respect to number reading, the triple-code model postulates that the visual parsing of digital numbers and the verbal production of number words are handled by separate processes, connected by a direct digit-to-verbal transcoding pathway that is at least partially separate from the access to number semantics. Indeed, several studies have shown that the visual analysis of numbers can be selectively impaired (Cohen & Dehaene, 1995; Friedmann, Dotan, & Rahamim, 2010; McCloskey et al., 1986; Noël & Seron, 1993), and so can the verbal production of numbers (Benson & Denckla, 1969; Cohen et al., 1997; Delazer & Bartha, 2001; Dotan & Friedmann, 2015; Dotan, Friedmann, & Dehaene, 2014; Marangolo, Nasti, & Zorzi, 2004; Marangolo, Piras, & Fias, 2005).

1.1.2. *McCloskey's number reading model*

The triple-code model, as well as many of the above studies, characterized the different number representations and transcoding pathways. Other studies, though fewer, were specifically concerned with offering a detailed cognitive model of number reading. Michael McCloskey and his colleagues (McCloskey, 1992; McCloskey et al., 1986) proposed a model where number reading – transcoding a digit string into number words – is mediated by a central semantic representation, which essentially reflects the number's decimal structure (e.g., $2{,}031 = 2 \times 10^3 + 0 \times 10^2 + 3 \times 10^1 + 1 \times 10^0$). Their model postulates that converting this representation to number words begins by creating a *syntactic frame*, which reflects the verbal structure of a number with a given number of digits – e.g., for 4-digit numbers, the syntactic frame is *[_:ones] [thousand:multiplier] [_:ones] [hundred:multiplier] [_:tens] [_:ones]* (the [_:] notation – [_:ones], [_:tens] – represents placeholders for a number word of the corresponding lexical class[1]). The

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[1] McCloskey and his colleagues experimented in English and mentioned ones, teens and tens as lexical classes for words. The specific lexical classes may depend on the characteristics of verbal numbers in a specific language. Our study was conducted in Hebrew, in which the number words for hundreds and thousands often introduce some verbal irregularity and may therefore be lexicalized. This would result in hundreds and thousands as two additional lexical classes. However, this question – whether hundreds and thousands are indeed lexical classes in Hebrew – was not in the scope of the present study.

syntactic frame is then “filled” with the specific digit identities. In the example above, this results in *[2:ones] [thousand:multiplier] [_:ones] [hundred:multiplier] [3:tens] [1:ones]*. Within this filled frame, each slot uniquely identifies a single word.

Some numbers have an irregular structure – e.g., the digit 0 is not spoken. Such situations result in unfilled slots (as in the example above), which are discarded from the frame after it has been filled. In English, another irregularity is that the digit 1 in the tens position yields a teen word. This too results in modifying the filled frame – e.g., *[1:tens] [3:ones]* would be changed into *[3:teens]*.

After these modifications were made, the filled frame becomes a plan for phonological retrieval: each combination of a lexical class and a digit, or the specification of a multiplier word, is used to retrieve the corresponding phonological form of a single word. McCloskey et al. suggested that this form is retrieved from the phonological output lexicon, but in Dotan and Friedmann (2015) we showed that number words are actually retrieved from a dedicated phonological store that is separate from the phonological output lexicon of words.

1.1.3. *Cohen and Dehaene's number reading model*

Cohen and Dehaene (1991) proposed a modified version of this reading model: they proposed that the visual analysis of the digit string is directly followed by verbal production, without the mediation of a central semantic representation.

The challenge for any number-reading model is explaining how the number’s decimal and verbal structure are handled; Cohen and Dehaene proposed that they are handled by two separate processes, one visual and one verbal. The visual process, which is a part of the visual analysis of numbers, is responsible for parsing the number's decimal structure, which consists of the number length (how many digits it has) and the positions of 0 and 1. The remaining digits (2–9) are identified by a separate process. Within the verbal mechanism, Cohen and Dehaene accepted McCloskey's notion of a syntactic frame, but proposed that it is quickly converted into what they termed a *number word frame*. Conceptually, the number word frame is the number's verbal structure. Concretely, it is a sequence of lexical classes (ones, teens, tens) of the number words to be produced (the frame for 24,013 is *[_:tens] [_:ones] [thousand] [and] [_:teens]*). The number word frame is generated based on the number's decimal structure: the number length determines the basic structure of the number word frame, the positions of 0 indicate entries in the frame that should be skipped, and the existence of 1 in the tens position differentiates teens from tens. The number word frame is then filled with specific digit values and goes on to phonological retrieval and articulation.

In terms of information flow, the concept of number word frame may seem like a small deviance from McCloskey's model: instead of filling the syntactic frame and only then modifying it according to 0's and 1's, as McCloskey proposed, Cohen and Dehaene propose that the syntactic frame is first modified by 0–1, resulting in a number word frame, and only then filled. Theoretically, however, the difference is important: Cohen and Dehaene propose a concrete representation of the number's verbal structure that is independent of specific digits or number words.

1.2. *An integrated model of number reading*

Here we propose another model, which is a mixture of the above models with few modifications and additions. Just like the previous models of number reading, it accounts only for the reading of positive, base-10 integers, and admittedly ignores very long numbers, whose reading may involve different processes (in this study we considered only numbers up to 6 digits). The model also does not focus on “lexicalized” numbers such as “1984”, which may be identified as a whole and be processed in different pathways (Cohen, Dehaene, & Verstichel, 1994). A general illustration of this model appears in Fig. 1, and we revisit it with more detail in the General Discussion.

The model postulates that within visual analysis, one process extracts the number's decimal structure, which consists of the number length, the positions of 0 (but not of 1), and the way the number is parsed into triplets (e.g., 24013 is parsed as 24 and 013). Two other processes encode the digit identities and their relative order, and can provide the 1–9 value of each digit in the correct order. Within the verbal system, the number's decimal structure is used to generate a number word frame, defined as in Cohen and Dehaene's model: a sequence of word specifiers, each of which can be a number word lexical class, a multiplier word (e.g., thousand, hundred; we hereby refer to them as “decimal words”[2]), or the function word “and”. In conjunction with the 1–9 digit values, these specifiers unambiguously define the sequence of words that forms the verbal number. These words are retrieved from a dedicated phonological store and sent to articulation.

The main components of this model are similar to the models presented above. As in those models, we also assume separate processes for visual parsing and verbal production, each of which is further divided into a “lexical” component (which handles single digits or words) and structural components (which handle the relations between digits or words). From Cohen and Dehaene's (1991) model, we adopted the assumption that no semantic representation mediates the digit-to-verbal transcoding. From McCloskey's model, we borrowed the notion that number words are retrieved according to the lexical class and the digit. However, our model also proposes some modifications and enhancements to the existing models. First, we propose a different internal organization of the decimal structure extraction. In our model, the decimal structure does not consist of number length and 0, 1

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[2] In English, number words such as “hundred” and “thousand” are special in two respects. First, semantically, they impact the quantity in a predictable manner – they are multiplied by the preceding word, such that the quantity of “three hundred” is three times hundred, hence the term “multiplier”. Second, lexically: each multiplier is a single lexical item, separate from the units word (they are the “building blocks” of multidigit verbal numbers, Cohen et al., 1997; Dotan & Friedmann, 2015). We wish to keep the term “multiplier” to refer to the semantic notion, and use the term “decimal word” to refer to the lexical notion. Indeed, in some languages such as Hebrew, not all multipliers are decimal words. For example, “hundred” is a multiplier in the semantic sense, yet it is not an independent word: apparently, it is not a separate lexical entry in the phonological storage of number words, and for some numbers it is not even a separate orthographic entry (e.g., 200 is a single word – מאתיים, /matayim/).

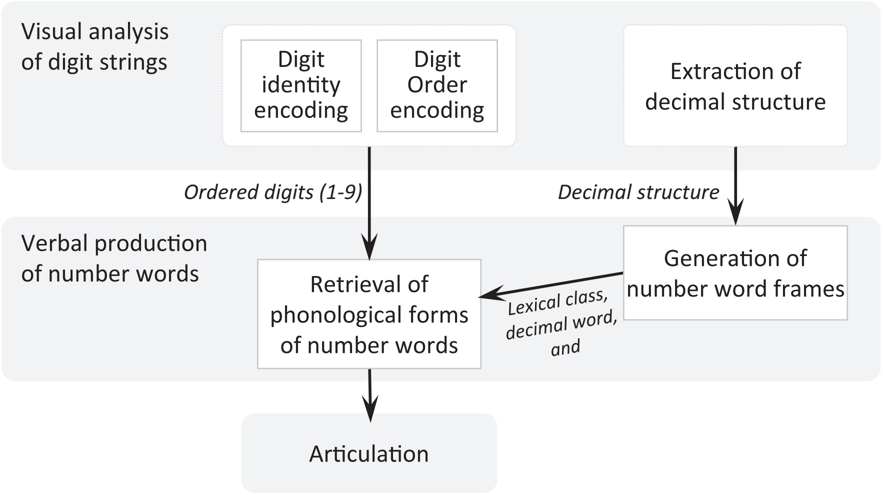

Fig. 1 – A proposed cognitive model of number reading. Separate processes handle the visual analysis of the digit string and the verbal production of the number words. The visual analyzer has several distinct sub-processes: the digit identity encoder and digit order encoder provide the identity of each digit (1–9) in their respective order. Another set of sub-processes extract the number's decimal structure, which is used to generate a number word frame – the number's verbal structure. The word frame is a sequence of one lexical class per number word (ones, teens, tens), and further specifies where decimal words ("thousand", "hundred") and the word "and" should be embedded in the number. Each entry in the number word frame, in conjunction with the corresponding digit value, is used to retrieve the phonological form of one number word at a time.

positions, but of number length, the positions of 0 (not 1), and the number's triplet structure. Below, we bring evidence from number reading impairments in support of these claims. Second, the process of digit identification was broken here into two – a digit identity encoder and a digit order encoder (Friedmann, Dotan, & Rahamim, 2010). Third, we accept Cohen and Dehaene's definition of the number word frame, however, how this frame is obtained is different in our model: we discarded the notion of a syntactic frame, and in the General Discussion we describe several specific processes involved in generating the number word frame.

1.3. *The present study*

The present study reports seven neuropsychological case studies with different types of number reading impairments, whose performance led us to propose the model above. We report individuals with selective impairments in three of the components depicted in Fig. 1: the encoding of digit order, the extraction of decimal structure, and the generation of number word frames. Previous studies have shown that selective impairments can occur also in the phonological retrieval of number words (Cohen et al., 1997; Delazer & Bartha, 2001; Dotan & Friedmann, 2015; Girelli & Delazer, 1999; Marangolo et al., 2004, 2005) and in the articulation of number words (Dotan et al., 2014; Shalev, Ophir, Gvion, Gil, & Friedmann, 2014). Furthermore, we report specific dissociations that support the separation of decimal structure extraction into three distinct sub-processes – encoding the number length, identifying the positions of zeros, and parsing the number into triplets.

2. Method

2.1. *Participants*

Seven adults with various number processing impairments participated in this study: HZ and OZ were undergraduate students. EY was a PhD candidate whose performance was reported in Friedmann, Dotan, and Rahamim (2010). MA was a self-employed woman with an undergraduate degree. ED and NL were sisters: ED had an undergraduate degree and worked in an administrative job, and NL was a BA student. Finally, UN was a retired lawyer who was recovering from a stroke that he had 3 months prior to our first meeting. All participants were native speakers of Hebrew, with normal or corrected-to-normal vision. Table 1 shows their background information.

EY was selected for the study because we previously diagnosed her as having a number reading deficit (Friedmann, Dotan, & Rahamim, 2010). The remaining participants were recruited through ads in the university and in social networks, which invited adults with difficulties in numbers to participate

Table 1 – The participants' background information.

	HZ	EY	MA	ED	NL	OZ	UN
Gender	F	F	F	F	F	M	M
Age	24	34	26	31	24	20	79
Dominant hand	R	R	R	R	R	R	R
Education years	13	20	15	15	13	14	20
Acquired/Developmental deficit	D	D	D	D	D	D	A

in a study. We selected for the study people who erred in reading at least 15% of the items in a list of 60 numbers with 3–5 digits, as long as at least one error type was observed in more than 10% of the items (the number reading errors were classified into types as described in Section 3.2 below). Because our goal was to detect dissociations, we avoided including participants who had multiple error types, except the first two participants that we encountered (HZ and UN).

All control participants were native speakers of Hebrew with at least 12 years of education and no reported cognitive disorders. They were compensated for participation. All participants and control participants gave informed consent to participate in the study. The Tel-Aviv University ethics committee approved the experimental protocol.

2.2. *General procedure*

The participants were tested individually in a series of 1- to 2-h sessions in a quiet room. All tests were conducted in Hebrew. Unless specified otherwise, EY read stimuli from the computer screen, where each stimulus was presented for 400 msec, and the other participants read the stimuli from paper, where they were printed as vertical lists. Each task is described in the text below, and additional methodological comments appear in the supplemental online material. Table S1 lists the tasks used in this study and the cognitive processes that each task can tap. When the participants made both a correct and an erroneous response to a single item, the response was classified as an error. Error percentages were calculated out of the total number of items in a task.

Appendix A shows the demographic details of the control participants in all experiments. Control participants with outlier error rates (higher than the 75th percentile by more than 150% the inter-quartile range) were excluded. Statistical comparisons of individual performance between conditions were done using chi-square test or Fisher's exact test. Individual participants were compared to control groups using Crawford and Garthwaite's (2002) one-tailed *t*-test. In cases of a control group ceiling effect (mean error rate $\leq 2\%$), the low variance does not allow for a reliable statistical comparison. In such cases, we set an arbitrary threshold for impaired performance. This threshold was set at 7% errors, in line with the recommendations of Willmes (1990) for analyzing performance in situations of ceiling effects.

We analyzed dissociation patterns according to the definition provided by Shallice (1988): when a participant's performance in condition A was worse than the control group and better than his own performance in condition B, this was defined either as a *strong dissociation* (if the performance in B was also worse than the control group) or as a *classical dissociation* (if the performance in B was comparable to the control group). In this dissociation analysis, participants were compared with the control group as defined above, and the participant's own performance was compared between conditions with a χ^2 test.[3]

3. Experimental investigation

3.1. *Background: language assessment*

The participants' language abilities and general cognitive abilities were examined using several tasks (Table 2): digit and word spans (FriGvi, Friedmann & Gvion, 2002; Gvion & Friedmann, 2012; comparison to control data was done using Crawford & Garthwaite's *t*-test, 2002); picture naming (SHEMESH, Biran & Friedmann, 2004); reading single words, nonwords, and word pairs (TILTAN screening test, Friedmann & Gvion, 2003); lexical decision (TILTAN, Friedmann & Gvion, 2003), in which they classified letter strings as words or nonwords without reading them aloud (the task included words, migratable nonwords, i.e., nonwords in which letter migration can yield an existing word, and non-migratable nonwords); nonword reading (TILTAN, Friedmann & Gvion, 2003); nonword repetition (BLIP, Friedmann, 2003); and writing single words to dictation (TILTAN, Friedmann, Gvion, & Yachini, 2007).

These tasks showed that HZ and EY had letter position dyslexia, a selective deficit in letter position encoding by the visual analyzer (Friedmann & Gvion, 2001; Friedmann & Rahamim, 2007; Friedmann, Dotan, & Rahamim, 2010; Kezilas, Kohnen, McKague, & Castles, 2014; Kohnen, Nickels, Castles, Friedmann, & McArthur, 2012). Both of them had a high rate of letter migration errors in word reading and in lexical decision – two tasks that have orthographic input so they involve the orthographic-visual analyzer. Conversely, they did not have migration errors in tasks that did not involve the orthographic-visual analyzer (i.e., tasks without orthographic input): neither had errors in spontaneous speech, and HZ did not have many migrations also in formal tasks without orthographic input – picture naming, nonword repetition, and sentence elicitation. HZ also had a mild surface dysgraphia (Barry, 1994; Weekes, 1996).

MA, ED, NL, and OZ had intact word reading, writing, and naming (for a detailed comparison of number reading with word reading, see Dotan & Friedmann, 2018). UN, the participant with acquired aphasia, had impairments in writing and naming and a low digit span (lower than that of the other participants). He also had some difficulty in word reading. Most of his reading errors were surface errors and vowel letter errors, which typically originate in processing stages later than visual analysis (Friedmann & Lukov, 2008; Gvion & Friedmann, 2016; Khentov-Krauss & Friedmann, in press).

EY, MA, ED, and NL had slightly low scores on memory span tasks that involved production, suggesting a slightly low capacity of phonological short-term memory (PSTM). For MA, ED, and NL, we tested and found normal scores in PSTM tasks not involving verbal production, indicating that this capacity limit was specifically in the production-related PSTM. This mild PSTM impairment did not seem to impact their speech: they performed well in naming and in nonword repetition, tasks that are typically sensitive to PSTM deficits (Friedmann et al., 2013). A deficit in production-related PSTM (the phonological output buffer) sometimes causes substitutions of number words during speech (Dotan & Friedmann, 2015). However, as we shall see below, here this was not the case for any participant except UN.

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[3] Due to control group ceiling effects, we could not compute the participants' standardized scores in each task as the basis for comparison (as required by the definition of Crawford, Garthwaite, & Gray, 2003).

Table 2 – Memory spans, and error percentages in language tasks.

	No. of items	HZ	EY	MA	ED	NL	OZ	UN
Memory spans								
Digit (free recall)		6	5^{*}	5^{*}	5^{*}	5^{*}	6	3^{**}
Digit (matching)		7		7	7	7	7	4
Word (free recall)		6		4^{*}	4½	5	6	3^{*}
Word (matching)		7		5	7	7	7	2^{**}
Picture naming	**100**	1		0	2	3	2	26^{***}
Word reading	**136**							
All errors		14^{***}	16^{***}	2	1	3	1	12^{***}
Migration errors[a]		24^{***}	30^{***}	0	0	0	0	0
Lexical decision	**60**							
Migratable nonwords	**15**	60^{+++}	65^{+++}	0	0	0	7^{+++}	0
Non-migratable nonwords	**15**	20^{+++}	5	0	0	13^{+++}	7^{+++}	7^{+++}
Nonword reading	**40**							
All errors		43^{***}	17^{**}	8	3	8	5	57^{***}
Migration errors		35^{***}	17^{***}	8	0	3	3	3
Nonword repetition	**48**	2		4	2	2	4	46^{***}
Word writing	**50**	14^{***}		4	2	2	4	28^{***}

Comparison *vs.* control group: $^{*}p < .05$ $^{**}p < .01$ $^{***}p < .001$. $^{+++}$Errors ≥ 7%, control group ≤ 2% errors.

[a] Percentage out of all words with a lexical potential for interior migration.

3.2. *Experiment 1: assessment of number reading*

The participants' ability to process symbolic numbers was first assessed with a number reading task, which involves digit input and verbal output.

3.2.1. *Method*

The participants read aloud from paper a randomly-ordered list of 120 Arabic numbers with 3, 4, 5, or 6 digits (30, 38, 47, and 5 numbers, respectively). The digit 0 appeared in 63 of the numbers, and the other numbers contained only the digits 2–9. The full list of numbers is enclosed as supplemental material. EY read a different list of 316 numbers with 3, 4, or 5 digits (100, 84, and 132 numbers, respectively), each presented on the computer screen for 400 msec. The digit 0 appeared in 134 of the numbers, and the others contained only the digits 2–9.

3.2.2. *Results*

All participants had many errors in the number reading task (Table 3). The errors were classified as follows: *transposition*, or a *digit order error*, is a change in the relative order of digits (e.g., 234 → 324). In word reading, transposition errors are the hallmark of letter position dyslexia, a deficit in letter position encoding by the visual analyzer (Friedmann & Gvion, 2001; Friedmann & Haddad-Hanna, 2012, 2014; Friedmann & Rahamim, 2007; Kezilas et al., 2014; Kohnen et al., 2012). A similar deficit also exists in the visual analyzer of numbers (Dotan & Friedmann, 2017; Friedmann, Dotan, & Rahamim, 2010).

A *decimal shift* is the production of a number word as if the corresponding digit was in a different decimal position (e.g., 2345 → “two thousand and thirty… sorry, three hundred and forty five” – in this example, the error was spontaneously corrected).

Importantly, whereas both error types – decimal shift errors and digit order errors – reflect situations where one or more digits appear in an incorrect decimal position, these are two different error types. Digit order errors are digit displacements that result in erroneous relative order of digits except 0 (which, in turn, causes erroneous order of the corresponding number words – e.g., 2345 → 2354). In contrast, decimal shifts are digit displacements that keep the relative order of digits (and hence do not result in erroneous order of number words).[4] The distinction between decimal shifts and digit order errors was demonstrated in our data by the finding of a double dissociation between the two error types: EY had only digit order errors, whereas MA, ED, NL, OZ, and UN had only decimal shift errors. The difference between order errors and decimal shifts was significant for all these participants ($\chi^2 = 5.81$, two-tailed $p = .02$ for NL; and $\chi^2 > 11.8$, two-tailed $p < .001$ for the others). Thus, for these participants, the difference between order errors and decimal shifts fulfills the criteria for classical dissociation (Shallice, 1988). This double dissociation indicates that the digit order errors and decimal shift errors have different cognitive origins. In the following sections, we confirm and clarify this dissociation and its implication for the number reading model.

Decimal shift errors are especially interesting when they occur in the leftmost digits of the number (e.g., reading 234 as 2034) – hereby, *first-digit shifts*. Such errors may indicate that the participant was processing the number structure incorrectly. For example, the above example may originate in the 3-digit number 234 being processed as if it has 4 digits. Decimal shifts were therefore analyzed by their position in the target number.

There were also *errors related with the decimal word “thousand”* (in 5- and 6-digit numbers) – omission of the word

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[4] Some errors can arguably be classified both as an order error and as a decimal shift: this is the case when a non-leftmost digit was transposed with a zero (e.g., 3,405 → 3,045). These errors were rare (only 9 errors for all participants pooled together), and were classified as decimal shifts.

Table 3 – Error percentages in number reading (Experiment 1). EY had many transpositions, without decimal shifts. MA, ED, NL, OZ, and UN had many decimal shift errors, with only a few transpositions. HZ had many errors of both types.

	Order	Decimal shift	Substitutions	Thousand[a]	All errors[b]
HZ	18^{+++}	30^{+++}	2	0	46^{***}
EY	17^{***}	0	2	0	27^{***}
MA	4	18^{+++}	0	0	20^{***}
ED	3	17^{+++}	1	6	23^{***}
NL	5	14^{+++}	2	1	23^{***}
OZ	2	22^{+++}	3	13^{+++}	32^{***}
UN	1	24^{+++}	17^{+++}	15^{+++}	44^{***}
Controls (SD)	0.5 (0.7)	1.1 (1.1)	0.7 (0.8)	0.9 (1.3)	2.8 (1.3)
EY Controls (SD)[c]	2.1 (1.5)	0.03 (0.1)	2.2 (2.3)	0	6.6 (4.3)

Comparison to the control group: $^{***}p \le .001$ $^{+++}$Errors ≥ 7%, control group ≤ 2% errors.

[a] The rate of errors related with the decimal word "thousand" was counted out of the 52 numbers that contained the word "thousand" (numbers with 5 or 6 digits).

[b] The percentage of items with any error.

[c] EY read a different list of numbers. Her control group had more errors than the other control group, perhaps because they saw each number for 400 msec (like EY) whereas the other control group had unlimited exposure.

"thousand" or addition of an excessive "thousand". Last, there were *substitutions* of a digit by another digit (e.g., 234 → 294).

The participants showed different error patterns: HZ and EY had high rates of digit order errors; all participants but EY had many decimal shift errors; OZ and UN, and to a lesser extent ED, had many errors in the decimal word "thousand"; and UN had many substitution errors. Importantly, even when making mistakes, participants rarely produced invalid number names (e.g., 2030 → "two thousand and three thousand" or "two, thirty"): none of them produced more than 2 invalid number names in this task.

3.2.2.1. Digit order errors. Digit order errors may result from impaired encoding of the digit order by the visual analyzer. Note that according to the model in Fig. 1, different visual analyzer sub-processes encode the order information of different digits: the order of 1–9 is encoded by the digit order encoder, whereas the presence of 0 and its positions are encoded by another, dedicated process, as part of extracting the number's decimal structure. A spared zero detector could potentially compensate for an impaired digit order encoder if the number has 0. To examine this, we analyzed the order errors of HZ and EY (the two participants who had order errors) in numbers with or without 0. EY had 25.8% order errors in numbers that included only the digits 2–9 (significantly more order errors than the control group, whose error rate was 2.9%, SD = 2.2%, $t(9) = 6.93$, $p < .001$); in contrast, she had only 6.0% order errors in numbers that included 0 (which did not pass our 7% criterion for impaired performance given the control group's ceiling effect, 0.9% errors). This difference was significant ($\chi^2 = 19.58$, one-tailed $p < .001$) – i.e., the difference between numbers with 0 and without 0 showed a pattern of classical dissociation (Shallice, 1988). This suggests that EY had a selective impairment in digit order encoding, yet her impairment spared the encoding of the positions of 0. In contrast to EY, HZ' order errors in numbers with 0 (35%) were as frequent as in numbers that included only the digits 2–9 (46%; $\chi^2 = 1.49$, one-tailed $p = .11$), suggesting that her impairment was not as selective as EY's: she was impaired both in the digit order encoder and in the decimal structure extractor's sub-process that detects the digit 0.

3.2.2.2. Decimal shift errors. All participants except EY had decimal shift errors. Some decimal shifts involved omissions of digits and number words, and in other cases a zero was omitted (so no word was omitted). Participants usually self-corrected their decimal shift errors (e.g., 2345 → "two thousand and thirty… sorry, three hundred and forty five"): 85% self-corrections for HZ, 100% for the other participants, but UN self-corrected only 32% of these errors. We assume that the spared digit identity encoding of all participants (except UN) provided cues that allowed them to detect and self-correct their mistakes.

Table 4 shows decimal shift errors according to their position in the target number: shifts of the leftmost digit or digits (e.g., 4,320 → 40,320 or 432), shifts of the first digit of the second triplet (e.g., 4,320 → 4,032), and shifts of other digits (e.g., 4,320 → 4,302). The table clearly shows that decimal shift errors were most frequent in the leftmost digits. We examined whether the erroneously produced number had more or fewer digits than the target number (rightmost columns in Table 4). No clear tendency was found – "longer" and "shorter" errors did not significantly differ for any of the participants (binomial test, $z \le 1.25$, two-tailed $p \ge .21$).

3.2.3. *Discussion of Experiment 1*

All the participants showed impaired oral reading of numbers, yet they showed different types of errors. Two participants – HZ and EY – had high rates of digit order errors. These errors may originate either in the visual analyzer, which encodes the digit order, or in the verbal production processes. In Section 3.3 we assess the exact locus of deficit underlying their order errors.

EY's order errors were almost completely absent from numbers that included the digit 0. Our explanation for this pattern is that EY's impairment selectively disrupted the processing of digit order, but the positions of 0 are processed by another mechanism, which was not impaired for EY. This issue is systematically examined in Section 3.4.

Table 4 – Decimal shift errors in number reading, classified by the decimal position of the target digit. For each participant, most of the errors occurred in the first (leftmost) digits. The table shows raw number of errors in reading 120 numbers in Experiment 1, and in parentheses, the percentage of each error out of the participant's total number of decimal shift errors.

	Position of decimal shift			First-digit-shift made the number… [a]	
	Leftmost digits	First digit of 2^{nd} triplet	Other digits	Longer	Shorter
HZ	33 (92%)	3 (8%)	1 (3%)	13	14
MA	18 (82%)	4 (18%)	0	9	7
ED	20 (100%)	0	0	11	5
NL	14 (82%)	4 (24%)	0	5	7
OZ	17 (65%)	10 (38%)	1 (4%)	6	8
UN	22 (76%)	6 (21%)	1 (3%)	6	11

[a] In the two right columns, the numbers sum to less than the total number of first-digit-shift errors because the longer/shorter classification of some errors was ambiguous.

All participants except EY had many decimal shift errors. These errors were not uniformly distributed across all decimal positions – most of them occurred in the leftmost digits. This pattern may have two explanations. One possibility is that the participants processed the number length incorrectly, i.e., they processed numbers (e.g., 4,320) as if they had more digits (reading it as 43,200) or fewer digits (432). Alternatively, the participants may have grouped the digits incorrectly to triplets (e.g., as 43,20 rather than 4,320). Under both interpretations, these first-digit shift errors indicate a deficit in a dedicated mechanism that handles the number structure. The reading task, however, cannot indicate whether this deficit was in the visual analysis or in the production stage. We further investigate the origin of these errors in Section 3.5. Note that a tendency to err in the leftmost digits is unlikely to result from a plain memory difficulty: serial recall tasks typically show *better* recall of the first items in the list (Baddeley, 1968; Gvion & Friedmann, 2012; Hanten & Martin, 2000; Jahnke, 1965).

OZ and UN, and marginally ED too, had errors related to the decimal word “thousand”, e.g., reading “20345” as “twenty, three hundred and forty five”. In the General Discussion, we propose a possible explanation for these errors.

Last, UN had many digit substitution errors. We will show in the next sections that his substitutions resulted from a deficit in the verbal output, and in the General Discussion we discuss his locus of deficit in more detail. No other participant had many substitution errors, indicating that they had no deficit in processing digit identities – neither in visual analysis nor in verbal production.

3.3. *Impaired encoding of digit order in the visual analyzer*

HZ and EY had many digit order errors in number reading, indicating a digit order processing deficit. To identify the functional locus of this deficit, we administered several tasks sensitive to digit order information in different processing stages. To tap the encoding of digit order by the visual analyzer, we used tasks with visual digit input and without verbal output. To tap the use of digit order information by the verbal production system, we used tasks with verbal number production and without visual digit input. A digit order encoding deficit in the visual analyzer should cause order errors in the visual input tasks but not in the verbal production tasks. (the results of each task are reported here in full, including decimal shift errors, but these decimal shifts will be discussed only below, in Section 3.5).

3.3.1. *Input tasks*

We administered three tasks that tap digit order encoding within the visual analyzer: sequence identification, same-different decision, and number matching.

3.3.1.1. EXPERIMENT 2: SEQUENCE IDENTIFICATION

3.3.1.1.1. METHOD. The participants saw 4-digit strings printed on paper, and were asked to circle strings that consisted of only consecutive digits (e.g., 3456). In these consecutive strings, digits always appeared in ascending order. The non-consecutive strings were derived from a consecutive string by transposing two adjacent digits (e.g., 3546) or by substituting a digit (e.g., 3496). A selective digit order encoding deficit in the visual analyzer should cause difficulty in the digit-transposition stimuli but not in the digit-substitution stimuli.

The task included 100 consecutive and 100 non-sequence digit strings: 53 digit-transposition strings and 47 digit-substitution strings. No digit string included 0 or 1. EY performed a computerized version of this task, with 150 consecutive-digit strings, 75 transposition strings, and 75 substitution strings: each stimulus was presented centered on the computer screen for 400 msec, and she clicked on one of two buttons with the mouse.

3.3.1.1.2. RESULTS. HZ and EY had significantly more errors than the control group in the transposition stimuli (Table 5), no more errors than the control group in the substitution stimuli, and more errors in the transposition stimuli than in the substitution stimuli ($\chi^2 = 62.21$ for HZ, 24.0 for EY; two-tailed $p < .001$ for both) – i.e., their performance satisfies the conditions for classical dissociation (Shallice, 1988) between digit position and digit identity encoding. Because the task involved the visual analyzer but not the verbal production of numbers, these results reaffirm that HZ and EY had a digit order encoding deficit in the visual analyzer.

OZ showed a similar pattern of errors – more errors in transposition stimuli than the control group, no more substitution errors than the control group, and more errors in the transposition stimuli than in the substitution stimuli ($\chi^2 = 6.11$, two-tailed $p = .01$). However, his transposition error rate was significantly lower than HZ's and EY's ($\chi^2 > 5.55$, two-tailed $p < .02$), and he had no transposition errors in the number reading task. Thus, it seems that he did not have a digit order encoding deficit, or at most – had a mild one. The other participants (MA, ED, NL, UN) performed well in all stimulus types, confirming that their digit order encoding in the visual analyzer, as well as their digit identity encoding, were intact.

3.3.1.2. Experiment 3: same-different decision. To further assess digit order encoding in the visual analyzer, we administered another task that involved the visual analyzer but not verbal production of numbers: participants were shown pairs of numbers and judged whether the numbers in each pair were identical or not. A digit order encoding deficit in the visual analyzer should create a difficulty in this task when the two numbers in a pair differ only in the order of digits.

3.3.1.2.1. Method. The participants saw 144 pairs of 4-digit numbers printed on paper, and were asked to circle pairs with two identical numbers. Half of the pairs were identical. In the remaining pairs, the second number was derived from the first number by transposing two adjacent digits (36 transposition pairs, e.g., 2345–3245) or by substituting a digit (36 substitution pairs, e.g., 2345–2347). Transpositions and substitutions were evenly distributed across all decimal positions. No number included 0 or 1. EY performed a computerized version of this task – each pair was presented on screen for 1300 msec, and she responded by clicking one of two buttons with the mouse. Her task included 120 identical pairs, 75 transposition pairs (50 ones-tens, 20 tens-hundreds, and 10 hundreds-thousands), and 75 substitution pairs (25, 25, 15, and 10 items with a substitution in the ones, tens, hundreds or thousands digit, respectively). UN did not perform this task because his memory span was lower than 4 digits (Table 2).

3.3.1.2.2. Results. Table 5 shows the results in this task. Notably, even the control group made some errors in the transposition pairs (4.7%) – significantly more than in the substitution pairs (paired $t(23) = 3.43$, one-tailed $p = .001$; for EY's control group, paired $t(9) = 2.62$, one-tailed $p = .01$).

Unlike the reading task, in which only HZ and EY had high rates of transposition errors, here ED and OZ also had many transpositions (albeit fewer than HZ and EY). However, a deeper analysis clearly showed that HZ and EY had much more transpositions than the other participants: each of them had significantly more transpositions than MA, ED, NL, and OZ ($\chi^2 > 15.91$, two-tailed $p < .001$), whose error rates were similar to each other (i.e., a strong dissociation between HZ, EY, who had many transposition errors, and MA, ED, NL, OZ, who had fewer transposition errors; there were no pairwise differences in the transposition error rate between MA, ED, and OZ, $\chi^2 < .84$, two-tailed $p > .36$; but NL had fewer errors). Moreover, HZ and EY were the only participants whose error rates exceeded those of the worst-

Table 5 – Error percentages in tasks that tap the visual analyzer (tasks that involve visual digit input but no production of verbal numbers). HZ and EY had high rates of transposition errors. MA, ED, NL, and UN had lower transposition error rates.

Experiment 2 – sequence identification				
	Sequence	Transposition	Substitution	
HZ	1	83^{***}	4	
EY	1	36^{+++}	4	
MA	1	0	0	
ED	3	4	0	
NL	4	2	0	
OZ	7	17^{***}	2	
UN	6	4	0	
Controls (SD)	3.3 (3.9)	2.7 (3.4)	0	
EY Controls (SD)	0.9 (1.0)	1.3 (1.3)	0.1 (0.4)	
Experiment 3 – same-different decision				
	Identical	Transposition	Substitution	
HZ	1	100^{***}	39^{+++}	
EY	0	63^{+++}	0	
MA	0	14^{+}	3	
ED	1	22^{**}	0	
NL	0	6	0	
OZ	0	19^{**}	3	
Controls (SD)	1.2 (2.1)	4.7 (5.5)	0.6 (2.3)	
EY Controls (SD)	0.1 (0.4)	1.9 (2.5)	0.4 (0.6)	
Experiment 4 – number matching				
	Equal	Transposition	Substitution	Number length
HZ	6	19^{+++}	4	29^{+++}
MA	0	3	0	5
ED	2	5	0	6
NL	7	0	0	0
OZ	6	2	0	2
Controls (SD)	3.1 (2.7)	0.3 (0.6)	0.2 (0.4)	0.5 (0.8)

Comparison with control group: $^{+}p \le .1$ $^{**}p \le .01$ $^{***}p \le .001$. $^{+++}$Errors ≥ 7%, control group ≤ 2%.

performing control participant; and they had more transposition errors than substitution errors ($\chi2 > 31.6$, one-tailed $p \leq .001$).

HZ had many substitution errors, but her predominant error type was still transpositions: they were more frequent than her substitutions, and they all went undetected by her, whereas she self-corrected all but 5 substitution errors.

The same-different decision task does not require verbal production of numbers. Thus, HZ's and EY's high transposition error rates clearly indicate that they have a digit order encoding deficit in the visual analyzer. The other participants had a more moderate (even if significant) transposition error rate in this task. One possibility is that they had a milder digit order encoding deficit. Another possibility, however, is that their transposition errors reflect the normal difference between the difficulty of transposition pairs and that of other pairs. This difference was observed even in the control group, and may have been amplified for the participants due to a general difficulty in number reading or in memory.

3.3.1.3. Experiment 4: number matching. In this variation of same-different decision, the participant was presented with a list of numbers, and compared each number in the list to a fixed sample number. This task too involves visual digit input with no verbal output, so it can be used to specifically assess the visual analyzer. Some numbers differed from the sample only in the order of digits; this allows to specifically assess the digit order encoding in the visual analyzer.

3.3.1.3.1. Method. The task was designed as 10 blocks. In each block, the participants saw a sample number and 49 target numbers printed underneath, and were instructed to circle all targets that were identical with the sample number, working as accurately and quickly as possible. The sample numbers consisted of a digit that repeated 4 or 5 times, and one different digit in an interior position (e.g., 22322, 777747, etc.). Of the 490 target numbers, 191 were identical with the sample, 100 were derived from the sample by transposing two digits (777747-777477), 100 were derived by adding or deleting a repeated digit (number-length difference, 228222-22822), and 99 were derived by substituting the non-repeated digit (33533–33933). This stimulus selection was motivated mainly by considerations of diagnosing impaired encoding of number length; these considerations will be explained in detail in Section 3.5.1. The numbers were printed on A4 paper, two blocks per sheet. EY and UN did not perform this task.

3.3.1.3.2. Results. Only HZ (and not MA, ED, NL, and OZ) had significantly more errors than the control group in the transposition targets (Table 5). She also had more errors in the transposition targets than in the substitution targets ($\chi^2 = 10.89$, $p < .001$), and no more substitutions than the control group (classical dissociation). This further indicates that she had a digit order encoding deficit in the visual analyzer, whereas MA, ED, NL, and OZ did not.

3.3.1.4. Interim summary: digit order errors in the visual input tasks. The tasks described above, all of which specifically examined digit order encoding in the visual analyzer, showed a consistent pattern: EY and HZ had many digit order errors, whereas MA, ED, NL, and UN did not (except the same-different task, where they had transposition errors, but still significantly fewer than HZ and EY). This indicates that EY and HZ, but not the other participants, have a digit order encoding deficit in the visual analyzer. The only inconsistent finding was OZ's high rate of transposition errors in the sequence identification task (which was still much lower than EY's and HZ's). It is therefore possible that OZ has a mild digit order encoding deficit.

3.3.2. *Output task: number repetition – Experiment 5*

The participants performed a number repetition task, which involves verbal production without visual digit input. This task involves the phonological retrieval mechanisms of number words (Dotan & Friedmann, 2015; McCloskey et al., 1986). If the transposition errors result from a deficit in phonological retrieval, they should be seen in this task too. The task may also expose a verbal production difficulty in other stages (e.g., the generation of a number word frame), but not necessarily: in a previous study we observed a patient with a deficit in verbal production of numbers, who nevertheless managed to repeat numbers correctly, apparently by using various strategies (Dotan et al., 2014).

3.3.2.1. Method. The experimenter said aloud each number and the participant repeated it. HZ, MA, and ED repeated the 120 numbers from Experiment 1. UN's digit span was very low, so he repeated 120 numbers in which only 2 or 3 digits were non-zero. The numbers had 3, 4, 5, or 6 digits (22, 39, 37, and 22 items per length, respectively). To allow for direct comparison of his number repetition with his number reading, in a separate session he also read the same numbers from paper. EY repeated 82 numbers – one block of 40 four-digit numbers, and another block of 42 five-digit numbers.

3.3.2.2. Results. All participants had almost no digit order errors in the repetition task (Table 6). This suggests that HZ's and EY's digit order errors in number reading (which we saw in Experiments 1–4) did not originate in an impaired production process, and certainly not in impaired phonological retrieval.

3.3.3. *Interim summary: the assessment of digit order errors*

The results of the experiments above are clear: HZ and EY had "digit order dyslexia" – a digit order encoding deficit in the visual analysis of Arabic numbers. They had high rates of digit order errors in all tasks that involved visual digit input – reading aloud, same-different decision, sequence identification, and number matching. Conversely, they had only few order errors in number repetition, a task that involved verbal output without visual digit input. They also did not have substitution errors, indicating that their digit order processing deficit did not affect the processing of digit identities. HZ's deficit was more severe than EY's, and indeed it caused her a real difficulty in real life situations. For example, she told us that she used to take bus number 24 to her office every day. By some vicious coincidence, bus number 42 was stopping on the same bus station, and several times HZ boarded the incorrect bus. For a graphic illustration of her perception of this

Table 6 – Error percentages in the number repetition task (Experiment 5), which involved production of verbal numbers but did not involve visual digit input. The rate of digit order errors was low for all participants.

Task		Digit order errors	1st digit shifts	Decimal shifts	Substitutions	"Thousand" errors[a]	All errors[b]
Number repetition	HZ	1	0	0	3	0	4
	EY[c]	0	0	0	9	0	12
	MA	3	0	0	4	0	9
	ED	1	0	0	8	0	11^{+}
	NL	1	1	1	11*	0	16**
	OZ	1	0	0	7	0	8
	UN[c]	0	13^{+++}	29^{+++}	27^{+++}	2	48^{+++}
	Controls (SD)	1.1 (1.2)	0.5 (1.2)	1.1 (1.0)	3.6 (3.5)	0	4.6 (3.6)
	EY controls (SD)	1.5 (1.1)	0	0	5.4 (5.4)	0	6.6 (5.8)
	UN controls (SD)	0.1 (0.2)	0.1 (0.3)	0.1 (0.3)	0.4 (0.4)	0	0.6 (0.7)
UN's reading of the same stimuli		0	48	48	7	0	52

Comparison to the control group: $^{+}p < .1$ $^{*}p \le .05$ $^{**}p \le .01$. $^{+++}$Errors $\ge$ 7%, control group $\le$ 2%.

[a] The rate of errors related to the decimal word "thousand" was counted out of the 52 numbers that contained the word "thousand"(numbers with 5 or 6).

[b] The percentage of items with any error.

[c] The stimuli lists of EY and UN were different from those of the other participants.

difficulty, see her art in Appendix B and on the cover of this volume, in a piece she called "frustration".

MA, ED, NL, and UN had relatively few digit order errors in all tasks, indicating that their digit order encoding was intact in all processing stages.

OZ had no digit order errors in the output-only tasks and in the number reading task, and he had relatively few digit order errors in two of the three input-only tasks. Nevertheless, he had many transpositions in a third input-only task (sequence identification), suggesting perhaps a mild impairment in digit order encoding in the visual analyzer.

3.4. *Impaired encoding of 0 positions in the visual analyzer*

In Experiment 1, EY showed an interesting performance pattern: she had many digit order errors when reading numbers that included only the digits 2–9, but virtually no errors when reading numbers that included the digit 0. This is an important finding, as it suggests the existence of another mechanism, separate from the digit order encoder, which selectively encodes the position of 0. Presumably, this mechanism was spared for EY, and this is what allowed her to avoid order errors when the number included the digit 0. We further tested this dissociation using two experiments in which the presence of 0 in the number was carefully controlled. These experiments were administered to the two participants with digit order encoding deficit, EY and HZ.

According to Cohen and Dehaene's (1991) model, a dedicated process encodes not only the positions of 0, but also of 1. To test this possibility, we also controlled for the presence of 1 in the number. Moreover, Cohen and Dehaene suggested that the importance of 1 is the verbal irregularity it creates when it appears in the tens position (it cues that the number should include a teen word). This may imply that 1 would have an effect only when appearing in the tens position. We therefore compared the participants' performance in numbers where the digit 1 appeared in different positions.

3.4.1. *Experiment 6: reading numbers with 0, 1, or neither*

3.4.1.1. METHOD. EY and HZ read 350 four-digit numbers: 100 numbers included the digit 0 in the hundreds or tens positions (x0xx and xx0x, 50 items per type), and 150 numbers included the digit 1 (xxx1, xx1x, and x1xx, 50 items per type). Additional 100 control numbers included neither 0 nor 1 and were derived from the xxx1 and xx1x numbers by substituting the digit 1 with another digit (xxx6 and xx3x). The 350 numbers were administered in random order in four blocks.

In Experiment 1, transpositions with 0 (e.g., 2304 → 2034) were classified as decimal shifts. Here, to avoid any bias that may artificially reduce order errors in numbers with 0, we classified transpositions with 0 as order errors.

3.4.1.2. RESULTS. Both participants had many digit order errors (Table 7). Importantly, EY had only a single order error in numbers with 0, more order errors in numbers with 1 ($\chi^2 = 16.6$, one-tailed $p < .001$), and even more order errors in numbers with neither 0 nor 1 ($\chi^2 = 25.5$, one-tailed $p < .001$). This replicates the dissociation she showed in Experiment 1 between numbers with and without 0. In the numbers with 1, she made fewer order errors involving the digit 1 than order errors not involving 1 ($\chi^2 = 4.62$, one-tailed $p = .02$). Her performance was unaffected by the position in which the digit 1 appeared: she had similar digit order error rates in xxx1 (14%), xx1x (20%), and x1xx (18%, $\chi^2(2) = .54$, two-tailed $p = .76$), and for both xxx1 and xx1x, the error rate in numbers with 1 was lower than in numbers with 2–9 ($\chi^2 > 10.31$, one-tailed $p < .001$).

HZ did not show this sensitivity to 0 and 1. In fact, she showed the opposite pattern – *more* digit order errors in numbers with 0 than in numbers without 0 and 1 ($\chi^2 = 5.78$, two-tailed $p = .02$). Table 7 shows that this pattern resulted from her high rate of transpositions of 0 with another digit (e.g., 4302 → 4032), suggesting that at least some of these errors were in fact decimal shifts rather than order errors. This interpretation is supported by two findings: first, when

Table 7 – Error percentages in Experiment 6 – reading aloud numbers with 0, with 1, or with only the digits 2–9. EY had fewer order errors in numbers with 0/1 than in numbers without these digits. HZ showed no such sensitivity to 0/1.

		Numbers with 0	Numbers with 1	Numbers with only 2–9
EY	Order errors	1	17	47
	Transpositions with 0/1	0	6	
	Only in the digits 2–9	1	13	
	All errors[a]	2	20	48
HZ	Order errors	59	35	42
	Transpositions with 0/1	50	18	
	Only in the digits 2–9	10	19	
	All errors[a]	62	42	45

[a] The percentage of items with any error.

excluding transpositions of 0 with another digit (and correspondingly excluding from the control numbers transpositions of 3 or 6 with another digit), HZ showed similar order error rates in numbers with 0 (10%) and without 0–1 (12%, $\chi^2 = .2$, one-tailed $p = .32$). Second, when we compared HZ's transpositions of 0 with another digit against her transpositions in the same decimal positions in the numbers without 0–1, we observed more transpositions with 0 (50% *vs* 21%, $\chi^2 = 18.36$, $p < .001$).

The different patterns exhibited by HZ and EY cannot be explained by the slightly different methods of stimulus presentation (EY read the numbers on a computer screen with limited exposure, HZ read them from paper): HZ's error pattern did not change when she re-read the Experiment 6 stimuli under EY's conditions (from a computer screen with 400 msec exposure). Crucially, HZ's and EY's different stimulus presentation methods cannot explain the main finding in the present experiment – the effect of 0 and 1 on EY's reading.

The results can also not be attributed to visual differences between 0 and 1 and the other digits. According to such a visual account, what helped EY was visual parameters such as the unique shape of 0 (circle) and 1 (line). To rule out this explanation, we administered EY a control experiment in which she saw a circle-shaped character that was not zero. We used EY's letter position dyslexia in word reading, and relied on the fact that the Hebrew letter Samekh (**O**, pronounced /s/) has a circle-like shape, similarly to the English letter O. EY read a list of 51 words with the letter **O** as a middle letter (because letter position dyslexia affects only the middle letters of a word), mixed with 51 words without **O**. The words were presented on the computer screen for 400 msec in Guttman-Yad font (ס). The visual account predicts that EY would have fewer transposition errors in words with **O** than in words without **O**, but this was not the case: she had 24% migration errors in words with **O** and 25% in words without **O** ($\chi^2 = 0.50$, one-tailed $p = .41$).

3.4.2. *Experiment 7: same-different decision in numbers with 0, 1, or neither*

Experiments 1 and 6 showed that EY can read numbers without digit order errors if the number includes 0. As we saw in Section 3.3, EY's digit order errors originate in a visual analyzer deficit. We therefore hypothesized that her ability to avoid order errors in numbers with 0 also originates in the visual analyzer. To examine this hypothesis, she completed a same-different decision task where the items did or did not include the digit 0. This task involves visual input but no verbal output, and as demonstrated in Experiment 3, EY's impaired digit order encoder fails in distinguishing between numbers that differ in the order of digits. If her visual analyzer can avoid digit order errors in numbers with 0, EY should be able to tell apart transposed pairs that contain 0. HZ performed the task too, as a control.

Table 8 – Error percentages in same-different decision (Experiment 7). EY had fewer errors in numbers with 0 than in other numbers, whereas HZ showed no sensitivity to 0.

	Pairs differing in digit order			
	Only 2–9	With 1	With 0	Identical pairs
HZ	68	73	60	20
EY	47	41	11	3

3.4.2.1. Method. HZ and EY saw 300 pairs of 4-digit numbers and decided, for each pair, whether the two numbers were identical (143 pairs) or differed in the order of two adjacent digits (157 pairs, e.g., 2345-3245). Of the transposition pairs, 53 pairs contained the digit zero, 51 pairs contained the digit 1, and 53 pairs contained only the digits 2–9. Both 0 and 1 appeared in the ones (19%), tens (15%) or hundreds (66%) position. The numbers in each pair appeared next to each other on the computer screen. The other methodological details were like in the same-different experiment described above (Experiment 3).

3.4.2.2. Results. Similar to her performance in number reading (Experiments 1, 6), EY had significantly fewer errors in detecting transpositions when the numbers included the digit 0 than when they included only the digits 2–9 (Table 8, $\chi^2 = 16.5$, one-tailed $p < .001$). Unlike Experiment 6, the existence of 1 in the number did not improve EY's performance ($\chi^2 = 0.27$, one-tailed $p = .30$). The specific position of the digit 0 or 1 had no significant effect on EY's error rate (Fisher's $p = .44$ for 0, Fisher's $p = .34$ for 1).

HZ did not show a boosting effect of 0 or 1 compared to the 2–9 pairs ($\chi^2 \leq 0.66$, one-tailed $p \geq .21$), replicating her performance pattern in the reading aloud task.

3.4.3. *Interim summary: the assessment of 0-position encoding*

The two number reading Experiments (1, 6) clearly show that EY had a highly selective deficit in number reading: she had difficulty in digit order encoding, but this difficulty had almost no impact on numbers that included the digit 0. A similar boosting effect of 0 was observed in a task with visual input and no verbal output (same-different, Experiment 7). Our best explanation for this pattern is that the visual analyzer has a dedicated sub-process that detects the presence of 0 in the number and encodes its position, as part of the decimal structure extraction (Fig. 1). EY had a selective impairment in the digit order encoding mechanism, but her zero detector was still intact. This allowed her, for numbers with 0, not only to identify the position of 0 but also to use it as a marker for ordering the remaining digits. HZ was impaired in both processes, so the presence of 0 in the number did not help her.

The findings were slightly different with respect to the digit 1. The presence of 1 in the number helped EY to avoid digit order errors in reading aloud but not in the input-only task (same-different). This suggests that 1 has a special status in the verbal production stage but not in the visual analysis stage. We elaborate further on the implications of this finding in the General Discussion.

3.5. *Impaired processing of the number's structural information*

In Experiment 1, all participants except EY had many decimal shift errors. These errors occurred mainly in the leftmost digits, a pattern that can potentially result from impairments in several possible sub-processes that handle the number's decimal or verbal structure. In the present section we identify, per participant, the locus of deficit underlying these first-digit shift errors. One possibility is that the errors result from erroneous encoding of the number length in the visual analyzer, which would make participants produce a number as if it had fewer digits or more digits (e.g., 4,320 → 43,200). This possibility is examined in Section 3.5.1. A second possibility, examined in Section 3.5.2, is that the errors result from impaired triplet parsing in the visual analyzer (e.g., 4320 → 43,20 → “forty three, twenty”). A third possibility, assessed in Section 3.5.3, is that first-digit shift errors result from impaired detection of 0's and their positions: ignoring a 0 or encoding an excessive 0 would change the perceived number of digits in the number (e.g., 4,320 → 43,200), and transposing a 0 would shift the decimal position of the transposed non-0 digit (e.g., 4,320 → 4,302). Finally, in Section 3.5.4 we examine the possibility that the decimal shift errors result from impaired generation of number word frames in the verbal production stage. Such an impairment could potentially distort the number length or the positions of 0's.

3.5.1. *Do decimal shift errors result from impaired number-length encoding in the visual analyzer?*

The participants performed two visual tasks without verbal production, which were sensitive to number length: same-different decision and number matching. If the participants have impaired number-length detection in the visual analyzer, they should have difficulties in these tasks.

Both tasks required the participants to judge whether visually presented numbers were identical or not. Pilot experiments suggested a major methodological challenge in designing this kind of task: participants often rely on alternative strategies rather than on number length information. For example, if we ask whether 234 and 2345 were identical, the participant could detect the difference by relying on the digit identities (only the second number has “5”). In the pair “234 = ? 2343”, they could rely on the order of 3 versus other digits. Thus, pairs such as 234-2345 and 234-2343 could yield good performance even if number length encoding is impaired. To prevent these alternative strategies, we used numbers in which all digits but one were identical (e.g., 99949). Number length was manipulated by changing the number of instances of the repeated digit, e.g., 99949-9949: both numbers contain only 4's and 9's and in the same relative order, so they are indistinguishable by digit identity and digit order. In the supplemental online material, we discuss more fine-grained methodological aspects of these tasks.

3.5.1.1. EXPERIMENT 8: SAME-DIFFERENT DECISION

3.5.1.1.1. METHOD. The participants saw 240 pairs of numbers with 3–6 digits, and decided whether the two numbers in each pair were identical or not. In all numbers, one digit was non-9 and the other digits were 9. There were 120 identical pairs and 120 different pairs. In the different pairs, the second number was derived from the first by adding or removing a single 9 (e.g., 99949-9949 or 99949-999499, 60 pairs), or by substituting the non-9 digit (e.g., 99949-99979, 60 pairs). The two numbers appeared in the center of the screen one after another for 1000 msec each, with a 500 msec delay between them. The participants responded using two keyboard keys. EY did not have decimal shift errors, so she did not perform this task. HZ had very high error rates in all stimulus types, suggesting impulsivity, so she later performed the task again while responding verbally rather than with the keyboard. We report her performance in both response modes.

3.5.1.1.2. RESULTS. If a participant has a selective deficit in the decimal structure analyzer, their error rate in the length-differing pairs should be higher than the control group's. It should also be higher than their own error rate in the substitution pairs. This pattern (strong dissociation) was observed for HZ and MA (Table 9): they had significantly more errors than the control group in the length-differing pairs (HZ had more errors in all stimulus types, indicating a general difficulty in this task, but the difference was most evident in the length-differing pairs). They also had more errors in length-differing pairs than in substitution pairs (HZ: $\chi^2 = 13.1$, one-tailed $p < .001$; MA: $\chi^2 = 9.84$, one-tailed $p < .001$). The control group had similar error rates in the length-differing pairs and in the substitution pairs (paired $t(19) = 0.62$, two-tailed $p = .54$).

Table 9 – Error percentages in Experiment 8 – same-different decision. If number-length detection in the visual analyzer is impaired, the participant's error rate in length-differing pairs should be higher than their error rate in the other types of pairs, and higher than the control group's error rate in length-differing pairs. Only HZ and MA showed this pattern.

	Length difference	Digit substitution	Identical
HZ (keyboard answer)	70^{+++}	23^{***}	15^{***}
HZ (verbal answer)	18^{+++}	8^{**}	3
MA	20^{+++}	2	5^{+}
ED	5	7^{*}	5^{+}
NL	0	0	1
OZ	3	2	8^{***}
UN	22^{+++}	37^{***}	18^{***}
Control group (SD)	1.9 (2.4)	2.3 (2.0)	2.5 (1.5)

Comparison to control group: $^{+}p < .1$ $^{*}p < .05$ $^{**}p < .01$ $^{***}p \leq .001$.
$^{+++}$Errors $\geq$ 7%, control group $\leq$ 2%.

The other participants – ED, NL, OZ, and UN – did not show this pattern of results. None of them had more errors in length-differing pairs than in substitution pairs, and none had more length errors than the control group (UN did have more errors than the control group, but in all pair types rather than just in the length-differing pairs). These findings indicate that HZ and MA, but not ED, NL, OZ, and UN, had a deficit in number length encoding in the visual analyzer.

3.5.1.2. NUMBER MATCHING. This task also involved visual digit input and no verbal production, and was designed to examine number length encoding in the visual analyzer. The task, described in Section 3.3.1.3 (Experiment 4), required the comparison of several numbers to a fixed sample number. The number could be either identical to the sample or differ from it in the number of digits (number length), the order of digits, or the identity of digits. People with impaired number length encoding in the visual analyzer are expected to show a higher error rate in length-differing items than in substitution items. We also expect their error rate in the length-differing items to be higher than the control group. Such a pattern (strong dissociation) was observed only for HZ (Table 5; length-differing items vs. substitution items: $\chi^2 = 11.05$, one-tailed $p < .001$). MA and ED showed a partial match to this pattern: they had more errors in length-differing items than in substitution items ($\chi^2 > 5.08$, one-tailed $p \leq .02$), but their length error rates did not exceed the 7% criterion that we set as the threshold for impaired performance. NL and OZ did not have many length-related errors ($p \geq .25$ for length vs. substitution, and neither had more length errors than the control group). Thus, this task indicates that HZ, and perhaps MA and ED too, had impaired number length encoding in the visual analyzer, but NL and OZ did not.

3.5.1.3. INTERIM SUMMARY: FIRST-DIGIT SHIFT ERRORS FOLLOWING IMPAIRED NUMBER-LENGTH ENCODING IN THE VISUAL ANALYZER. We examined number-length encoding in the visual analyzer using two tasks. HZ had many number-length errors in both tasks, and MA had many number-length errors in the same-different task. The other participants – ED, NL, OZ, and UN – did not have many number-length errors. Together, the tasks indicate that HZ and MA have a visual analyzer impairment that disrupts the encoding of number length. The other participants have good number-length encoding in the visual analyzer.

3.5.2. *Do decimal shift errors result from impaired triplet parsing in the visual analyzer?*

Another possible origin of first-digit shift errors is a deficit in triplet parsing. For example, if the digits of 65432, which should be grouped as 65,432, were grouped as 654,32, the result would be a first-digit shift – saying "six hundred" instead of "sixty". We reasoned that if this was the source of the participants' decimal shift errors in number reading, the errors should disappear if we provide them with explicit cues about the correct way to parse the number into triplets. As a parsing cue, we used a comma separator between the hundreds and thousands digits.

3.5.2.1. EXPERIMENT 9: READING NUMBERS WITH A COMMA SEPARATOR. The participants read aloud the 120 numbers that were presented in Experiment 1, but unlike Experiment 1, here they were presented with a comma separator between the thousands and hundreds digits (e.g., 65,432, whereas in Experiment 1 it was 65432). If a participant's decimal shifts in reading aloud originate in a triplet parsing deficit, the comma separator should provide her/him with a direct visual cue to improve the parsing to triplets, and the participant should therefore make fewer first-digit shift errors here than in Experiment 1. The comma separator may also help participants with impaired number-length encoding in the visual analyzer, because this visual cue could help estimating the number length. In contrast, the visual manipulation of adding a comma is not expected to help participants whose first-digit shift errors originate in impaired production processes. We also hypothesized that the comma separator would have no effect on digit order encoding, and consequently would not decrease the rate of transpositions.

Table 10 compares the participants' reading with a comma separator versus their reading without it, using the McNemar test. Only the 90 numbers that can include a comma (4–6 digits) were analyzed. The addition of the comma separator clearly reduced the rate of first-digit shifts for HZ, MA, and ED, but not for NL, OZ and UN. This indicates that HZ, MA, and ED had a visual analyzer deficit, either in encoding the number length or in parsing the triplets. Because we already concluded above that ED did not have a number-length encoding deficit in the visual analyzer, the present results indicate that she had impaired parsing of triplets in the visual analyzer.

NL, OZ, and UN did not gain from the addition of a comma separator. This finding can be interpreted in two ways: either their deficit that caused decimal shifts was not visual, or they had a double deficit – a visual deficit and another deficit – and the second deficit made them err even when the numbers were presented with a comma separator. We resolve this

Table 10 – Error percentages in reading numbers with 4–6 digits with a comma separator (e.g., 2,345, Experiment 9) and without a comma (2345, Experiment 1). The visual manipulation of adding the comma, which presumably affects only the visual analyzer, reduced the number of 1st-digit shifts in the reading of HZ, MA, and ED, but did not help NL, OZ, and UN.

	With comma (Experiment 9)				Without comma (Experiment 1)			
	1st-digit shift	Order	Thousand[a]	All	1st-digit shift	Order	Thousand[a]	All
HZ	8^{***}	18	0	51	33	21	0	52
MA	4^{***}	4	4	13^{*}	20	4	0	24
ED	0^{***}	2	1	4^{***}	21	3	6	28
NL	9	3	0	16	11	4	1	20
OZ	16	0	3	29	19	1	8	39
UN	18	1	16	39	18	1	8	49

Comparison between conditions: $^{*}p \leq .05$ $^{***}p \leq .001$.

[a] The rate of errors related to the decimal word “thousand” was counted out of the 52 numbers that had a sufficient number of digits (5 or 6).

ambiguity later below (Section 3.5.4.2) by considering the results in the present task in conjunction with other tasks.

HZ's performance in this task is interesting also from another perspective. The addition of a comma separator significantly decreased her first-digit shift errors but not her digit order errors. The difference between the commas' effect on the two error types was significant (an analysis of the Experiment × Error Type × Success contingency table showed a three-way interaction: $\chi^2(4) = 7.64$, one-tailed $p = .05$). This within-participant dissociation between order errors and first-digit shifts further supports our earlier conclusion that order errors and first-digit shift errors originate in two distinct processes.

3.5.3. *Do decimal shift errors result from an impaired 0 detection in the visual analyzer?*

An alternative account that we examined and ruled out is that our participants' decimal shift errors result from an impaired 0 detection in the visual analyzer. Incorrect encoding of 0 could result in three possible kinds of errors. Two of them would end in a first-digit shift: the elimination of a 0 that existed in the target number, resulting in producing a number shorter than the target number; or the addition of a 0 that was not there, resulting in producing a number longer than the target. The third kind of error is a transposition of 0 with another digit, which would result in a decimal shift in middle digits. We derive several specific predictions from this view, but as we shall see, none of the participants fulfilled these predictions, indicating that their decimal shifts did not originate from a zero-encoding deficit.

First, two of the error types above, omission and transposition of 0, can only occur when the target number includes the digit 0. Thus, for a given task, a person with a zero-encoding deficit should show more decimal shift errors in numbers with 0 than in numbers without 0. In the number reading task (Experiment 1), no participant showed this pattern (Table 11, left columns).

Second, in numbers without 0, a zero-encoding deficit can only add an excessive 0, which would result in producing a number longer than the target. Thus, if a person's decimal shifts result from a zero-encoding deficit, their decimal shifts in numbers without 0 should yield numbers longer than the target. The first-digit shift errors didn't support this prediction: no participant had more number-lengthening errors than number-shortening errors (Table 11, right columns). If anything, the trend was opposite.

Table 11 – Error percentages of decimal shift errors in Experiment 1 (number reading), indicating that these errors do not result from erroneous encoding of 0 positions. No participant showed: (a) more decimal shifts in numbers with 0; or (b) a tendency of 1st digit shift errors to lengthen numbers without 0 (which could be explained as an addition of 0) rather than shorten these numbers (which is unexplained as a 0 effect).

	Decimal shifts in numbers…		Numbers without 0: 1st-digit-shifts resulted in…	
	with 0	without 0	longer number	shorter number
HZ	25	30	11	17
MA	9	21	10	11
ED	14	19	10	10
NL	9	14	5	10
OZ	16	14	6	6
UN	16	21	6	11

Third, in the same-different decision (Experiment 8) and number matching (Experiment 4) tasks, participants with a deficit in zero-encoding should not miss the difference between numbers that differ in aspects that do not relate to 0, such as 9949-949. However, two participants did show impaired performance in these experiments: MA had number-length errors in Experiment 8, and HZ had number-length errors in both experiments. These errors cannot be explained as resulting from incorrect encoding of 0. They can only be explained by other deficits (such as impaired encoding of the number length).

Finally, explaining decimal shifts as resulting from poor 0 detection does not explain why, for some participants (HZ, MA, ED), decimal shifts were nearly eliminated by the addition of a comma separator (Experiment 9): there is no clear reason why the comma separator should improve the detection of the presence of 0 in the number or the detection of the positions of 0 (especially given that the comma did not improve the order encoding for other digits).

All these findings indicate that erroneous 0 detection does not account for the decimal shift errors of any of the participants in this study. However, it is still possible that other individuals, who may have a selective deficit in 0 detection in the visual analyzer, would make decimal shift errors as a result of this impairment. In such cases, we would expect error patterns different from the ones observed in our participants: (1) there would be more decimal shift errors in numbers with 0 than in numbers without 0; (2) only lengthening errors would occur in numbers without 0; (3) the person would succeed in the number comparison tasks that we used here; and (4) the addition of a comma separator would not reduce the number of decimal shifts. Finally, our findings still allow the possibility of a double deficit – i.e., participants in this study, whose decimal shifts are explained by another impairment, may still have a 0 detection deficit on top of that impairment.

3.5.4. *Do decimal shift errors result from impaired generation of number word frames in verbal production?*

The last locus of deficit we considered as a possible origin for first-digit shift errors was the generation of number word frames in verbal production. The generation of incorrect frames could result in decimal shift errors of all kinds, including first-digit shifts. We assessed this frame-generation process with two tasks that allow for first-digit shift errors. One task – multiply/divide by 10 (Experiment 10) – involved verbal production of number words that were not presented visually. Individuals with impaired generation of number word frames should perform poorly in this task, but individuals with a selective deficit in the visual analyzer should perform well. The other task (Experiment 11) was number reading, with a manipulation that was designed to improve the reading of participants with impaired generation of number word frames.

3.5.4.1. Experiment 10: multiply/divide by 10. In this task, the participants saw simple exercises of multiplication or division by 10, read the exercise aloud, and then said the result. This task specifically examines the production processes, for several reasons. First, we presented the numbers with a comma separator, which helps an impaired visual analyzer. Second, because the participants read the exercise aloud before saying the result, we could rely on correct reading of the numbers as an index for the good visual analysis of these numbers. Third, because the produced number was different from the one printed on paper, the number that was to be said aloud did not come directly from the visual analyzer, but from the calculation mechanism (i.e., the task did not involve the direct digit-to-verbal transcoding pathway). To reduce the calculation difficulty, we used very simple calculations – multiplication and division by 10. In the supplementary online material, we provide additional methodological aspects of this task.

3.5.4.1.1. Method. The participants saw a list of 28 multiplication exercises mixed with 28 division exercises. The numbers were printed with a comma separator between the hundreds and thousands digits. The first operand had 3-5 digits, with two non-zero digits and then zeros, and the second operand was always 10 (e.g., “6,500 × 10”, “740 ÷ 10”), i.e., the results had 2-6 digits. The participants read aloud each exercise and then said the result.

3.5.4.1.2. Results. The participants had some errors in reading the exercises, but most of these errors were self-corrected prior to providing the answer. Importantly, except UN, there was not even a single case of an uncorrected reading error followed by an incorrect answer. For UN there were 8 such cases, but in none of them could the erroneous result be explained by the reading error. Thus, the errors reported below originate in a production difficulty and not in a visual analysis difficulty.

NL, OZ, and UN had high rates of first-digit shift errors (Table 12), indicating that they have a number-length processing deficit in verbal production. The other participants did not have high rates of first-digit shift errors, indicating spared verbal production processes.

3.5.4.2. Experiments 11 and 12: reading numbers as triplets. Another experiment that assessed verbal production was a variation of number reading. In a way, it is the verbal correlate of Experiment 9, where we used a visual manipulation (comma separator) to help participants with

Table 12 – Error percentages in the multiply/divide-by-10 task (Experiment 10), which specifically taps the verbal output processes. To examine the processing of number length, we inspected the first-digit shift errors. NL, OZ, and UN had high rates of such errors, whereas the other participants did not.

	First-digit shift	Substitution	Thousand[a]	Transposition	All errors[b]
HZ	4	0	3	2	7^{+}
MA	2	0	5	0	5
ED	5	0	0	0	5
NL	20^{***}	0	0	0	20^{***}
OZ	9^{**}	0	0	2	11^{**}
UN	43^{***}	20^{+++}	5	0	57^{***}
Controls (SD)	2.0 (2.7)	0	0.1 (0.6)	0	2.9 (3.0)

Comparison to control group: $^{+}p < .1$ $^{**}p \le .01$ $^{***}p \le .001$ $^{+++}$Errors ≥ 7%, control group ≤ 2%.

[a] The rate of errors related to the decimal word “thousand” was counted out of the 37 numbers that had a sufficient number of digits (5 or 6).

[b] The percentage of items with any error.

Table 13 – Error percentages in reading numbers with 4–6 digits – as a whole number ("12 thousand, 345", Experiment 1) or when saying each number as two shorter numbers ("12 and then 345", Experiment 11). This manipulation, designed to ease on an impaired production process, helped OZ but not ED and NL. HZ had fewer errors in reading as triplets than in whole-number reading, but still more errors than the control group.

	As triplets, no comma (Experiment 11)[a]		As triplets, with comma (Experiment 12)		Whole number (Experiment 1)	
	1st digit shift	All	1st digit shift	All	1st-digit shift	All
HZ	20^{+++}	41	4	24	33^{+++}	52
ED	13^{+++}	21	3	7	21^{+++}	28
NL	18^{+++}	19	1	6	11^{+++}	20
OZ	6	10	7	8	19^{+++}	39
Controls (SD)	1.5 (1.7)	3.2 (2.5)	–	–	1.4 (1.2)	3.5 (1.6)

Comparison to the control group: $^{+++}$Errors $\geq$ 7%, control group $\leq$ 2% errors.

[a] Comparison of 1st-digit shifts between reading whole numbers (Experiment 1) and reading the same numbers divided into shorter numbers (Experiment 11): $p \leq .01$ for HZ, $p = .004$ for OZ, no significant difference for ED and NL.

impaired visual analyzer. Here, Experiment 11 used a verbal manipulation designed to help participants with impaired generation of the number word frame (but not participants with impaired visual analysis). Numbers were presented exactly as in the number reading experiments, but the required verbal output was simplified: participants were asked to read each number as two shorter numbers, up to 3 digits long (e.g., the number 65432 was to be read as "sixty five *and then* four hundred and thirty two"). In this reading mode, participants never had to produce any number longer than 3 digits, so they would only need to generate short number word frames (e.g., for a 5-digit number they would generate a 2-digit frame and a 3-digit frame). The visual analysis in this task, however, is as demanding as in Experiment 1. Thus, if a participant has decimal shift errors in standard reading (Experiment 1) but not here, this would indicate that their decimal shift errors originate in a verbal production deficit.

Note that even if a person has a production deficit that causes first-digit shifts, the verbal manipulation of Experiment 11 may fail to eliminate decimal shift errors if this person has an additional impairment that causes such errors – e.g., a visual analyzer deficit in the number length encoder or in parsing the number to triplets. To address such cases, Experiment 12 combined the manipulations of Experiments 11 and 9: the participants saw the numbers with a comma separator (which should help with a visual impairment) and read them as pairs of shorter numbers (which should help with a verbal impairment).

3.5.4.2.1. METHOD. The participants saw the 120 numbers from Experiment 1 and read aloud each number as described above: 3-digit numbers were read like in Experiment 1, and each longer number was produced as two shorter numbers, separated by "and then" (the single word /ve-az/ in Hebrew). Participants were instructed to split the numbers in two such that the second number would have 3 digits; we verified that they understood these instructions, and gave them examples for each number length between 3 and 6 digits. In Experiment 11 the numbers were presented without a comma separator (like in Experiment 1), and in Experiment 12 they were presented with a comma separator (like in Experiment 9). The results were compared against the participants' reading in Experiment 1 using the McNemar test. Only the 90 numbers with 4–6 digits were analyzed, because shorter numbers are produced in the same manner here and in Experiment 1. The 90 numbers in Experiments 1 and 11/12 created minimal pairs in the phonological-surface level: they were the exact same stimuli, and the verbal production of 5-digit and 6-digit numbers was even identical except for a single word, which was phonologically parallel.[5]

3.5.4.2.2. RESULTS. Reading numbers as triplets clearly helped OZ (Table 13): his first-digit shift error rate in Experiment 11 was not significantly higher than the control group, and was lower than when reading the same digit strings as whole numbers (in Experiment 1). This pattern (classical dissociation) indicates that OZ's first-digit shift errors originated in a production deficit.

Reading as triplets also helped HZ, but to a lesser extent: she had fewer first-digit shift errors in Experiment 11 than in Experiment 1, indicating that at least some of her first-digit shift errors originate in impaired production processes. Nevertheless, even when reading as triplets, she still had more first-digit shift errors than the control group, indicating that some of her first-digit shift errors originated in another process – presumably in her visual analyzer deficit. Indeed, in Experiment 12, where we used both the visual and verbal easing manipulations, her error rate was even lower (Experiment 11 *vs* 12: McNemar $\chi^2 = 9.8$, one-tailed $p = .001$). Thus, HZ had a double deficit, in the visual analyzer and in the production stage. Her remaining errors in Experiment 12 can be explained by the severity of her deficit. Indeed, even when we asked her to read 120

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[5] For 4–6 digit numbers, the number of words was identical between the conditions: the whole-number reading (Experiment 1) involved the decimal word "*elef*", "thousand" and the split-triplets condition (Experiment 11) involved the word *ve-az* ("and then"), in the same position. For 5 and 6 digit numbers, both the decimal word and "ve-az" included 2 syllables and 4 phonemes; for 4-digit numbers, the decimal word (in Experiment 1) was the plural of "thousands", "*alafim*", a 3-syllable word. Restricting the analyses to the 52 numbers with 5–6 digits yielded essentially the same results.

numbers with only 1–3 digits, she had 6% first-digit shifts, similar to her error rate in Experiment 12 ($\chi^2 = 1.43$, one-tailed $p = .12$).

ED did not gain from reading as triplets – her first-digit shift error rate in Experiment 11 was not significantly lower than in Experiment 1, and was higher than the control group. This indicates that her first-digit shifts originate in another, pre-production process, in line with our earlier conclusion that she has a visual analyzer deficit. The present results cannot indicate whether she had a production deficit on top of her visual deficit or not. Based on her good performance in the other production task (Experiment 10), we assume that she did not.

NL too did not gain from reading as triplets (Experiment 11). Remember that unlike ED, she also did not gain from the visual manipulation of displaying the number with comma separator (Experiment 9). However, when both manipulations were used in conjunction – i.e., when the numbers were presented with a comma separator and she read them as triplets (Experiment 12), she had merely one first-digit shift error (significantly fewer than in Experiment 1, McNemar $\chi^2 = 10.29$, $p = .001$) and no other decimal shift. This pattern indicates that NL's first-digit shift errors originated in a double deficit: in the visual analyzer and in verbal production. The comma separator helped the visual deficit and reading as triplets helped the verbal deficit. Neither manipulation on its own was sufficient to improve her performance, because neither addressed the full problem; only applying both manipulations in conjunction helped her.

3.5.4.3. Number repetition. The number repetition task was already described above (Section 3.3.2, Experiment 5). The existence of decimal shift errors in this task would indicate a deficit in the verbal production stage. Conversely, good performance in repetition cannot be taken as an indication for intact processing of the number's structure, because the repetition task may be too easy and may allow for alternative strategies (in this task, the participants hear the number's verbal structure and do not have to produce it on their own). Indeed, in a previous study we observed an aphasic patient with a deficit in verbal production of numbers, who nevertheless managed to repeat numbers correctly (Dotan et al., 2014).

UN had many first-digit shift errors in number repetition (Table 6), suggesting he had a deficit in verbal production. Unlike the reading task, here UN's errors were not restricted to first-digit shifts (13%) – he also had many shifts in the beginning of the second triplet (18%, and only 5% errors in mid-triplet digits).

The other participants performed well in this task – either because their verbal production was spared, or because they used strategies to bypass their impaired verbal production processes. Possibly, UN was unable to use strategies such as word-by-word repetition (perhaps because of his severely impaired short-term memory) or morphological cueing (perhaps due to his morphological deficit). Another possibility is that UN, but not the other participants, had a later deficit – in phonological retrieval – which corrupted the lexical class information (ones, tens, teens etc., Dotan & Friedmann, 2015; patient JG in McCloskey et al., 1986). We revisit these possibilities in the General Discussion.

4. Summary: the participants' impairments, dissociations, and loci of deficit

All participants in this study had difficulties in number reading. Number reading impairments are dissociable from word reading impairments (Dotan & Friedmann, 2018), so we propose *dysnumeria* as a general term to describe impaired processing of symbolic numbers. We used several number processing tasks to identify the functional locus of deficit underlying the number reading difficulties of each participant. The results of these tasks for each participant, and our

Table 14 – The locus of deficit for each participant.

	Visual analysis					Verbal production	
	Digit identity	Digit order	Number length	0 positions	Triplet structure	Verbal structure	Phonological retrieval
EY	✓	✗	✓	✓	✓	✓	✓
HZ	✓	✗	✗	✗	?	Mild(?)	✓
MA	✓	✓	✗	?	?	✓	✓
ED	✓	✓	✓	?	✗	✓	✓
NL	✓	✓	✓	?	✗	✗	✓
OZ	✓	Mild(?)	✓	✓	✓	✗	✓
UN	✓	✓	✓	✓	✓	✗	✓

conclusions about the functional locus of deficit of each participant, are summarized below and in Table 14. As we can see, different participants had different impairments, i.e., different types of dysnumeria.

EY had many digit order errors when she read numbers aloud. She made digit order errors in number reading and in tasks that involved visual digit input without verbal production (hereby "visual input tasks") – sequence identification (Experiment 2) and same-different decision (Experiment 3). Conversely, she did not have order errors in the number repetition task (Experiment 5), which involved verbal production without visual digit input (hereby "verbal production task"). This pattern indicates that she had impaired digit order encoding in the visual analyzer (*digit order dysnumeria*). She did not commit order errors in numbers that included 0, i.e., the digit order encoding deficit did not interfere with her ability to encode the positions of 0. This pattern suggests the existence of an additional process that specifically detects zeros and their positions. EY did not have decimal shift errors, indicating that her processing of the number's decimal and verbal structure was intact. In particular, her digit order encoding deficit did not interfere with her ability to encode the number length.

HZ had many digit order errors. Similarly to EY, these errors occurred in number reading and in the visual input tasks – sequence identification (Experiment 2), same-different decision (Experiment 3), and number matching (Experiment 4), but not in the verbal production task (number repetition, Experiment 5). This indicates that she too had a *digit order dysnumeria* – impaired digit order encoding in the visual analyzer. Unlike EY, she had digit order errors in numbers with 0, indicating that in her case, the "zero detector" process (whose existence was demonstrated by EY's dissociation) was impaired.

HZ also had many first-digit shift errors in number reading – decimal shifts of the first (leftmost) digits of the numbers. These decimal shift errors originated in a visual analyzer deficit: they occurred in visual input tasks (same-different decision, Experiment 8, and number matching, Experiment 4), and the visual manipulation of adding a comma separator (Experiment 9) reduced the rate of decimal shifts. The specific deficit that can explain her decimal shifts is an impairment in number length encoding (*number length dysnumeria*) or in triplet parsing (*triplet parsing dysnumeria*). Of these two possibilities, a *number-length dysnumeria* is the likely one, because HZ made errors in the visual input tasks that specifically examined number length. Our findings are insufficient to tell whether she had impaired triplet parsing too. HZ did not have decimal shifts in the verbal production tasks (number repetition, Experiment 5, and multiply/divide by 10, Experiment 10), but her decimal shift rate was significantly decreased by a verbal manipulation aimed to improve reading in case of impaired production (Experiment 11). Thus, it is possible that some of her decimal shift errors originated in a mild production deficit.

MA had many first-digit shift errors in number reading. Her performance indicates that her first-digit shifts originated in a visual analysis deficit: the errors appeared in a visual input task (number-length errors in same-different decision, Experiment 8), and their rate dropped when we introduced the visual manipulation of adding a comma separator (Experiment 9). Conversely, she did not make decimal shifts in verbal production tasks (number repetition, Experiment 5, and multiply/divide by 10, Experiment 10), and she did not gain from the verbal manipulation designed to help in case of impaired production (Experiment 11). The type of errors – first-digit shifts in the reading task, and number length errors in the visual input tasks – indicates that the impaired visual analyzer sub-process was the number length encoder (*number length dysnumeria*). Our findings are insufficient to tell whether MA had impaired triplet parsing too. Importantly, she only had decimal shift errors (similarly perhaps to Noël & Seron, 1993), and did not have many errors of other types, in particular digit order errors. Thus, her digit order encoding was spared – a dissociation pattern opposite to EY's. Together, MA and EY show double dissociation between two visual analyzer sub-processes: digit order encoding and number-length encoding.

ED had many first-digit shift errors in number reading. These errors did not originate in a production deficit: she performed well in the verbal production tasks (number repetition, Experiment 5, and multiply/divide by 10, Experiment 10), and she did not gain from reading the numbers separated into triplets (Experiment 11) – a verbal manipulation designed to ease on impaired production processes. Her first-digit shifts also did not originate in impaired encoding of number length by the visual analyzer, because she succeeded in the visual input tasks that investigated number length encoding (same-different decision, Experiment 8, and number matching, Experiment 4. Her first-digit shift error rate dropped when she read numbers with a comma separator (Experiment 9) – a visual manipulation designed to ease on impaired parsing of triplets in the visual analyzer. We concluded that her deficit was in a visual analyzer sub-process that parses digit strings into triplets (*triplet parsing dysnumeria*).

NL, ED's sister, also had many first-digit shift errors in number reading. At least some of these errors originated in a production deficit: she made first-digit shift errors in a verbal production task (multiply/divide by 10, Experiment 10). Her success in the visual input tasks clearly indicates that her visual analyzer was intact in terms of processing the digit identity, digit order, and number length. She did not gain from the visual manipulation of adding a comma separator (Experiment 9), which was designed to ease on a visual analyzer deficit in number length encoding or parsing to triplets, nor did she gain from the verbal manipulation of reading the numbers separated to triplets (Experiment 11), which was designed to ease on impaired processing of the number's verbal structure in the production stage; but she had no first-digit shifts when both manipulations were present (Experiment 12). We concluded that she had a double deficit: in parsing the number to triplets in the visual analyzer (*triplet parsing dysnumeria*), and in the number word frame generation in verbal production (*frame generation dysnumeria*). Adding a comma separator addressed the former deficit, reading the number as triplets addressed the latter, and only when both manipulations were present was there a reduction in her decimal shift errors.

OZ also had mainly decimal shift errors in number reading. These errors did not originate in impaired number-length encoding in the visual analyzer, because he succeeded in the visual input tasks that tap this process without requiring verbal production (same-different decision, Experiment 8, and number matching, Experiment 4). His errors also did not originate in impaired triplet parsing in the visual analyzer: the rate of his decimal shifts did not decrease following the visual manipulation of adding a comma separator (Experiment 9). Rather, his decimal shifts originated in impaired production processes: he had many decimal shift errors in a verbal production task (multiply/divide by 10, Experiment 10), and the rate of decimal shifts dropped when he read each number as two shorter numbers (Experiment 11) – a verbal manipulation designed to ease on number reading in case of impaired production. Within verbal production, OZ's deficit was not in the phonological retrieval process. Impaired phonological retrieval should cause random substitution of number words, which should result in decimal shifts in all decimal positions, as well as in producing invalid number names (e.g., 32 → "thirty and twenty"). This was not the case for OZ: his decimal shifts occurred almost exclusively in the first digit/s of a triplet. Thus, his impairment was not in phonological retrieval, but in the generation of number word frames (*frame generation dysnumeria*).

OZ had many digit order errors in one of the visual input tasks (sequence identification, Experiment 2), but we believe that he did not have a digit order impairment, or at least that it was very mild: first, his digit order error rate in this task, although higher than the control group, was lower than the other order-impaired participants (EY and HZ). Second, OZ did not have digit order errors in any other task, neither visual nor verbal: number reading (Experiment 1), same-different decision (Experiment 3), number matching (Experiment 4), number repetition (Experiment 5), and multiply/divide by 10 (Experiment 10).

UN had many first-digit shift errors in number reading and in a verbal production task (number repetition, Experiment 5), but not in a visual input task (sequence identification, Experiment 2), indicating that his first-digit shifts originated in a production deficit. Like OZ, he did not have mid-triplet decimal shifts and rarely produced invalid number names, indicating that his deficit was not in phonological retrieval but in the generation of number word frames (*frame generation dysnumeria*).

UN also had a high rate of digit substitution errors, which appeared in number reading and in verbal production tasks but not in visual input tasks. Thus, his substitution errors originated in impaired production processes. UN's type of errors, substitution of the digit value in production tasks only, resembles patient HY reported by McCloskey et al. (1986). It is possible that UN had, on top of his deficit in number word frame generation, a deficit similar to HY's – in transferring the digit identities to the phonological retrieval stage.

5. General discussion

5.1. *Processes involved in number reading: conclusions from the participants' performance patterns*

This study investigated the number reading of seven individuals with impaired reading aloud of multi-digit Arabic numbers. A series of experiments showed that different participants had different number reading impairments - different types of *dysnumeria* - resulting from deficits in different processes of number reading. The identification of the selective deficit of each of the participants, summarized in the previous section, leads to the following conclusions, for which any cognitive model of number reading should be able to account:

(1) The visual analysis of digit strings and the verbal production of number words are handled by separate processes, as concluded by several previous studies (Benson & Denckla, 1969; Cohen & Dehaene, 1995; Cohen et al., 1997; Dehaene & Cohen, 1995; Delazer & Bartha, 2001; Dotan & Friedmann, 2015; Dotan et al., 2014; Friedmann, Dotan, & Rahamim, 2010; Marangolo et al., 2005, 2004; McCloskey et al., 1986; Noël & Seron, 1993). In support of this assertion, we observed that visual analysis and verbal production can be selectively impaired: EY, MA, and ED were impaired only in visual analysis, whereas OZ and UN were impaired only in verbal production (except perhaps for a minor visual analysis deficit for OZ, in digit order encoding).

(2) Within visual analysis, separate sub-processes encode digit order and digit identity (Friedmann, Dotan, & Rahamim, 2010). This is shown by the dissociation in EY's and HZ's impairments – good digit identity encoding and impaired digit order encoding.

(3) Within visual analysis, separate sub-processes encode the digit order and the number length. This conclusion is supported by the double dissociation between EY and MA: EY showed impaired digit order encoding and spared number-length encoding, and MA showed the opposite pattern. Number production was intact for both of them, indicating that both digit order encoding and number-length encoding exist as a part of the visual analysis stage.

This dissociation has direct implications for the error analysis in number processing tasks. It shows that decimal shifts and digit transpositions (digit order errors) should be treated as two different types of errors rather than as two exemplars of the same error type (a digit in an incorrect decimal position). Our findings further indicate that even this distinction between two error types is not the end of the story, because decimal shifts can originate in several different loci of impairments: number length encoding in the visual analyzer, as is the case for MA; triplet parsing in the visual analyzer, ED; verbal production, OZ; and perhaps also from impaired zero detection in the visual analyzer (Section 3.5.3).

(4) The digit order encoder identifies the relative order of digits rather than their absolute positions. This can be deduced from MA's performance: her digit order encoder was intact (Section 3.3), but even with the digit order information available, she did not successfully distinguish between numbers such as 9949 and 99499 (same-different task, Experiment 8). If the visual analyzer was encoding absolute positions, MA should have been able to distinguish between 9949 and 99499 by relying on the spared position encoding of the digit 4 as tens or hundreds (or between 9949 and 99949, if the position is encoded relative to the left side of the number). Her inability to do so suggests that the visual analyzer does not encode the absolute positions of the digits but rather their relative order. Furthermore, the digit order encoder is not a

general visual-spatial mechanism but specifically encodes the positions of digits in a digit string. This is shown by dissociations between impaired encoding of the order of letters in a word and spared encoding of the order of digits in digit strings (Friedmann, Dotan, & Rahamim, 2010).

(5) A dedicated visual analyzer sub-process is responsible for parsing the digit string into triplets. Supporting this conclusion, ED had a selective deficit in parsing the digit string to triplets, with spared encoding of the digit order and the number length. Her deficit was in the visual analyzer, as revealed by her sensitivity to the visual manipulation of adding a comma separator (Experiment 9).

(6) The presence and position of 0 are encoded by a visual process that is separate from the one that encodes the order of the digits 1–9. This conclusion is supported by EY's performance pattern: she had impaired digit order encoding but this did not affect numbers with 0, indicating that 0 positions were encoded by another, spared process. This 0-position encoder is a part of the visual analyzer, because we observed the boosting effect of 0 not only when she read numbers aloud, but also in a task that required only visual analysis, without verbal production (same-different decision, Experiment 7).

(7) The reading system handles 0 and 1 in different ways. The positions of 0 are encoded by a dedicated visual analyzer sub-process, as described in the previous paragraph, but the positions of 1 are not. This conclusion is again supported by EY's performance: the presence of 0 in a number eliminated her digit order errors, but the presence of 1 did not have this effect: in the visual-only task (same-different) the digit 1 had no effect at all, and in the reading task the boosting effect of 1 was much smaller than that of 0.

(8) Within verbal production, there is a process dedicated to handling the number structure. This conclusion is supported by OZ's and UN's performance pattern: they had many decimal shift errors, indicating a deficit in a process that handles the number structure; and these errors occurred exclusively in tasks that involved verbal production, indicating that this structural process was a verbal production process. The specific type of errors, first-digit shifts, further indicates that this verbal production process specifically represents the number length and/or the triplet structure as part of the number's verbal structure. The first-digit shift errors may result either from errors in the process that uses the parsed visual decimal structure to construct the verbal structure, or from errors in the processes that use this verbal structure to produce the number words.

5.2. A revised model of number reading

On the basis of these findings and their implications for the number reading process, we propose the following cognitive model of number reading (Fig. 2) – a refinement of the model presented in the Introduction. Within visual analysis, the model postulates several processes: digit identity encoding, digit order encoding (which encodes the relative order of digits), and three processes that extract the number's decimal structure – length encoder, zero detector, and triplet parser. The output of these processes is sent to the verbal production stage. The decimal structure (length, triplets, 0's) is used to generate a number word frame, and the constituents of the number word frame are bound with the ordered digits to retrieve the phonological forms of each number word. We will now describe in detail each of the components in the model.

5.2.1. *Visual analysis*

The assumption of separate processes that encode the digit identity and order on the one hand, and the number's decimal structure (length, 0, triplets) on the other hand, is based on finding a double dissociation between the two kinds of information: EY showed impaired digit order and spared decimal structure; MA, ED, and NL showed spared digit order and a deficit in specific parts of the decimal structure (number length, triplet structure). These dissociations clearly refute the possibility that the decimal structure is extracted from the digit order information, and show that the decimal structure and the digit order are encoded by separate processes. This separation is rigid: an intact encoder of digit identity and order cannot compensate for an impaired decimal structure analyzer, and vice versa, even when the impairment is developmental, as is the case for most of the participants in the present study.

The model assumes that the number's decimal structure is not just sent to the verbal production processes, but is also used within the visual analyzer, to help the digit identity and order encoders. For example, the positions of 0, encoded by the zero detector, may help the digit encoders skip 0's and avoid sending them as digit identities to the production stage.

5.2.2. *Generating the number word frame*

5.2.2.1. THE NUMBER WORD FRAME AS A HIERARCHICAL STRUCTURE. The number word frame (hereby, NWF) represents the number's verbal structure. It specifies the sequence of words in the verbal number, excluding the information about specific digit identities. Concretely, the NWF is a sequence of word specifiers/features of 3 different types: lexical classes of number words (ones, teens, tens, etc.), decimal words ("thousand", "hundred", etc.), and function words (the word "and"). For example, the NWF of 5,050 is *{_:ones} [thousand] [and] {_:tens}* (the notation [...] indicates a lexical, pre-phonological precise specification of a word; the notation { ... } indicates a specification of a lexical class without the digit). The NWF, in conjunction with the ordered digits 1–9, provide sufficient information for the next processing stages to retrieve the phonological forms of all words in the verbal number.

Two main findings from our participants illuminate on how the NWF is generated. First, a deficit in NWF generation (OZ and UN) resulted in many errors related to the decimal word "thousand" (mostly omissions), whereas such errors were rare in the control group and in the number production of participants with other impairments. Second, the pattern of decimal shift errors depended on their origin: the participants with a pure visual analyzer deficit (HZ, MA, ED) had mainly first-digit shift errors, whereas OZ, whose decimal shifts originated in a pure production deficit, had many decimal shifts also in the beginning of the second triplet (Table 4) – e.g., reading 1200 as "one thousand and twenty".

Both findings can be explained if the NWF is generated in a hierarchical manner. For example, the NWF of a 6-digit number may consist of two "sub-NWFs", one per triplet.

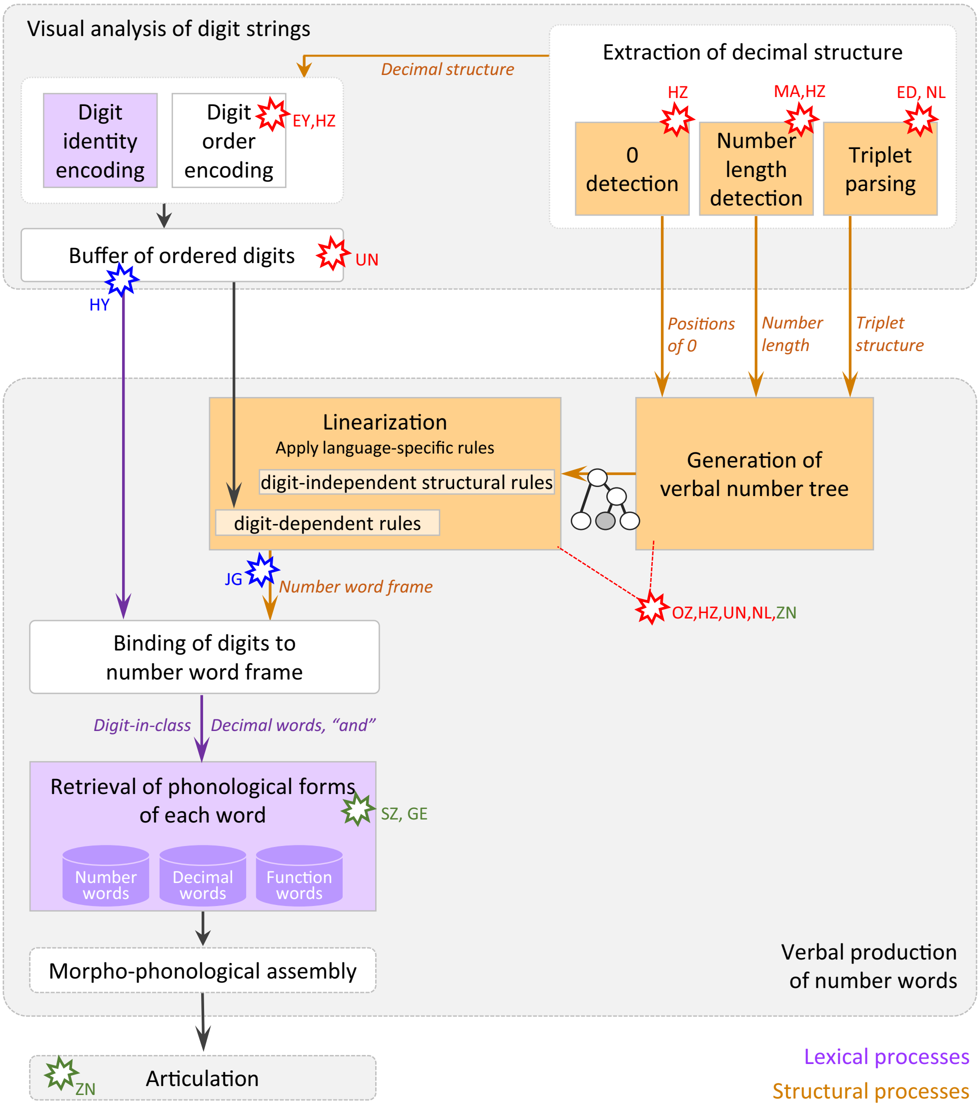

Fig. 2 – A proposed model for reading aloud multi-digit numbers. The extraction of decimal structure in the visual analyzer involves 3 sub-processes: detecting 0's and their positions, detecting the number length, and parsing the number into triplets. This information is sent not only to verbal production, but also to the encoders of digit identity and order. The number's decimal structure is used to create a verbal, language-independent, representation of the number in the form of a syntactic tree. This representation is then serialized into a linear form – the number word frame – according to language-dependent rules: some rules depend only on the language and are independent of the specific digit (e.g., in German and Arabic, the ones word precedes the tens word), and some depend also on specific structure-modifying digits in the number (e.g., in English, 1 in the tens position yields a teen word). The number word frame is bound with the ordered digits, and this information is used to retrieve the phonological form of each word from dedicated phonological stores. This phonological string is sent to morpho-phonological assembly and then to articulation. Blast icons indicate the locus of deficit of each participant: red = participants in this study; green = individuals we reported previously (Dotan & Friedmann, 2015; Dotan et al., 2014); blue = individuals from McCloskey et al. (1986). A PowerPoint version of this figure is included as supplementary online material.

Certain impairments in verbal production may prevent the person from generating the full NWF of the 6-digit number, but still allow him to generate shorter NWFs. In such cases, the person may resort to processing long numbers in segments – e.g., one triplet at a time – because this method requires shorter NWFs, which he can still create. This approach could result in omissions of the decimal word "thousand": this word is a part of the 6-digit number NWF, but it is not a part of either triplet's NWF. Furthermore, if the person's impaired verbal production causes first-digit shifts, and each triplet is being processed as a separate NWF (and a separate number), then decimal shifts could occur not only in the beginning of the first triplet but also in the beginning of the second triplet. Importantly, both findings (thousand errors, OZ's errors in the 2^{nd} triplet) show that multi-triplet NWFs result in errors specifically around the triplet boundaries, indicating that the

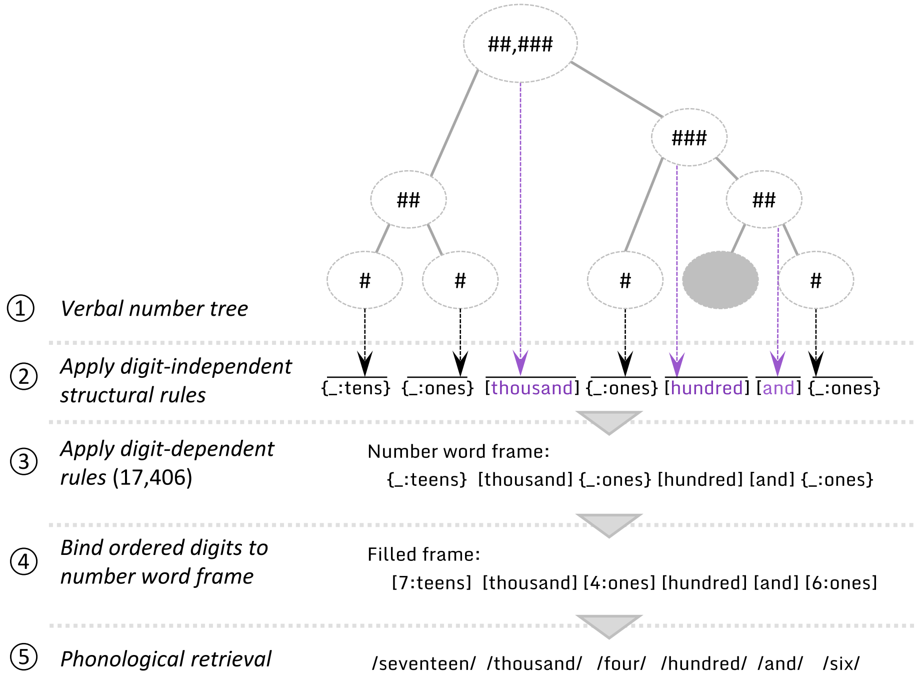

Fig. 3 – The verbal number representations during production according to our proposed model, demonstrated here for the number 17,406 in English. ① First, a tree-like representation is created based on the number's decimal structure: length and triplet structure. The presence of 0 in the number results in some nodes being disabled (grey color). ②③ The tree is converted into a number word frame – a linear representation of the number's verbal structure. The linearization is done by applying language-specific rules. Some of these rules require only the tree representation (e.g., the order of words). Other rules depend also on information about specific structure-modifying digits in specific decimal positions (e.g., 1 tens yields a teens word). In English, this linearization converts each 1st level (bottom) node into a number word lexical class, each 3rd level node (triplet node) into "hundred and" or "hundred", and each 4th level node into "thousand". The linearization results in a number word frame, in which each element is a lexical class of a number word, a decimal word (thousand, hundred, etc.), or "and". ④ The lexical classes are then bound with the number's ordered digits, resulting in information sufficient to ⑤ retrieve the phonological form of each number word. A PowerPoint version of this figure is included as supplementary online material.

verbal production system was not just splitting long numbers randomly, but specifically into triplets.

We hypothesize that the hierarchical processing in NWF generation is not merely in the separation to triplets, but involves a fully hierarchical representation of the number's verbal structure. Specifically, we propose that NWF generation is done in two stages: first, a hierarchical representation of the verbal number is created as a tree-like structure, analogous in a way to the syntactic trees that represent the syntactic structure of sentences. Then, this tree is serialized into a linear form, which is the NWF. This numerical-verbal syntactic tree is hereby explained in detail.

5.2.2.2. Creating the tree. The first stage is creating the tree, which reflects the number's verbal structure (Fig. 3). For example, a two-digit number would be represented by three nodes: a tens node, a ones node, and a higher-level node that merges them. A three-digit number would be represented by three nodes for hundreds, tens, and ones, which are merged by two higher-level nodes. The tree of a 5-digit number such as 17,406 would include one sub-tree for 17, another sub-tree for 406, and a top-level node that merges the two sub-trees (stage ① in Fig. 3). This tree structure mirrors an assumption from the model of McCloskey et al. (1986): they assumed that the syntactic frame of a number, i.e., its structural representation, indicates the scope of each multiplier word – i.e., that the thousands multiplier applies to the triplet preceding it, and the hundreds multiplier applies to the hundreds digit.

The tree is a purely structural representation, and its creation requires only the number's decimal structure. The number length determines the height of the tree and the size of the leftmost triplet's sub-tree (for a 5-digit number such as the one in Fig. 3, the leftmost triplet yields a sub-tree of a 2-digit number). The triplet structure determines the internal structure of the tree, and the positions of zeros determines the disabled nodes (which appear grey in Fig. 3).

The numerical-verbal syntactic tree does not reflect language-specific properties such as the order of words, neither does it reflect language-specific irregularities such as teens. However, the structure of the tree does depend on the way the target language organizes verbal numbers into groups. In English and Hebrew, for example, verbal numbers are organized in triplets. In Chinese and Japanese, verbal numbers are structured in myriads (4-digit chunks). The number 10,000 is a single Japanese word (万, /man/), and a number such as 200,000 is verbalized /ni-jū man/, literally "two ten ten-thousand". In several languages used in India (e.g., Hindi, Urdu, Bengali, Indian English), there is a decimal word not only for 1,000 but also for 100,000 (/lakh/ in Indian English). These different organizations are reflected in the node-organization of the tree: a particular number would have the same numerical-verbal syntactic tree in all triplet-based languages (English, French, Hebrew, etc.), a different tree in Chinese and Japanese, and yet another tree for Bengali numerals. Note that the numerical-verbal syntactic tree is a verbal representation that is used only for processing verbal numbers. Thus, it is not the abstract representation hypothesized by some number processing models (Cipolotti & Butterworth, 1995; McCloskey, 1992).

5.2.2.3. Converting the tree to a number word frame. The next stage (② and ③ in Fig. 3) is linearization – applying structural rules that convert the numerical-verbal syntactic tree into a linear representation, the number word frame. These conversion rules, which partly correspond with Power and Dal Martello's (1997) digit-to-verbal transcoding algorithm, depend on the specific language. Some of them are general rules of the language (②): they require knowledge of the syntactic tree, but do not rely on specific digits. An example for such a rule is the ordering of words: in many languages, the order of number words is identical with the order of digits (e.g., for three digit numbers, the hundreds word precedes the tens word, which precedes the ones word), but in some languages the ones word precedes the tens word (26 → "six and twenty") – e.g., in German, in Arabic, and in old English (Berg & Neubauer, 2014). The linearization that creates the NWF will be dictated by the language-specific rules of ordering the different decimal roles. Another example for a language-specific, digit-independent rule is the proper placement of the function word "and", which is required only in some languages, and only in specific positions in the tree.

Other linearization rules (③) depend on specific digits in specific decimal positions in the number – structure-modifying digits. One example for such a rule is the teens irregularity: a tens + ones sub-tree usually translates into *{_:tens} {_:ones}*, but when the tens digit is 1, it translates into *{_:teens}*. Another example is the French rule that converts a tens + ones sub-tree into *{_:tens} {_:teens}* when the tens digit is 7 or 9.

We assume that structure-modifying digits, which are language-dependent, are assigned a special status in verbal production but not in the visual analyzer. This is because structure-modifying digits are relevant only for verbal production, and are irrelevant for tasks such as digit-to-quantity conversion or number comparison. Thus, the model correctly predicts that structure-modifying digits would not have a special status in nonverbal tasks, where the NWF generation processes are inactive (this is the pattern observed with respect to the effect of 1 on EY's performance, see item 7 in Section 5.1).

5.2.2.4. Deficits in creating the number word frame. Turning to impairments in this part of the model, a deficit in the creation of the tree may impair a person's ability to represent high-level tree nodes, and limit their representational ability to trees up to a certain level of complexity (a certain height). An analogous impairment was described in aphasia, where individuals with agrammatic aphasia cannot construct the syntactic tree of a sentence up to its highest node. The outcome of this sentence-processing impairment is inability to produce certain complex sentences that require the high nodes of the syntactic tree, and preserved ability to produce simple sentences that can be represented with shallower trees (Friedmann & Grodzinsky, 1997; Friedmann, 2001, 2006). In number production, such tree-pruning may limit the size of the tree that a person can handle, and consequently limit the length of the numbers they can handle. For example, if a person is able to represent trees of up to 3-digit numbers, they would have errors when reading numbers with more digits. This may have been the case for OZ and UN. A more severe deficit may restrict the person to very simple trees – e.g., to single-node trees – rendering the person completely unable to produce multi-word numbers. This may have been the case for ZN (Dotan et al., 2014), who could hardly produce even two-digit numbers. Intermediate stages in development may also be described in terms of limited tree size. For example, Power and Dal Martello (1997) reported that 7-year olds tend to fragment numbers into shorter numbers. This can be accounted for if children at this stage are able to generate only small trees. As a result, when they are required to read numbers longer than their representational ability, they truncate them or produce them as several small trees.

Such numerical-verbal tree pruning clearly explains why it was easier for OZ to produce a digit string as two separate 3-digit numbers than as a 6-digit number, and why he had many "thousand"-related errors. When, instead of reading a 6-digit number as a whole, he was requested to read it as separate triplets (saying "and then" between the triplets rather than with the decimal word "thousand", Experiment 11), he was actually allowed to construct two small trees, which he could construct, instead of one large tree (like the one depicted in Fig. 3), which he could not construct. The omission of the word "thousand" in his normal reading of multi-digit numbers is another result of his inability to construct a tree high enough: when he resorted to producing the number via two smaller trees, he discarded the top-level node, which is the one dictating the use of the decimal word "thousand" (Fig. 3).

The model makes predictions about specific performance patterns that should be observed given impairments in the different stages of NWF generation. A deficit in creating the numerical-verbal syntactic tree should result in incorrect verbal structure. The specific deficit may be an inability to represent trees with sufficient levels (similar to "tree pruning" of syntactic trees, Friedmann & Grodzinsky, 1997; Friedmann, 2001; 2006). This may result in representing long numbers as several smaller trees, and consequently – omitting the decimal word "thousand". Alternatively, such a deficit may result in the creation of undersized trees (e.g., a 3-digit tree for a 5-digit number), yielding too-short NWFs with which only some of the digits can bebound – e.g., 23,456 may be read as 356. In some cases, the deficit at the tree construction level may reflect errors in merely a single type of information provided by the decimal structure analyzer. This may have been the impairment of OZ and UN: they had errors in the number length information, but not in other structural aspects of the number (e.g., 0 positions).

A deficit in the language-specific (digit-independent) linearization rules may yield errors in function words (such as "and") and in their position, and may also result in language-specific ordering errors such as failing to reorder the tens and ones words in German and Arabic.

A deficit in the digit-dependent linearization rules should yield errors in the language-specific irregularities handled by this process – e.g., failing to apply a teen word when the tens digit is 1.

5.2.3. *Binding the number word frame with the digits*

Verbal numbers include three types of words: number words ("five", "eleven"), decimal words ("thousand"), and function words ("and"). The NWF identifies unambiguously the decimal words and the function words. Number words, however, are underspecified – the NWF merely specifies their lexical class. To obtain a full specification of the number word, the lexical class must be bound with the corresponding digit. This binding process combines the NWF, provided by the tree linearization process, and the sequence of ordered digits, provided by the digit identity and order encoders in the visual analyzer. The bound NWF contains the full information required for retrieving the phonological forms of each word.

The existence of a dedicated digit-NWF binding process within verbal production solves a potential problem of synchronization. Number words are retrieved from the phonological store one at a time, based on two parameters – the identity of a digit (1–9) and a lexical class (ones, tens, teens, etc.). The digits arrive from the digit identity and order encoders, but the lexical class arrives via a different pathway – from the NWF. For successful retrieval, the two pathways must be synchronized. This synchronization challenge can be easily solved if a single process (binding) activates both the digit and the corresponding lexical class. The dedicated binding process may also be responsible for a particular modification needed for the French numbers 71–79 and 91–99: saying 75 as *soixante-et-quinze*, "sixty and fifteen", requires not only creating the irregular NWF *{_:tens} {_:teens}*, but also changing the digit 7 into 6 (to get the word *soixante*, sixty), and similarly, the digit 9 into 8.

The model postulates that the binding process is triggered by the verbal system. Namely, the verbal system does not passively receive the ordered digits from the visual analyzer, but rather it actively picks the digits in the appropriate order. This verbal-triggered architecture allows picking the digits in the order imposed by the particular language, which is presumably unknown to the visual analyzer. An alternative possibility, that the number words are ordered *after* phonological retrieval, is unlikely: the retrieved phonological forms are sent directly to articulation, without the mediation of a phonological short-term memory store that might have taken care of reordering the words (Dotan & Friedmann, 2015; Shalev et al., 2014).

To allow for this architecture, where binding is triggered by the verbal system, the model proposes a digit buffer, a short-term storage of digits. The digit identity and order encoders in the visual analyzer feed this buffer, and the binding process picks digits from the buffer. The NWF linearization also picks some digits from this buffer – the structure-modifying digits. The existence of such a buffer could explain UN's high rate of digit substitution errors: his low short-term memory capacity (Table 2) may have affected this short-term buffer too, resulting in a high rate of digit substitutions.

We are inclined to assume that this buffer is visual rather than verbal, for two reasons. First, the buffer is presumably updated by the visual analyzer and not by any verbal process, so the information it contains reflects the ordered digits per-se, and in this sense the information is visual. Second, a visual buffer can explain a peculiar pattern in EY's performance: the presence of the digit 1 in the number reduced her digit order errors in reading numbers aloud but not in number comparison. Presumably, when reading numbers aloud, the linearization stage explicitly looked up "1 tens" in the digit buffer. Being a part of the visual mechanism, the buffer could easily interact with the visual analyzer and prompt it to improve the position encoding when the number contained 1. When the task was nonverbal, this feedback loop was inactive, so the presence of 1 did not reduce EY's order errors.

5.2.4. *Phonological retrieval and articulation*

The digit-NWF binding process produces a full specification of the sequence of words in the verbal number. This specification is used to retrieve the phonological form of each word. Unlike ordinary content words, which are retrieved from a phonological output lexicon (Butterworth, 1992; Friedmann et al., 2013; Levelt, 1992), the phonological forms of the verbal number's words are retrieved from a dedicated phonological store at the level of the phonological output buffer (Dotan & Friedmann, 2015). Verbal numbers include three types of words: number words, decimal words (e.g., "thousand", "hundred"), and the function word "and". Each of these word types is stored in a separate phonological store (or alternatively, there is a single phonological store that is strictly organized in a category-based manner, Dotan & Friedmann, 2015).

The dedicated phonological stores maintain number words and function words that are already phonologically assembled, so they can be directly sent to the articulation mechanisms – they do not require an additional stage of phoneme assembly (Dotan & Friedmann, 2015). Still, the phonological forms may undergo *morpho-phonological* assembly – e.g., in Hebrew, "and" is a clitic, a bound function word, and should be assembled as the prefix of a number word (the pronunciation of this clitic varies according to the word to which it is bound). After this assembly, the phonological sequences are sent to the articulation mechanisms, which prepare all types of verbal material for articulation and are not specific to numbers (Shalev et al., 2014).

5.2.5. *Additional processes*

Our model postulates that information is directly sent from visual processes to verbal processes. Our findings can be fully explained without resorting to an additional conversion process that transforms the data from one format to another. However, such an intermediate transformation process is still possible, and this could resemble the mechanisms that convert graphemes to phonemes – letters to sounds – in word reading (Coltheart et al., 2001). Future studies may specifically examine this point.

5.3. *Relation to other studies and specific types of impairments*

Our model can account for several cases reported in the literature. SZ and GE, two individuals who we reported previously (Dotan & Friedmann, 2015), made substitutions of number words in number reading and in verbal production tasks. We diagnosed their deficit as localized in the retrieval from the phonological storage of number words.

In another study we reported ZN, an aphasic patient with apraxia of speech and complete inability to say multi-digit numbers (Dotan et al., 2014). Interestingly, ZN's difficulty was observed only when he had to generate a number word frame (e.g., when reading multi-digit numbers aloud or when saying a calculation result), whereas he performed quite well when the number word frame was explicitly provided to him (e.g., in a number repetition task). We therefore diagnosed his deficit as localized in the NWF generation process – similarly to OZ and UN, yet more severe.

McCloskey et al.'s (1986) patient JG, who made only class errors, was impaired in handling the number word lexical class information – presumably either during NWF-digit binding or in transferring the class information from the NWF to the binding stage. Their patient HY, who had only within-class errors (such as 17 → 13), was apparently impaired either in the digit buffer or in transferring the digit identities from this buffer to the binding stage.

Cipolotti (1995) reported patient SF, who made errors in number reading but not in comprehension-only or production-only tasks. His errors were mainly first-digit shifts, but he also made other decimal shifts and substitutions. Cipolotti concluded that SF was impaired in the digit-to-verbal transcoding pathway, with spared visual analyzer and verbal production. Translating this conclusion to our model would point to the *decimal structure analyzer → tree generation* pathway as SF's locus of deficit.

Benavides-Varela et al. (2016) examined number reading and writing in a group of 22 patients with a right-hemisphere damage. More than 60% of the errors these patients made in reading involved the digit 0: omissions of 0, additions of 0, and number fragmentations that involve producing the word "zero" (e.g., 203 → "two hundred zero three"). Benavides-Varela et al. proposed that numbers with 0 are harder because their reading involves processes that are not required by numbers without 0. Our model can precisely point at these processes: for example, one possible way to explain the participants' error pattern is as a selective deficit in a mechanism that extracts the number's decimal structure in the visual analyzer – e.g., in number length detection or zero processing.

5.4. *Number reading and other cognitive processes*

5.4.1. *Word reading*

The reading processes described by our model are specific to number reading, and do not support word reading. This is exemplified by ED and NL who, in spite of their number reading deficits, could read words flawlessly; as well as by several other dissociations between reading of words and numbers. In Dotan and Friedmann (2018) we review these dissociations, and compare in detail ED's and NL's word reading to their number reading. It is possible that very early stages of feature identification and character shapes are shared by letter and digit processes (McCloskey & Schubert, 2014), but the later stages that we discussed here are separate for word reading and number reading.

5.4.2. *Lexical versus syntactic processes*

The elements of our model can be classified into lexical pathways and processes, which handle single digits or number words (purple color in Fig. 2), and syntactic/structural pathways and processes, which handle the decimal and verbal structure (orange color in Fig. 2). The model therefore complies with the classic lexical-syntactic distinction theorized in several previous number transcoding models (Cappelletti et al., 2005; Cipolotti, 1995; Cipolotti et al., 1995; Delazer & Bartha, 2001; Deloche & Seron, 1982; McCloskey et al., 1985; Noël & Seron, 1993, 1995; Sokol & McCloskey, 1988). Nevertheless, our model goes beyond the lexical-syntactic distinction by offering a finer level of granularity: it distinguishes between visual and verbal processes, and for each of those, it separates between specific lexical and syntactic processes. It further describes the internal structure of each of these processes: the lexical processes in the visual stage, which involve digit identity and digit order encoders; the decimal structure processes in the visual stage, which involve length encoding, triplet parsing, and zero-encoding; the verbal structure processes, which involve tree building, linearization, and the binding of the digits and NWF. Finally, our model proposes a detailed account of the information flow. Using this model we aimed to propose a concrete and precise definition of what the term "number syntax" might mean.

5.4.3. *Number writing*

We hypothesize that a verbal tree-like representation of the number is created not only during number reading, but also when transcoding a verbal number to a digit string. The tree would be generated whenever we hear (or read) a verbal number. In order to write the number in digit format, the tree would be converted to a digit string by a set of linearization processes. Different tree nodes would invoke different linearization processes, partly corresponding with Power and Dal Martello's (1990) rules of concatenation (e.g., concatenating 4 and 00 to write *four hundred*) and overwriting (e.g., when writing *four hundred and six*, overwriting the last 0 of 400 with 6). For example, when hearing the number 17,406, the person would create the verbal tree depicted in Fig. 3. Inability to generate a tree high enough or failure in the overwriting process could yield errors such as writing "four thousand and twenty" as 400020 (Benavides-Varela et al., 2016; Cipolotti, Butterworth, & Warrington, 1994; Power & Dal Martello, 1990; Seron & Fayol, 1994).

5.4.4. *Number comprehension or production without reading*

Visually presented numbers are used not only in the context of reading aloud. Perhaps more often than not, we merely need to comprehend what they represent – a quantity, a concept (e.g., "100%"), or a proper name (e.g., bus numbers). Dissecting these comprehension processes was not in the scope of the present study. Nevertheless, the processes described by our model – the visual analyzer and the verbal production – are presumably used whenever we need to comprehend a digit string (in a process that feeds the output of the visual analysis to the comprehension system) or say a verbal number (in a process that starts from a concept or quantity and feeds into the verbal production system). This assumption is supported by our participants' pattern of performance, at least with respect to several simple tasks (same-different decision, number matching, sequence identification, saying the answer for a math calculation).

In particular, the number-length encoder in the visual analyzer may play an important role in converting digit strings to quantity. Dotan and Dehaene (2017) used a task that required individuals to convert multi-digit strings to quantity. They concluded that participants were creating a quantity-oriented representation of the number's structure, a "syntactic frame", which serves to assign each digit the appropriate relative weight when converting it to quantity. Crucially, creating a syntactic frame is the first stage in quantifying the number, and the only information it requires is how many digits the number has. The number-length encoder, which explicitly extracts this information, may be the process that allows for quick and accurate initiation of the syntactic frame. The zero detector may also play a role in digit-to-quantity conversion: the quantification of multi-digit strings involves a stage of quantifying single digits (Dotan & Dehaene, 2017; Meyerhoff, Moeller, Debus, & Nuerk, 2012; Moeller, Nuerk, & Willmes, 2009; Nuerk & Willmes, 2005), and the digit 0 may have a special status within this single-digit quantification (Pinhas & Tzelgov, 2012).

5.4.5. *Subitizing and digit grouping*

The separation between the visual and verbal representations of numbers is supported by a large number of studies (Benson & Denckla, 1969; Cohen & Dehaene, 1995; Cohen et al., 1997; Delazer & Bartha, 2001; Dotan & Friedmann, 2015; Dotan et al., 2014; Friedmann, Dotan, et al., 2010; Marangolo et al., 2005, 2004;; McCloskey et al., 1986; Noël & Seron, 1993). Our model conforms to this separation by proposing that the visual analyzer is separate from verbal production. The visual analyzer could be independent of any specific language, because it can be used to transcode digit strings into any language that the person speaks (indeed, the visual analyzer's output is language-independent). Moreover, the visual analyzer may be independent of *any* verbal knowledge, because it also serves non-verbal processes – converting digit strings to quantities and comprehension of lexicalized numbers (indeed, the visual analyzer's output presumably serves these processes – e.g., the number length and the positions of 0 may play a special role in digit-to-quantity conversion, as explained in Section 5.4.4). The only exception is the number's triplet-structure, which is encoded by the visual analyzer, yet it is language-specific and does not seem to be relevant in nonverbal context.[6]

Why should the visual analyzer handle digit grouping, which is apparently verbal and language-specific? One possibility is that the grouping of digits is not motivated by language factors, but by a general cognitive factor that limits the digit processing capacity to 3–4 digits at a time. All languages group digits in groups of 3 or 4, and this limit of 3–4 items also exists in other cognitive operations. For example, the precise number of objects in a given set can be detected by a process called *subitizing*, which can enumerate up to 3–4 objects (Cipolotti, Butterworth, & Denes, 1991; Feigenson, Dehaene, & Spelke, 2004; Revkin, Piazza, Izard, Cohen, & Dehaene, 2008). This is not to say that reading a digit string involves subitizing, but to propose that both processes may be capped by the same general cognitive limit – for example, working memory capacity. Indeed, Piazza, Fumarola, Chinello, and Melcher (2011) suggested that subitizing relies on visual working memory mechanisms, and that the subitizing range reflects a visual working memory limit.

The congruency between digit grouping and the specific language can be plausibly discarded as cultural convention (given that the digits need to be grouped in *some* manner, why not agree with your verbal system). However, our model can propose another possible explanation for this congruency: the verbal system extracts data from a visual digit buffer, and it may be easier to synchronize this process when the size of this visual buffer agrees with the structure of the verbal number sub-tree. The completion of processing a verbal number sub-tree could trigger the visual analyzer to fill the digit buffer with the next triplet. This explanation predicts that individuals who speak two grouping-mismatched languages (e.g., Chinese as L1 and English as L2) will have a relative difficulty when reading a specific digit string in the incongruent language (e.g., reading 10,0000 in English).

5.5. *Terminology*

We propose the term *dysnumeria* for deficits in processing symbolic numbers (essentially dyslexia or dysgraphia for numbers). This term would differ from the term *anarithmetia*, which refers to deficits in calculation (Benson & Weir, 1972; Dehaene & Cohen, 1997; Hécaen, Angelergues, & Houillier, 1961). It would also differ from *dyscalculia* and *acalculia*, which are sometimes used to refer to deficits in calculation (Benson & Weir, 1972; Dehaene & Cohen, 1997; World Health Organization, 1992), sometimes to a wider array of difficulties in number processing (American Psychiatric Association, 2013; Gross-Tsur, Manor, & Shalev, 1996; McCloskey, 1992), and seem to suffer some degree of vagueness (Butterworth, 2008).

Selective impairments in specific number reading processes may be termed according to the impaired process. Visual analysis deficits: digit identity dysnumeria, digit order dysnumeria, number length dysnumeria, zero detection dysnumeria,

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[6] This peculiarity is not a derivative of our specific model, because the existence of triplet structure as part of the digit representation is clearly mirrored in our cultural conventions: when writing digit strings we use a separator symbol to group digits, and we place it in agreement with our verbal number system. When the verbal system runs in cycles of 3 decimal roles (e.g., English: ten – ten thousand – ten million – etc.), a separator is placed every 3rd digit (10,000,000); when the verbal system runs in cycle of 4 (e.g., Japanese), a separator is placed every 4th digit (1000,0000); and in Indian languages the separators are placed in agreement with their unique verbal number system (1,00,00,000). There are rare exceptions to this language-separator congruency (e.g., Korean, with a 4-cycle verbal system but a 3-digit grouping of digit strings). In contrast, the other structural aspects of verbal numbers – e.g., the teen structure, the structure-modifying digits, and the fact that the digit 0 is not verbalized – are not reflected transparently in the digit strings and are not represented in the visual analyzer.

and triplet parsing dysnumeria. Production deficits: lexical retrieval dysnumeria (deficit in the phonological retrieval stage), digit-to-frame binding dysnumeria, and frame generation dysnumeria. The latter refers to any deficit in generating the number word frame; in the future, it may be further refined into several subtypes of dysnumeria (e.g., deficits in creating the verbal number tree versus deficits in linearizing the tree).

5.6. *Using a single cognitive model to explain developmental and acquired deficits*

This study presented six adults with developmental deficits and one adult (UN) with an acquired deficit following a stroke. The model we proposed here could further account for the performance of several previously-published cases of acquired deficits in number processing. Including both developmental and acquired cases in a single neuropsychological study is not common, but we believe that it is justified, in particular in the present study. We hereby address two possible issues with respect to this method.

The first issue concerns the use of developmental disorders as evidence for a cognitive model. Conceivably, even a selective developmental deficit may disrupt the development of additional processes that were not directly impaired. In such a case, although the cognitive impairment was specific, it would manifest as a non-specific functional/behavioral pattern (resembling the deficits resulting from a non-focal brain lesion). However, even if this may *sometimes* be the case, it is not *necessarily* the case: highly specific developmental impairment may remain specific and not necessarily disrupt the development of additional processes. This is the situation with our participants, all of whom showed highly specific performance patterns: the functional deficit of four of them (EY, MA, NL, OZ) was localized in merely a single cognitive process; two others (HZ and ED) were impaired in more than one process, but even for these two individuals, the deficit was no less specific than other number-reading deficits reported in the literature.

A second issue concerns the mixture of developmental and acquired case studies with respect to the same cognitive model. If, as we showed above, a developmental deficit gives rise to a performance pattern specific enough to provide evidence about a specific cognitive process, we see no reason why this information cannot be used in conjunction with information from acquired deficits. Specifically in this present study, the mix between developmental and acquired data is not a problem for another reason: all our conclusions would hold even if we exclude the single participant with acquired deficit (UN), because UN's deficit was very similar to OZ's. In fact, the finding of very similar performance patterns for UN and OZ strongly suggests that both participants had a similar cognitive impairment (in the generation of the number word frame) and a similar set of unimpaired processes. Namely, UN's acquired deficit and OZ's developmental deficit can be explained by the same cognitive model.

High similarity between developmental and acquired impairments was reported also with respect to another process involved in number reading – phonological retrieval. Some aphasic patients, whose phonological retrieval is impaired, substitute number words by other number words during speech (e.g., 24 → “sixty four”), but when saying non-number words they make *phonological* rather than lexical substitutions (e.g., carry → marry). This phenomenon (STEPS, the Stimulus Type Effect on Phonological and Semantic errors) exists not only in aphasic patients (Cohen et al., 1997; Dotan & Friedmann, 2015), but also in some children and adults with an equivalent developmental impairment (Guggenheim & Friedmann, 2015).

Similarities between developmental and acquired deficits were reported in other domains too. For example, several specific types of dyslexia were observed both as a developmental deficit and as the result of a focal brain damage, with highly similar performance patterns between developmental and acquired cases. As it turns out, the detailed cognitive model of word reading, which was developed based on acquired deficits, can just as well account for the developmental deficits (Castles & Friedmann, 2014; Friedmann & Coltheart, 2017; Marshall, 1984; Temple, 2006). Similarities between developmental and acquired deficits were also reported for specific disorders of naming (Friedmann et al., 2013) and syntax (Szterman & Friedmann, 2014).

5.7. *The role of peripheral versus central processes in implementing cognitive operations*

Reading multi-digit numbers is a complex process. The conversion of numbers from one representation to another is not merely a simple symbol-to-word conversion: the existence of 0, 1, and other structure-modifying digits in the number creates a structural complexity, often referred to as “syntactic”. The model we presented here provides a detailed account of how this syntactic complexity is addressed by the cognitive system. Note that the model puts a lot of weight on the encoding stage. This architecture is not trivial: hypothetically, it could have been the case that the visual analyzer extracts only the minimum required information – the identity and position of each digit – and a later, more central mechanism identifies the number's structure in order to convert it from digit format to verbal format. Based on the performance of several individuals with impairments in the visual analysis stage, the present study suggests that this is not the case: the visual analyzer, although a visual process, allocates dedicated mechanisms to extract the number's structure. Similarly, even peripheral stages in verbal production allocate dedicated mechanisms to handle numbers as high-level representations: the phonological output buffer, the last processing stage before articulation, handles sequence of phonemes for most words, but whole-word representations in the case of number words (Dotan & Friedmann, 2015). The existence of these higher-level representations in peripheral processes is perhaps useful as they may simplify the process of converting numbers from one format to another and make it more efficient.

High-level representations in peripheral stages exist in other domains too. In word reading, like in number reading, the visual analyzer, together with the orthographic input buffer following it, not only encodes letter identities and positions but also performs morphological analysis on the basis of the morphological structure of the word (Beyersmann, Castles, & Coltheart, 2011; Friedmann & Gvion, 2012; Friedmann, Gvion, & Nisim, 2015; Friedmann, Kerbel, & Shvimer, 2010; Longtin & Meunier, 2005; Rastle & Davis, 2008; Rastle, Davis, & New, 2004; Reznick & Friedmann, 2015;

Sternberg & Friedmann, 2007; Taft & Forster, 1975; Velan & Frost, 2011). Morphological structure also has dedicated processing mechanisms in the peripheral-orthographic stages of writing (Badecker, Hillis, & Caramazza, 1990; Badecker, Rapp, & Caramazza, 1996; Yachini & Friedmann, 2008), in the mechanisms that parse verbal auditory input (Bacovcin, Goodwin Davies, Wilder, & Embick, 2017; Kouider & Dupoux, 2009), and in the peripheral post-lexical stages of speech (Dotan & Friedmann, 2015; Job & Sartori, 1984; Kohn & Melvold, 2000; Patterson, 1982). These post-lexical stages of speech also handle lexical information in the case of function words (the conversion of syntactic features to lexical entries, Dotan & Friedmann, 2015), and may even participate in the linearization of sentence-level operations such as verb movement (Chomsky, 1995, 2001; Dotan & Friedmann, 2015; Friedmann et al., 2013; Zwart, 2001). Taken together, this body of research does not suggest a centralized system with sophisticated central processes and simple peripheral processes. Rather, it suggests a distributed system, in which peripheral processes communicate high-level information to one another.

5.8. *Conclusion*

An examination of individuals with number reading deficits, dysnumeria, revealed a series of dissociations between specific sub-processes involved in number reading, in particular with respect to how these cognitive mechanisms handle the structure of numbers. Future studies may elaborate further on these mechanisms, e.g., corroborate the notion of a syntactic tree for numbers. From a clinical perspective, we showed various ways in which the number reading mechanisms can be impaired, and we created a set of tasks for assessment of such impairments. These tasks should find clinical application for assessment of dysnumeria, and we hope that they will pave the way to developing new treatments for different impairments in number processing.

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Supplementary data

Supplementary data related to this article can be found at https://doi.org/10.1016/j.cortex.2017.10.025.

Appendix A. Details of the control participants

Table A1 – Control participants per experiment

Experiment		No. of participants	Outliers	Age		
				Range	Mean	SD
1	Number reading	20	3	20; 7–30; 4	25; 5	2; 7
	Number reading (EY)	10	–	21; 3–42; 4	27; 0	5; 10
2	Sequence identification	20	1	20; 9–42; 0	31; 7	8; 4
	Sequence identification (EY)	10	–	23; 0–35; 5	29; 2	5; 6
3	Same-different decision	24	1	20; 9–42; 0	30; 0	6; 11
	Same-different decision (EY)	10	–	27; 8–35; 0	28; 8	4; 10
4	Number matching	20	1	21; 3–42; 6	26; 1	4; 4
5	Number repetition	20	2	20; 7–42; 4	26; 1	4; 8
	Number repetition (EY)	10	–	22; 10–28; 8	25; 2	2; 0
	Number repetition (UN)	15	–	21; 9–30; 1	25; 0	2; 4
8	Same-different (length)	20	1	20; 10–43; 4	29; 6	7; 3
10	Multiply/divide by 10	20	2	21; 4–44; 4	27; 10	5; 4
11	Read numbers as triplets	20	–	24; 8–49; 4	35; 1	7; 7

The “outliers” column indicates the number of control participants who were excluded as outliers – i.e., their error rate exceeded the 75th percentile of error rates by more than 150% the inter-quartile distance.

Some control groups were run for experiment versions used for a specific participant. These cases are indicated in the “Experiment” column by parentheses with the participant's initials.

Appendix B. Artwork

Fig. 4 – "Frustration" - a piece of art by the participant HZ, illustrating her perception of her dyslexia and dysnumeria.

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