The concept of mass (mass, energy, relativity)

1989 Sov. Phys. Usp. 32 629

(http://iopscience.iop.org/0038-5670/32/7/A05)

The concept of mass (mass, energy, relativity)

L. B. Okun'

Institute of Theoretical and Experimental Physics, Moscow
Usp. Fiz. Nauk **158**, 511–530 (July 1989)

Present-day ideas concerning the relationship between mass and energy are presented. The history of the origin of archaic terms and concepts that are widely used in the literature in discussing the problem of mass and energy is related, and arguments are presented for the necessity of abandoning these archaic terms and concepts.

1. A small test instead of an introduction

Einstein's relation between the mass of a body and the energy contained in it is undoubtedly the most famous formula of the theory of relativity. It permitted a new and deeper understanding of the world that surrounds us. Its practical consequences are vast and, to a large degree, tragic. In a certain sense this formula has become the symbol of science in the 20th century.

What is the need for yet another paper on this famous relation, on which thousands of papers and hundreds of books have already been written?

Before I answer this question, let us think about the form in which, in your opinion, the physical meaning of the relation between mass and energy is most adequately expressed. Here are four relations:

$$E_0 = mc^2, \tag{1.1}$$

$$E = mc^2, \tag{1.2}$$

$$E_0 = m_0c^2, \tag{1.3}$$

$$E = m_0c^2; \tag{1.4}$$

where c is the speed of light, E is the total energy of the body, m is its mass, E_0 is the rest energy, and m_0 is the rest mass of the same body. Please write the numbers of these relations in the order in which you regard them as most "correct." Now continue reading.

In the popular scientific literature, school textbooks, and the overwhelming proportion of university textbooks the dominant relation is (1.2) (and its consequence (1.3)), which is usually read from the right to the left and interpreted as follows: The mass of a body increases with its energy—both internal and kinetic.

The overwhelming majority of serious monographs and scientific papers on theoretical physics, particularly on the theoretical physics of elementary particles, for which the special theory of relativity is a working tool, does not contain relations (1.2) and (1.3) at all. According to these books, the mass of a body m does not change when it is in motion and, apart from from the factor c, is equal to the energy contained in the body at rest, i.e., the relation (1.1) is valid. The implication is that both the term "rest mass" and the symbol m_0 are superfluous and therefore are not used. Thus, a kind of pyramid exists where base is formed by the popular science books and school textbooks with press runs of millions, and at whose apex are monographs and papers on the theory of elementary particles with press runs in the thousands.

Between the apex and base of this theoretical pyramid we find a significant number of books and papers in which all three (and even four!) relations coexist peacefully in a mysterious manner. Responsible in the first place for this situation are the theoretical physicists who have not yet explained to large circles of educated people this absolutely simple matter.

The aim of this paper is to explain as simply as possible why the relation (1.1) adequately reflects the essence of the theory of relativity while (1.2) and (1.3) do not, and thus to foster the adoption in the textbook and popular scientific literature of a clear terminology that does not introduce confusion and misunderstandings. In what follows I shall call such terminology the correct terminology. I hope that I shall succeed to convince the reader that the term "rest mass" m_0 is superfluous, that instead of speaking of the "rest mass" m_0 one should speak of the mass m of a body which for ordinary bodies is the same, in the theory of relativity and in Newtonian mechanics, that in both theories the mass m does not depend on the reference frame, that the concept of mass dependent on velocity arose at the beginning of the twentieth century as a result of an unjustified extension of the Newtonian relation between momentum and velocity to the range of velocities comparable to the velocity of light in which it is invalid, and that at the end of the twentieth century one should bid a final farewell to the concept of mass dependent on velocity.

The paper consists of two parts. In part I (Secs. 2–12) the role played by mass in Newtonian mechanics is discussed. We then consider the basic relations of the theory of relativity that connect the energy and momentum of a particle to its mass and velocity, establish the connection between acceleration and force, and give the relativistic expression

 0038-5670/89/070629-10$01.80

for the gravitational force. We show how the mass of a system consisting of several particles is defined and consider examples of physical processes that result in a change in the mass of a body or system of bodies, this change being accompanied by the absorption or emission of particles carrying kinetic energy. The first part of the paper ends with a brief account of modern attempts to calculate theoretically the masses of the elementary particles.

In Part II (Secs: 13–20) we discuss the history of the development of the notion of the mass of a body that increases with its energy, the so-called relativistic mass. We show that the use of this archaic concept does not correspond to the four-dimensionally symmetric form of the theory of relativity and leads to numerous confusions in the textbook and popular scientific literature.

I. FACTS

2. Mass in Newtonian mechanics

In Newtonian mechanics, mass possesses a number of important properties and presents, as it were, several faces:

1. Mass is a measure of the amount of matter.
2. The mass of a composite body is equal to the mass of the bodies that constitute it.
3. The mass of an isolated system of bodies is conserved—it does not change with time.
4. The mass of a body does not change on the transition from one system of coordinates to another; in particular, the mass is the same in different inertial systems of coordinates.
5. The mass of a body is a measure of its inertness (inertia).
6. The masses of bodies are the sources of their gravitational attraction to each other.

We discuss in more detail the last two properties of mass.

As a measure of the inertia of a body, the mass m appears in the formula that connects the momentum $\mathbf{p}$ of the body to its velocity $\mathbf{v}$:

$$\mathbf{p} = m\mathbf{v} . \tag{2.1}$$

The mass also occurs in the expression for the kinetic energy $E_{\rm kin}$ of the body:

$$E_{\rm kin} = \frac{\mathbf{p}^2}{2m} = \frac{m\mathbf{v}^2}{2} . \tag{2.2}$$

By virtue of the homogeneity of space and time the momentum and energy of a free body are conserved in an inertial system of coordinates. The momentum of a given body changes with the time only under the influence of other bodies:

$$\frac{d\mathbf{p}}{dt} = \mathbf{F}, \tag{2.3}$$

where $\mathbf{F}$ is the force that acts on the body. If it is borne in mind that in accordance with the definition of the acceleration $\mathbf{a}$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} , \tag{2.4}$$

and allowance is made for the relations (2.1) and (2.3), then

$$\mathbf{F} = m\mathbf{a}. \tag{2.5}$$

In this relation the mass again appears as a measure of inertia. Thus, in Newtonian mechanics mass as a measure of inertia is determined by two relations: (2.1) and (2.5). Some authors prefer to define the measure of inertia by the relations (2.1), others by the relation (2.5). For the subject of our paper it is only important that these two definitions are compatible in Newtonian mechanics.

We now turn to gravitation. The potential energy of the attraction between two bodies with masses M and m (for example, the earth and a stone) is

$$U_g = -\frac{GMm}{r} , \tag{2.6}$$

where $G = 6.7\cdot 10^{-11}\,{\rm N\cdot m^2\cdot kg^{-2}}$ (we recall that $1N = 1\,{\rm kg\cdot m\cdot sec^{-2}}$). The force with which the earth attracts the stone is

$$\mathbf{F}_g = -\frac{GMm\mathbf{r}}{r^3} , \tag{2.7}$$

where the radius vector $\mathbf{r}$, which joins the centers of mass of the bodies, is directed from the earth to the stone. (The stone attracts the earth with a force of the same magnitude but opposite direction.) It follows from (2.7) and (2.5) that the acceleration of a body falling freely in a gravitational field does not depend on its mass. The acceleration in the field of the earth is usually denoted by g:

$$\mathbf{g} = \frac{\mathbf{F}_g}{m} = -\frac{GM\mathbf{r}}{r^3} . \tag{2.8}$$

Substituting the mass and radius of the earth ($M_\oplus \simeq 6\cdot 10^{24}$ kg, $R_\oplus \simeq 6.4.10^6$ m) in (2.9), we readily obtain the estimate $g \approx 9.8$ m/sec.

The universality of g was first established by Galileo, who concluded that the acceleration of a falling sphere depended neither on its mass nor the material of which it was made. This independence was verified to a very high degree of accuracy at the beginning of the 20th century by Eötvös and in a number of recent experiments. The fact that the gravitational acceleration does not depend on the mass of the accelerated body is usually characterized in school physics courses as equality of the inertial and gravitational masses, by which it is meant that one and the same quantity m occurs in the relation (2.5), on the one hand, and (2.6) and (2.7), on the other.

We shall not discuss here the other properties of mass listed at the beginning of this section, since they appear obvious from the point of view of common sense. In particular, no one doubts that the mass of a vase is equal to the sum of its fragments:

$$m = \sum_i m_i . \tag{2.9}$$

Nor does anyone doubt that the mass of two automobiles is equal to the sum of their masses irrespective of whether they are parked or race toward each other at maximal speed.

3. Galileo's principle of relativity

If one does not go into the actual formulas, one can say that the quintessence of Newtonian mechanics is the principle of relativity.

In one of Galileo's books there is a brilliant argument about the fact that in the cabin of a ship with portholes closed it is not possible to detect by any mechanical experiments a uniform and rectilinear motion of the ship relative to the shore. In giving this example, Galileo emphasized that no mechanical experiments could distinguish one inertial frame of reference from another. This assertion became known as Galileo's principle of relativity. Mathematically, this principle is expressed by the fact that the equations of Newtonian mechanics do not change on the transition to new coordinates: $\mathbf{r}\to\mathbf{r}'=\mathbf{r}-\mathbf{V}t \quad t\to t'=t$, where $\mathbf{V}$ is the velocity of the new inertial system with respect to the original one.

4. Einstein's principle of relativity

At the beginning of the 20th century a more general principle, which became known as Einstein's principle of relativity, was formulated. According to this principle not only mechanical but also all other experiments (optical, electrical, magnetic, etc.) are incapable of distinguishing one inertial system from another. The theory constructed on this principle has become known as the theory of relativity, or relativistic theory.

Relativistic theory, in contrast to nonrelativistic (Newtonian) mechanics, takes account of the fact that there exists in nature a limiting speed c of the propagation of physical signals: $c=3\cdot10^8$ m/sec.

Usually, c is called the speed of light in vacuum. Relativistic theory makes it possible to calculate the motion of bodies (particles) with all speeds v up to $v=c$. Nonrelativistic Newtonian mechanics is the limiting case of Einstein's relativistic mechanics as $v/c\to0$. Formally, in Newtonian mechanics there is no limit to the speed of propagation of signals, i.e., $c=\infty$.

The introduction of Einstein's principle of relativity required a modification in our view of fundamental concepts such as space, time, and simultaneity. It was found that, considered separately, the distances between two events in space, $\mathbf{r}$, and in time, t, do not remain unchanged on the transition from one inertial frame of reference to another but behave like the components of a four-dimensional vector in a four-dimensional Minkowski spacetime. All that remains unchanged, invariant, is the interval s: $s^2=c^2t^2-\mathbf{r}^2$.

5. Energy, momentum, and mass in the theory of relativity

The fundamental relations of the theory of relativity for a freely moving particle (system of particles, body) are

$$E^2-\mathbf{p}^2c^2=m^2c^4, \tag{5.1}$$

$$\mathbf{p}=\frac{\mathbf{v}E}{c^2}, \tag{5.2}$$

where E is the energy, $\mathbf{p}$ is the momentum, m is the mass, and $\mathbf{v}$ is the velocity of the particle (or system of particles, or body). It should be emphasized that the mass m and the velocity $\mathbf{v}$ for a particle or a body are the same quantities with which we deal in Newtonian mechanics. Like the four-dimensional coordinates t and $\mathbf{r}$, the energy E and the momentum $\mathbf{p}$ are the components of a four-dimensional vector. They change on the transition from one inertial system to another in accordance with the Lorentz transformations. The mass, however, is not changed—it is a Lorentz invariant.

It should be emphasized that, as in Newtonian mechanics, in the theory of relativity there are laws of conservation of the energy and momentum of an isolated particle or an isolated system of particles.

In addition, as in Newtonian mechanics, the energy and momentum are additive—the total energy and total momentum of n free particles are, respectively,

$$E=\sum_{i=1}^{n}E_i, \qquad \mathbf{p}=\sum_{i=1}^{n}\mathbf{p}_i. \tag{5.3}$$

With regard to the mass, in theory of relativity the mass of an isolated system is conserved (does not change with the time), but does not possess the property of additivity (see below).

The most important difference of the theory of relativity from nonrelativistic mechanics is that the energy of a massive body does not vanish even when the body is at rest, i.e., for $\mathbf{v}=0$, $\mathbf{p}=0$. As can be seen from (5.1), the rest energy of a body (it is usually denoted by E_0) is proportional to its mass:

$$E_0=mc^2. \tag{5.4}$$

Indeed, the assertion that inert matter at rest hides within it a huge (by virtue of the square of the limiting velocity c) store of energy, made by Einstein in 1905, is the main practical consequence of the theory of relativity. All nuclear energy and all military nuclear technology is based on the relation (5.4). It may not be quite so well known that all ordinary energy production is based on the same relation.

6. Limiting cases of the relativistic equations

It is a remarkable property of Eqs. (5.1) and (5.2) that they describe the motion of particles in the complete interval of speeds: $0\leqslant v\leqslant c$. In particular, for $v=c$ it follows from (5.2) that

$$pc=E. \tag{6.1}$$

Substituting this relation in (5.1), we conclude that if a particle moves with speed c, then its mass is equal to zero, and vice versa. For a massless particle there is no coordinate system in which it is at rest—it "can only dream" of rest.

For massive particles (as we shall call all particles with nonzero mass, even if they are very light) the relations for the energy and momentum can be conveniently expressed in terms of the mass and velocity. For this we substitute (5.2) in (5.1):

$$E^2\left(1-\frac{v^2}{c^2}\right)=m^2c^4, \tag{6.2}$$

and, taking the square root, we obtain

$$E=mc^2\left(1-\frac{v^2}{c^2}\right)^{-1/2}. \tag{6.3}$$

Substituting (6.3) in (5.2), we obtain

$$\mathbf{p}=m\mathbf{v}\left(1-\frac{v^2}{c^2}\right)^{-1/2}. \tag{6.4}$$

It is obvious from (6.3) and (6.4) that a massive body (with $m\neq0$) cannot move with the speed of light, since then the energy and momentum of the body would have to be infinite.

In the literature on the theory of relativity it is customary to use the notation

$$\beta=\frac{v}{c}, \tag{6.5}$$

and

$$\gamma=(1-\beta^2)^{-1/2}. \tag{6.6}$$

Using γ, we can express E and $\mathbf{p}$ in the form

$$E=mc^2\gamma, \tag{6.7}$$

$$\mathbf{p}=m\mathbf{v}\gamma. \qquad (6.8)$$

We define the kinetic energy $E_{\rm kin}$ as the difference between the total energy E and the rest energy E_0:

$$E_{\rm kin}=E-E_0=mc^2(\gamma-1). \qquad (6.9)$$

In the limit when $v/c \ll 1$, only the first terms in the series in β need be retained in the expressions (6.8) and (6.9). Then we return in a natural manner to the equations of Newtonian mechanics:

$$\mathbf{p}=m\mathbf{v}, \qquad (6.10)$$

$$E_{\rm kin}=\frac{\mathbf{p}^2}{2m}=\frac{m\mathbf{v}^2}{2}, \qquad (6.11)$$

from which it can be seen that the mass of the body in Newtonian mechanics and the mass of the same body in relativistic mechanics are the same quantity.

7. Connection between the force and acceleration in the theory of relativity

One can show that in the theory of relativity the Newtonian relation between the force $\mathbf{F}$ and the change in the momentum remains the same:

$$\mathbf{F}=\frac{d\mathbf{p}}{dt}. \qquad (7.1)$$

Using the relation (7.1) and the definition of acceleration,

$$\mathbf{a}=\frac{d\mathbf{v}}{dt}, \qquad (7.2)$$

we readily obtain

$$\mathbf{F}=m\gamma\mathbf{a}+m\gamma^3\boldsymbol{\beta}(\boldsymbol{\beta}a). \qquad (7.3)$$

We see that, in contrast to the nonrelativistic case, the acceleration in the relativistic case is not directed along the force but also has a component along the velocity. Multiplying (7.3) by $\mathbf{v}$, we find

$$\mathbf{a}\mathbf{v}=\frac{\mathbf{F}\mathbf{v}}{m\gamma(1+\gamma^2\beta^2)}=\frac{\mathbf{F}\mathbf{v}}{m\gamma^3}. \qquad (7.4)$$

Substituting this in (7.3), we obtain

$$F-(F\boldsymbol{\beta})\boldsymbol{\beta}=m\gamma\mathbf{a}. \qquad (7.5)$$

Despite the unusual appearance of Eq. (7.3) from the point of view of Newtonian mechanics (we should say, rather, precisely because of this unusual appearance), this equation correctly describes the motion of relativistic particles. From the beginning of the century it was frequently submitted to experimental verification for different configurations of the electric and magnetic fields. This equation is the foundation of the engineering calculations for relativistic accelerators.

Thus, if $\mathbf{F}\perp\mathbf{v}$, then

$$\mathbf{F}=m\gamma\mathbf{a}, \qquad (7.6)$$

but if $\mathbf{F}\parallel\mathbf{v}$, then

$$\mathbf{F}=m\gamma^3\mathbf{a}. \qquad (7.7)$$

Thus, if one attempts to define an "inertial mass" as the ratio of the force to the acceleration, then in the theory of relativity this quantity depends on the direction of the force relative to the velocity, and therefore cannot be unambiguously defined. Consideration of the gravitational interaction leads to the same conclusion with regard to the "gravitational mass."

8. Gravitational attraction in the theory of relativity

Whereas in Newtonian theory the force of the gravitational interaction is determined by the masses of the interacting bodies, in the relativistic case the situation is much more complicated. The point is that the source of the gravitational field is a complicated quantity possessing ten different components—the so-called energy-momentum tensor of the body. (For comparison we point out that the source of the electromagnetic field is the electromagnetic current, which is a four-dimensional vector and has four components.

We consider a very simple example, when one of the bodies has a very large mass M and is at rest (for example, the sun or the earth), and the other has a very small or even zero mass, for example, an electron or photon with energy E. On the basis of the general theory of relativity, one can show that in this case the force acting on a light particle is

$$\mathbf{F}=-GM\frac{E}{c^2}[(1+\boldsymbol{\beta}^2)\mathbf{r}-(\mathbf{r}\boldsymbol{\beta})\boldsymbol{\beta}]r^{-3}. \qquad (8.1)$$

It is easy to see that for a slow electron, with $\beta \ll 1$, the expression in the square bracket reduces to $\mathbf{r}$, and, bearing in mind that $E_0/c^2=m$, we return to Newton's nonrelativistic formula. However, for $v/c\sim 1$ or $v/c=1$ we encounter a fundamentally new phenomenon, namely, the quantity that plays the role of the "gravitational mass" of the relativistic particle depends not only on its energy but also on the mutual direction of the vectors $\mathbf{r}$ and $\mathbf{v}$. If $\mathbf{v}\parallel\mathbf{r}$, then the "gravitational mass" is E/c^2, but is $\mathbf{v}\perp\mathbf{r}$, it is $(E/c^2)(1+\beta^2)$, and for a photon $2E/c^2$.

We use the quotation marks to emphasize that for a relativistic body the concept of gravitational mass is invalid. It is meaningless to speak of the gravitational mass of a photon if for a vertically falling photon this quantity is half that for one traveling horizontally.

Having discussed different aspects of the dynamics of a single relativistic particle, we now turn to the question of the mass of a system of particles.

9. Mass of a system of particles

We have already noted above that in the theory of relativity the mass of a system is not equal to the mass of the bodies that make up the system. This assertion can be illustrated by several examples.

1. Consider two photons moving in opposite directions with equal energies E. The total momentum of such a system is zero, and the total energy (it is the rest energy of the system of the two photons) is $2E$. Therefore, the mass of this system is $2E/c^2$. It is easy to show that a system of two photons will have zero mass only when they move in the same direction.

2. We consider a system consisting of n bodies. The mass of this system is determined by

$$m=\left[\left(\sum_{i=1}^{n}\frac{E_i}{c^2}\right)^2-\left(\sum_{i=1}^{n}\frac{\mathbf{p}_i}{c}\right)^2\right]^{1/2}, \qquad (9.1)$$

where $\Sigma_i E_i$ is the sum of the energies of these bodies, and $\Sigma\,\mathbf{p}_i$ is the vector sum of their momenta.

The first two examples are characterized by being systems of free particles; the sizes of these systems increase without limit with the time as the particles that constitute them move away from each other. We now consider systems whose sizes remain unchanged.

3. We consider a hydrogen atom consisting of a proton and an electron. To a good accuracy, the rest energy E_0 of the atom can be represented as a sum of four terms:

$$E_0 = m_p c^2 + m_e c^2 + E_{kin} + U, \tag{9.2}$$

where m_p is the mass of the proton, m_e is the mass of the electron, and E_{kin} and U are the kinetic and potential energies of the electron.

The potential energy U is due to the mutual attraction of the electric charges of the proton and electron, which prevents the electron from leaving the proton. From a theory exhaustively tested by experiment it follows that

$$E_{kin} + U = -E_{kin} = -\frac{1}{2} m_e v_e^2, \tag{9.3}$$

where $v_e \approx c/137$ is the speed of the electron in the hydrogen atom. Hence

$$m_H = \frac{E_0}{c^2} = m_p + m_e - \frac{m_e v_e^2}{2c^2}. \tag{9.4}$$

Thus, the mass of the hydrogen atom is less than $m_p + m_e$ by a few millionths of the electron mass.

4. Let us consider the deuteron, the nucleus of the heavy isotope of hydrogen, consisting of a proton and a neutron. The proton and neutron attract each other more strongly and move more rapidly than the electron in the hydrogen atom. As a result, the mass of the deuteron is about 0.1% less than the sum of the masses of the proton and neutron.

Essentially, we treated the last two examples on the basis of nonrelativistic mechanics, since the considered mass differences, or, as they are called, the mass defects, are, although important, fairly small compared with the masses themselves.

Now is the time to recall the broken vase mentioned in Sec. 2. The sum of the masses of the fragments is equal to the mass of the vase to the accuracy with which the binding energy of these fragments is small compared with their rest energy.

10. Examples of transformations into each other of rest energy and kinetic energy

In nuclear or chemical reactions the rest energy must, by virtue of the law of conservation of energy, be transformed into the kinetic energy of the reaction of products if the total mass of the particles that interact is greater than the total mass of the reaction products. We consider four examples:

1. When an electron and positron annihilate into two photons, the entire rest energy of the electron and positron is transformed into the kinetic energy of the photons.

2. As a result of thermonuclear reactions taking place in the sun, there are transformations of two electrons and four protons into a helium nucleus and two neutrinos:

$$2e + 4p \rightarrow {}^4He + 2\nu + E_{kin}. \tag{10.1}$$

The energy released is $E_{kin} = 29.3$ MeV. Remembering that the mass of the proton is 938 MeV and the mass of the electron 0.5 MeV, the relative decrease of the mass is of the order of a percent ($\Delta m/m = 0.8 \cdot 10^{-2}$).

3. If a slow neutron collides with a ^{235}U nucleus, the nucleus breaks up into two fragments and also emits two or three neutrons capable of striking other uranium nuclei, and an energy $E_{kin} \approx 200$ MeV is released. In this case, as is readily seen, $\Delta m/m = 0.9 \cdot 10^{-3}$.

4. In the combustion reaction of methane in the gas burner of a kitchen stove,

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O \tag{10.2}$$

an energy equal to 35.6 MJ per cubic meter of methane is released. Since the density of methane is 0.89 kg/m^3, we can readily see that in this case $\Delta m/m = 10^{-10}$. In chemical reactions $\Delta m/m$ is 7–8 orders of magnitude less than in nuclear reactions, but the essence of the mechanism of energy release is the same—rest energy is transformed into kinetic energy.

To emphasize that the mass of a body changes whenever its internal energy changes, we consider two common examples:

1) if a flat iron is heated to 200°, its mass increases by $\Delta m/m = 10^{-12}$ (this is readily estimated using the specific heat 450 J·kg^{-1}·deg^{-1} of iron);

2) if a certain amount of ice is transformed completely into water, $\Delta m/m = 3.7 \cdot 10^{-12}$.

11. Comparison of the role played by mass in the theories of Einstein and Newton

Summarizing what was said above, it is expedient to compare the role played by mass in the mechanics of Einstein and of Newton.

1. In the theory of relativity, in contrast to Newtonian mechanics, the mass of a system is not a measure of the amount of matter. In relativistic theory the very concept of matter is much richer than in nonrelativistic theory. In relativistic theory there is no fundamental difference between matter (protons, neutrons, electrons) and radiation (photons).

Protons, neutrons, electrons, and photons are the most commonly encountered representatives in nature of the large family of so-called elementary particles. It is possible that the photons are not the only particles having zero mass. For example, certain types of neutrinos could also have zero mass. It is also possible that there exist other massless particles that have not yet been detected because of the great difficulty of detecting them by means of existing instruments.

2. In nonrelativistic theory, the more individual particles (atoms) a system (a scale weight) contains, the greater its mass. In relativistic theory, when the energies of particles are very large compared with their masses, the mass of a system of particles is determined not so much by their number as by their energies and mutual orientations of their momenta. The mass of a composite body is not equal to the sum of the masses of the bodies that constitute it.

3. As in Newtonian mechanics, the mass of an isolated system of bodies is conserved, i.e., does not change with time. However, it is now necessary to include among the bodies not only "matter," say atoms, but also "radiation" (photons).

4. As in Newtonian mechanics, in the theory of relativity the mass of a body does not change on the transition from one inertial frame of reference to another.

5. The mass of a relativistically moving body is not a

measure of its inertia. Indeed, a single measure of inertia for relativistically moving bodies does not exist at all, since the resistance of a body to the force accelerating it depends on the angle between the force and the velocity.

6. The mass of a relativistically moving body does not determine its interaction with the gravitational field. This interaction is determined by an expression that depends on the energy and momentum of the body.

Despite these four "noes" the mass of a body is also an extremely important property in the theory of relativity. A vanishing mass means that the "body" must always move with the speed of light. A nonvanishing mass characterizes the mechanics of a body in a frame of reference in which it moves slowly or is at rest. This frame of reference is distinguished compared with other inertial systems.

7. According to the theory of relativity, the mass of a particle is a measure of the energy "sleeping" in the particle at rest; it is a measure of the rest energy: $E_0 = mc^2$. This property of mass was unknown in nonrelativistic mechanics.

The mass of an elementary particle is one its most important characteristics. Attempts are made to measure it as accurately as possible. For stable or long-lived particles the mass is determined by independent measurement of the energy and momentum of the particle and application of the formula $m^2 = (E^2/c^4) - (\mathbf{p}^2/c^2)$. The masses of short-lived particles are determined by measuring the energies and momenta of the particles produced by their decay or of particles that are "present" when they are produced.

Information about the masses of all elementary particles together with their other properties (lifetime, spin, decay modes) is contained in reference collections that are regularly updated.

12. The nature of mass: Question No. 1 of modern physics

During recent decades great progress has been made in understanding the properties of elementary particles. We have seen the construction of quantum electrodynamics—the theory of the interaction of electrons with photons, and the foundations have been laid of quantum chromodynamics—the theory of the interaction of quarks with gluons and of the theory of the electroweak interaction. In all these theories the particles that transmit the interactions are the so-called vector bosons—particles that have spin equal to unity: the photon, gluons, and the W and Z bosons.

As regards the masses of the particles, the achievements here are much more modest. At the turn of the 19th and 20th centuries it was believed that mass could have a purely electromagnetic origin, at least for the electron. We know today that the electromagnetic fraction of the mass of the electron is of the order of a percent of its total mass. We know that the main contribution to the masses of the protons and neutrons are made by the strong interactions mediated by gluons and not the masses of the quarks that are present in protons and neutrons.

But we know absolutely nothing of what produces the masses of the six leptons (electron, neutrino, and four further particles analogous to them) and six quarks (of which the first three are significantly lighter than the proton, the fourth is somewhat lighter than the proton, the fifth is five times heavier, while the sixth is so massive that the attempts to produce and detect it have hitherto failed).

There are theoretical guesses that hypothetical particles with spin zero play a decisive role in creating the masses of the leptons and quarks, and also of the W and Z bosons. The searches for these particles represent one of the fundamental problems of high-energy physics.

II. ARTIFACTS

13. At the turn of the century: Four masses

Everything that has been said in the first part of this paper is well known to any theoretical physicist who has ever dealt with the special theory of relativity. On the other hand, any physicist (and not only a physicist) has heard of Einstein's "famous" relation $E = mc^2$. It is therefore natural to ask how it comes about that there is a peaceful coexistence of mutually exclusive formulas in the literature and in the minds of readers:

$$E_0 = mc^2,$$

$$E = mc^2.$$

Before we seek to answer this question, we recall once more that in accordance with the first formula the rest energy E_0 corresponds to the mass of a body at rest, while according to the second any body with energy E has mass E/c^2. According to the first, the mass of a body does not change when it is in motion. According to the second, the mass of the body increases with increasing velocity of the body. According to the first, the photon is massless, but according to the second it has a mass equal to E/c^2.

To answer the question we have posed about the coexistence of the formulas, we must examine the history of the creation, interpretation, and recognition of the special theory of relativity.

In discussions of the connection between mass and energy, the starting point is usually taken to be the paper of J. J. Thomson[1] published in 1881. In this paper, the first attempt was made to estimate the contribution to the inertial mass of an electrically charged-body made by the energy of the electromagnetic field of this body.

The creation of the theory of relativity is usually associated with Einstein's 1905 paper[2] in which the relativity of simultaneity was clearly formulated. But, of course, the work on the creation and interpretation of the theory began long before 1905 and continued long after that date.

If one speaks of interpretation, the process must still be regarded as continuing today. Otherwise it would not be necessary to write this paper. As regards recognition, one can say that even at the end of 1922, when Einstein was awarded the Nobel prize, the theory of relativity was not generally accepted.

The secretary of the Swedish Academy of Sciences wrote to Einstein that the Academy had awarded him the Nobel prize for the discovery of the law of the photoelectric effect "but without taking into account the value which will be accorded your relativity and gravitation theories after these are confirmed in the future" (quoted from Pais's book[3]).

The formula $E = mc^2$ appeared in 1900 before the creation of the theory of relativity. It was written down by Poincaré whose point of departure was that a plane light wave carrying energy E has a momentum $\mathbf{p}$ of absolute magnitude that, in accordance with Poynting's theorem, is E/c. Using Newton's nonrelativistic formula for the momentum,

$\mathbf{p} = m\mathbf{v}$, and the fact that for light $v = c$c, Poincaré[4] concluded that a photon must possess an inertial mass $m = E/c^2$.

Already a year before this, in 1899, Lorentz[5] had first introduced the concept of longitudinal and transverse masses of ions, the first of which increases with the velocity as γ^3, the other as γ. He arrived at this conclusion by using the Newtonian relation between the force and the acceleration, $\mathbf{F} = m\mathbf{a}$. A detailed consideration of these masses for electrons is contained in his paper[6] published in 1904.

Thus, at the turn of the century, there arose, through, as we now understand, the incorrect use of nonrelativistic equations to describe relativistic objects, a family of "masses" that increase with the energy of the body:

"relativistic mass" $m = E/c^2$,

"transverse mass" $m_t = m\gamma$,

"longitudinal mass" $m_l = m\gamma^3$.

Note that for $m \neq 0$ the relativistic mass is equal to the transverse mass, but, in contrast to the latter, it also exists for massless bodies, for which $m = 0$. We here use the letter m in the usual sense, since we used it in the first part of this paper. But all physicists during the first five years of this century, i.e., before the creation of the theory of relativity, and many after the creation of that theory called the relativistic mass the mass and denoted it by the letter m, as did Poincaré in his 1900 paper. And then there must necessarily arise, and did arise, a further, fourth term: the "rest mass," which was denoted by m_0. The term "rest mass" came to be used for the ordinary mass, which, in a consistent exposition of the theory of relativity, is denoted m.

This is the origin of the "gang of four,' which successfully established itself in the incipient theory of relativity. Thus were created the prerequisites for the confusion that continues to the present day.

From 1900 special experiments were made with β rays and cathode rays, i.e., energetic electrons, beams of which were deflected by magnetic and electric fields (see Miller's book[7]).

These experiments were called experiments to measure the velocity dependence of the mass, and during almost the whole of the first decade of our century their results did not agree with the expressions for m_t and m_l obtained by Lorentz and, essentially, refuted the theory of relativity and were in good agreement with the incorrect theory of Abraham. Subsequently, agreement with the formulas of Lorentz was established, but it can be seen from the letter of the secretary of the Swedish Academy of Sciences quoted earlier that it did not appear absolutely convincing.

14. Mass and Energy in Einstein's papers in 1905

In Einstein's first paper on the theory of relativity,[2] he, like everyone at that time, used the concepts of longitudinal and transverse mass, but did not denote them by special symbols, while for the kinetic energy W he obtained the relation

$$W = \mu V^2 \left\{ \frac{1}{[1-(v^2/V^2)]^{1/2}} - 1 \right\},$$

where μ is the mass, and v is the speed of light. Thus, he did not use the concept of "rest mass."

In the same 1905 Einstein published a short note[8] in which he concluded that "the mass of a body is a measure of the energy contained in it.' If we use modern notation, this conclusion is expressed by the formula

$$E_0 = mc^2.$$

Actually, the symbol E_0 occurs already in the first phrase with which the proof begins: "Suppose that in the system (x,y,z) there is a body at rest whose energy, referred to the system (x,y,z) is E_0." This body radiates two plane light waves with equal energies $L/2$ in opposite directions. Considering this process in a system moving with velocity v using the circumstance that in this system the total energy of the photons is $L(\gamma - 1)$, and equating it to the difference of the kinetic energies of the body before and after the emission, Einstein concluded that "if the body gives up energy L in the form of radiation, its mass is reduced by L/V^2," i.e., $\Delta m = \Delta E_0/c^2$. Thus, in this paper he introduced the concept of rest energy of the body and established equivalence between the mass of the body and the rest energy.

15. "Generalized Poincaré formula"

If in the 1905 paper Einstein was completely clear, in his subsequent paper[9] of 1906 the clarity is somewhat lost. Referring to the 1900 paper of Poincaré that we mentioned earlier, Einstein proposed a more transparent proof of Poincaré's conclusion and asserted that to every energy E there corresponds an inertia E/V^2 (inertial mass E/V^2, where V is the speed of light), and he ascribed "to the electromagnetic field a mass density (ρ_e) that differs from the energy density by a factor $1/V^2$." Moreover, it can be seen from the text of the paper[9] that he regards these assertions as a development of his 1905 paper. And although in the paper[10] that appeared in 1907 Einstein again clearly speaks of the equivalence of mass and the rest energy of a body (§11), he does not draw a clear distinction between the relativistic formula $E_0 = mc^2$ and the prerelativistic formula $E = mc^2$ and in his paper[11] "On the influence of gravitation on the propagation of light" he wrote: "... If the increment of the energy is E, then the increment of the inertial mass is E/c^2."

At the end of the first decade of this century, Planck[12,13] and Minkowski[14] played an important part in creating the modern unified four-dimensional spacetime formalism of the theory of relativity. At about the same time, in the papers of Lewis and Tolman[15,16] the "prerelativistic mass," equal to E/c^2, was finally elevated to the throne of the theory of relativity. It received the title "relativistic mass" and, most unfortunate of all, usurped the simple name of "mass." Meanwhile, the true mass suffered Cinderella's fate and was given the nickname "rest mass." Lewis and Tolman based their papers on the Newtonian definition $\mathbf{p} = m\mathbf{v}$ of momentum and the law of conservation of "mass," which in essence was the law of conservation of energy, divided by c^2.

It is remarkable that in the literature on the theory of relativity this "palace revolution" has remained unnoted, and the development of the theory of relativity has been represented as a logically consistent process. In particular, the historians of physics (for example, in the books of Refs. 3, 7, 17, and 18) do not note the fundamental difference between Einstein's paper of Ref. 8, on the one hand, and the papers of Poincaré[4] and Einstein[9] on the other.

I once saw a cartoon representing the process of scientific creativity. A scientist, with a back like Einstein, is writing at the blackboard. He has written $E = ma^2$, crossed it out, below that $E = mb^2$, and again crossed it out, and, finally, still lower; $E = mc^2$. A humorous trifle, but this cartoon may be nearer the truth than the received description of the process of scientific creation as a continuous logical development.

It is not by chance that I mentioned Cinderella. A mass that increased with speed—that was truly incomprehensible and symbolized the depth and grandeur of science, bewitching the imagination. Compared with it, what was ordinary mass, so simple, so comprehensible!

16. A thousand and two books

The title of this section is not to be taken literally as giving the total number of books which discuss the theory of relativity, which I do not know. The number is certainly greater than several hundred, and may be a thousand. However, two books that appeared at the beginning of the twenties need to be considered especially. They are both very famous and have been admired by more than one generation of physicists. The first is the encyclopaedic monograph of the 20-year-old student Wolfgang Pauli "Relativitätstheorie",[19] which appeared in 1921. The second is the *The Meaning of Relativity*,[20] published in 1922 by the creator of the special and general theories himself—Albert Einstein. In these two books the question of the connection between energy and mass is treated in radically different ways.

Pauli decisively rejects, as obsolete, the longitudinal and transverse masses (and with it the formula $\mathbf{F} = ,m\mathbf{a}$) but regards it as "expedient" to use the formula $\mathbf{p} = m\mathbf{v}$ and, therefore, the concept of a velocity-dependent mass, to which he devotes several sections. He gives much space to the "law of equivalence of mass and energy" or, as he calls it, "the law of inertia of energy of any form," according to which "to every energy there corresponds a mass $m = E/c^2$."

In contrast to Pauli, Einstein denotes the ordinary mass by the letter m. Expressing the four-dimensional energy-momentum vector in terms of m and the velocity of the body, Einstein then considers a body at rest and concludes that "the energy, E_0, of a body at rest is equal to its mass." It should be noted that he had earlier adopted c as the unit of velocity. Further, he wrote: "Had we chosen the second as our unit of time, we would have obtained

$$E_0 = mc^2. \qquad (44)$$

Mass and energy are therefore essentially alike; they are only different expressions for the same thing. The mass of a body is not a constant; it varies with changes in its energy." The two last phrases acquire an unambiguous meaning through the introductory word "therefore" and the circumstance that they follow directly after the equation $E_0 = mc^2$. Thus, a mass that depends on the velocity is not found in the book *The Meaning of Relativity*.

It is possible that if Einstein had commented in more detail and systematically on his equation $E_0 = mc^2$, the equation $E = mc^2$ would have disappeared from the literature already in the twenties. But that he did not do, and the majority of subsequent authors followed Pauli, and a velocity-dependent mass captured the majority of popular scientific books and brochures, encyclopaedias, school and university textbooks on general physics, and also monographs, including books of eminent physicists specially devoted to the theory of relativity.

One of the first monograph textbooks in which the theory of relativity was given a systematically relativistic exposition was the *The Classical Theory of Fields* of Landau and Lifshitz.[21] It was followed by a number of other books.

The diagram method of Feynman, which he created in the middle of this century,[22] occupied a central position in the systematically relativistic four-dimensional formalism of quantum field theory. But the tradition of using a velocity-dependent mass was so ingrained that in his famous lectures published at the beginning of the sixties[23] Feynman made it the basis of the chapters devoted to the theory of relativity. It is true that the discussion of the velocity-dependent mass ends in Chap. 16 with the two following sentences:

"That the mass in motion at speed v is the mass m_0 at rest divided by $\sqrt{1 - v^2/c^2}$, surprisingly enough, is rarely used. Instead, the following relations are easily proved, and turn out to be very useful:

$$E^2 - \mathbf{p}^2c^2 = M_0^2c^4 \qquad (16.13)$$

and

$$\mathbf{p}c = \frac{E\mathbf{v}}{c}. \qquad (16.14'')$$

In the last lecture published in his life (it was read in 1986, is dedicated to Dirac, and is called "The reason for antiparticles,"[24]) Feynman mentions neither a velocity-dependent mass nor a rest mass and speaks merely of mass and denotes it by m.

17. Imprinting and mass culture

Why is the formula $m = E/c^2$ so tenacious? I cannot give a complete explanation. But it seems to me that here the popular scientific literature has played a fatal role. For from it we draw our first impressions about the theory of relativity.

In ethology there is the concept of imprinting. An example of imprinting is the way chicks learn to follow a hen in a short period after their hatching. If at this period the chick is palmed off with a moving child's toy, it will subsequently follow the toy and not the hen. It is known from numerous observations that the result of imprinting cannot be subsequently changed.

Of course, children and, still less, young people, are not chicks. And, having become students, they can learn the theory of relativity in covariant form, "according to Landau and Lifshitz," so to speak, and without a mass that depends on the velocity and all the nonsense that goes with it. But when, having grown up, they start to write booklets and textbooks for young people, the imprinting takes over again.

The formula $E = mc$ long ago became an element of mass culture. This gives it a particular tenacity. Sitting down to write about the theory of relativity, many authors start with the assumption that the reader is already familiar with this formula, and they attempt to exploit this knowledge. Thus a self-sustaining process arises.

18. Why it is bad to call E/c^2 the mass

Sometimes one of my physicist friends says to me: "Now why do you bother about this relativistic mass and rest mass? At the end of the day nothing terrible can happen if a certain combination of letters is denoted by some one letter and described by one or two words. After all, even although they use these concepts, which are indeed archaic, engineers correctly design relativistic accelerators. The important thing is that the formulas should not contain mathematical errors."

Of course, one can use formulas without fully understanding their physical meaning, and one can make correct calculations despite having a distorted idea of the essence of the science that these formulas represent. But, first, distorted concepts can sooner or later lead to an incorrect result in some unfamiliar situation. Second, a clear understanding of the simple and beautiful foundations of science is more important than the unthinking substitution of numbers in equations.

The theory of relativity is simple and beautiful, but its exposition in the language of two masses is confused and ugly. The formulas $E^2 - \mathbf{p}^2 = m^2$ and $\mathbf{p} = E\mathbf{v}$ (I now use units in which $c = 1$) are among the most transparent, elegant, and powerful formulas of physics. Quite generally, the concepts of a Lorentz vector and a Lorentz scalar are very important, since they reflect a remarkable symmetry of nature.

On the other hand, the formula $E = m$ (I again set $c = 1$) is ugly, since it represents an extremely unfortunate designation of the energy E by a further letter and term, these being, moreover, the letter and term associated in physics with another important concept. The only justification of the formula is the historical justification—at the beginning of the century it helped the creators of the theory of relativity to create this theory. From the historical point of view, this formula and everything associated with it can be regarded as the remnants of the scaffolding used in the construction of the beautiful edifice of modern science. But to judge from the literature, it is today regarded as almost the principal portal of this edifice.

If the first argument against $E = mc^2$ can be called aesthetic—beautiful as against ugly—the second can be called ethical. Teaching the reader this formula usually entails deceiving him, hiding from him at least part of the truth and provoking in his mind unjustified illusions.

First, it is hidden from the inexperienced reader that this formula is based on the arbitrary assumption that the Newtonian definition $\mathbf{p} = m\mathbf{v}$ of the momentum is natural in the relativistic domain.

Second, it creates implicitly in the reader the illusion that E/c^2 is a universal measure of inertia and that, in particular, proportionality of the inertial mass to γ is sufficient to ensure that a massive body cannot be accelerated to the speed of light, even if its acceleration is defined bv the formula $\mathbf{a} = \mathbf{F}/m$. But from

$$\frac{dv}{dt} = \frac{F}{m_0}\left(1 - \frac{v^2}{c^2}\right)^{1/2} \qquad (18.1)$$

it follows that

$$\int_0^c dv \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \frac{1}{m_0}\int_0^T F\,dt. \qquad (18.2)$$

Assuming that the force F is constant, we readily find that the time T required for the body to reach the speed c is

$$T = \frac{\pi m_0}{2Fc}. \qquad (18.3)$$

This incorrect result is due to the fact that it is necessary to substitute in the formula $a = F/m$, not the "relativistic mass," but the "longitudinal mass," which is proportional to γ^3, a fact that, as a rule, modern authors do not remember.

Third, an illusion is created for the reader that E/c^2 is the universal gravitational mass. In reality, as we have seen, in the relativistic case, in contrast to the nonrelativistic case, there is no universal gravitational mass—for the force that acts on a photon traveling horizontally is twice the force that acts on a photon falling vertically.

Fourth, in calling this Einstein's formula, one hides Einstein's true formula, $E_0 = mc^2$, from the reader.

A third argument can be called philosophical. After all, the definition $E = mc^2$ provides the basis for tens of pages of profound philosophical discussions about the complete equivalence of mass and energy, the existence of a single essence "mass-energy," etc., whereas, according to the theory of relativity, an energy does indeed correspond to any mass but the opposite is by no means true—a mass does not correspond to every energy. Thus, there is not a complete equivalence between mass and energy.

A fourth argument is terminological. The literature on the theory of relativity contains such a confusion in the notation and terminology that is resembles a city in which the transport must simultaneously observe the rule of driving on the right and driving on the left. For example, in the Great Soviet Encyclopaedia, in various physics encyclopedias, and in handbooks the letter m is used to denote the mass and the relativistic mass; the ordinary mass is sometimes called the mass, but more often the rest mass, while the relativistic mass is also called the kinetic mass, but frequently simply the mass. In some papers the authors adhere to a mainly consistent relativistic terminology, in others to a consistently archaic one. It is difficult for an inexperienced reader who wishes to compare, say, a "mass" paper with a "relativity" paper.

The same mixing of notation and terms can also be found in many textbooks and monographs. And all this confusion flourishes at a time in which there is in the theory of relativity essentially just one term, mass, and all the others come "from the devil."

A fifth argument is pedagogical. Neither a school pupil, nor a school teacher, nor a student of first courses who has learnt dogmatically that the mass of a body increases with its velocity can truly understand the essence of the theory of relativity without then spending considerable efforts on reeducation.

As a rule, someone who has not subsequently become a professional relativist has the most false ideas about mass and energy. At times the formula $m = m_0[1 - (v^2/c^2)]^{-1/2}$ is all that remains in his memory, together, of course, with the formula $E = mc^2$.

It is clear that any independently thinking student must feel an intellectual discomfort when studying the theory of relativity using a standard school textbook.

19. "Does mass really depend on velocity, dad?"

Such is the title of a paper published by C. Adler[24] published in the American Journal of Physics in 1987. The question posed in the title was put to the author by his son. The answer was: "No!" "Well, yes...", "Actually, no, but don't tell your teacher." The next day his son dropped physics.

Adler writes that with every year the concept of a relativistic mass plays an ever decreasing role in the teaching of the special theory of relativity. He illustrates this assertion with quotations from four successive editions of a textbook widely used in the United States, *University Physics*, from 1963 through 1982.

Speaking of the opinions of Einstein, Adler gives extracts from an unpublished letter of Einstein to Lincoln Barnett, written in 1948:

"It is not good to introduce the concept of the mass $M = m[1-(v^2/c^2)]^{-1/2}$ of a body, for which no clear definition can be given. It is better to introduce no other mass than the rest mass' m. Instead of introducing M, it is better to mention the expression for the momentum and energy of a body in motion."[2)]

Viewed historically, Adler regards the relativistic mass as an inheritance from the prerelativistic theories of Lorentz and Poincaré. He criticizes this concept and expresses optimism with regard to the decrease of its use.

20. Fizika v shkole (Physics at School)

It so happened that in the same year 1987 in which Adler's paper appeared I had to work in a commission created by the former Ministry of Education of the USSR to determine the winners of the All-Union competition for the best textbooks on physics. Having become acquainted with about 20 submitted textbooks, I was struck by the fact that they all treated a velocity-dependent mass as one of the central points of the theory of relativity.

My surprise increased still more when I found that the majority of the members of the commission—pedagogues and specialists on methods of teaching—had not even heard that a different point of view existed. In a brief improvised talk I told them about the two basic formulas $E^2 - \mathbf{p}^2c^2 = m^2c^4$ and $\mathbf{p} = (E/c^2)\mathbf{v}$. One of them then said to me: "Now you know about this and we know, but nobody else knows. You must write an article about mass for the journal *Fizika v shkole*. Then 100,000 physics school teachers will know about it."

Somewhat flippantly, as it subsequently turned out, I assured them that everything I had said was known not only to all professional physicists but also to students of nonpedagogical universities. But I promised to write the paper.

A few days later, encountering at the next meeting of the commission the deputy of the editor-in-chief of the journal *Fizika v Shkole*, I told her about the proposal that had been made and asked if the journal would commission from me a paper on the concept of mass in the theory of relativity. For about two months there was no answer, and then the person to whom I had spoken rang me up and said that the editorial board had decided not to commission such a paper. It appears that the imprinting that I wrote about earlier had been at work.

This refusal only strengthened my conviction of the need for such a paper. Working on it, I studied more than 100 books and about 50 papers. I saw that the school textbooks were not much worse than the university textbooks, and I became interested in the history of the question. The material expanded, and the work began to absorb me. And there appeared no end to it.

I then decided to sit down and write this short text, putting away in a separate file the detailed bibliography and pages with an analysis of different papers and books.

Time does not wait. Every year books are published in millions of copies that hammer into the heads of the young generations false ideas about the theory of relativity. This process must be stopped.

I am grateful to the members of that competition commission for initiating the writing of this paper. I am also grateful for helpful discussions and comments to B. M. Bolotovskiĭ, M. B. Voloshin, P. A. Krupchitskiĭ, and I. S. Tsukerman.

1) (Editor's Note. Essentially the same material by the same author appears in Phys. Today **40**(6), 31 (1989).

2) In connection with the extract from the letter to Barnett, it is appropriate to give extracts from the "Autobiographical notes" by Einstein[26] published in 1949 (in Vol. 4 of the Soviet edition of Einstein's scientific works these come directly after the foreword to the book of L. Barnett: *The Universe and Dr Einstein* (New York, 1949)). In these notes, recalling the initial stage of work on the creation of the relativistic theory of gravitation, Einstein writes: "... the theory had to combine the following things:

1) From general considerations of special relativity theory it was clear that the *inert* mass of a physical system increases with the total energy (therefore, e.g., with the kinetic energy).

2) From very accurate experiments ... it was empirically known with very high accuracy that the gravitational mass of a body is exactly equal to its *inert* mass."

This extract confirms that in the work on the creation of the general theory of relativity the concept of a mass that increases with increasing kinetic energy was the point of departure for Einstein. The extract may also indicate that, recalling in 1949 this concept without any reservations, Einstein was not completely consistent. It is possible that he assumed that in this way he would be understood by a larger number of readers.

1. J. J. Thomson, Philos. Mag. **11**, 229 (1881).
2. A. Einstein, Ann. Phys. (Leipzig) **17**, 891 (1905).
3. A. Pais, *Subtle is the Lord. The Science and the Life of Albert Einstein* (Clarendon Press, Oxford, 1982).
4. H. Poincaré, *Lorentz Festschrift*, Arch. Neerland **5**, 252 (1900).
5. H. Lorentz, Proc. R. Acad. Sci. Amsterdam **1**, 427 (1899).
6. H. A. Lorentz, Proc. R. Acad. Sci. Amsterdam **6**, 809 (1904).
7. A. I. Miller, *Albert Einstein's Special Theory of Relativity: Emergence (1905) and Early Interpretation (1905–1911)*, Addison-Wesley, Reading, 1981.
8. A. Einstein, Ann. Phys. (Leipzig) **18**, 639 (1905).
9. A. Einstein, Ann. Phys. (Leipzig) **20**, 627 (1906).
10. A. Einstein, Ann. Phys. (Leipzig) **23**, 371 (1907).
11. A. Einstein, Ann. Phys. (Leipzig) **35**, 898 (1911).
12. M. Planck, Verh. Dtsch. Phys. Ges. **4**, 136 (1906).
13. M. Planck, Sitzungsber. Akad. Wiss. Berlin **13**, 542 (1907).
14. H. Minkowski, Phys. Z. **10**, 104 (1909).
15. G. Lewis and R. Tolman, Philos. Mag. **18**, 510 (1909).
16. R. Tolman, Philos. Mag., **23**, 375 (1912).
17. M. Jammer, *Concepts of Mass in Classical and Modern Physics*, Harvard University Press, Cambridge, Mass., 1961 [Russ. Transl. Progress, M., 1967].
18. E. A. Whittaker, *History of the Theories of Aether and Electricity*, Vol. 2, Nelson, London, 1953.
19. W. Pauli, "Relativistätstheorie," in: *Enzykl. Math. Wiss.*, Bd 19 Teubner, Leipzig, 1921. [Russ. transl., Gostekhizdat, M., 1947, Nauka, M. 1973; Engl. transl., Pergamon Press, Oxford, 1958].
20. *Einstein The Meaning of Relativity: Four Lectures Delivered at Princeton University May 1921*, Princeton Univ. Press, Princeton, 1970.
21. L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (2nd. ed. Pergamon Press, Oxford, 1962. [Russ. original, Gostekhizdat, M., 1955].
22. R. Feynman, Phys. Rev. **76**, 749, 769 (1949).
23. R. P. Feynman, R. B. Leighton, and M. Sands, *Feynman Lectures on Physics*, Vols. 1 and 2, Addison-Wesley, 1963, 1964, Chaps. 15, 16, and 28. [Russ. transl., Mir, M., 1966].
24. R. P. Feynman, "The reason for antiparticles," in: *Elementary Particles and the Laws of Physics. The 1986 Dirac Memorial Lectures*, Cambridge University Press, Cambridge, 1987. [Russ. transl., Usp. Fiz. Nauk **157**, 163 (1989)].
25. C. Adler, Am. J. Phys. **55**, 739 (1987).
26. A. Einstein, "Autobiographical notes," in: *Albert Einstein: Philosopher-Scientist*, (Ed.) P. A. Schilpp, Tudor, N.Y., 1949, reprinted, Harper, N.Y., 1959.

Translated by Julian B. Barbour