Synthetic Aperture Radar

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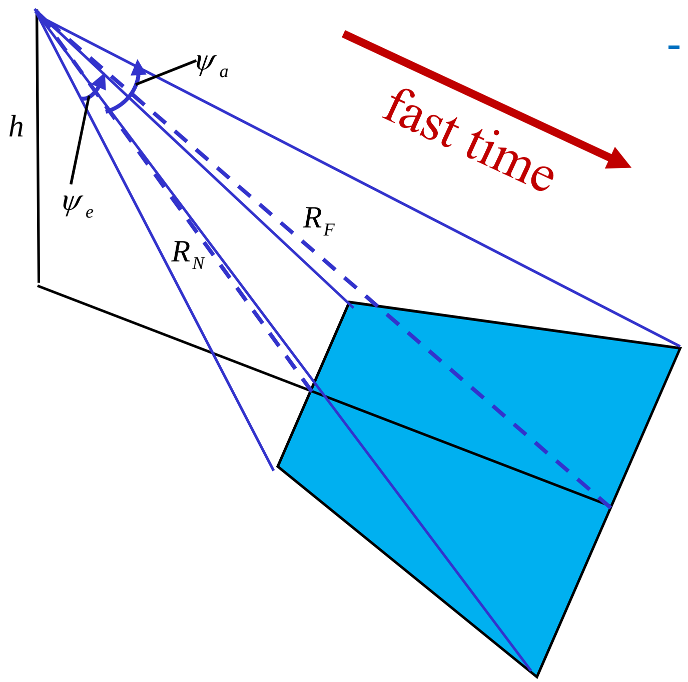
Single pulse radar echo



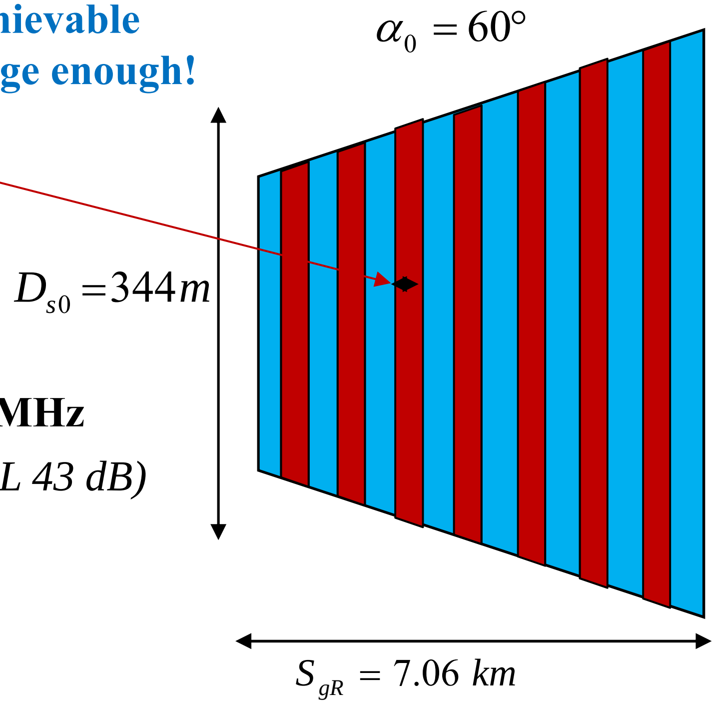
- **Any desired value is achievable using a pulse with B large enough!**

$$\delta_{gR} = \frac{k}{\sqrt{3}} \frac{c}{B}$$

Example: B = 450 MHz

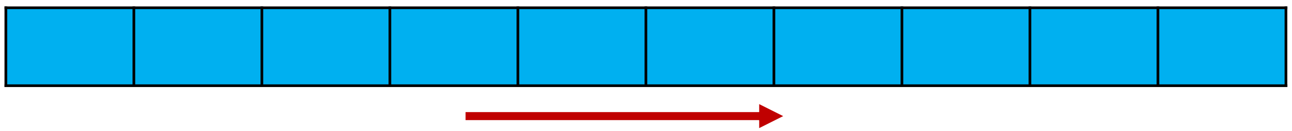
k= 1.3 Hamming (PSL 43 dB)

$\boldsymbol{\delta_{gR}}$**=0.5 m**



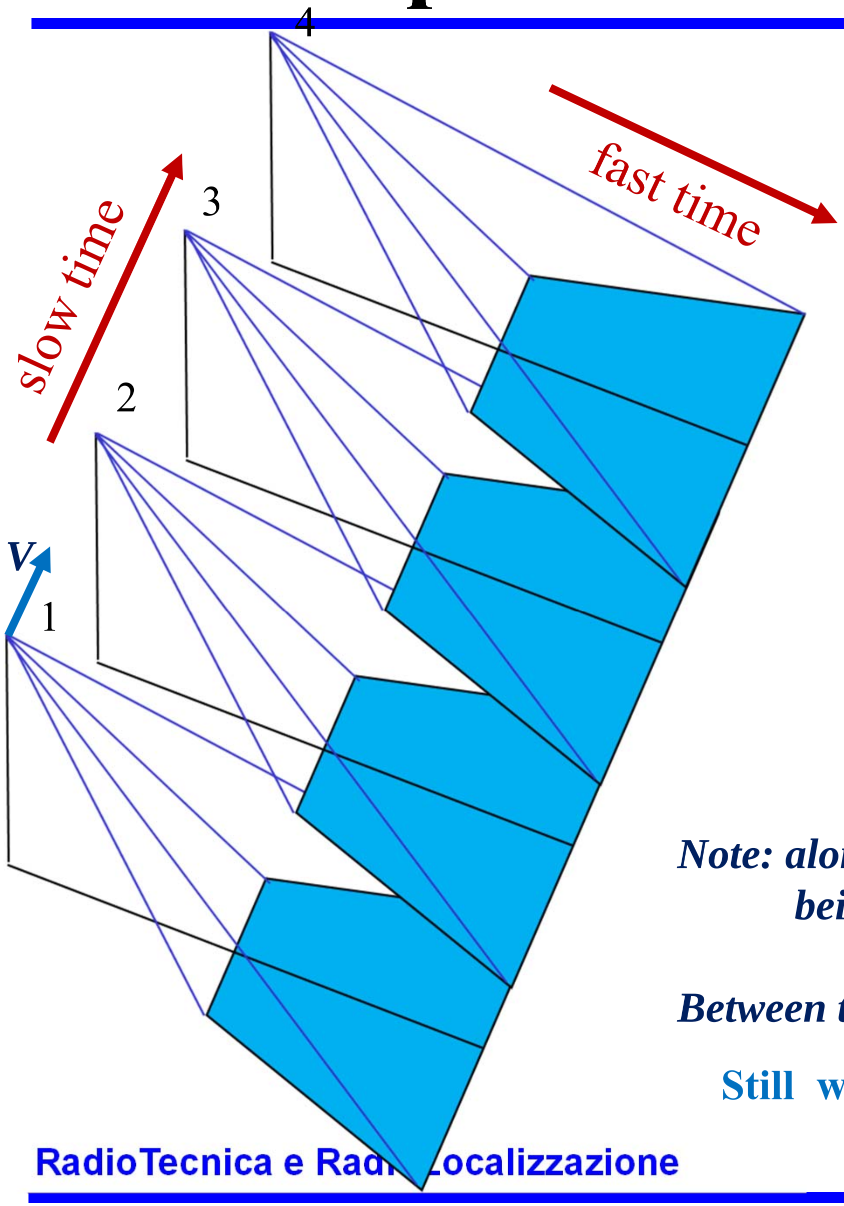
Two problems:

- **1 pixel represents a ground patch of: 0.5 m × 344 m !!!!**
- **Vector collecting "fast time" samples: not a matrix – not an image !!!!**



fast time

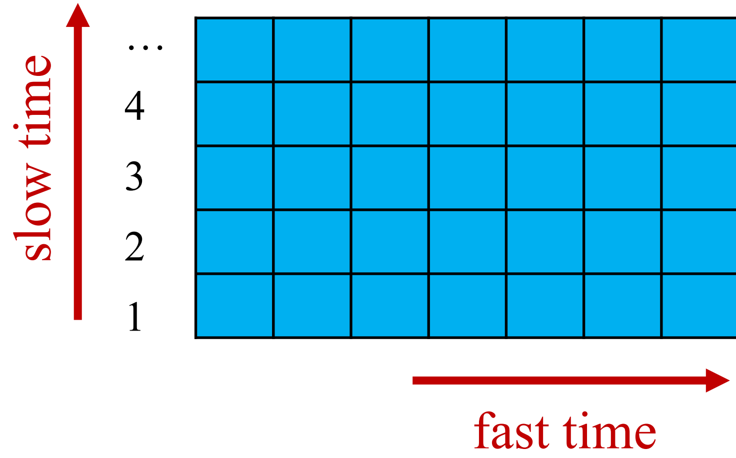
Real Aperture Radar



Exploit platform motion in "slow time"

→ **pulses collected from different positions**

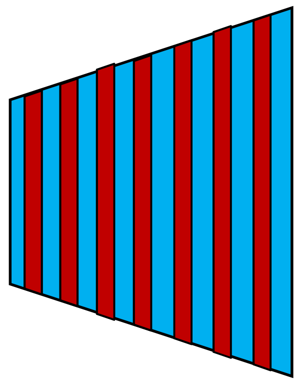
→ **a matrix → an image !!!!**



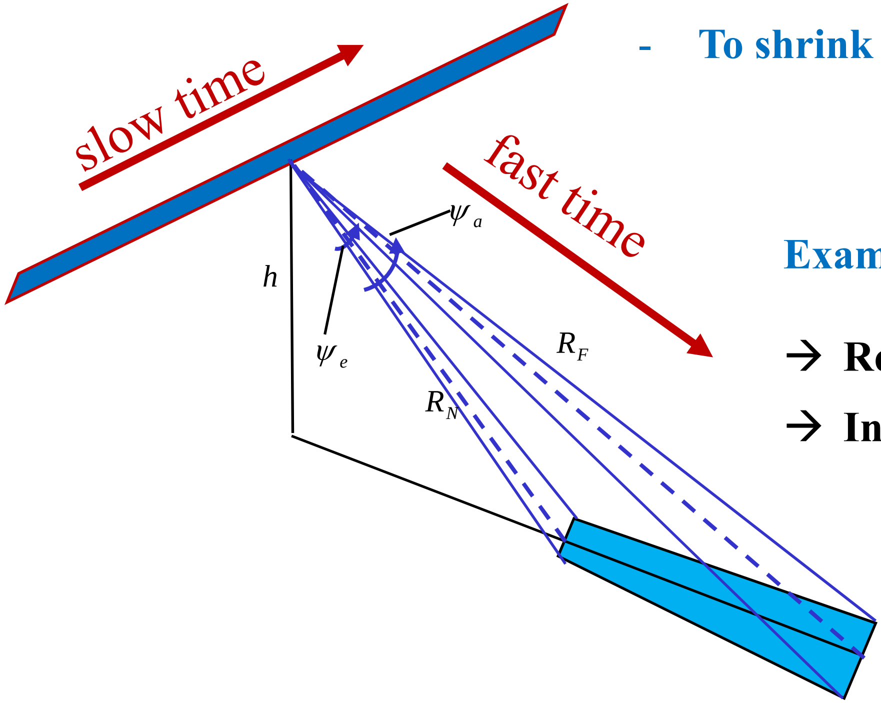
Note: along slow time …   space = V * time
being V the platform velocity

Between two pulses displacement of V PRT

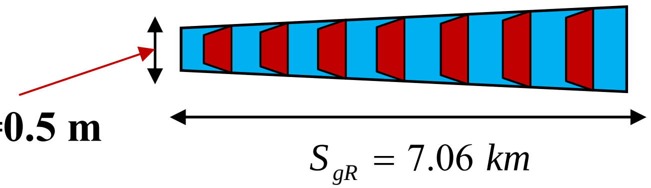
Still wide pixel along slow time !!!



Real Aperture Radar (II)



- To shrink resolution cell → increase antenna length d_a



Example: to achieve D_{s0}=0.5 m

→ Reduce beamwidth 344/0.5=688 times!

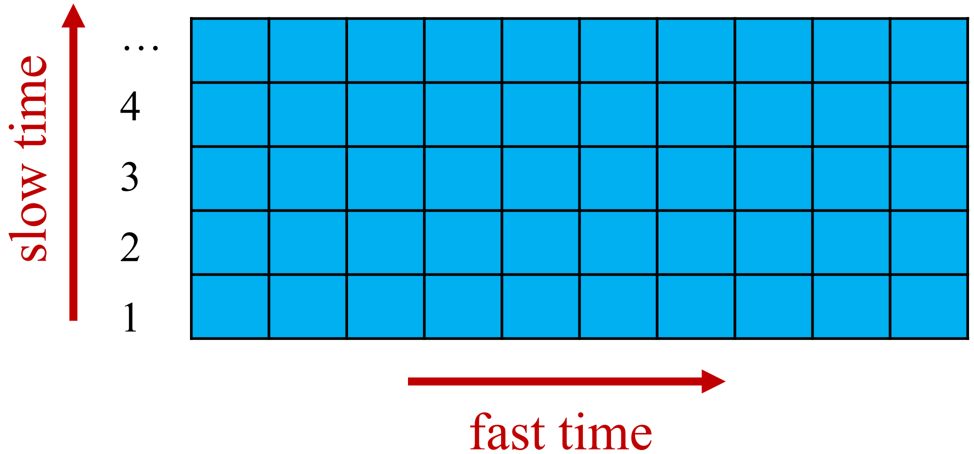
→ Increase antenna length da 688 times:

d_a=1.8*688= 1238.4 dm !!!!!!!! IMPOSSIBLE!!!

Using platform motion:

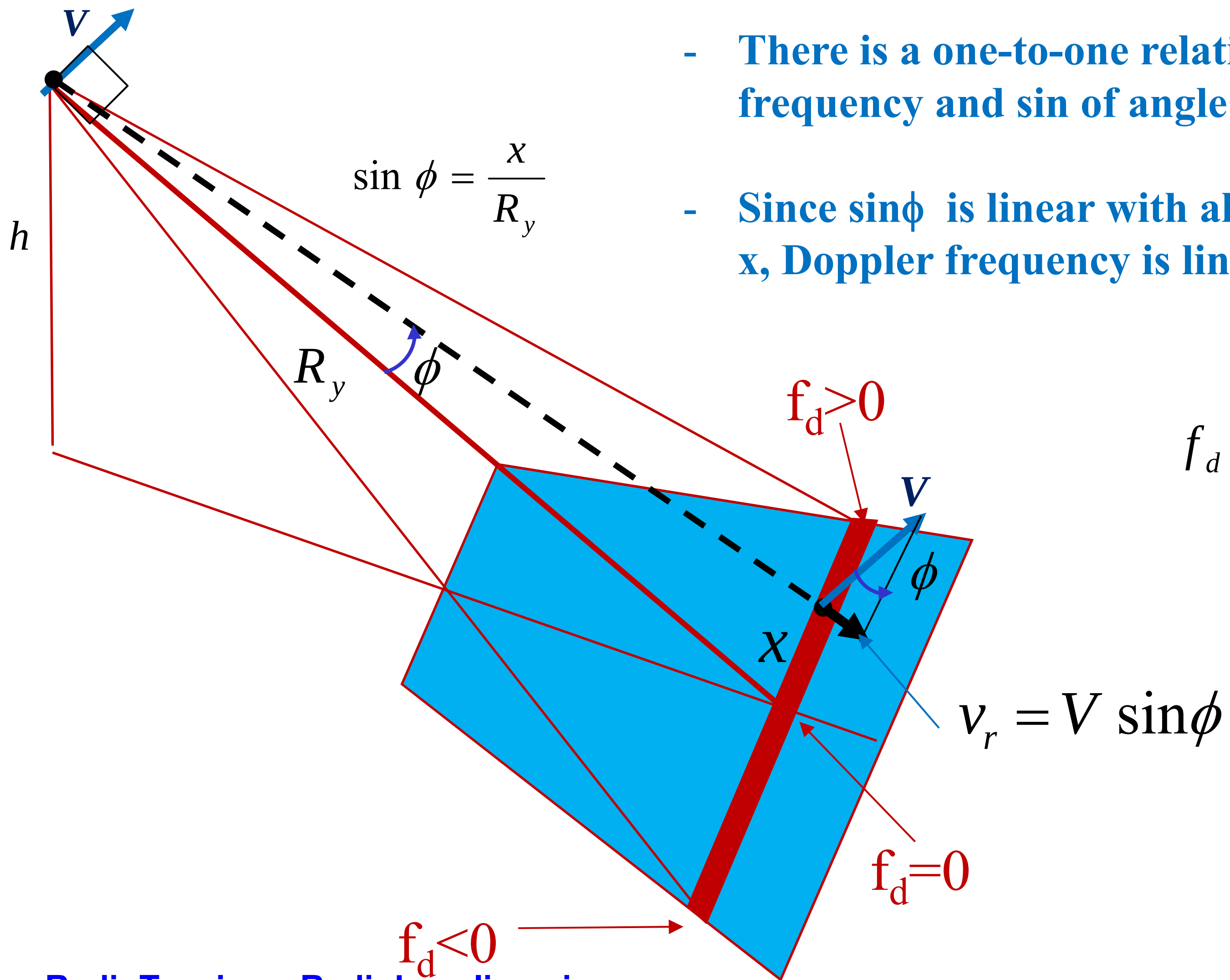
→ a matrix → an image !!!!

with 0.5 m × 0.5 m ground resolution



Angle-Doppler frequency relationship

- There is a one-to-one relationship between Doppler frequency and sin of angle ϕ
- Since $\sin\phi$ is linear with along-track displacement x, Doppler frequency is linear with x

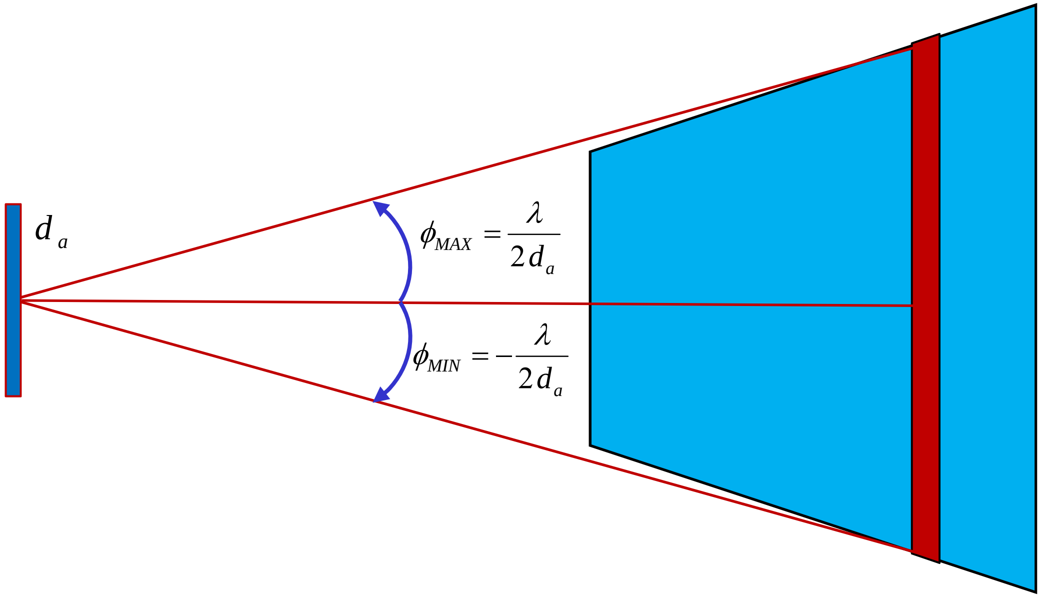


$$f_d = \frac{2}{\lambda} v_r = \frac{2}{\lambda} V \sin\phi$$

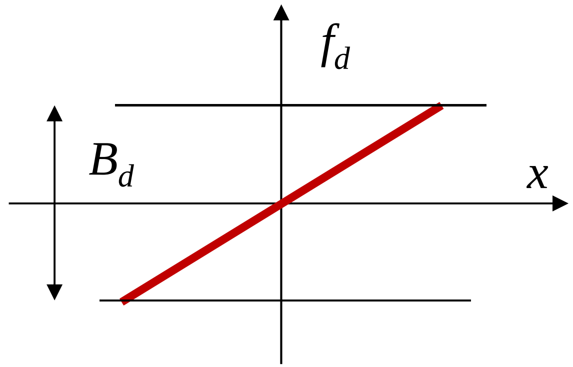
$$f_d \cong \frac{2V}{\lambda R_y} x$$

Doppler frequency bandwidth

$$f_{d\,MAX} = \frac{2}{\lambda} V \sin\phi_{MAX} = \frac{2}{\lambda} V \sin\left(\frac{\lambda}{2d_a}\right) \cong \frac{2}{\lambda} V \frac{\lambda}{2d_a} = \frac{V}{d_a}$$



$$f_{d\,MIN} = \frac{2}{\lambda} V \sin\phi_{MIN} \cong -\frac{V}{d_a}$$



Doppler frequency bandwidth:

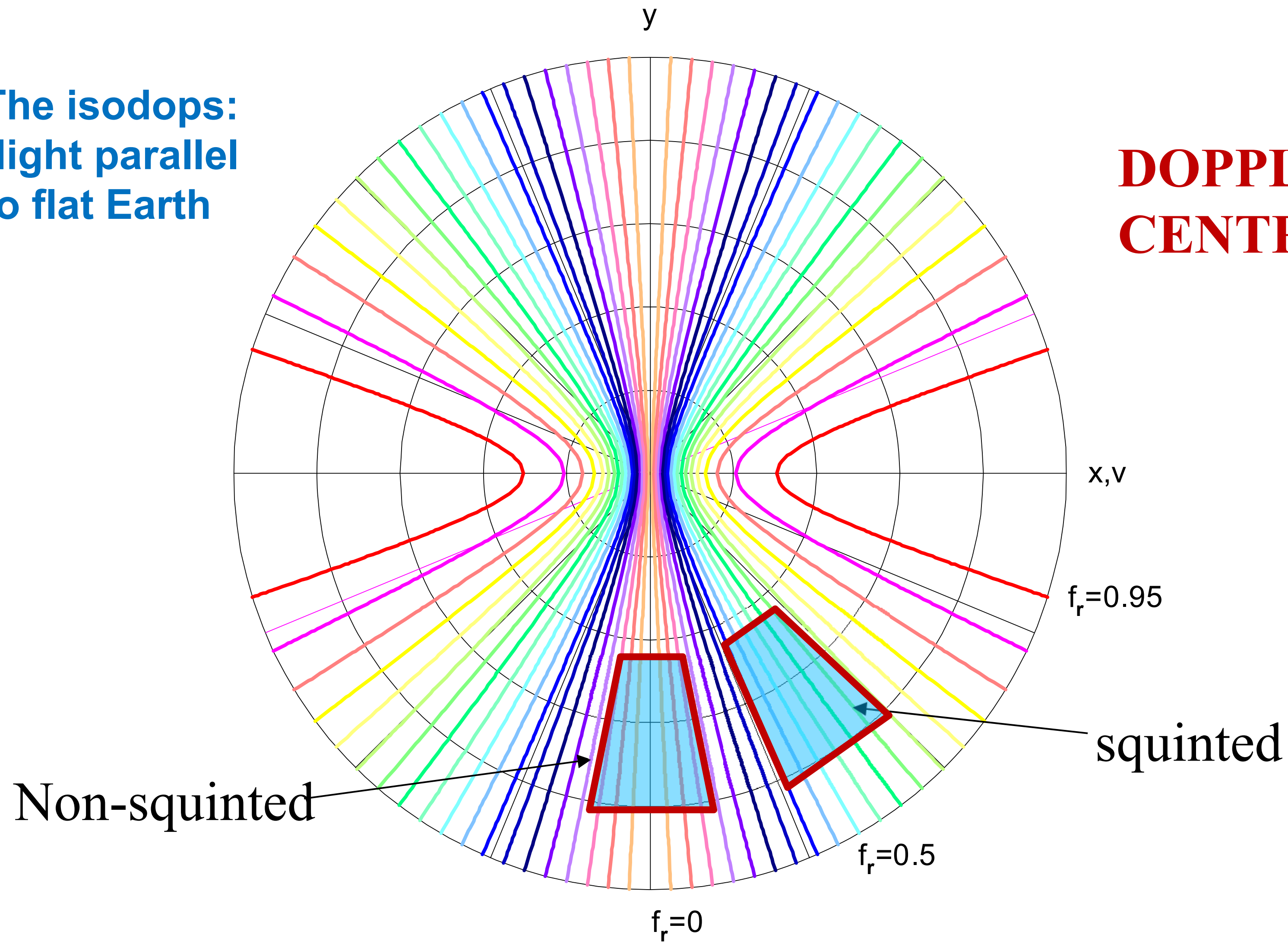
$$B_d = f_{d\,MAX} - f_{d\,MIN} = \frac{2V}{d_a}$$

- **Minimum PRF**

$$PRF \geq B_d = \frac{2V}{d_a}$$

Frequency approach to SAR

The isodops: flight parallel to flat Earth



DOPPLER CENTROID

Along-track resolution by Doppler

- **Doppler frequency resolution *(Fourier Transform)***

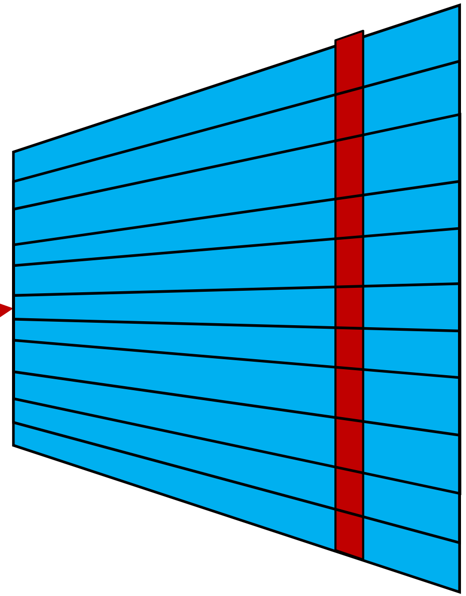
$$\Delta f_d = \frac{1}{T_{oss}} = \frac{1}{N \cdot PRT} = \frac{PRF}{N}$$

$$\begin{cases} \Delta f_d = \dfrac{1}{T_{oss}} = \dfrac{2}{\lambda} V \;\delta \sin\phi \\ \Delta f_d = \dfrac{1}{T_{oss}} = \dfrac{2V}{\lambda R_y} \;\delta x \end{cases}$$

$$\begin{cases} \delta \sin\phi = \dfrac{\lambda}{2V}\Delta f_d = \dfrac{\lambda}{2V}\dfrac{1}{T_{oss}} = \dfrac{\lambda}{2V}\dfrac{PRF}{N} \\ \delta x = \dfrac{\lambda R_y}{2V}\Delta f_d = \dfrac{\lambda R_y}{2V}\dfrac{1}{T_{oss}} = \dfrac{\lambda R_y}{2V}\dfrac{PRF}{N} \end{cases}$$

- **N pulses at min PRF: FFT provides N Doppler filters**

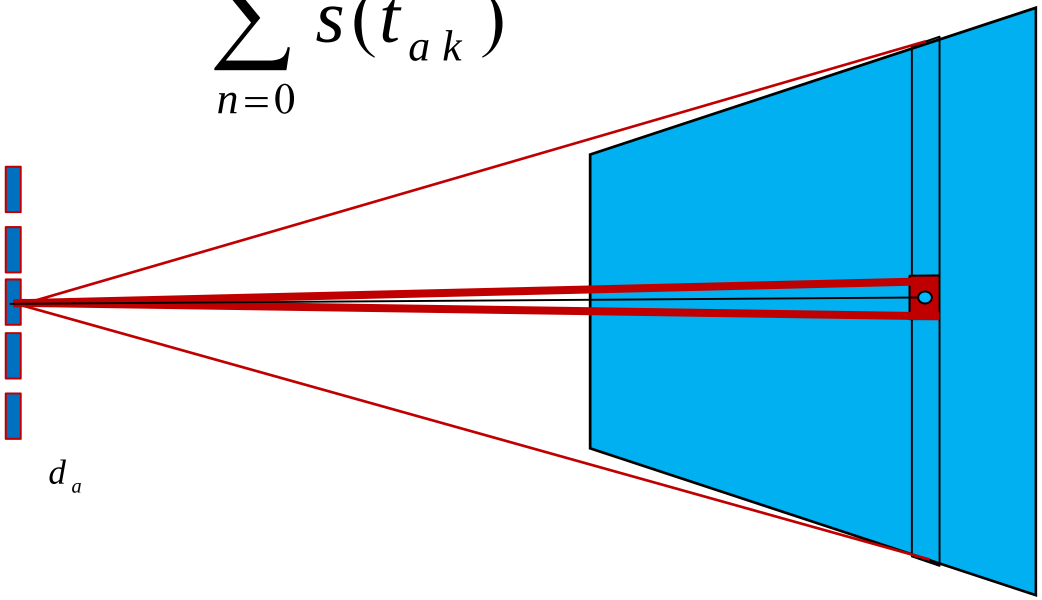
$$\begin{cases} \delta \sin\phi \geq \dfrac{\lambda}{2V}\dfrac{2V}{N d_a} = \dfrac{1}{N}\dfrac{\lambda}{d_a} = \dfrac{\psi_a}{N} \\ \delta x \geq \dfrac{\lambda R_y}{2V}\dfrac{2V}{N d_a} = \dfrac{1}{N}\dfrac{\lambda}{d_a} R_y = \dfrac{D_{sy}}{N} \end{cases}$$

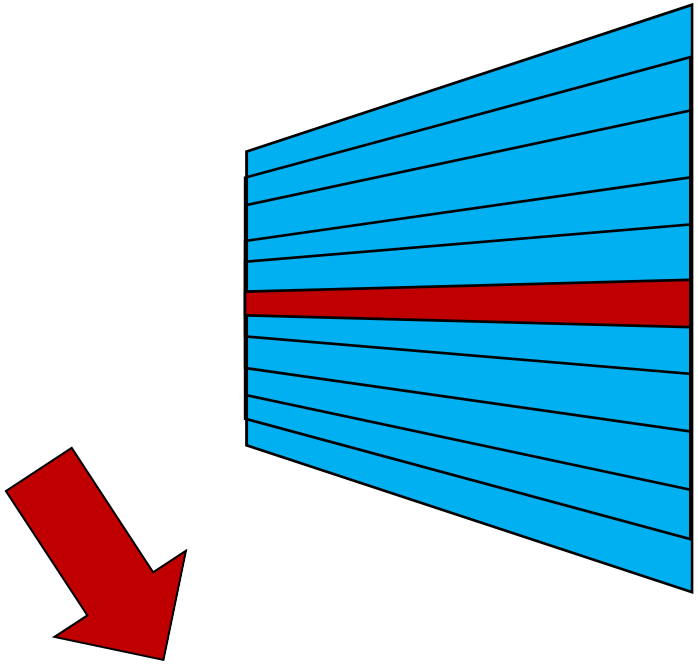


synthetic antenna principle

- By exploiting platform motion emulate "synthetic antenna array"

$$\sum_{n=0}^{N-1} s(t_{ak})$$





-Using $V\,PRT = d_a / 2$:

$$\frac{2V\;PRT}{\lambda}\,\delta\sin\phi = \frac{1}{N} \quad \rightarrow \delta\sin\phi = \frac{\lambda}{2\,V\cdot PRT}\,\frac{1}{N} = \frac{\lambda}{d_a}\,\frac{1}{N} = 2\,\frac{\psi_a}{N}$$

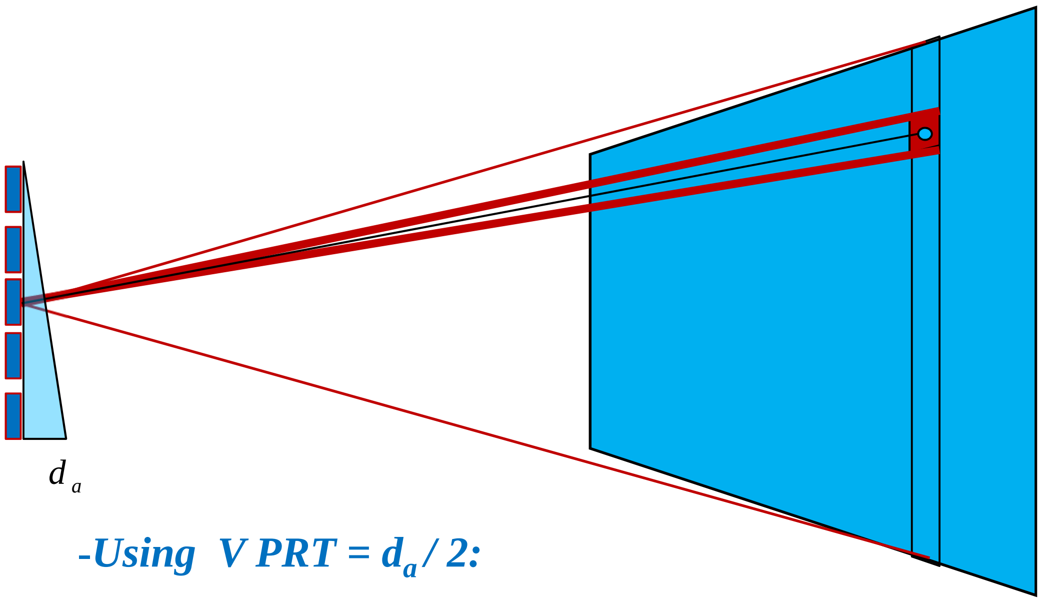
Synthetic antenna beam N times narrower than real antenna beam

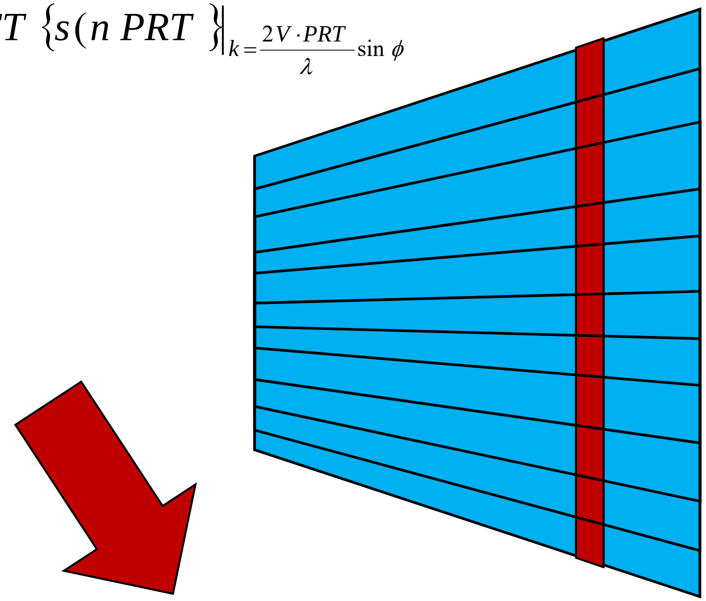
synthetic antenna principle (II)

- **By exploiting platform motion emulate "synthetic antenna array"**

-To steer in direction ϕ, add all returns of the N pulses after compensating a linear phase term $\Delta\phi = 2\pi k \frac{d}{\lambda}\sin\phi$ ***both*** *in TX and in RX (twice as in standard array):*

$$\sum_{n=0}^{N-1} s(t_{ak})\, e^{-j2\left[2\pi \frac{nd}{\lambda}\sin\phi\right]} = \sum_{n=0}^{N-1} s(n\,PRT)\, e^{-j4\pi \frac{nV\cdot PRT}{\lambda}\sin\phi} = FFT\left\{s(n\,PRT)\right\}\Big|_{k=\frac{2V\cdot PRT}{\lambda}\sin\phi}$$



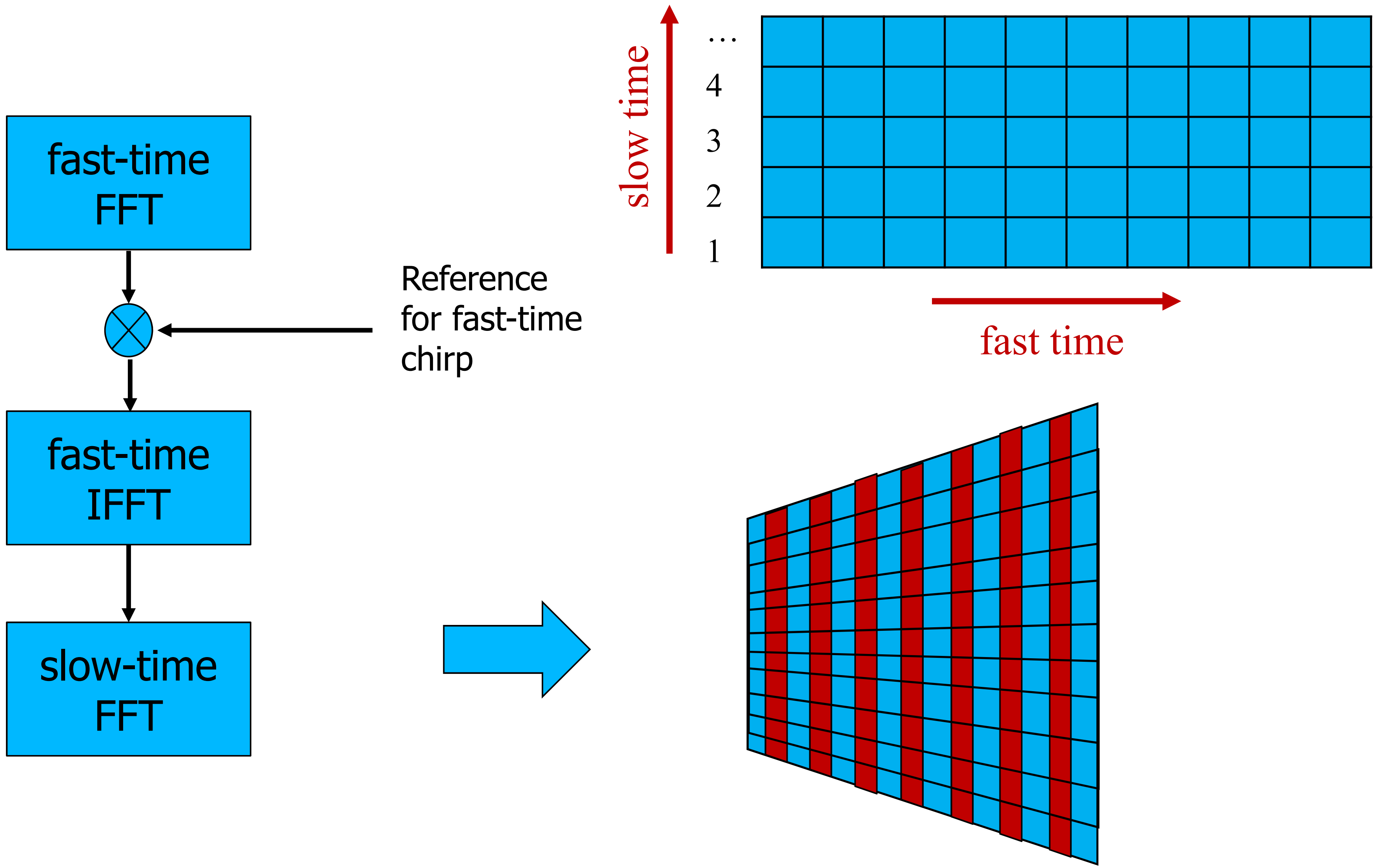


-Using $V\,PRT = d_a/2$:

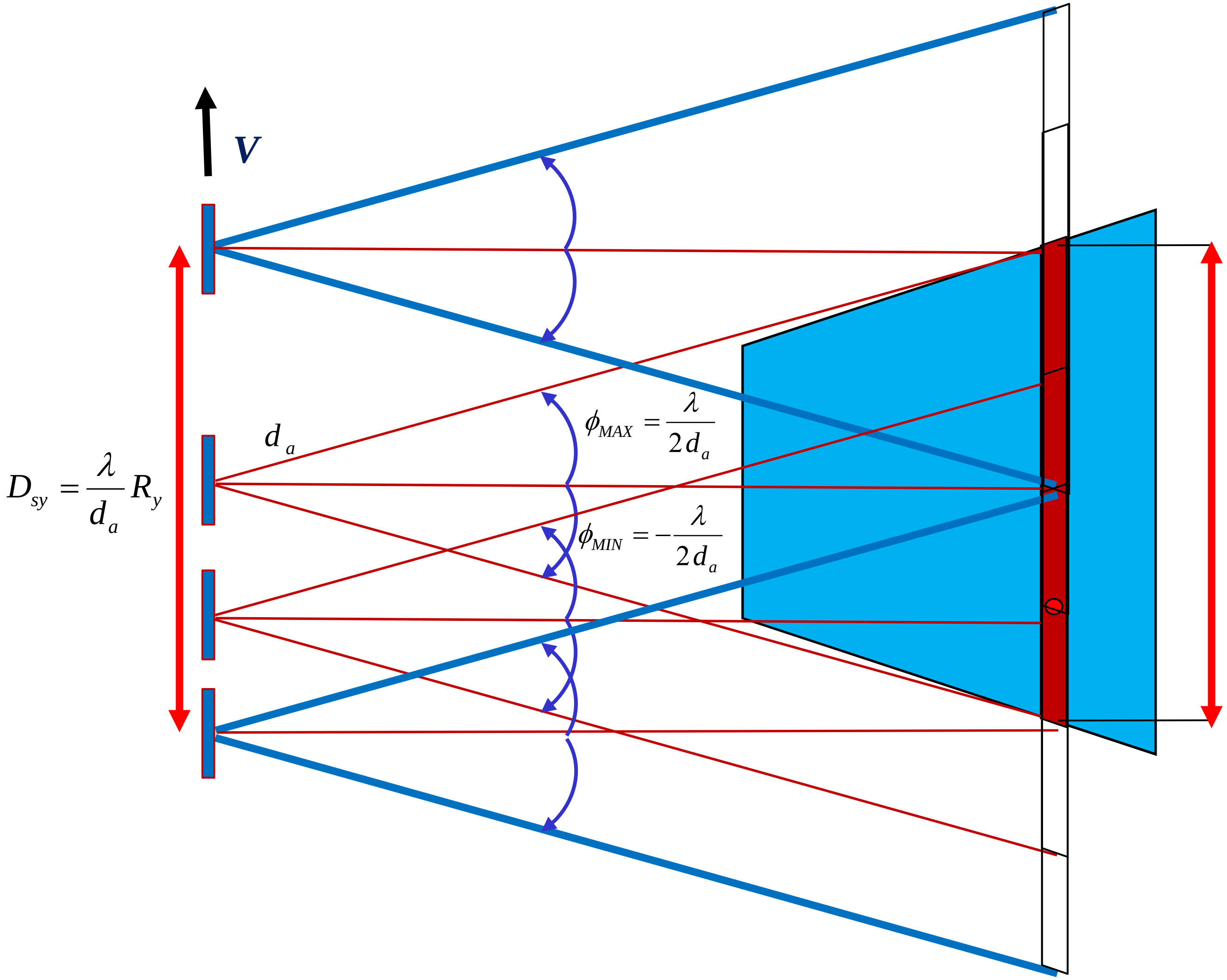
$$\frac{2V\,PRT}{\lambda}\delta\sin\phi = \frac{1}{N} \quad \rightarrow \delta\sin\phi = \frac{\lambda}{2V\cdot PRT}\frac{1}{N} = \frac{\lambda}{d_a}\frac{1}{N} = 2\frac{\psi_a}{N}$$

Synthetic antenna beam N times narrower than real antenna beam

Unfocused SAR Processing scheme



Maximum observation time for point target



$$D_{sy} = \frac{\lambda}{d_a} R_y$$

- **Maximum T_{obs} for the echo to be received inside the antenna beam**

$$T_{obs} = \frac{D_{sy}}{V} = \frac{\lambda}{d_a}\frac{R_y}{V}$$

SAR azimuth resolution

To exploit long T_{oss} we can think in terms of:

- **Narrow Doppler filter at zero Doppler using the whole T_{oss}**

$$T_{oss} = \frac{D_{sy}}{V} = \frac{\lambda}{d_a}\frac{R_y}{V}$$

$$\delta x = \frac{\lambda R_y}{2V}\Delta f_d = \frac{\lambda R_y}{2V}\frac{1}{T_{oss}} = \frac{\lambda R_y}{2V}\frac{1}{\frac{D_{sy}}{V}} = \frac{\lambda R_y}{2V}\frac{1}{\frac{\lambda}{d_a}\frac{R_y}{V}} = \frac{d_a}{2}$$

- **To achieve high resolution -→ Small-sized ANTENNA appears better !**

Limit to Doppler frequency resolution

Longer T_{oss} = longer pulse sequence → Higher Doppler frequency resolution

$$\Delta f_d = \frac{1}{T_{oss}} = \frac{PRF}{N}$$

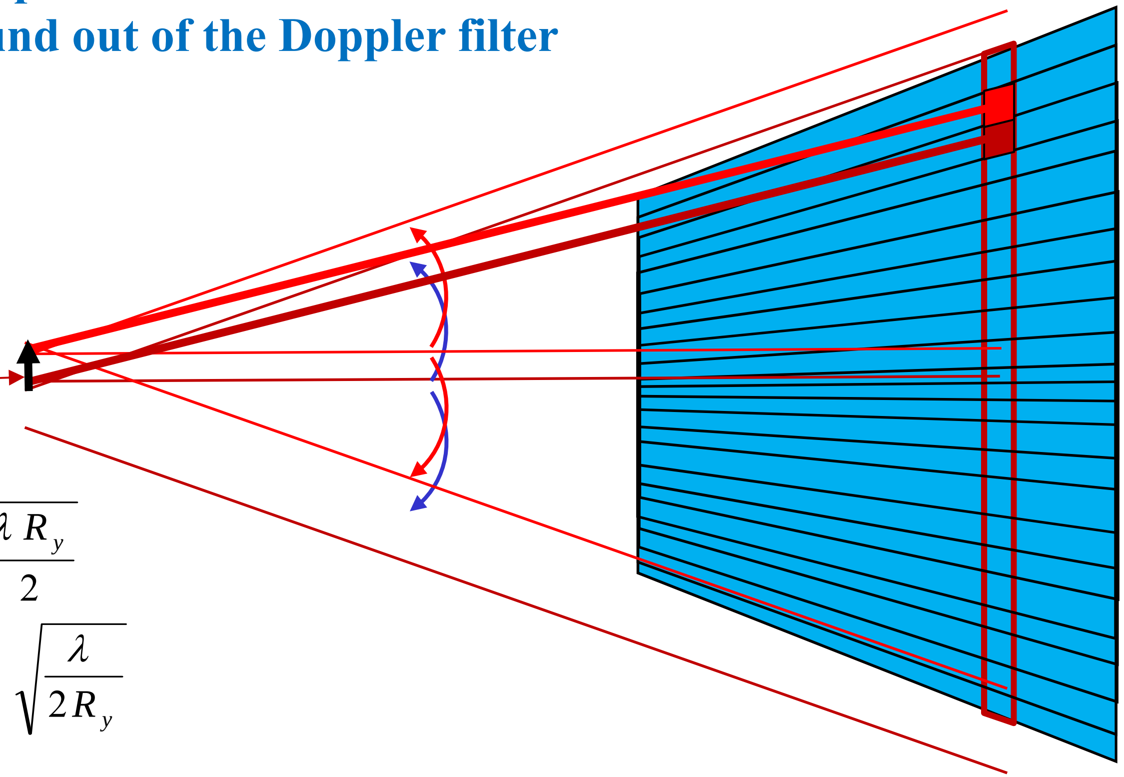
$$\begin{cases} \delta \sin\phi = \dfrac{\lambda}{2V}\dfrac{1}{T_{oss}} \\ \delta x = \dfrac{\lambda R_y}{2V}\dfrac{1}{T_{oss}} \end{cases}$$

- **This all applies as long as the platform motion does not force motion of point on ground out of the Doppler filter**

$$V\,T_{oss} \le \delta x = \frac{\lambda R_y}{2V}\frac{1}{T_{oss}}$$

$$(V\,T_{oss})^2 \le \frac{\lambda R_y}{2}$$

$$V\,T_{oss} \le \sqrt{\frac{\lambda R_y}{2}}$$



Maximum resolution:

$$\delta x \ge \sqrt{\frac{\lambda R_y}{2}}$$

$$\delta \sin\phi \ge \sqrt{\frac{\lambda}{2R_y}}$$

Max unfocused SAR resolution

Longer T_{oss} = longer pulse sequence → Higher Doppler frequency resolution

$$\Delta f_d = \frac{1}{T_{obs}} = \frac{PRF}{N}$$

$$N = T_{obs}\, PRF = T_{obs}\, B_d = T_{obs} \frac{2V}{d_a} \le \frac{2}{d_a}\sqrt{\frac{\lambda R_y}{2}}$$

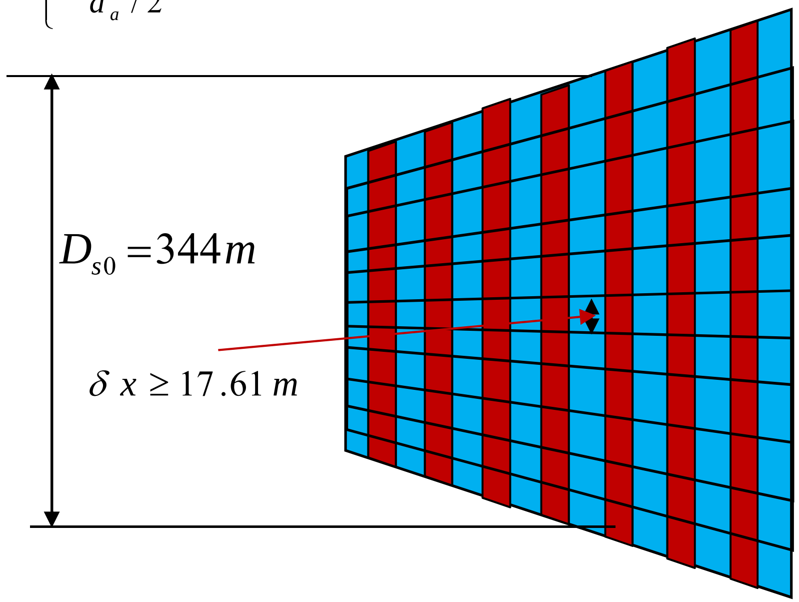
$$V\, T_{oss} \le \sqrt{\frac{\lambda R_y}{2}} = \begin{cases} \sqrt{\lambda R_N / 2} = 16.45\ m \\ \sqrt{\lambda R_0 / 2} = 17.61\ m \\ \sqrt{\lambda R_F / 2} = 19.13\ m \end{cases}$$

$$N \le \begin{cases} \dfrac{\sqrt{\lambda R_N / 2}}{d_a / 2} = 18.28 \\ \dfrac{\sqrt{\lambda R_0 / 2}}{d_a / 2} = 19.56 \\ \dfrac{\sqrt{\lambda R_F / 2}}{d_a / 2} = 21.25 \end{cases}$$

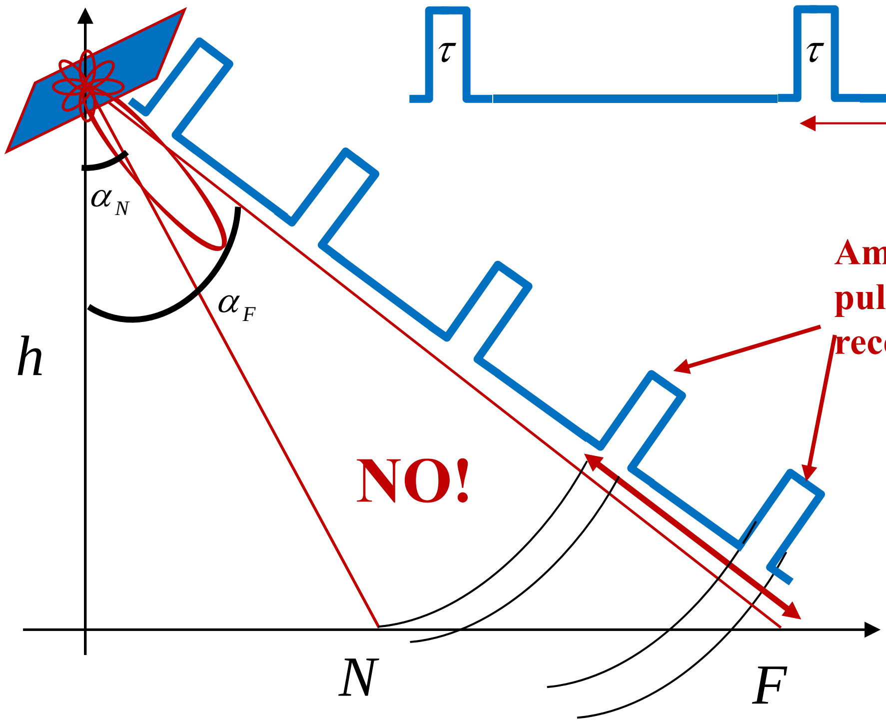
Maximum resolution:

$$\delta x \ge \sqrt{\frac{\lambda R_y}{2}} = \begin{cases} \sqrt{\lambda R_N / 2} = 16.45\ m \\ \sqrt{\lambda R_0 / 2} = 17.61\ m \\ \sqrt{\lambda R_F / 2} = 19.13\ m \end{cases}$$

$$\delta \sin\phi \ge \sqrt{\frac{\lambda}{2 R_y}} = \begin{cases} \sqrt{\dfrac{\lambda}{2 R_N}} \rightarrow \phi \cong 0.0540\ ° \\ \sqrt{\dfrac{\lambda}{2 R_F}} \rightarrow \phi \cong 0.0464\ ° \end{cases}$$



Range ambiguities



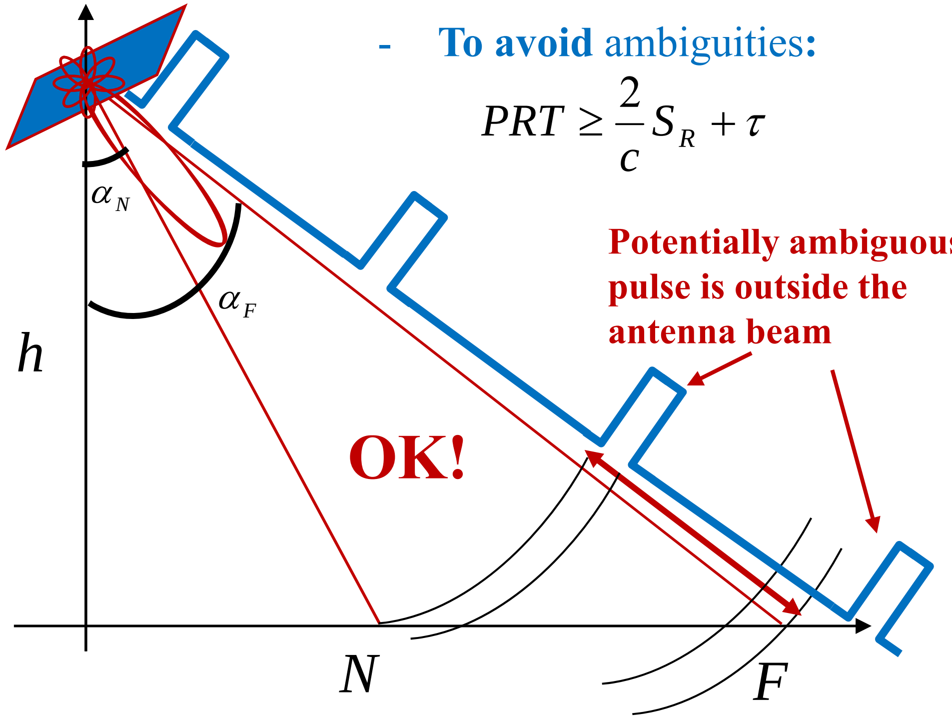
Ambiguities: echoes to subsequent pulses from these ranges reach the receiver at the same time

- **Pulse Repetition Frequency (PRF)**
  (assume $\tau \ll 2S_R/c$ negligible)

$$PRF = \frac{1}{PRT} < \frac{c}{2S_R}$$

- **To avoid** ambiguities**:**

$$PRT \geq \frac{2}{c} S_R + \tau$$



Potentially ambiguous pulse is outside the antenna beam

Fundamental limitation of SAR

Avoidance of Range Ambiguities: $1/PRF > 2\,S_R/c$

Avoidance of Azimuth Ambiguities: $PRF > 2v/\lambda$ *Antenna beamwidth AZ

Range Swath: $S_R = \psi_e\, R_o / \cos\alpha = \lambda/d_e\, R_o / \cos\alpha$

Antenna beamwidth AZ $\psi_a = \lambda/d_a$

$$\frac{2v}{\lambda}\frac{\lambda}{d_a} < PRF < \frac{c}{2}\,\frac{d_e\cos\alpha}{\lambda R_o} \quad\Rightarrow\quad \frac{2v}{\lambda}\frac{\lambda}{d_a} < \frac{c}{2}\,\frac{d_e\cos\alpha}{\lambda R_o}$$

$$\frac{S_R}{d_a/2} < \frac{c}{2v}$$

$$d_e\, d_a > \frac{4v}{c}\,\frac{\lambda R_o}{\cos\alpha}$$