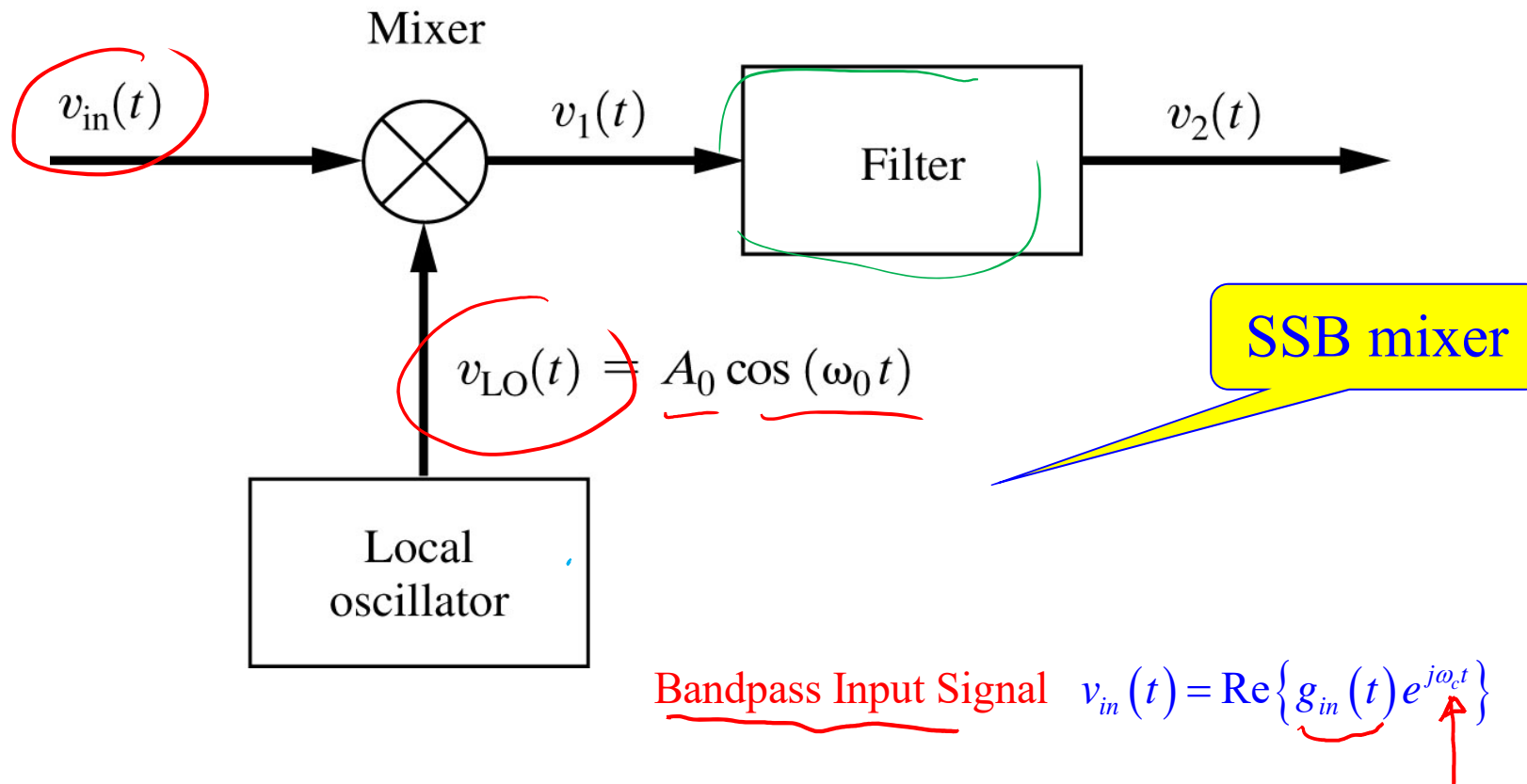

MIXER

Pierfrancesco Lombardo

Mixer ideale

- **Ideal mixer** is a mathematical multiplier of two input signals. One of the signals is sinusoidal generated by a local oscillator. Mixing results in frequency translation.



Mixer Up e Down conversion

Bandpass Input Signal

$$v_{in}(t) = \text{Re}\{g_{in}(t)e^{j\omega_c t}\}$$

Mixer Output

$$v_1(t) = \left[A_0 \text{Re}\{g_{in}(t)e^{j\omega_c t}\} \right] \cos \omega_0 t$$

$$= \frac{A_0}{4} \left[g_{in}(t)e^{j\omega_c t} + g_{in}^*(t)e^{-j\omega_c t} \right] (e^{j\omega_0 t} + e^{-j\omega_0 t})$$

$$= \frac{A_0}{4} \left[g_{in}(t)e^{j(\omega_c+\omega_0)t} + g_{in}^*(t)e^{-j(\omega_c+\omega_0)t} + g_{in}(t)e^{j(\omega_c-\omega_0)t} + g_{in}^*(t)e^{-j(\omega_c-\omega_0)t} \right]$$

$$\text{Re}\{\cdot\} = \frac{1}{2}\{\cdot\} + \frac{1}{2}\{\cdot\}^*$$

$$v_1(t) = \frac{A_0}{2} \text{Re}\{g_{in}(t)e^{j(\omega_c+\omega_0)t}\} + \frac{A_0}{2} \text{Re}\{g_{in}(t)e^{j(\omega_c-\omega_0)t}\}$$

$$f_u = f_c + f_0$$

UPCONVERSION

$$f_d = f_c - f_0$$

DOWNCONVERSION

BANDPASS FILTER

BASEBAND OR BANDPASS FILTER



Mixer Up e Down conversion

$$v_1(t) = \frac{A_0}{2} \operatorname{Re} \left\{ \underbrace{g_{in}(t) e^{j(\omega_c + \omega_0)t}}_{\text{Up-conversion}} \right\} + \frac{A_0}{2} \operatorname{Re} \left\{ \underbrace{g_{in}(t) e^{j(\omega_c - \omega_0)t}}_{\text{Down-conversion}} \right\}$$

$f_u = f_c + f_0$ $f_d = f_c - f_0$

↑

Bandpass Filter

↑

Baseband/bandpass Filter ($f_c - f_0$)

- If $(f_c - f_0) = 0$ ➔ Low Pass Filter gives baseband spectrum
- If $(f_c - f_0) > 0$ ➔ Bandpass filter ➔ Modulation is preserved

Filter Output:

$$v_2(t) = \operatorname{Re} \left\{ g_2(t) e^{j(\omega_c - \omega_0)t} \right\} = \frac{A_0}{2} \operatorname{Re} \left\{ g_{in}(t) e^{j(\omega_c - \omega_0)t} \right\}$$

- If $f_c > f_0$ ➔ modulation on the mixer input is preserved

Mixer Up e Down conversion (II)

$$v_1(t) = \frac{A_0}{2} \operatorname{Re} \left\{ \underbrace{g_{in}(t) e^{j(\omega_c + \omega_0)t}}_{\text{Up-conversion}} \right\} + \frac{A_0}{2} \operatorname{Re} \left\{ \underbrace{g_{in}(t) e^{j(\omega_c - \omega_0)t}}_{\text{Down-conversion}} \right\}$$

$f_u = f_c + f_0$ $f_d = f_c - f_0$

Bandpass Filter Baseband/bandpass Filter ($f_c - f_0$)

negative & $f_c < f_0$

If $f_c < f_0$ \rightarrow $v_1(t) = \frac{A_0}{2} \operatorname{Re} \left\{ g_{in}(t) e^{j(\omega_c + \omega_0)t} \right\} + \frac{A_0}{2} \operatorname{Re} \left\{ \underbrace{g_{in}^*(t) e^{j(\omega_0 - \omega_c)t}}_{\text{positive}} \right\}$
‘ ω ’ needs to be positive

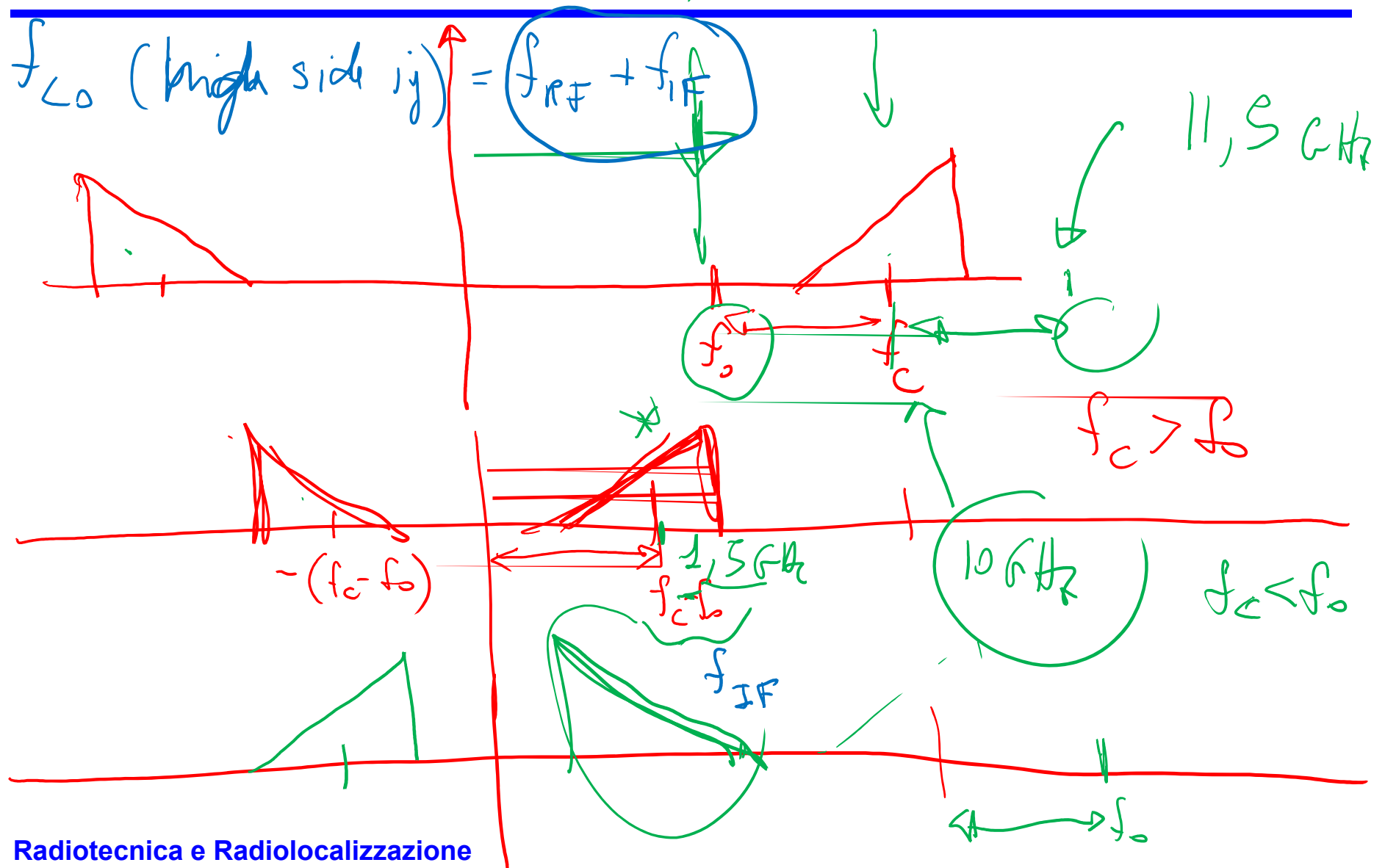
Complex envelope is conjugated ~ **sidebands are exchanged**

$$F[g_{in}^*(t)] = \int_{-\infty}^{\infty} g_{in}^*(t) e^{-j\omega t} dt = \left[\int_{-\infty}^{\infty} g_{in}(t) e^{-j(-\omega)t} dt \right]^* = G_{in}^*(-f)$$

$-f \rightarrow$ Upper & lower sidebands are exchanged

$*$ \rightarrow Phase spectrum is inverted

$$f_{LO} \text{ (low side inj.)} = f_{RF} - f_{IF} \quad 8,5 \text{ GHz}$$



High-Side e Low-Side Injection

- Complex envelope of an *Up Converter*:

$$g_2(t) = \frac{A_0}{2} g_{in}(t); \quad f_u = f_c + f_0 > 0 \quad - \text{Amplitude is scaled by } A_0/2$$

- Complex envelope of a *Down Converter*:

$f_d = f_c - f_0 > 0$ i.e., $f_0 < f_c$ → down conversion with low-side injection

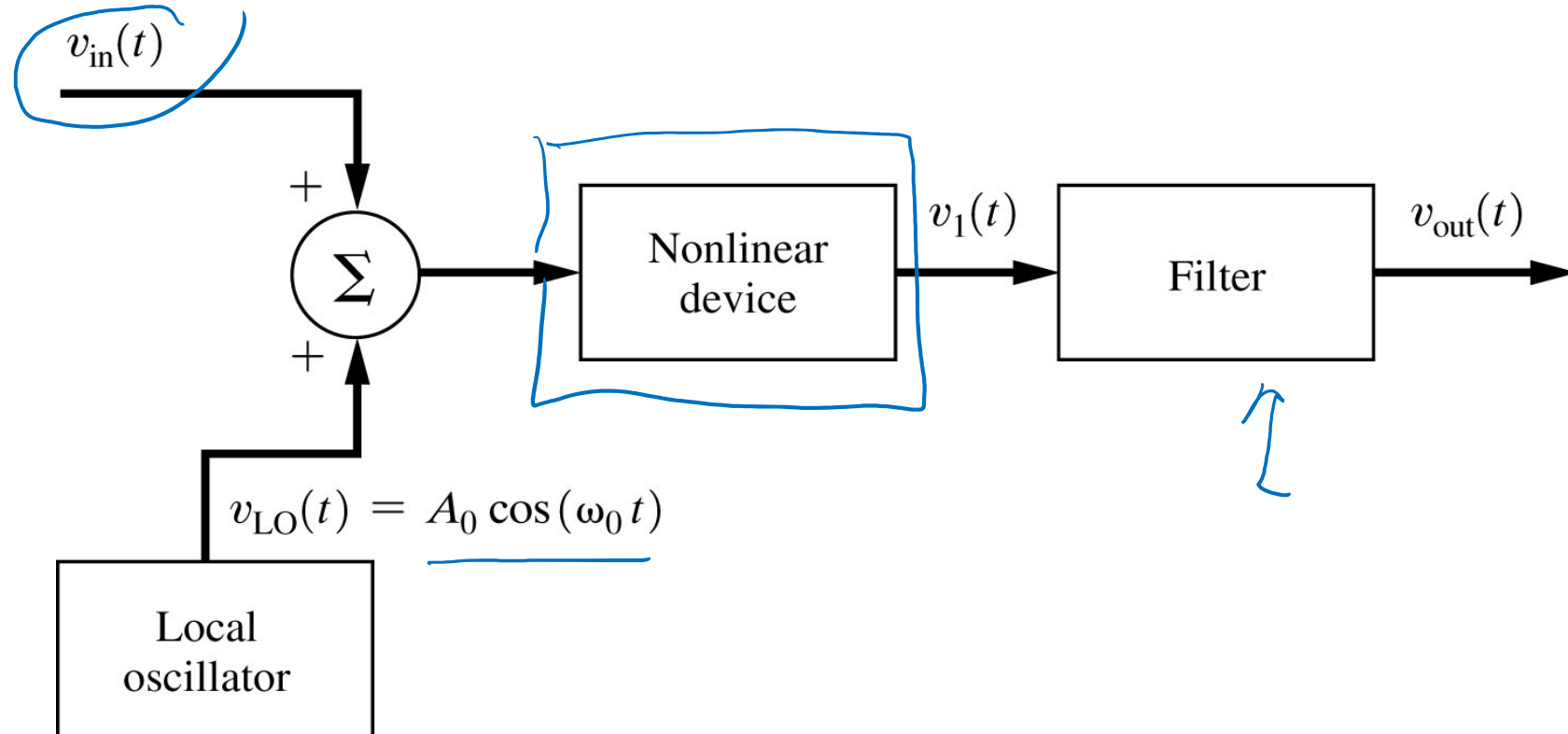
$$g_2(t) = \frac{A_0}{2} g_{in}(t) \quad - \text{Amplitude is scaled by } A_0/2$$

$f_d = f_0 - f_c > 0$ i.e., $f_0 > f_c$ → down conversion with high-side injection

$$g_2 = \frac{A_0}{2} g_{in}^*(t) \quad - \text{Amplitude is scaled by } A_0/2 \quad - \text{Sidebands are reversed from those on the input}$$

Mixer reale - con nonlinearità

- Multiplication operation needed by mixers can be obtained by using a nonlinear device together with a summer.

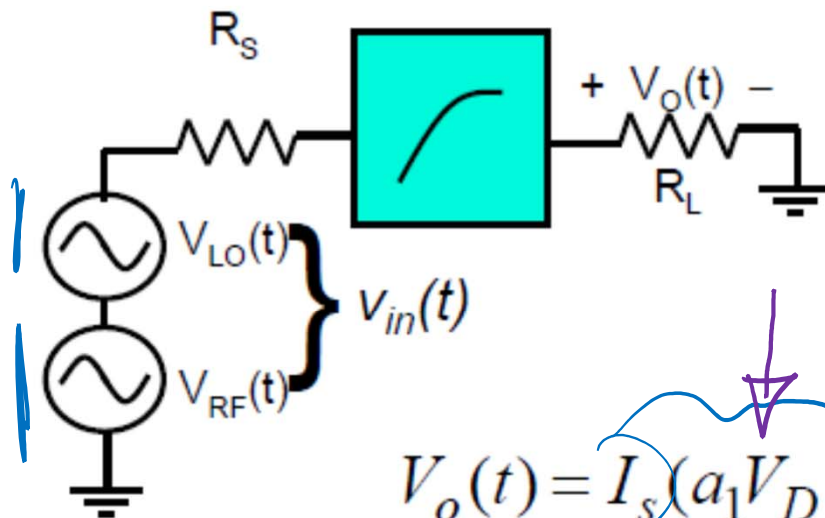


Nonlinear device used as a mixer.

Mixer reale - con nonlinearità (II)

Any diode or transistor will exhibit nonlinearity in its transfer characteristic at sufficiently high signal levels.

$$V_o(t) = a_1 V_{in}(t)$$



$$V_o(t) = I_s (a_1 V_D + a_2 V_D^2 + a_3 V_D^3 + \dots) R_L$$

$V_{RF}(t) + V_{LO}(t)$

second-order product term

Mixer reale - con nonlinearità (III)

We see that our output may contain a DC term, RF and LO feedthrough, and terms at all harmonics of the RF and LO frequencies. Only the second-order product term produces the desired output. Let's suppose that V_R and V_L are the fraction of V_{RF} and V_{LO} that appear across the nonlinear device (possibly a diode).

$$a_2 V_D^2 = a_2 \left[V_R^2 \sin^2(\omega_R t) + V_L^2 \sin^2(\omega_L t) + 2V_R V_L \sin(\omega_R t) \sin(\omega_L t) \right]$$

The product term produces the desired mixer output:

$$V_R V_L \left[\cos(\omega_R - \omega_L)t - \cos(\omega_R + \omega_L)t \right]$$

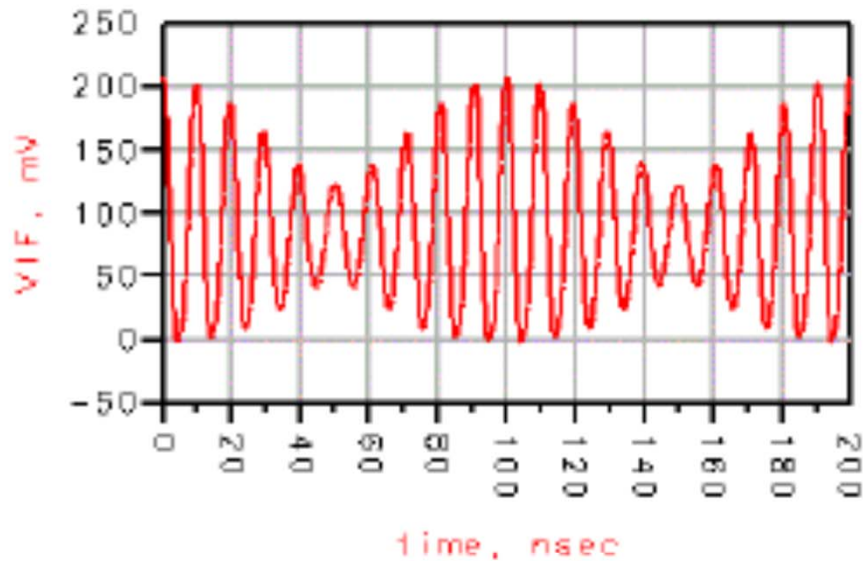
DC and second harmonic terms are also present. Recalling

$$V_R^2 \sin^2(\omega_R t) = \frac{V_R^2}{2} [1 - \cos(2\omega_R t)]$$

If the application is frequency multiplication rather than mixing, the harmonic term can be useful. The DC term is proportional to RF input power, so can be used as a power meter.

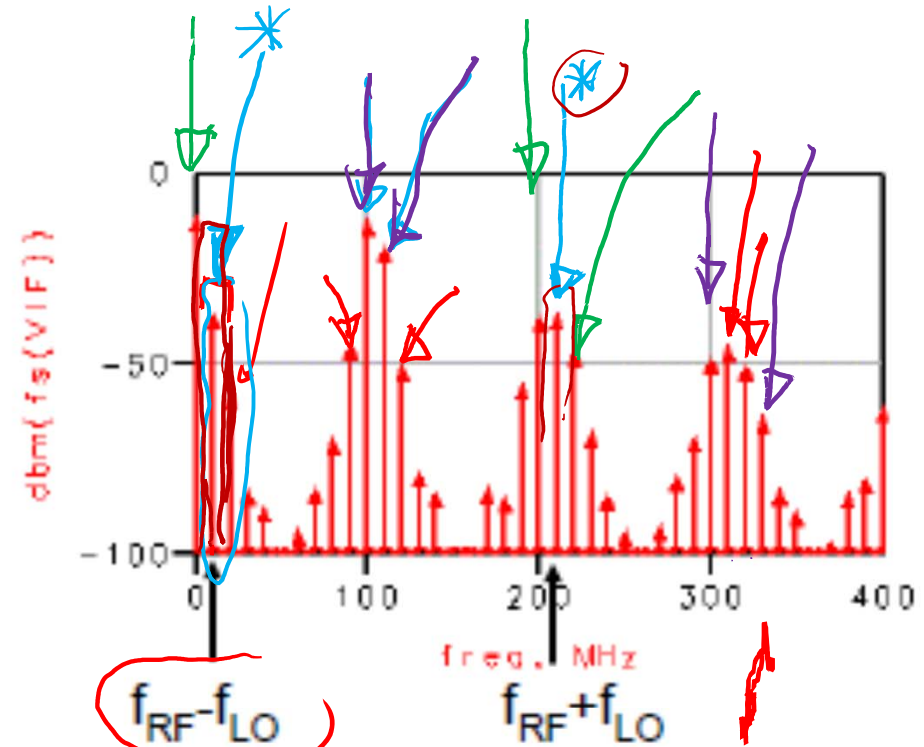
In addition, when v_{RF} consists of multiple carriers, the power series also will produce cross-products that make the desired output products dependent on the amplitude of other inputs. Spurious output signal strengths can be decreased when devices that are primarily square-law, such as FETs with longer gate lengths, are used in place of diodes or bipolar transistors.

Mixer reale - con nonlinearità (IV)



$F_{RF} = 110 \text{ MHz}$ $|V_{RF}| = 0.1V$
 $F_{LO} = 100 \text{ MHz}$ $|V_{LO}| = 0.2V$

$$[RF + LO]^3 = [RF^3 + 3RFLO + 3RFLO^2 + LO^3]$$



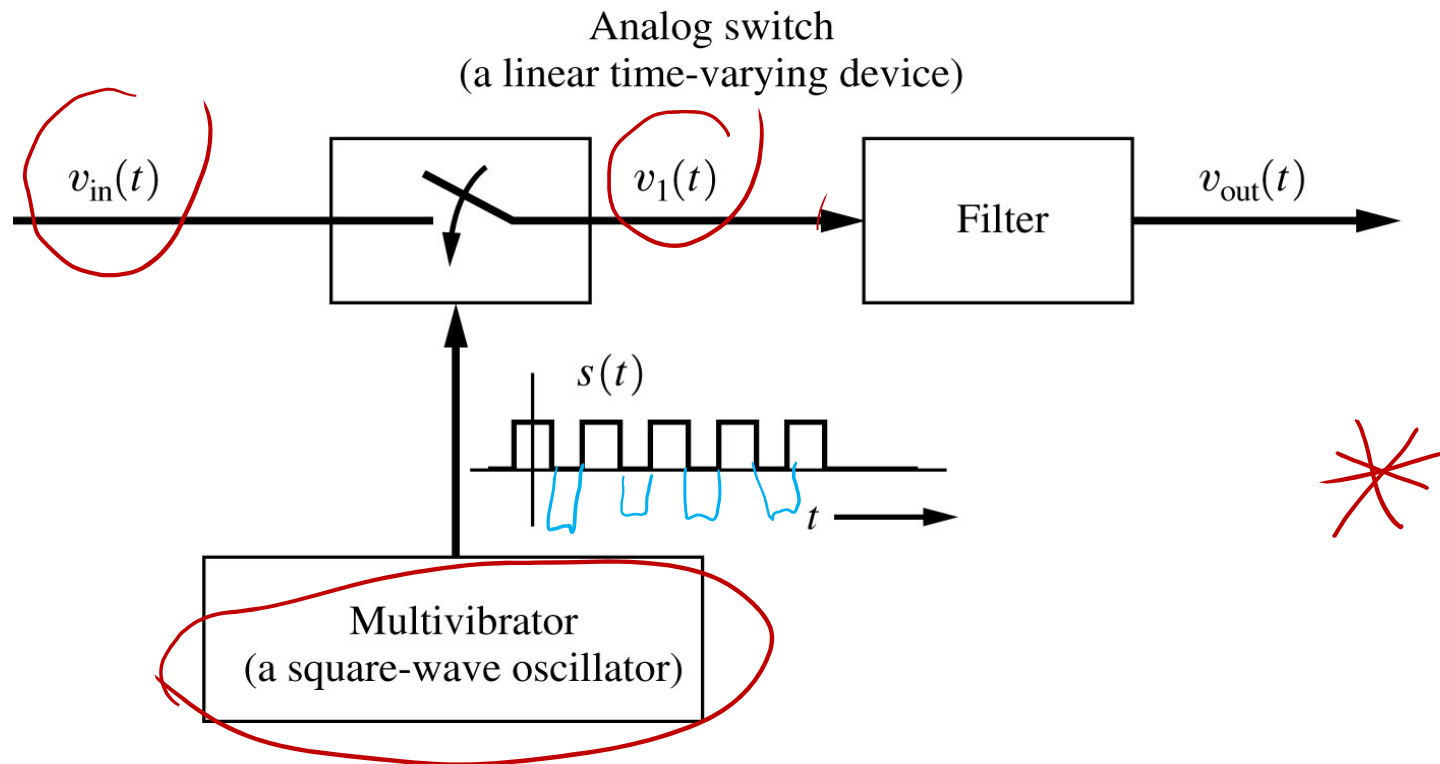
$V_{DC} = 0.6V$

$RF + 2LO$
 $RF - 2LO$

$$[2RF - LO] [2RF + LO] [RFLO^2] [LO^3]$$

Mixer reale – tipo switching

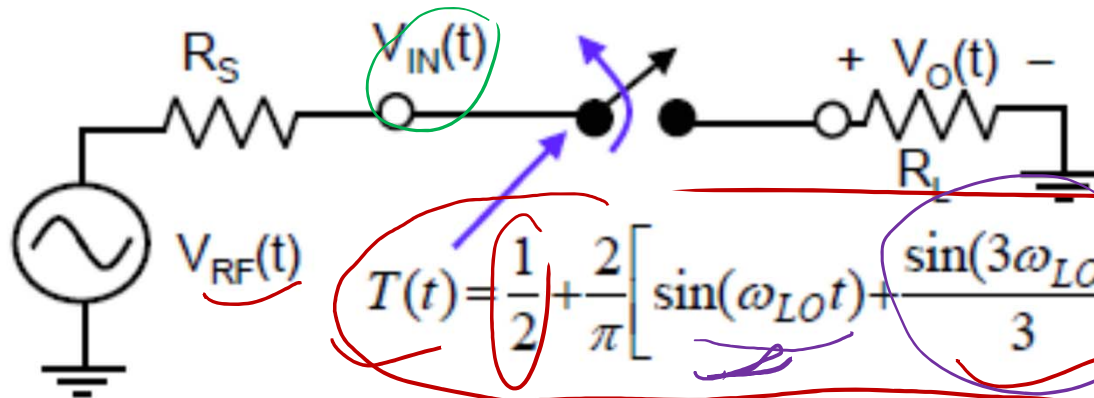
- Multiplication operation needed by mixers can also be obtained by using an analog switch.



Linear time-varying device used as a mixer.

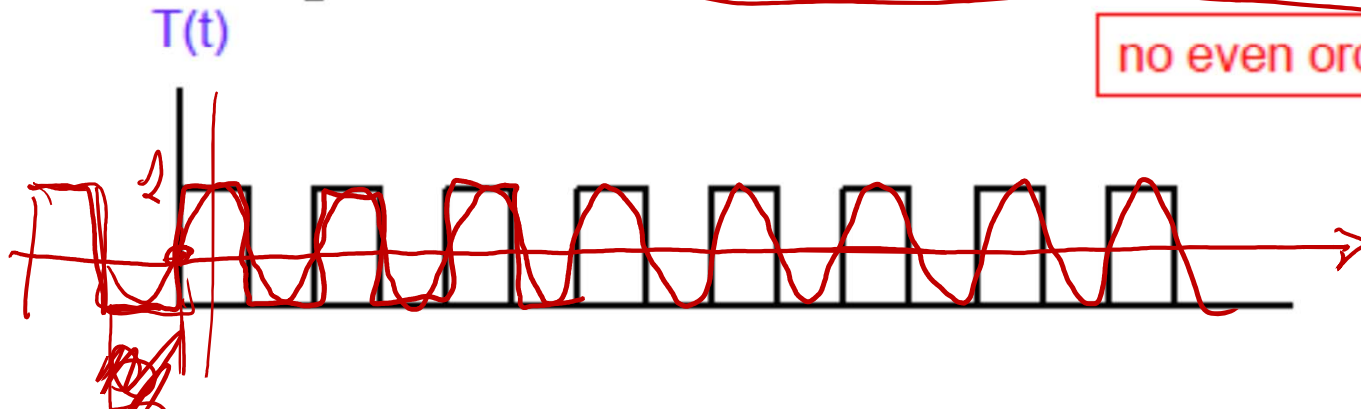
Mixer reale – tipo switching (II)

- Let $V_{IN}(t) = V_R \cos(\omega_{RF}t)$
- Multiply by the LO switching function $T(t)$



$$T(t) = \frac{1}{2} + \frac{2}{\pi} \left[\sin(\omega_{LO}t) + \frac{\sin(3\omega_{LO}t)}{3} + \frac{\sin(5\omega_{LO}t)}{5} + \dots \right]$$

no even order harmonics

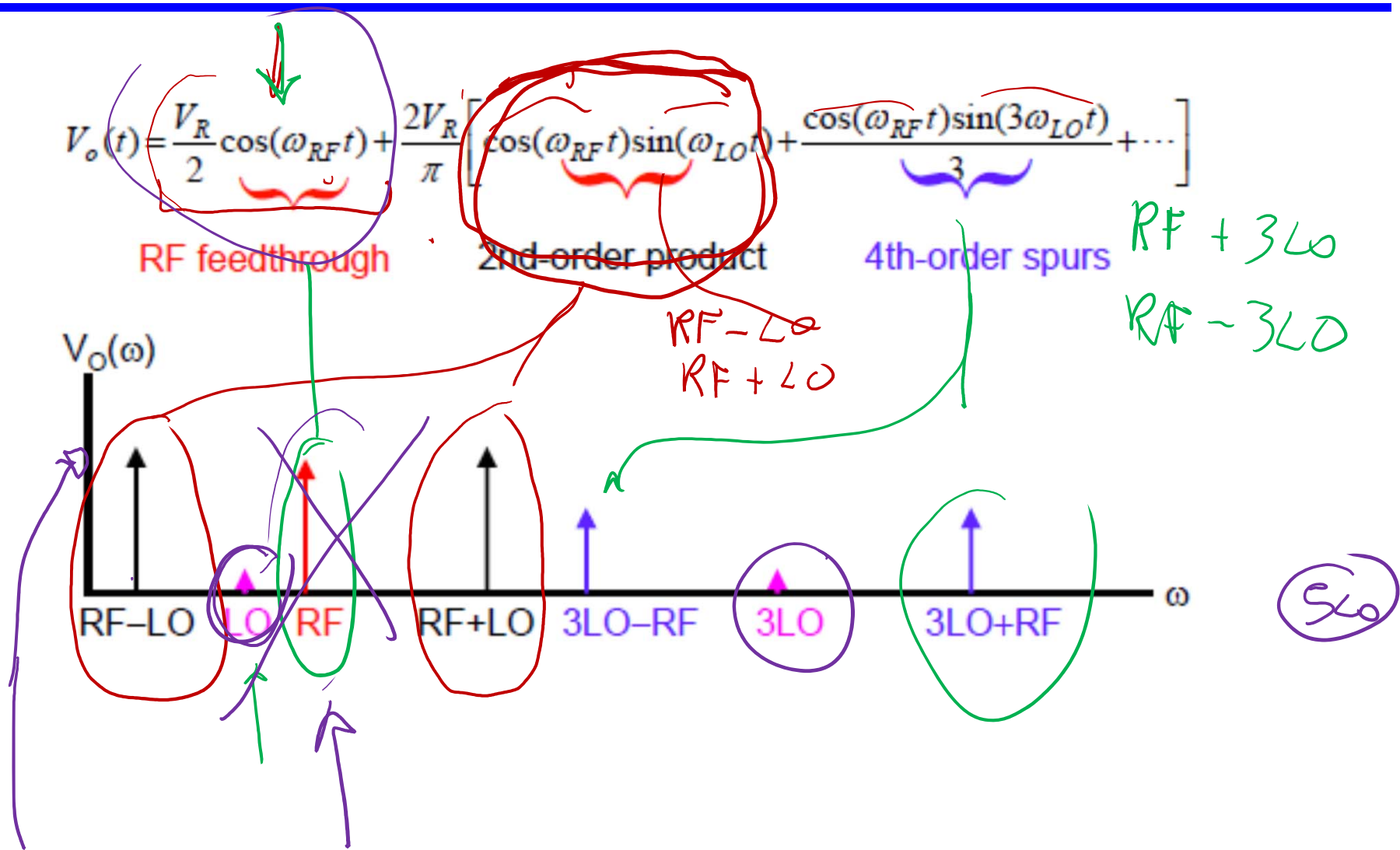


Mixer reale – tipo switching (III)

This simple switch is operated by the LO. If the LO is a square wave with 50% duty cycle, it is easily represented by its Fourier Series. The symmetry causes the even-order harmonics to drop out of the LO spectrum. When multiplied by a single frequency cosine at ω_{RF} the desired sum and difference outputs will be obtained as shown in the next slide. Note that everything is single-ended; there is no balancing on this design.

There will be harmonics of the LO present at $3\omega_{LO}$, $5\omega_{LO}$, etc. that will also mix to produce outputs called “*spurs*” (an abbreviation for spurious signals). These harmonics also convert broadband noise that is generated internal to the mixer (or that is allowed into the mixer input in the absence of a preselection filter) into the IF output band.

Mixer reale – tipo switching (IV)



Mixer reale – tipo switching (V)

The product of $V_{RF}(t)T(t)$ produces the desired output frequencies at

$$\omega_{RF} - \omega_{LO} \text{ and } \omega_{RF} + \omega_{LO} \text{ from the second order product.}$$

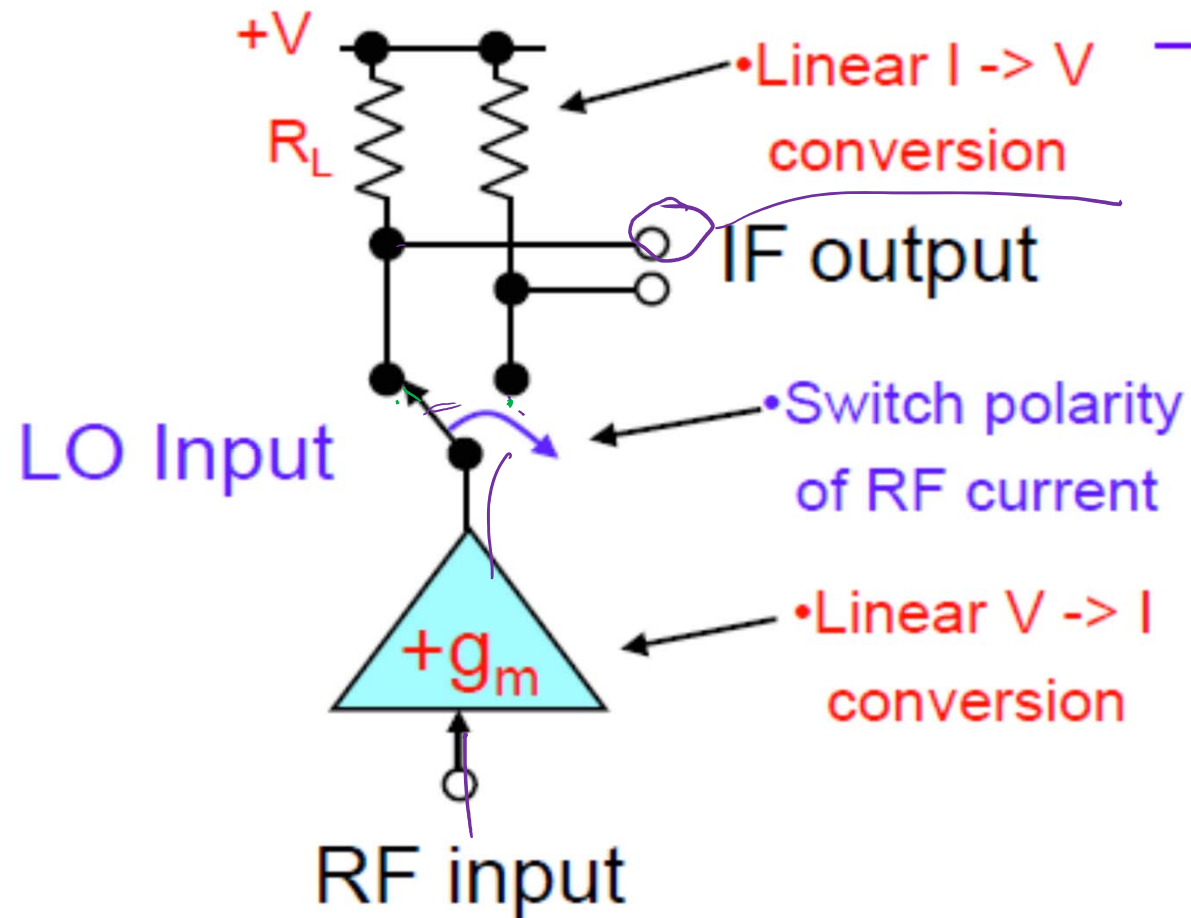
Odd harmonics of the LO frequency are also present since we have a square wave LO switching signal. These produce spurious 4th, 6th, ... order products with outputs at

$$n\omega_{LO} - \omega_{RF} \text{ and } n\omega_{LO} + \omega_{RF} \text{ where } n \text{ is odd.}$$

We also get *RF feedthrough* directly to the output.

None of the LO signal should appear in the output if the mixer behaves according to this equation. But, if there is a DC offset on the RF input, there will be a LO frequency component in the output as well. This requirement is not unusual, since many mixer implementations require some bias current which leads to a DC offset on the input.

Mixer reale – switching **single balanced**

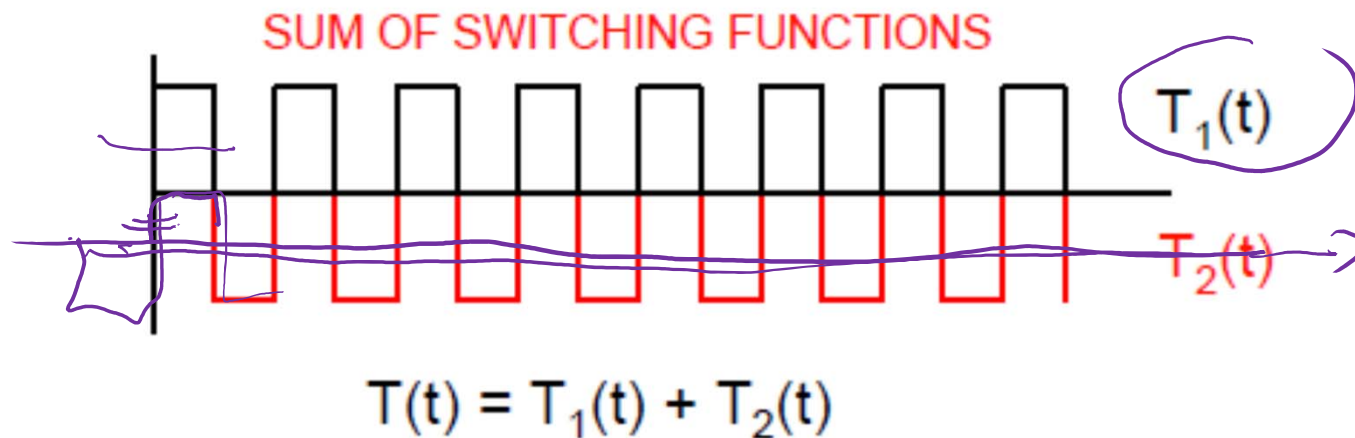


The RF feedthrough can be eliminated by using a differential IF output and a polarity reversing LO switch.

Mixer reale – switching single balanced

$$T_1(t) = \frac{1}{2} + \frac{2}{\pi} \left[\sin(\omega_{LO}t) + \frac{1}{3} \sin(3\omega_{LO}t) + \dots \right]$$

$$T_2(t) = -\frac{1}{2} + \frac{2}{\pi} \left[\sin(\omega_{LO}t) + \frac{1}{3} \sin(3\omega_{LO}t) + \dots \right]$$



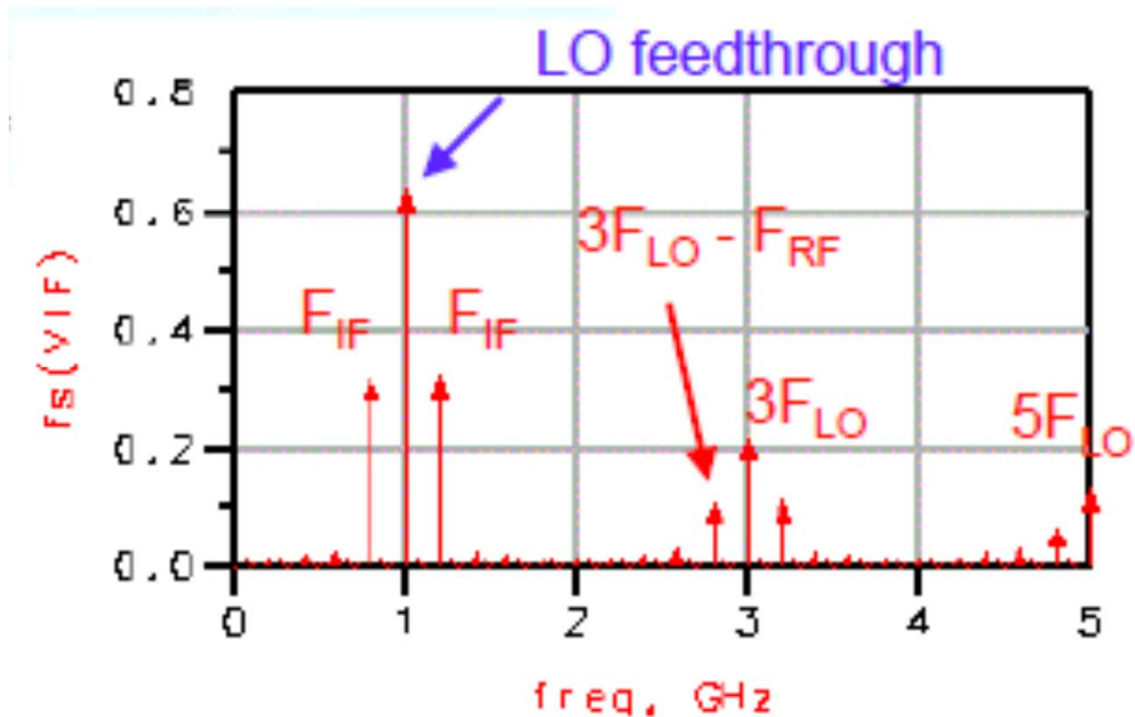
LO Feedthrough

$$V_{IF}(t) = R_L [I_{DC} + g_m V_R \cos(\omega_{RF} t)] \times$$
$$\frac{4}{\pi} \left[\sin(\omega_{LO} t) + \frac{1}{3} \sin(3\omega_{LO} t) + \frac{1}{5} \sin(5\omega_{LO} t) + \dots \right]$$
$$= \frac{4R_L}{\pi} \left\{ I_{DC} \sin(\omega_{LO} t) + \leftarrow \text{Present even with no RF input} \right.$$
$$\left. \frac{1}{2} g_m V_R [\sin(\omega_{RF} + \omega_{LO})t + \sin(\omega_{RF} - \omega_{LO})t] \right\}$$

But, we can still get LO feedthrough if we take a single-ended output or if there is a DC current in the signal path. There is often DC present since the output of the transconductance amplifier will have a DC current component. This current shows up as a differential output.

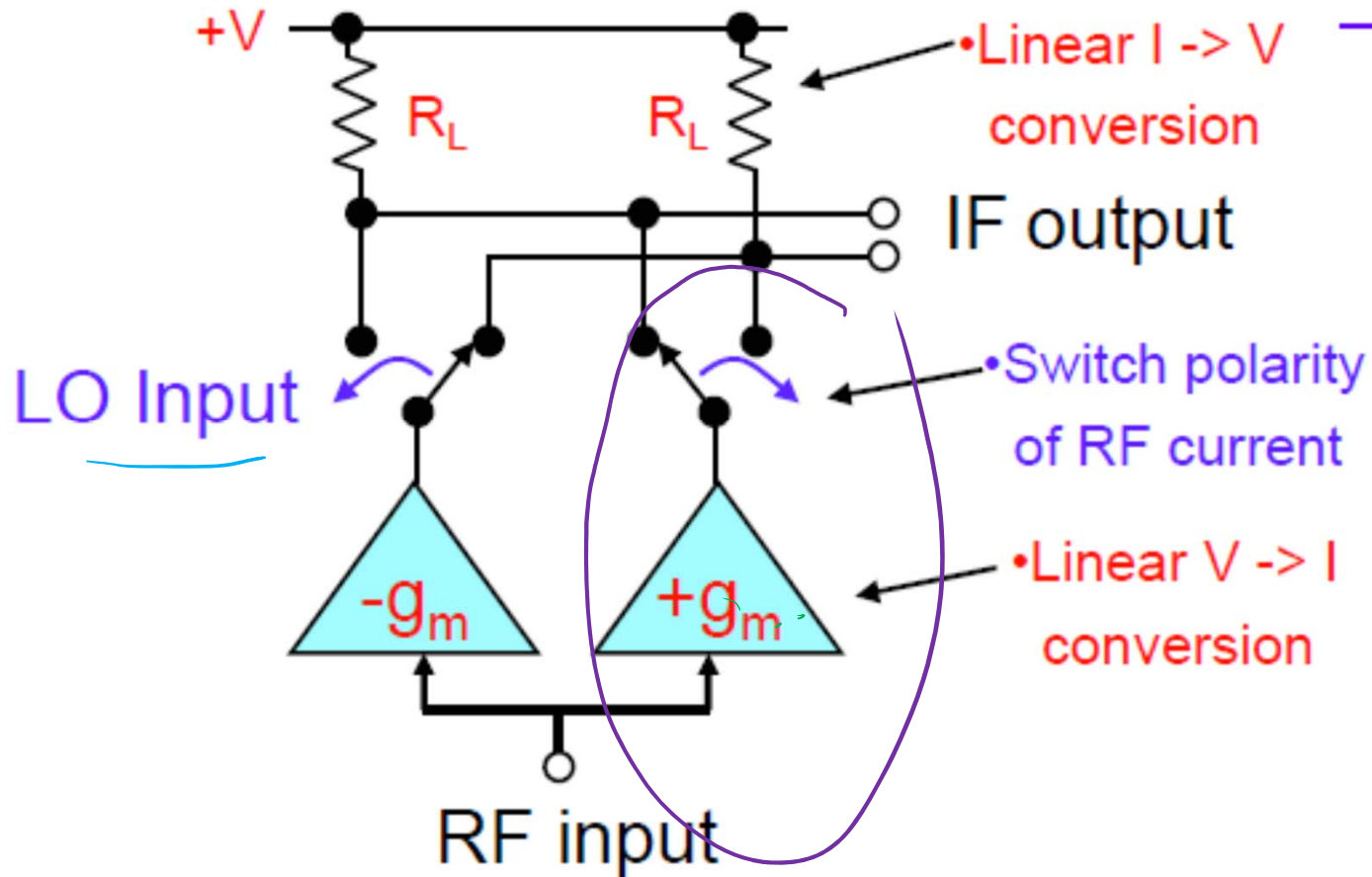
Mixer reale – switching single balanced

$F_{RF} = 200 \text{ MHz}$
 $F_{LO} = 1.0 \text{ GHz}$
 $F_{IF} @ 800 \text{ \& } 1200 \text{ MHz}$



As you can see, the output spectrum of the single-balanced switching mixer is much less cluttered than the nonlinear mixer spectrum.

Mixer reale – switching double balanced



Mixer reale – switching double balanced (II)

An ideal double balanced mixer consists of a switch driven by the local oscillator that reverses the polarity of the RF input at the LO frequency[1] and a differential transconductance amplifier stage. The polarity reversing switch and differential IF cancels any output at the RF input frequency since the DC term cancels as was the case for the single balanced design. The double LO switch cancels out any LO frequency component, even with currents in the RF to IF path. The LO is typically suppressed by 50 or 60 dB if the components are well matched and balanced.

An IF balun, either active (a differential amplifier) or passive (a transformer or hybrid), is often used, however, so that the conversion gain will be maximized.

To get the highest performance from the mixer we must make the RF to IF path as linear as possible and minimize the switching time of the LO switch. The ideal mixer above would not be troubled by intermodulation distortion (IMD) at the high end of the operating signal range since the ideal transconductors and resistors are linear and the switches are ideal.

Mixer reale – switching double balanced (III)

Two single-balanced mixers – difference cancels LO feedthrough

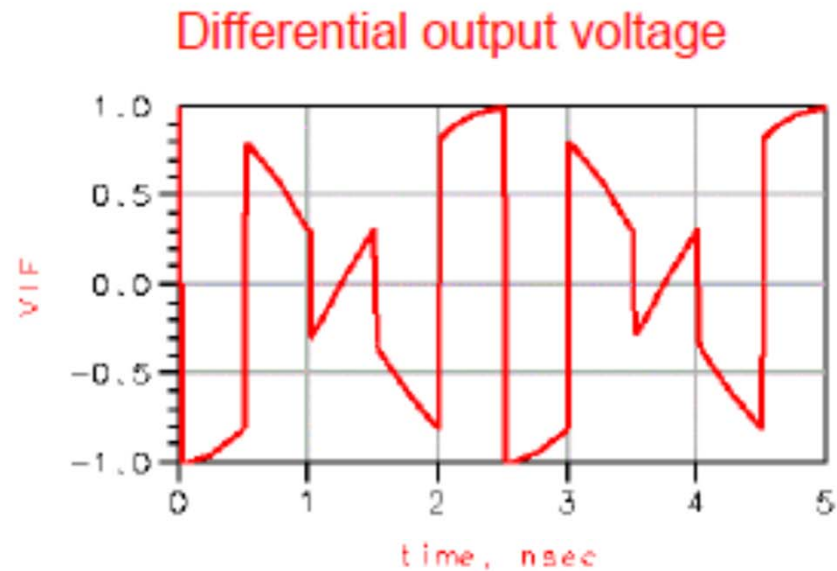
$$V_o = R_L [I_{DC} + I_{RF} \cos(\omega_{RF}t)] \frac{4}{\pi} \left[\sin(\omega_{LO}t) + \frac{1}{3} \sin(3\omega_{LO}t) + \dots \right] -$$

$$R_L [I_{DC} - I_{RF} \cos(\omega_{RF}t)] \frac{4}{\pi} \left[\sin(\omega_{LO}t) + \frac{1}{3} \sin(3\omega_{LO}t) + \dots \right]$$

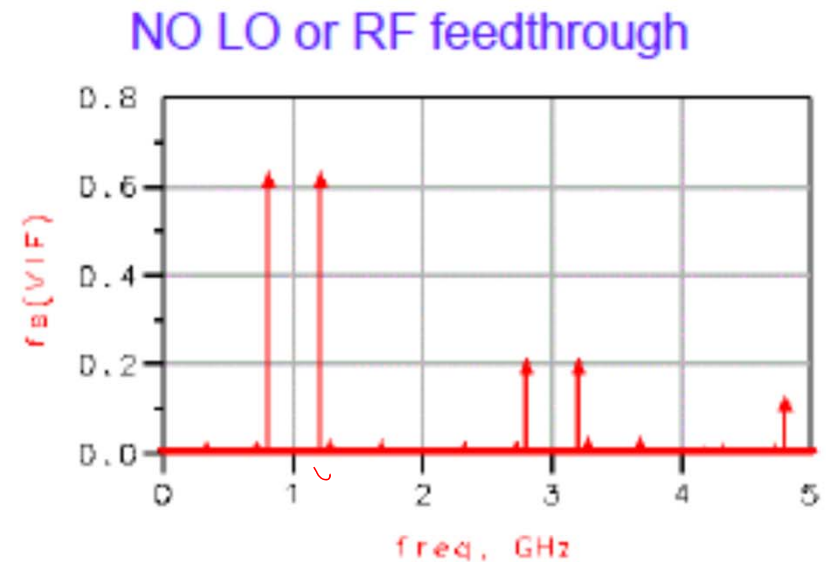
From - gm side of mixer (desired output adds)

Single balancing got rid of the RF feedthrough which was caused by the average DC value of the switching function. Double balancing removes the LO feedthrough as well, since the DC term cancels.

Mixer reale – switching double balanced (IV)



Polarity reversing switch function
is easily seen here



$$F_{RF} = 200 \text{ MHz}$$

$$F_{LO} = 1.0 \text{ GHz}$$

$$F_{IF} @ 800 \text{ \& } 1200 \text{ MHz}$$

Mixer reale – switching double balanced (V)

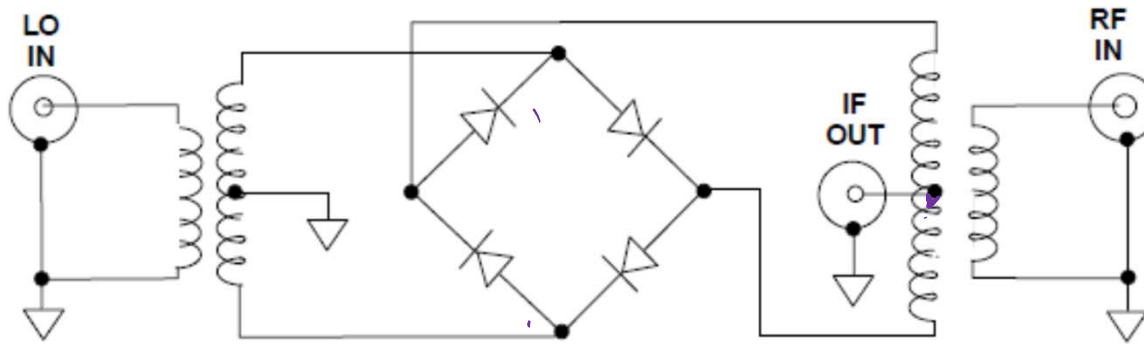


Figure 9: Diode Ring Mixer

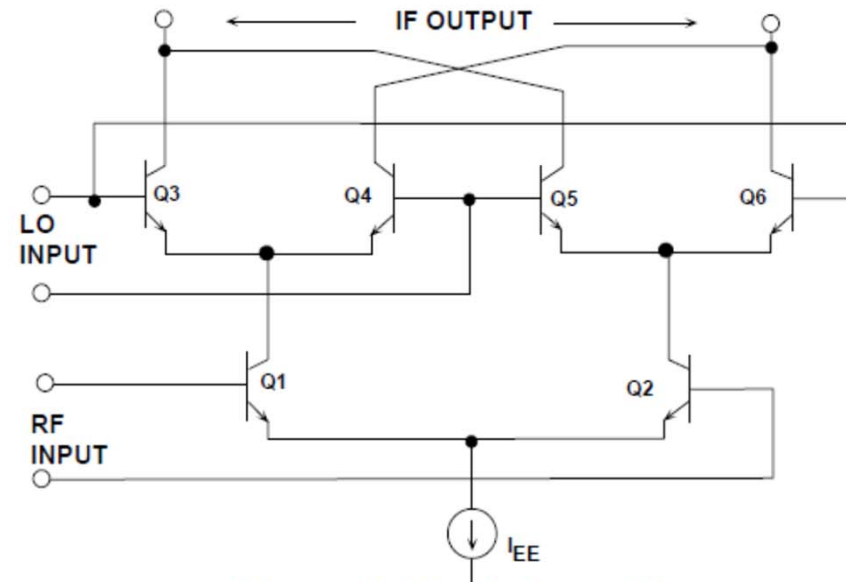
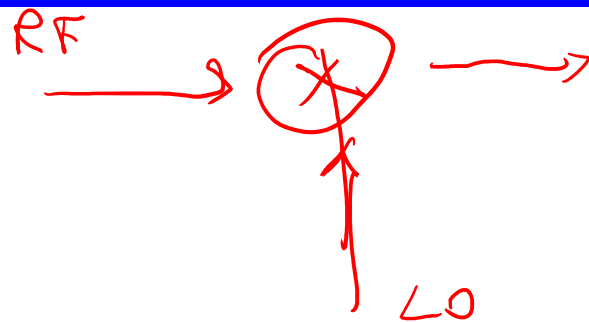


Figure 10: Classic Active Mixer

Mixer reale – Isolamento fra le porte

- The mixer is not perfectly unilateral - leakage between:
 - LO to IF = LO power at IF port/LO power at LO
 - LO to RF = LO power at RF port/LO power at LO
 - RF to IF = RF power at IF port/RF power at RF
- Determine the magnitude of these leakage components at the IF and RF ports using harmonic balance.
- Use the mix function to select frequencies.



$RF - LO$
 $RF + LO$

RF
 $2RF$
 $3RF$
 $4RF$
 \vdots

LO
 $2LO$
 $3LO$
 $4LO$
 \vdots

Spurteile

$n \cdot RF \pm m \cdot LO$
 n, m interi
 $\in \mathbb{N}$

\pm armonici
 $m=1, n=1$

Ordine $|n| + |m|$

$$\pm n RF \pm m LO$$

① $\pm RF \pm LO$

② $\pm 2RF, \pm 2LO$
 $\pm RF \pm LO$

③ $\pm 3RF, \pm 3LO$

$\pm 2RF \pm LO$
 $\pm RF \pm 2LO$

④ $\pm 4RF \pm 4LO$

$\pm 4RF$ $\pm 4LO$

$IF = RF - LO$

$2RF - LO$
 $-2RF + LO$

$2RF - LO$

$RF - 2LO$
 $-RF + 2LO$

$-RF + 2LO$

$\pm RF \pm 3LO$

$\pm 2RF \pm 2LO$

$2RF - 2LO = 2(RF - LO)$

$\pm 3RF \pm LO$

$RF \pm LO$

~~RF~~ RF 88 - 108 MHz

→ 103.3

B = 200 kHz

IF ?
LO ?

$IF = RF - LO$
 $IF = LO - RF$

400 kHz

250, 290 kHz

$IF = RF - LO$

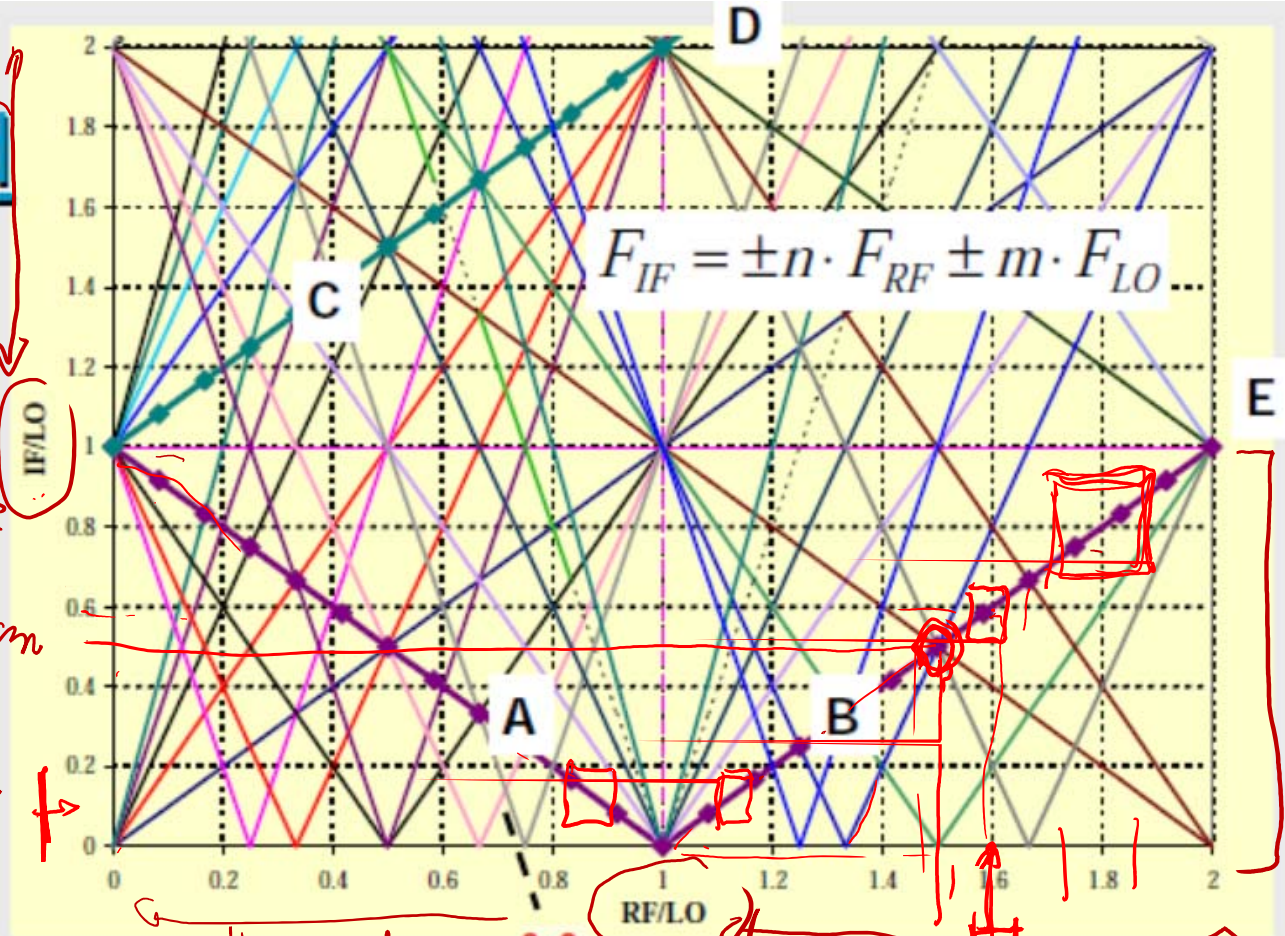
Spur e $= \pm n RF \pm m LO$

$\frac{IF}{LO}$

$\frac{\text{Spur e}}{LO} = \pm n \frac{RF}{LO} + m$

Generalised Spur Chart

- Visualisation of the mixer signal relationships
- Each is an equation of a straight line
- The dotted lines show the major signal relationships



Down conversion, High side
 Down conversion, Low side
 Up conversion, High side
 Up conversion, Low side

Output	Image	Opposite
A	B	C
B	A	D
C	A	E
D	B	E

$$\frac{IF}{LO} = \frac{RF}{LO} \pm 1$$

$$n = 1 \quad m = -1$$

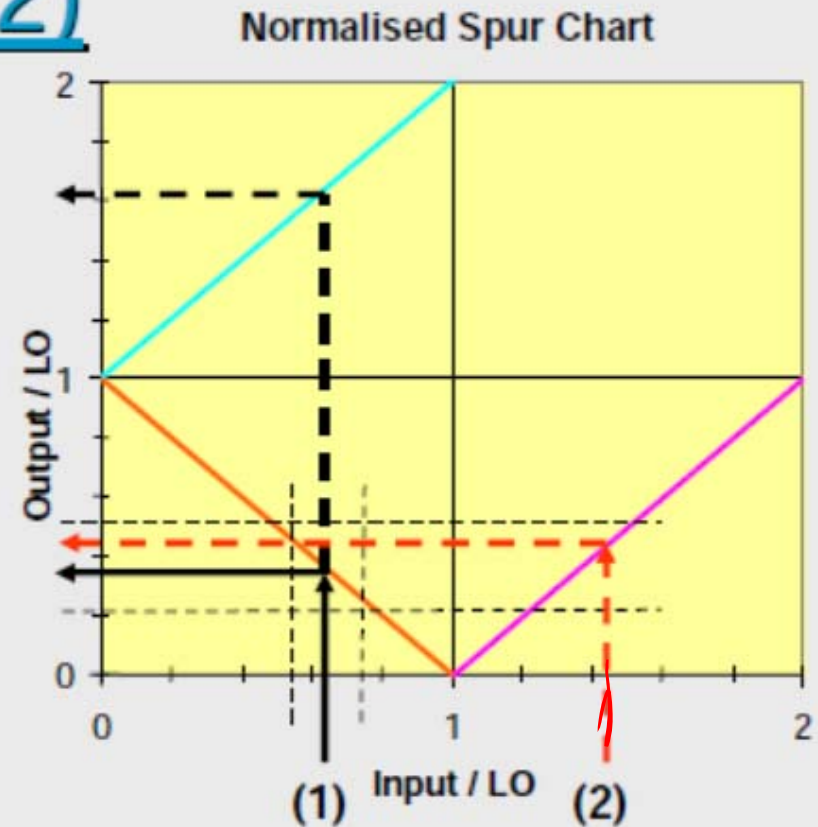
Mixer Spur Charts

$$F_{Out} = n \cdot F_{In} + m \cdot F_{LO}$$

- There are many others possible spurs corresponding to other values of m and n.
 - *These are often referred to as m x n or (m by n)*
- The power levels vary, generally being lower for higher values of m and n
 - But the levels are also influenced by which ports on the mixer are balanced
 - *Balance generally enhances performance (reduces spurs) for even values of the signal entering at that port*
- All spur level suppression are proportional to n and the level relative to the LO
 - Because LO drive is fixed, it is vital to ensure that the order of m products is high (i.e. low signal levels)
- LO frequency availability and spur analysis form the basis of most of the architectural aspects of radio system design

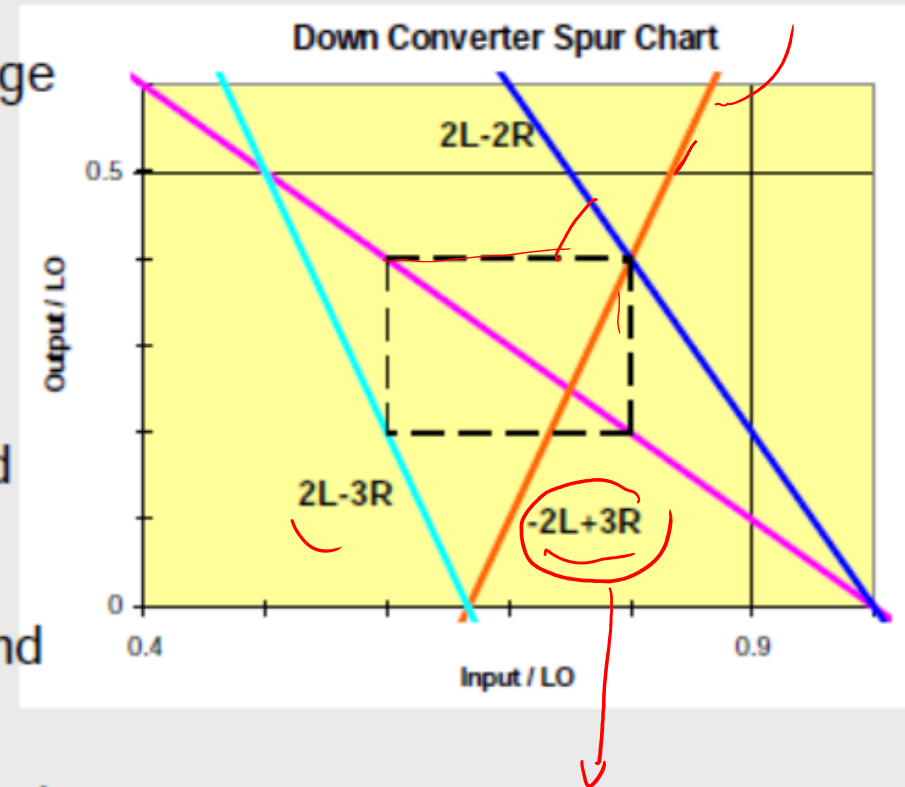
Using Spur Charts (2)

- 1) Wanted inputs produce
Outputs at the desired IF and
Undesired products (e.g. the sum)
Vertical range represents
frequency axis of the IF filter
 - 2) Unwanted signals at the input produce
Outputs at the IF (e.g. the image)
Horizontal range represents
frequency axis of the RF filter
- Spur level calculations consider
 - The RF, LO and IF frequencies
 - *i.e. where you are on the chart*
 - For a particular mixer the levels are determined by
 - the (m,n) coefficients
 - the mixer input levels relative to LO (taking RF filtering into account)
 - the output levels (taking IF filtering into account)



Using Spur Charts (3)

- Filters have a finite roll off rate
 - Signals close to the conversion box will not get much attenuation
- $2L-2R$
 - RF signals above the RF band edge will be converted back in band (e.g. 0.85)
 - In band RF signals produce additional output for IF filtering (e.g. 0.75)
- $2L-3R$
 - Similar situation below the RF and IF band edge
- $-2L+3R$ In band spur
 - In band RF signals produce in band IF products (e.g. 0.9)
 - LO drive and mixer performance must keep these to acceptable level



Estimating Spur Levels

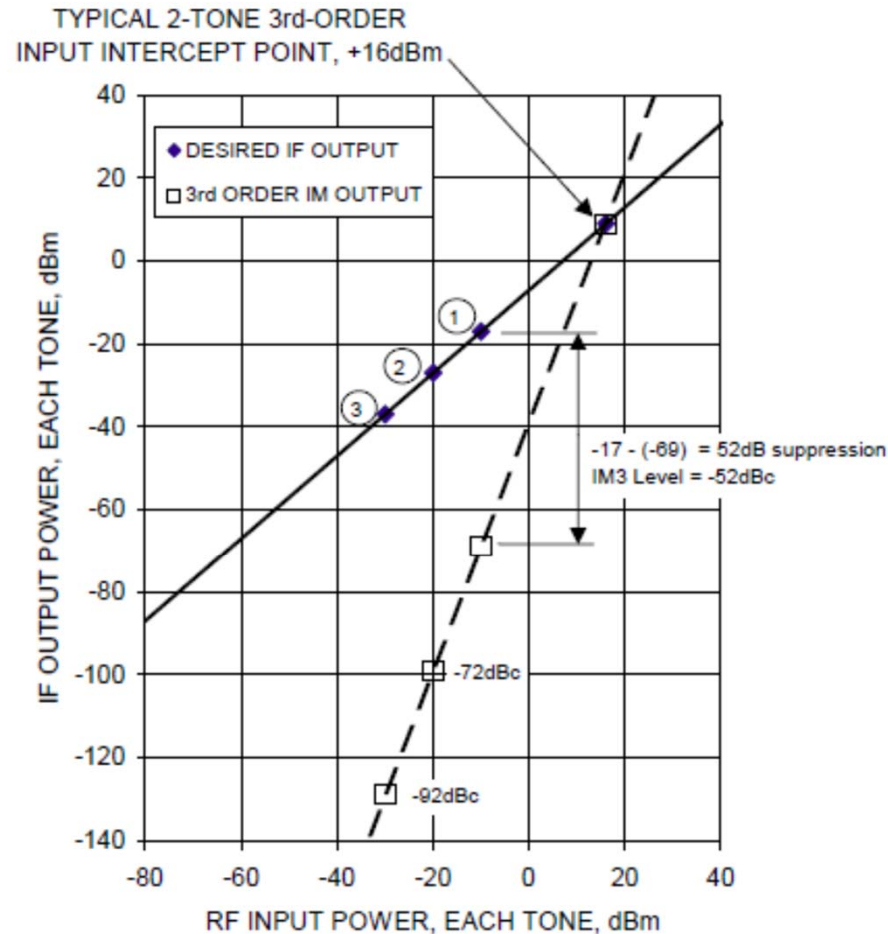
- The table shows typical spur levels relative to the wanted signal (dBc) for a double balanced mixer
- Many manufacturers provide similar information for there products
- Most spur levels vary with the level of the RF relative to the LO drive (ΔP)
 - This determines the maximum input for a given spur threshold and hence the SFDR
- IM levels also vary due to:
 - Port terminations
 - Diode balance, unit to unit variation
 - LO drive
- Notes
 1. $n=1$ products do not go down with signal level
 2. Others reduce at the rate of $\Delta P * (n-1)$
 3. Balance makes even ordered products lower than corresponding odd products

LO (m)	RF (n)	Suppression (dBc)	Notes
1	1	0	
1	2	$\Delta P+41$	3
1	3	$2\Delta P+28$	
2	1	35	3
2	2	$\Delta P+39$	2
2	3	$2\Delta P+44$	2
3	1	10	3
3	2	$\Delta P+32$	3
3	3	$2\Delta P+18$	
4	1	35	1
4	2	$\Delta P+39$	
5	1	14	1
5	3	$2\Delta P+14$	
6	1	35	1
6	2	$\Delta P+39$	
7	1	17	1
7	3	$2\Delta P+11$	

Mixer performance specifications

- Image rejection ✓
- Conversion gain: voltage or power ✓
- Port-to-port isolation: dBc
- Large signal performance:
 - gain compression: P_{1dB}
 - intermodulation distortion spec: third-order intercept (TOI)
- Small signal performance: noise figure
- Operating range: Spurious-free dynamic range

Mixer intermodulation



Example of Mixer Third-Order Intermodulation Intercept determination, illustrating derivation in the text.

Mixer intermodulation (II)

A convenient way to introduce the IP3 concept is to describe how intermodulation products vary relative to desired output signal level. In the example of Fig. 1, -10 dBm RF input produces -17 dBm IF output (Point 1 on the graph), so that conversion loss is 7 dB. The third-order intermodulation (IM3) power is -69 dBm, and the 52-dB difference is called the third-order suppression. We say that the mixer suppresses the third-order products by 52 dB. Another way to express this is to say that the IM3 level is -52 dBc, which means it is 52 dB below the “carrier” (the desired IF output). Whichever way IM3 is expressed, it is important to state along with it the corresponding RF input power, in this case -10 dBm.

Now, if the RF input power were reduced by 10 dB to -20 dBm (Point 2 in Fig. 1), the third-order product level would decrease by a factor of three, or 30 dB. The difference between the desired IF and the IM3 would increase by 20 dB; thus, the mixer would offer 72-dB suppression with two -20 dBm signals at its input. With another 10-dB drop in RF input to -30 dBm (Point 3 in Fig. 1), third-order products would drop another 30 dB and the difference would increase by 20 dB, so that the two -30 dBm input signals would produce third-order products suppressed 92 dB.

Mixer intermodulation (III)

When will the desired IF output and the third-order products theoretically become equal? The original input signals were -10 dBm (Point 1 in Fig. 1); the output signals were -17 dBm and the third-order products were 52 dB lower, at -69 dBm. If the input is now raised 26 dB to +16 dBm, the theoretical power of the desired IF output will be 16 dBm - 7dB (the conversion loss) = +9 dBm. The third-order products at the output will increase by a factor of three: $3 \times 26 = 78$ dB, making them $(-69 \text{ dBm} + 78 \text{ dB}) = +9$ dBm. Since the conversion loss is 7 dB, referencing the third-order-product power to the input of the mixer yields $9 + 7 = +16$ dBm, thus establishing the Input Third-Order Intercept Point as +16 dBm where the desired signals and the third-order products are equal.

Graphically, the intercept point is obtained by linearly extending both the desired signal curve (past the 1-dB compression point) and the third-order curve until they intersect.

The formula for calculating the intercept point at the output, given the output power of the desired IF signal (P_{OUT} , each of the two tones) and the IM3 suppression value ($|IM3 - P_{\text{OUT}}|$, which is the absolute value of the difference in dB between the intermodulation level at the output and the desired IF output), is:

$$IP3 \text{ (dBm)} = |IM3 - P_{\text{OUT}}| \text{ (dB)} / 2 + P_{\text{OUT}} \text{ (dBm)}$$

To refer IP3 to the input of the mixer, add the conversion loss value in dB.

Mixer intermodulation (IV)

A rule-of-thumb method for determining the intermodulation level of a mixer is as follows:

- (1) Find the 1-dB compression level (this is the RF input power level that causes the conversion loss to increase by 1 dB).
- (2) Determine the intercept point. For diode mixers, at the low end of the frequency range, this point is about 15 dB above the 1-dB compression point. As the mid- to upper frequency range is approached, the intercept point drops to about 10 dB above the 1-dB compression point. For FET mixers, the intercept point is about 10 dB above the 1-dB compression point.
- (3) Multiply the difference between the intercept point and the per-tone RF input power (equal RF tones) by the sum of the harmonic orders (in this case, $2 + 1 = 3$).
- (4) Subtract this number from the intercept point. This yields the intermodulation level in dBm.