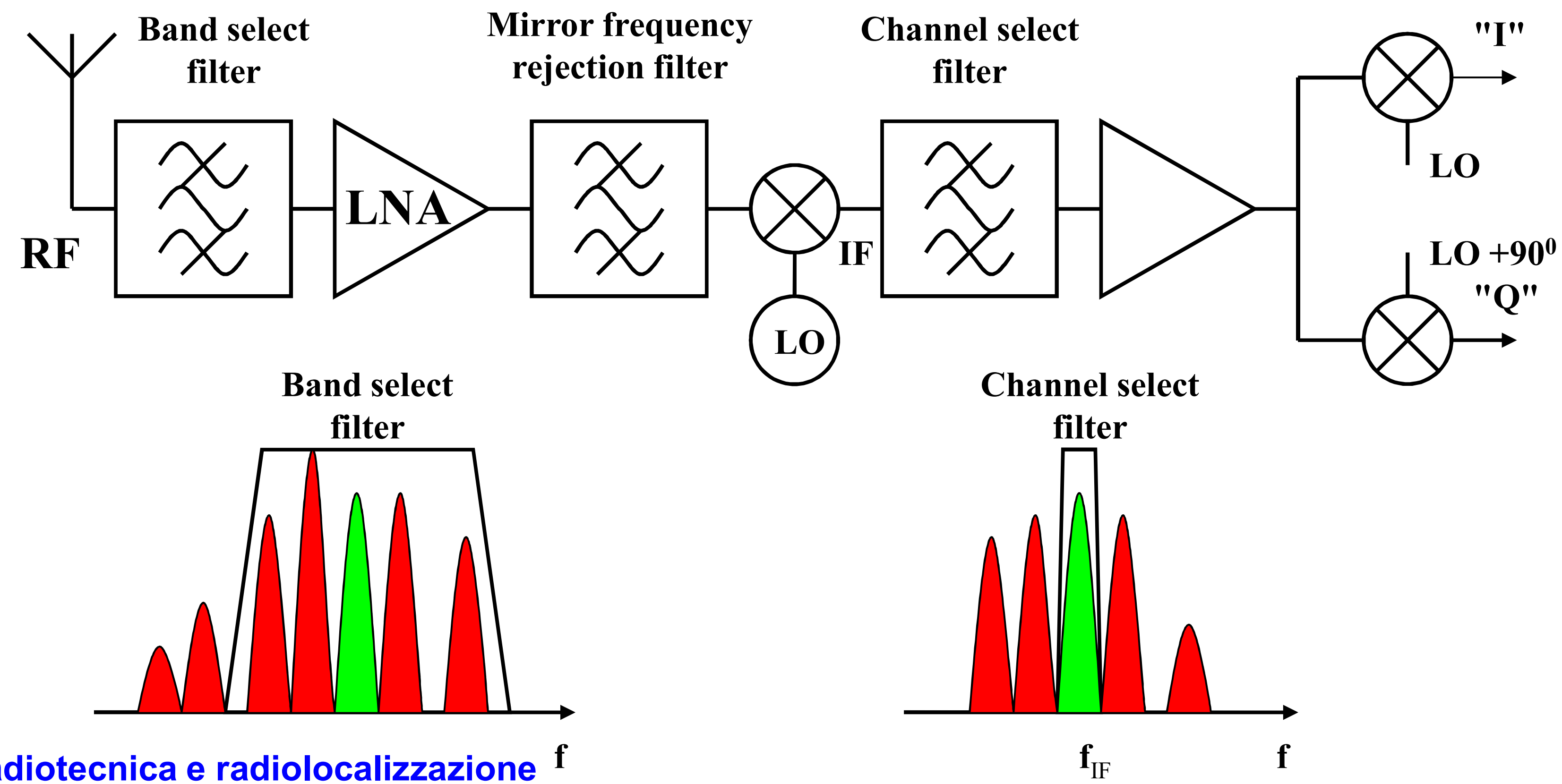
Echi appaiati

Pierfrancesco Lombardo

Schema di ricevitore eterodina

Due stadi di conversione:

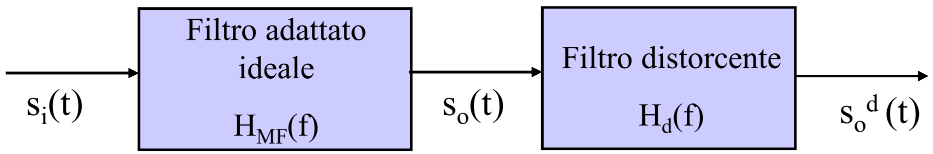
- **2° stadio ad IF :** ***amplificazione e filtraggio a frequenza costante***
- **1° stadio a RF :** ***capacità di selezionare una frequenza da una banda più ampia***



Distorsioni lineari (I)

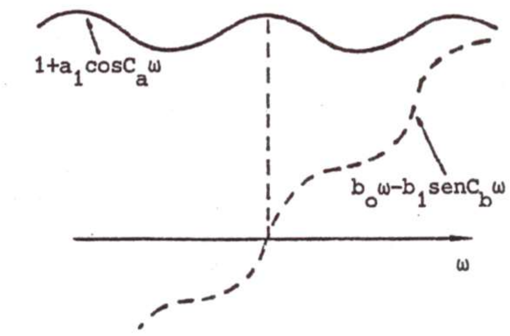
Effetto delle distorsioni

- Il sistema reale sarà affetto da distorsioni (non sarà esattamente uguale a quello ideale): tutte le distorsioni di sistema possono essere sintetizzate in un filtro distorcente posto in cascata al filtro adattato ideale:



- Nell'ipotesi di piccole distorsioni la $H_d(f)$ può essere sviluppata in serie arrestandosi al primo termine

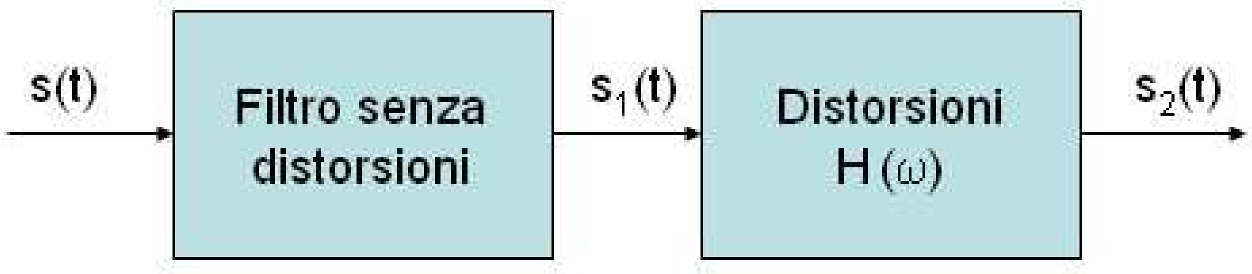
$$H_d(f)=A(f)e^{jB(f)} \rightarrow \begin{cases} A(f)=1+a_1\cos(2\pi C_a f) \\ e^{jB(f)}=e^{jb_1 sin(2\pi C_b f)} \cong 1+jb_1 sin(2\pi C_b f) \end{cases}$$



a_1: valore di picco della componente di ampiezza;
b_1: valore di picco della componente di fase;
C_a: frequenza ripple di ampiezza;
C_b: frequenza ripple di fase;

Echi appaiati (I)

Catena ricevente complessivamente



$$H(\omega) = A(\omega) \cdot e^{-jB(\omega)}$$

$$A(\omega) = a_0 + a_1 \cos(C_1\omega)$$
$$B(\omega) = b_0\omega - b_1 \sin(C_1\omega)$$

$$s_2(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega) \cdot [a_0 + a_1 \cos(C_1\omega)] \cdot e^{[j\omega(t-b_0)+jb_1 \sin(C_1\omega)]} d\omega$$

Echi appaiati (II)

$$s_2(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)\cdot[a_0 + a_1\cos(C_1\omega)]\cdot e^{[j\omega(t-b_0)+jb_1\sin(C_1\omega)]}d\omega$$

Distorsione di ampiezza (b_1=0)

$$s_2(t) = a_0 s_1(t') + \frac{a_1}{2}[s_1(t'+C_1) + s_1(t'-C_1)]$$

Distorsione di fase (a_1=0)

valori di b_1 piccoli (<0.5 radianti)

$$e^{jb_1\sin x} = J_0(b_1) + \sum_{n=1}^{\infty} J_n(b_1)\cdot\left\{e^{jnx} + (-1)^n\cdot e^{-jnx}\right\} \approx 1 + \frac{b_1}{2}\left\{e^{jx} - e^{-jx}\right\}$$

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$$s_2(t) = a_0\left\{s_1(t') + \frac{b_1}{2}[s_1(t'+C_1) - s_1(t'-C_1)]\right\}$$

Echi appaiati (III)

$$s_2(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)\cdot[a_0 + a_1\cos(C_1\omega)]\cdot e^{[j\omega(t-b_0)+jb_1\sin(C_1\omega)]}d\omega$$

Distorsione di fase (a_1=0)

$$e^{jb_1\sin x} = J_0(b_1) + \sum_{n=1}^{\infty} J_n(b_1)\cdot\left\{e^{jnx} + (-1)^n\cdot e^{-jnx}\right\}$$

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$$s_0(t) = J_0(b_1)\bar{s}_1(t)\cos(\omega_0 t) + \sum J_n(b_1)\left\{\bar{s}_1\left(t + \frac{n\omega_{m2}T}{\Delta\omega}\right)\cos\left((\omega_0 + \frac{\omega_{m2}}{2})t + n\theta_0\right) + \right.$$

$$\left. +(-1)^n\bar{s}_1\left(t - \frac{n\omega_{m2}T}{\Delta\omega}\right)\cos\left((\omega_0 - \frac{\omega_{m2}}{2})t - n\theta_0\right)\right\}$$

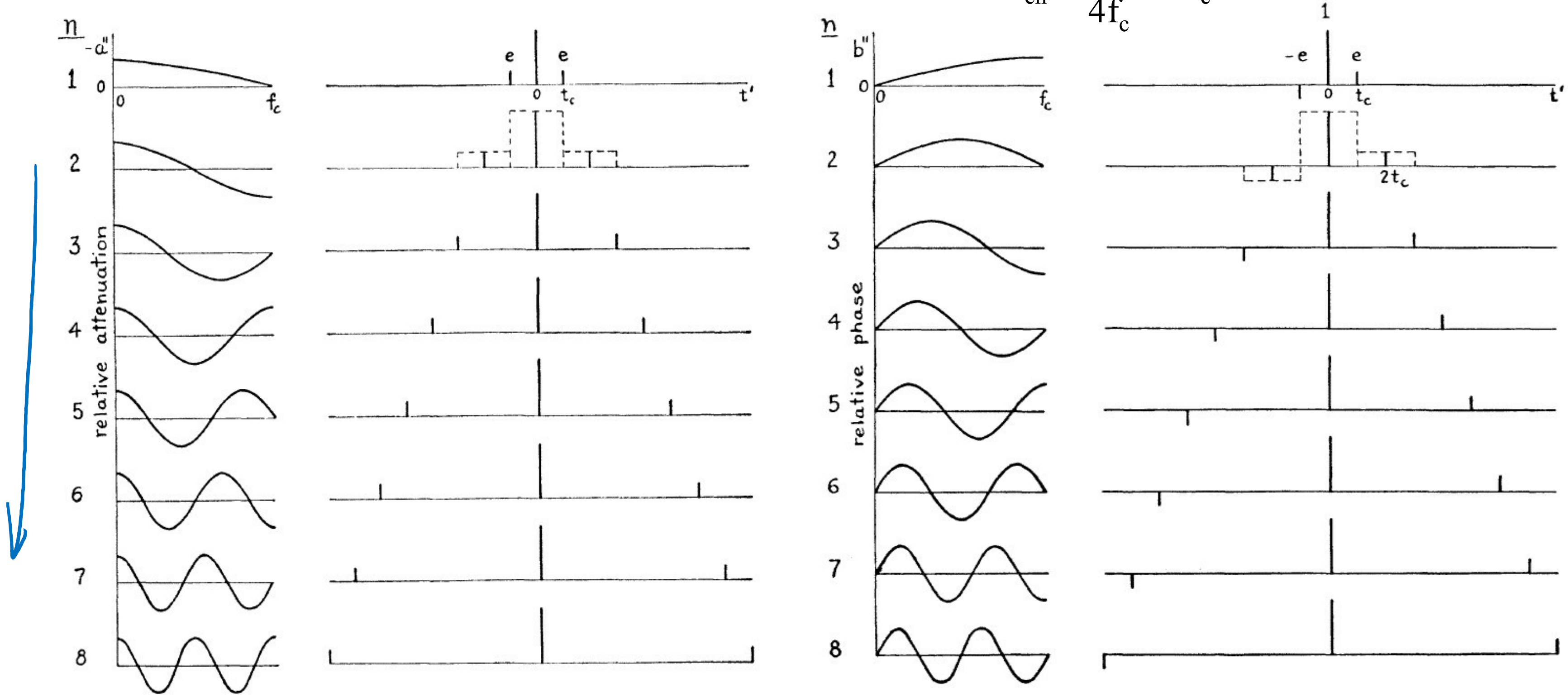
Echi appaiati (IV)

$$a_n = -2e_n \cos\left(\frac{n\pi f}{2f_c}\right)$$

Distorsioni di ampiezza (a) e fase (b) e corrispondenti coppie di echi appaiati

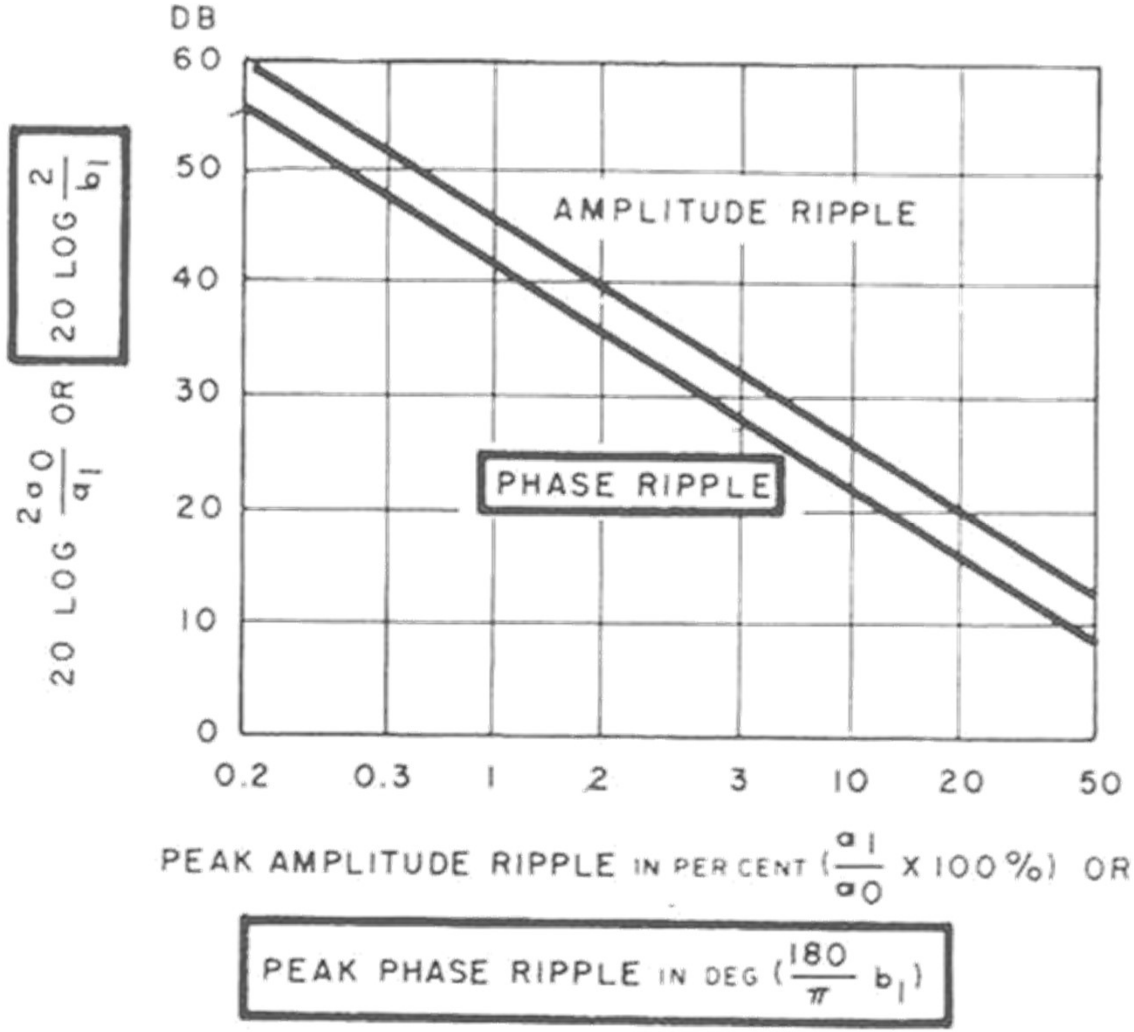
$$b_n = 2e_n \sin\left(\frac{n\pi f}{2f_c}\right)$$

$$t_{en} = \frac{n}{4f_c} = nt_c$$



Echi appaiati (V)

Distorsioni di ampiezza (a) e fase (b) e corrispondenti coppie di echi appaiati



Echi appaiati (VI)

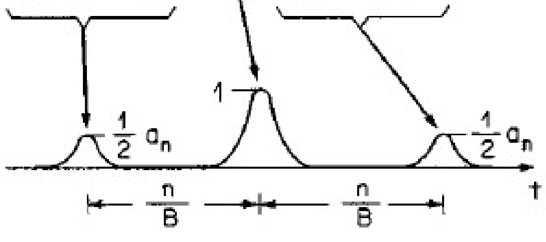
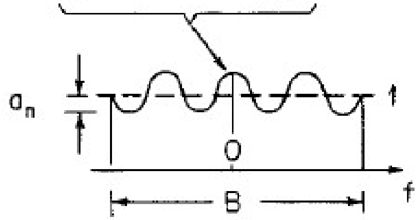
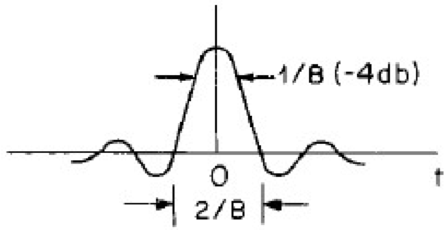
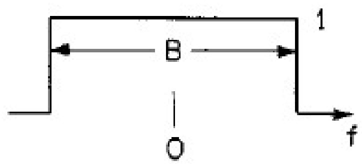
Paired Echoes and Weighting. A description of the weighting process is facilitated by the application of paired-echo theory.[36–39] The first seven entries in Table 10.7 provide a step-by-step development of Fourier transforms useful in frequency and time weighting, starting with a basic transform pair. The last entry pertains to phase-distortion echoes. The spectrum $G(f)$ of the time function $g(t)$ is assumed to have negligible energy outside the frequency interval $-B/2$ to $+B/2$, where B is the bandwidth in hertz. The transform pairs of Table 10.7 are interpreted as follows:

Pair 1. Cosinusoidal amplitude variation over the passband creates symmetrical paired echoes in the time domain in addition to the main signal $g(t)$, whose shape is uniquely determined by $G(f)$. The echoes are replicas of the main signal, delayed and advanced from it by n/B s and scaled in amplitude by $a_n/2$.

Pair 2. The rectangular frequency function $W_0(f)$, that is, uniform weighting over the band, leads to a $(\sin x)/x$ time function $w_0(t)$ with high-level sidelobes, which can be objectionable in some cases. A normalized logarithmic plot of the magnitude of this time function is shown by curve A in Fig. 10.15. (All functions illustrated are symmetrical about $t = 0$.) The sidelobe adjacent to the main lobe has a magnitude of -13.2 dB with respect to the main-lobe peak. The sidelobe falloff rate is very slow.

Echi appaiati (VII)

TABLE 10.7 Paired-Echo and Weighting Transforms

$g(t)=\int_{-\infty}^{\infty} G(f)\exp(j2\pi ft)\,df$	$G(f)=\int_{-\infty}^{\infty} g(t)\exp(-j2\pi ft)\,dt$
PAIRED ECHOES: 1. $\frac{a_n}{2}g\left(t+\frac{n}{B}\right)+g(t)+\frac{a_n}{2}g\left(t-\frac{n}{B}\right)$ 	n AMPLITUDE RIPPLES: $G(f)\left[1+a_n\cos 2\pi n\frac{f}{B}\right]$  (REFS. 36–39)
HIGH SIDELOBES (−13.2db): 2. $w_0(t)=B\frac{\sin \pi Bt}{\pi Bt}$ 	UNIFORM WEIGHTING: $W_0(f)=\begin{cases}1 & \lvert f\rvert < \frac{1}{2}B \\ 0 & \lvert f\rvert > \frac{1}{2}B\end{cases}$ 

Echi appaiati (VIII)

Pair 3. Taper is applied by introducing one amplitude ripple ($n = 1$) in the frequency domain to form $W_1(f)$. By pairs 1 and 2, the time function is the superposition of the three time-displaced and weighted $(\sin x)/x$ functions.[39] Low time sidelobes are attainable in the resultant function $w_1(t)$ by the proper choice of the coefficient F_1. In particular, $F_1 = 0.426$ corresponds to Hamming weighting[40–42] and to the time function whose magnitude is represented by the solid curve *B* in Fig. 10.15.

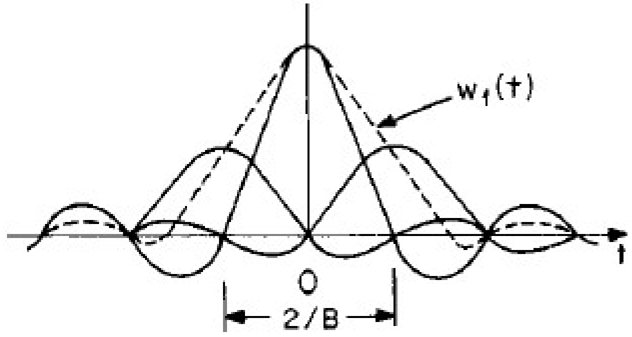
Pair 4. The frequency-weighting function includes a Fourier series of $\bar{n} - 1$ cosine terms, where the selection of $\bar{n}$ is determined by the required compressed pulse width and the desired sidelobe falloff. By pairs 1 and 2, the time function includes the superposition of $2(\bar{n} - 1)$ echoes that occur in $\bar{n} - 1$ symmetrical pairs. If the coefficients F_m are selected to specify the Taylor weighting function[39,42,43] $W_{\mathrm{Tay}}(f)$, the corresponding resultant time function $w_{\mathrm{Tay}}(t)$ exhibits good resolution characteristics by the criterion of small main-lobe width for a specified sidelobe level. Taylor coefficients chosen for a -40 dB sidelobe level, with $\bar{n}$ selected as 6, lead to the main-sidelobe structure indicated by curve *C* of Fig. 10.15.

Echi appaiati (IX)

LOW SIDELOBES:

3. $w_1(t) =$

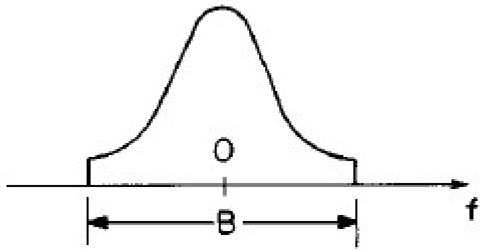
$$F_1 w_0\left(t+\frac{1}{B}\right) + w_0(t) + F_1 w_0\left(t-\frac{1}{B}\right)$$



TAPER:

$W_1(f) =$

$$W_0(f)\left[1+2F_1\cos 2\pi\frac{f}{B}\right]$$



(REFS. 39 – 42)

4. $$w_{Tay}(t) = \sum_{m=-\infty}^{\infty} F_m w_0\left(t-\frac{m}{B}\right)$$

where

$F_0 = 1$, $F_m = 0$ for $|m| \geqq \bar{n}$

and

$F_m = F_{-m}$

TAYLOR WEIGHTING:

$W_{Tay}(f) =$

$$W_0(f)\left[1+2\sum_{m=1}^{\bar{n}-1} F_m \cos 2\pi m\frac{f}{B}\right]$$

(REFS. 39,42,43)

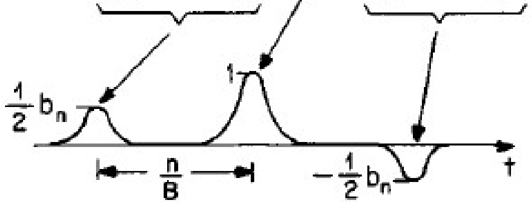
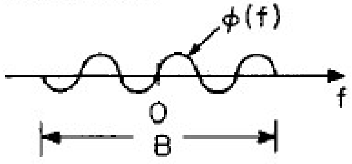
Echi appaiati (X)

Pairs 5 to 7. The duality theorem 5 permits the interchange of time and frequency functions in each of the preceding pairs. Functions may be interchanged if the sign of the parameter t is reversed. Examples are pairs 6 and 7 obtainable from pairs 2 and 4 with the substitution of T s for B Hz. Taylor time weighting is applied in pair 7 to achieve good frequency resolution when the coefficients are selected for a specified sidelobe level.

Pair 8. Similarly to the amplitude variations of pair 1, sinusoidal phase variation over the passband creates symmetrical paired echoes in the time domain in addition to the main signal $g(t)$. The echoes are replicas of the main signal, delayed and advanced from it by n/B s, scaled in amplitude by $b_n/2$, and opposite in polarity.

Echi appaiati (XI)

DUALITY THEOREM:

5. $G(-t)$	$g(f)$
6. $W_0(t) = \begin{cases} 1 & \lvert t \rvert < \frac{T}{2} \\ 0 & \lvert t \rvert > \frac{T}{2} \end{cases}$	$w_0(f) = T \frac{\sin \pi f T}{\pi f T}$
7. $W_{Tay}(t) =$ $W_0(t)\left[1 + 2\sum_{m=1}^{\bar{n}-1} F_m \cos 2\pi m \frac{t}{T}\right]$	$w_{Tay}(f) = \sum_{m=-\infty}^{\infty} F_m\, w_0\left(f - \frac{m}{T}\right)$ (SEE PAIR No. 4)
PAIRED ECHOES: 8. $\frac{b_n}{2} g\left(t + \frac{n}{B}\right) + g(t) - \frac{b_n}{2} g\left(t - \frac{n}{B}\right)$ 	n PHASE RIPPLES: $G(f)\, e^{\,j b_n \sin 2\pi n \frac{f}{B}} \cong$ $\left[1 + j b_n \sin 2\pi n \frac{f}{B}\right] G(f)$ $\lvert b_n \rvert < 0.4$ radian  (REFS. 36 − 39)