Equazione Radar e filtro adattato

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Matched filtering (I)

- If the radar transmission is coherent, it is possible to transmit a long duration pulse (with the advantage of reduced peak power) and compress the echoes received to resemble the echoes from a very short pulse (with the advantages of good range resolution).

$|S_{out}(t)|^2$

Matched filter characteristics

1) A matched filter gives the best possible Signal/Noise power ratio (S/N) at Receiver output. This S/N ratio is proportional to the energy contained in the signal waveform.

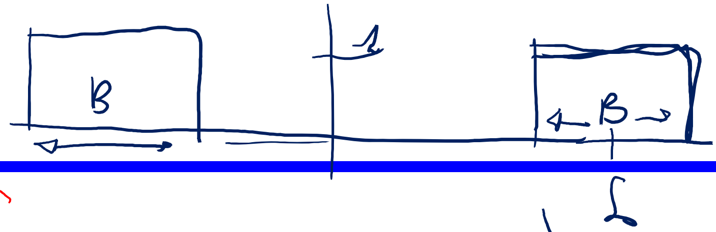
2) A receiver can be matched to a transmitted waveform by making its impulse response the time reverse of that waveform.

Ie. $y(t)_{FILTER} = s^*(-t)_{SIGNAL}$ it follows that: $Y(\omega)_{FILTER} = S^*(\omega)_{SIGNAl}$

(Receiver frequency response is complex conjugate of transmit signal spectrum)

3) The output waveform of a matched filter receiver is the delayed autocorrelation of the transmitted signal waveform.

Matched filtering (II)



$s(t)$ Input signal

$y(t)$ Output signal

$h(t)$ Impulse response

t_1 time of maximum

$B_n = B$

Signal output power at time t_1

$$P_{S_{out}} = |y(t_1)|^2 = \left| \int_{-\infty}^{+\infty} Y(f) e^{j2\pi f t_1} df \right|^2 = \left| \int_{-\infty}^{+\infty} S(f) H(f) e^{j2\pi f t_1} df \right|^2$$

Noise output power

$$N_{out} = \int_{-\infty}^{+\infty} N(f) |H(f)|^2 df = \frac{N_0}{2} \int_{-\infty}^{+\infty} |H(f)|^2 df$$

With noise power spectral density $N(f)$

$P_n = \frac{KT_0}{2} \cdot 2 B_n$

Matched filtering (III)

$$\left.\frac{S}{N}\right|_{out} = \frac{1}{2}\frac{\left|\int_{-\infty}^{+\infty} S(f)H(f)e^{j2\pi f t_1}df\right|^2}{\int_{-\infty}^{+\infty} N(f)|H(f)|^2 df} \Rightarrow \left.\frac{S}{N}\right|_{out} = 2\frac{\left|\int_{-\infty}^{+\infty} S(f)H(f)e^{j2\pi f t_1}df\right|^2}{N_0\int_{-\infty}^{+\infty} |H(f)|^2 df}$$

$$N(f) = \frac{N_0}{2}$$

$$\left|\int f \cdot g \, dt\right|^2 \leq \int |f|^2 dt \cdot \int |g|^2 dt$$

Schwartz inequality:

$$\left.\frac{S}{N}\right|_{out} \leq \frac{2\left(\int_{-\infty}^{+\infty} |S(f)|^2 df\right)\cancel{\left(\int_{-\infty}^{+\infty} |H(f)|^2 df\right)}}{N_0 \cancel{\int_{-\infty}^{+\infty} |H(f)|^2 df}} = \frac{2E}{N_0}$$

$$f = g^*$$

$$\left|\int |f|^2 dt\right|^2 = \int |f|^2 dt \cdot \int |f|^2 dt$$

Matched filtering (IV)

$$\left.\frac{S}{N}\right|_{out} = \frac{\left|\int_{-\infty}^{+\infty} S(f)H(f)e^{j2\pi f t_1}df\right|^2}{N_0\int_{-\infty}^{+\infty}|H(f)|^2 df} = \frac{E}{N_0}$$

Obtained for

$$\boxed{H(f) = G\cdot S^*(f)e^{-j2\pi f t_1}}$$

$$h(t) = G\cdot s^*(t_1 - t)$$

$$N_{out} = N_0\int_{-\infty}^{+\infty}|H(f)|^2 df = G^2 N_0\int_{-\infty}^{+\infty}|S(f)|^2 df = G^2 E_s\ N_0$$

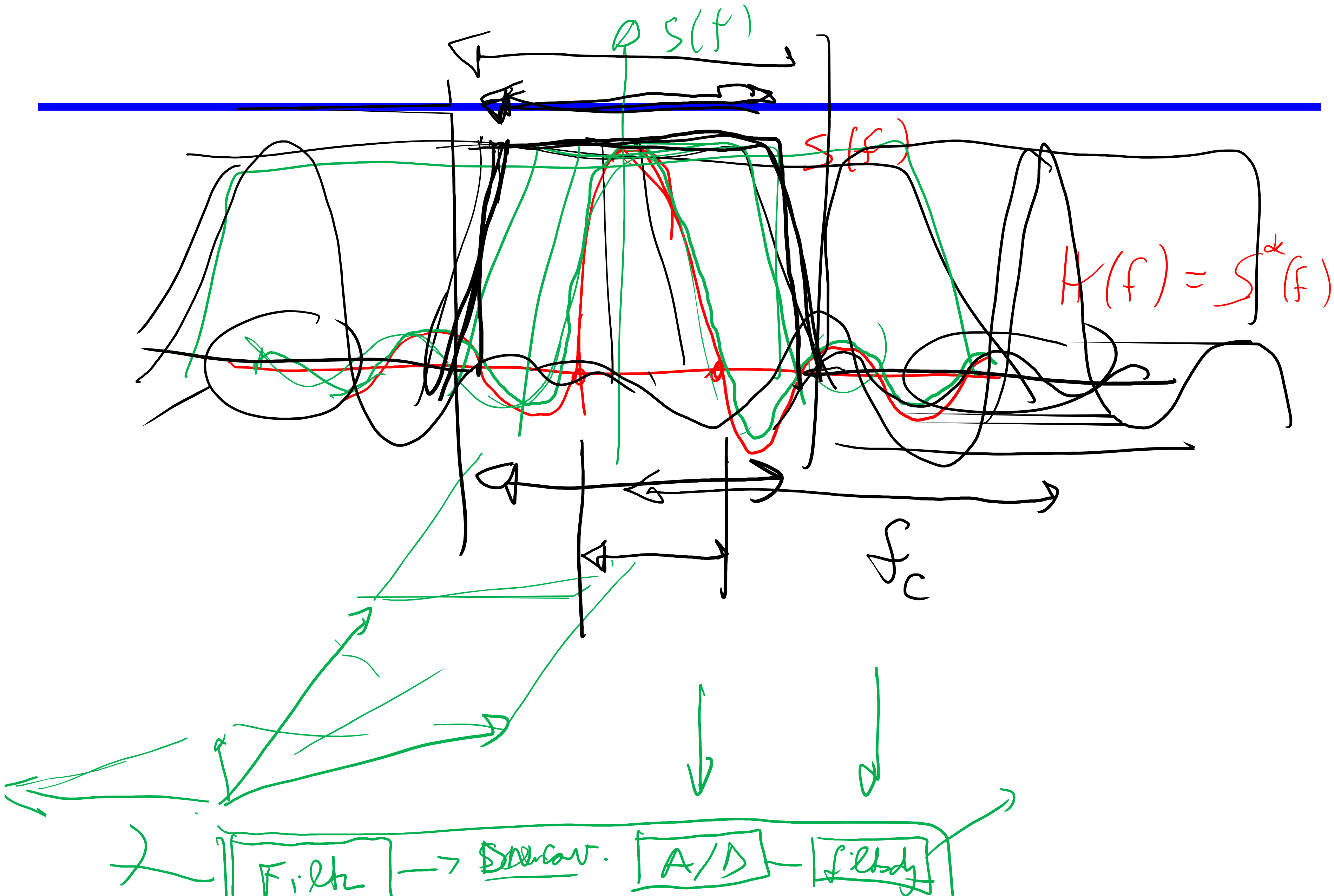
$$|y(t_1)|^2 = \left|\int_{-\infty}^{+\infty} S(f)H(f)e^{j2\pi f t_1}df\right|^2 = G^2\left|\int_{-\infty}^{+\infty}|S(f)|^2 df\right|^2 = G^2|E_s|^2$$

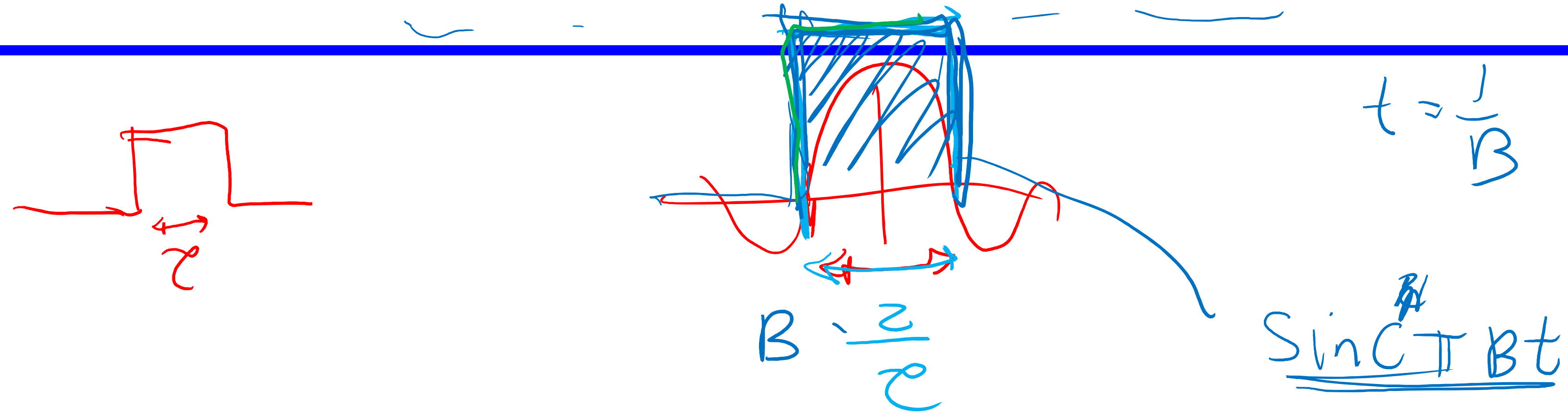
To maintain output noise power level equal to input use

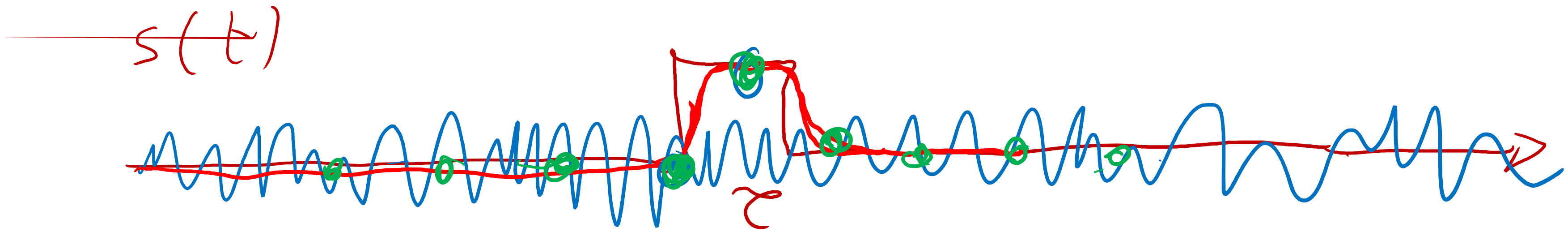
- especially important in digital matched filter implementation to use bit dynamics

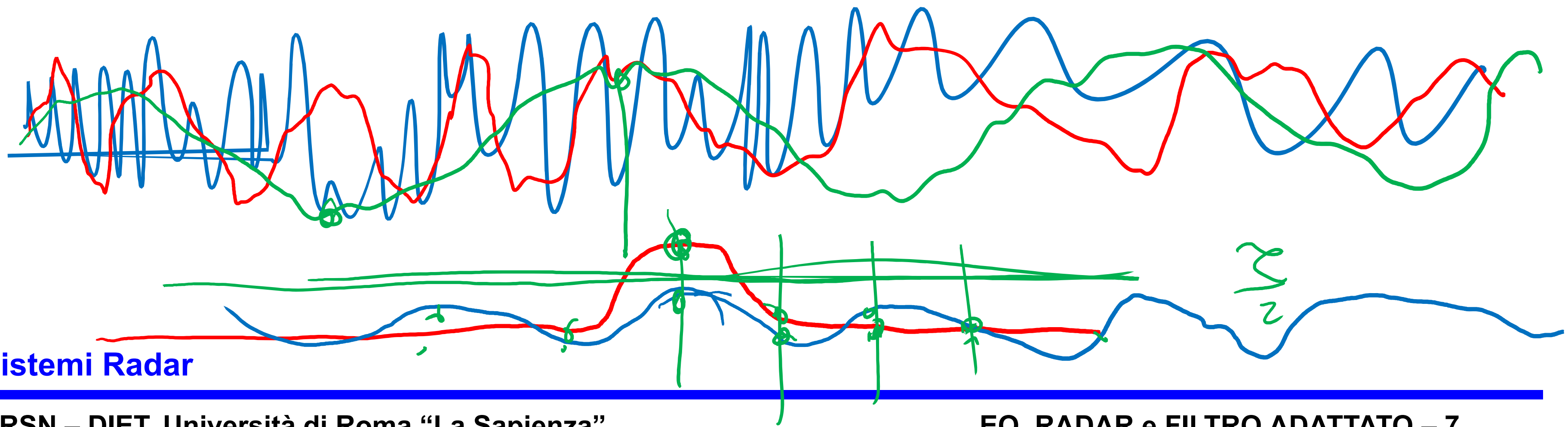
$$G^2 E_s = 1$$

$$\Rightarrow\ G = \sqrt{E_s}$$

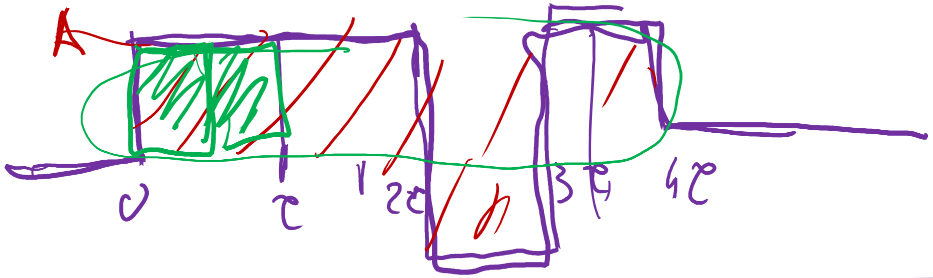

S(f)
S(f)
H(f) = S*(f)
f_c
Filtr
A/D

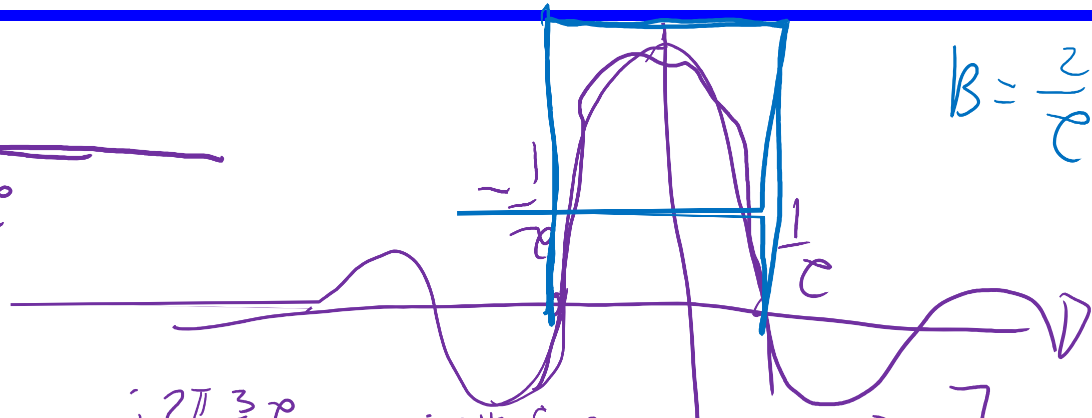

τ
B ≈ 2/τ
t = 1/B
sinc πBt


s(t)
τ


τ/2

$s(t)$ $\qquad\qquad\qquad$ $S(f)$

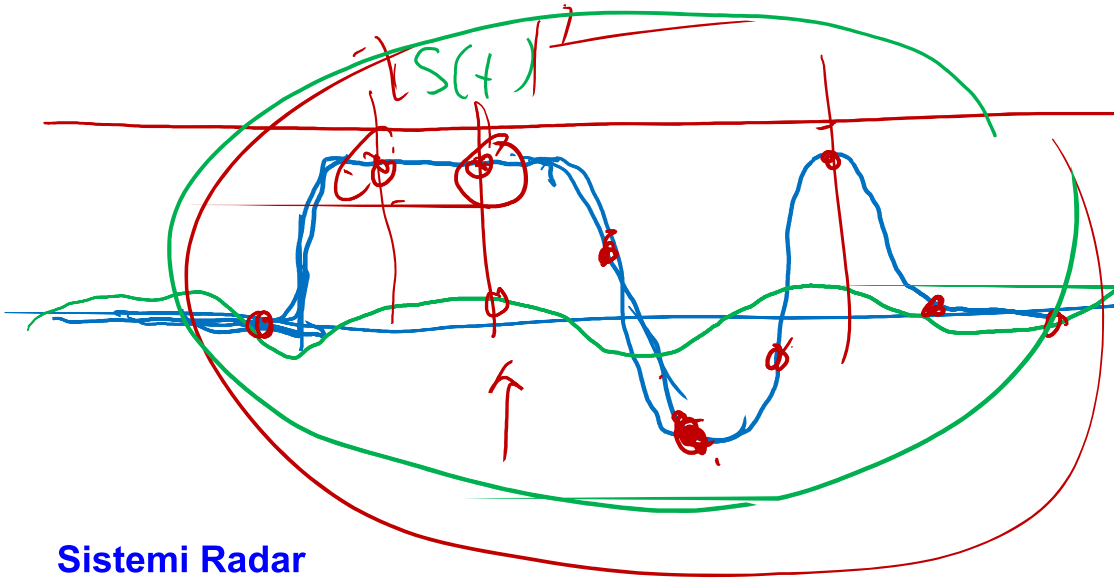




$B = \frac{2}{\tau}$

$$\mathrm{sinc}(\pi f \tau)\cdot\left[e^{-j2\pi\frac{\tau}{2}} + e^{-j2\pi\frac{3}{2}\tau} - e^{-j2\pi\frac{5}{2}\tau} + e^{-j\pi\frac{7}{2}\tau}\right]$$

$A = \sqrt{P_r}$



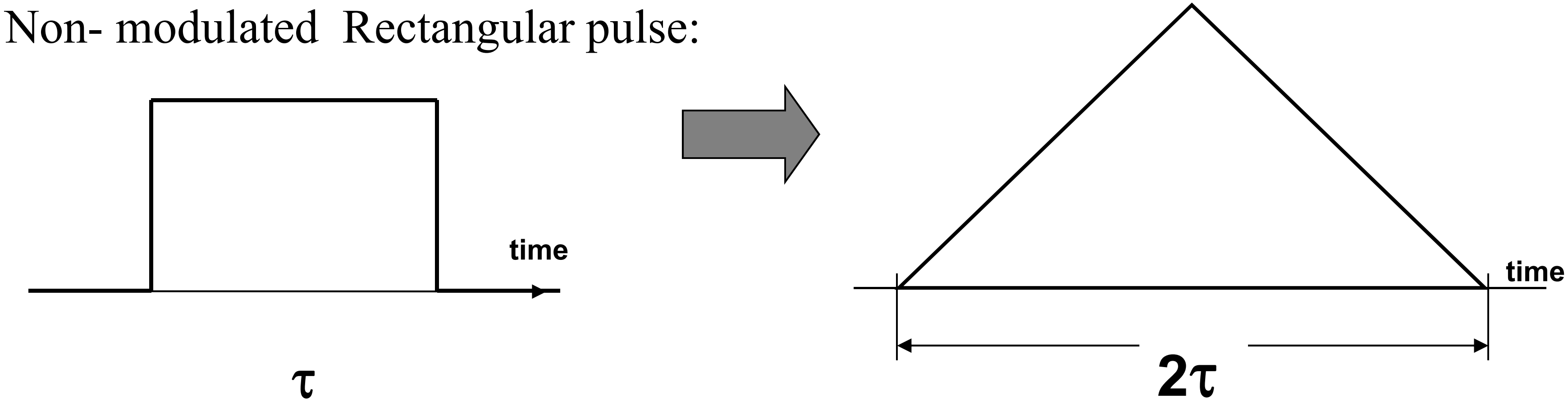
$$SNR = \frac{P_{ist}}{P_{nois}} = \frac{2A^2}{N_0 B}$$

$$SNR = \frac{P_{ist}}{P_{noise}} = \frac{\cancel{2}A^2\tau}{N_0\,\cancel{2}}$$

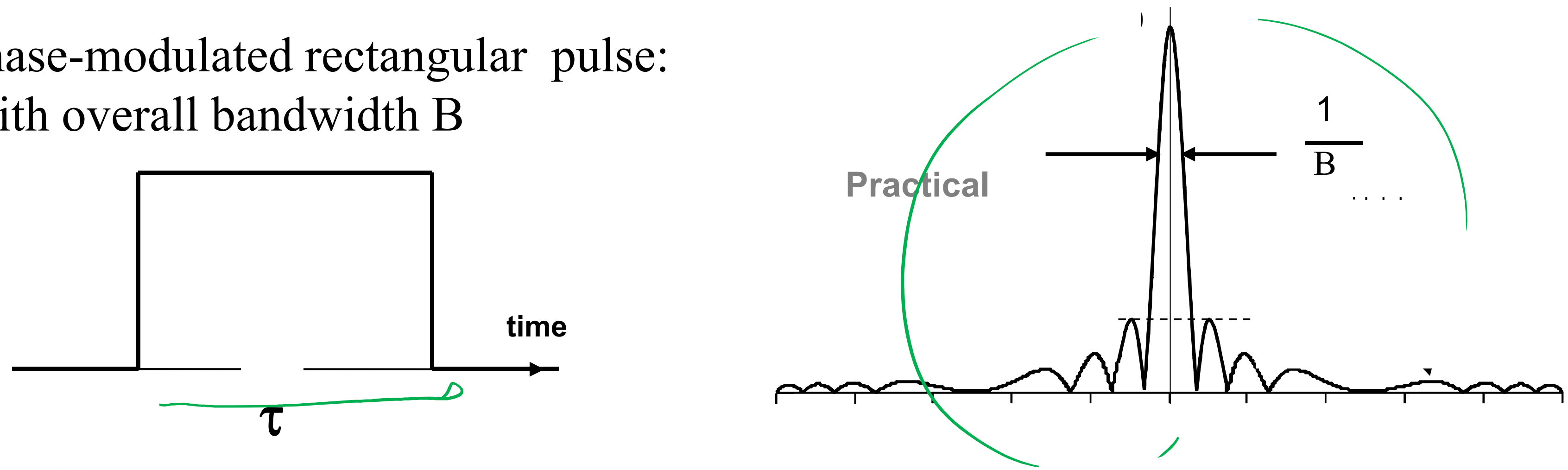
Sistemi Radar

Matched filtering (V)

Non- modulated Rectangular pulse:

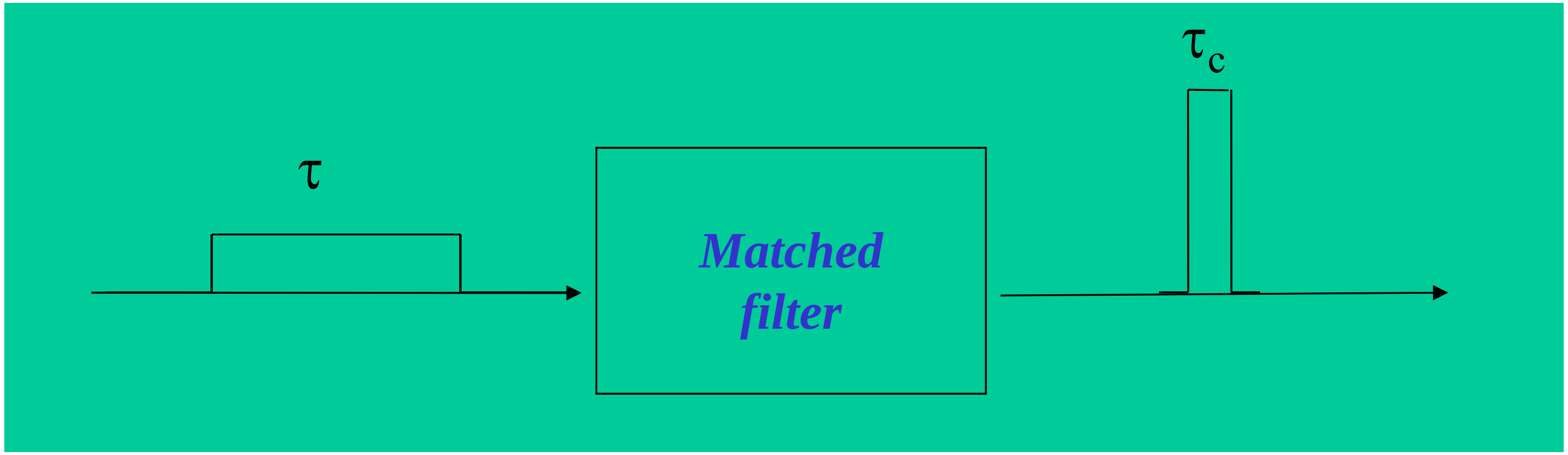


Phase-modulated rectangular pulse:
With overall bandwidth B



Matched filtering (VI)

- We want to transmit a long pulse (length τ) to get energy on target
- But we want a short compressed output (length $\tau_c \approx 1/B$) and larger amplitude:



Passive Filter $\longrightarrow$ principle of energy conservation:

$$E_o = E_i \quad \Longrightarrow \quad P_{po}\,\tau_c = P_{pi}\,\tau$$

Matched filtering (VII)

Passive Filter

principle of energy conservation:

$$E_o = E_i \Longrightarrow P_{po}\, \tau_c = P_{pi}\, \tau$$

$$P_{po} = P_{pi} \quad \tau / \tau_c = P_{pi} \quad B\tau$$

Noise power level does not change

Signal power improved with respect to noise power of

$$\tau / \tau_c = P_{pi} \quad B\tau$$

COMPRESSION RATIO

Equazione Radar con compressione

Varie forme dell'Equazione Radar

L'equazione radar, e di conseguenza l'espressione della portata, può essere particolarizzata in dipendenza delle applicazioni

— E

$$\boxed{\left(\frac{S}{N}\right)_r = \frac{P_r}{P_n} = \frac{P_t G A_e \sigma}{\left(4\pi R^2\right)^2 k T_0 \cancel{B} F\, L}\, B\tau = \frac{P_t G A_e \sigma\ \tau}{\left(4\pi R^2\right)^2 k T_0 F\, L}}$$

1 Si suppone di aver fissato il massimo valore del guadagno d'antenna G (vincolo la larghezza del fascio e quindi la risoluzione angolare): utilizzando $G=4\pi A_e/\lambda^2$

$$\left(\frac{S}{N}\right)_r = \frac{P_t G^2 \lambda^2 \sigma\, \tau}{\left(4\pi\right)^3 R^4 k T_0 F\, L} \Rightarrow R_{\max} = \left[\frac{P_t G^2 \lambda^2 \sigma\ \tau}{\left(4\pi\right)^3 k T_0 B F\, L \left(S/N\right)_{\min}}\right]^{1/4}$$

Preferibili le basse frequenze

2 Si suppone di aver fissato il massimo valore dell'area geometrica e quindi efficace d'antenna A_e: utilizzando $A_e = \lambda^2 G/ 4\pi$

$$\left(\frac{S}{N}\right)_r = \frac{P_t A_e^2 \sigma\, \tau}{4\pi R^4 \lambda^2 k T_0 F\, L} \Rightarrow R_{\max} = \left[\frac{P_t A_e^2 \sigma\, \tau}{4\pi \lambda^2 k T_0 F\, L \left(S/N\right)_{\min}}\right]^{1/4}$$

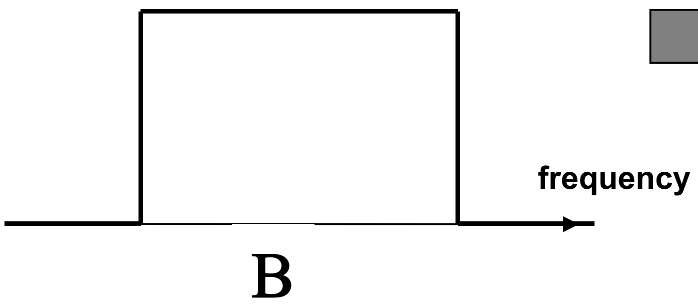
Preferibili le alte frequenze

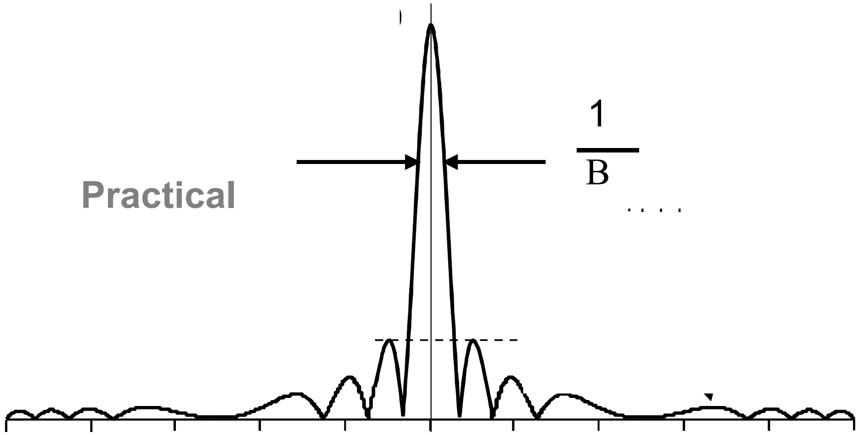
Pulse compression yields Sidelobes

Output is signal autocorrelation:

$$|y(t)|^2 = \left| \int_{-\infty}^{+\infty} S(f)H(f)e^{j2\pi ft}df \right|^2 = G^2 \left| \int_{-\infty}^{+\infty} |S(f)|^2 e^{j2\pi f(t-t_1)}df \right|^2 = G^2 |R_{ss}(t-t_1)|^2$$

Approximately fully occupied spectrum with overall bandwidth B

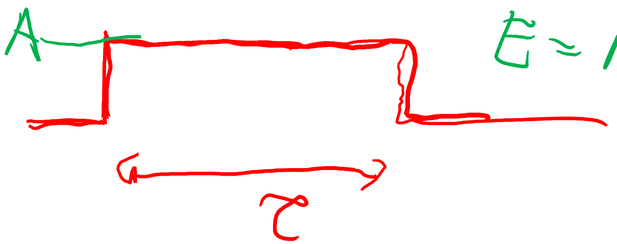




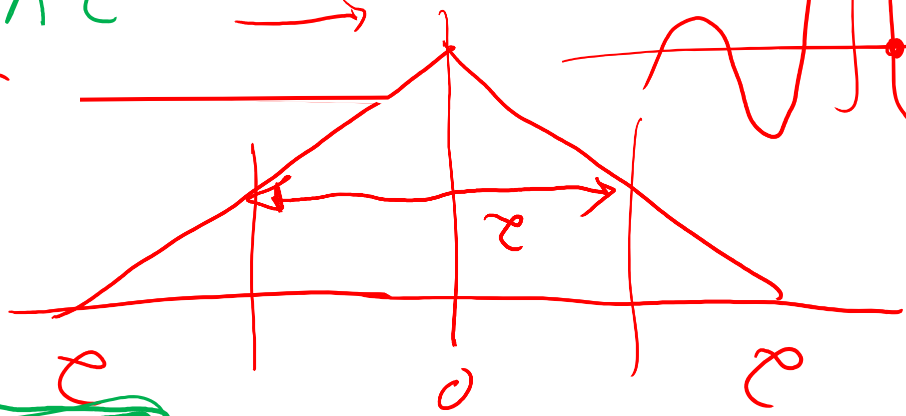
E' necessario un controllo dei lobi laterali: pesatura?

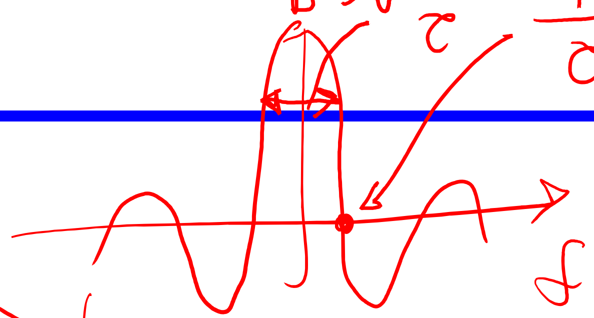
$s(t) = \mathrm{rect}_\tau(t)$

$B \simeq \frac{1}{\tau}$ $\quad \frac{1}{\tau}$



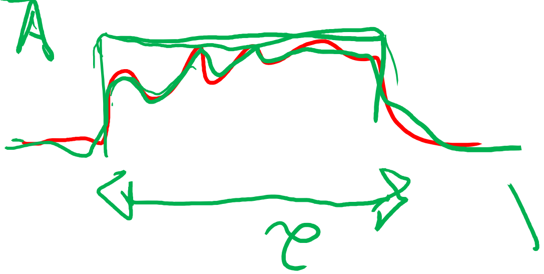
$E \simeq A^2\tau$





FA $\longrightarrow$

$$\boxed{s(t) = \mathrm{rect}_\tau(t)\, e^{j\phi(t)}}$$



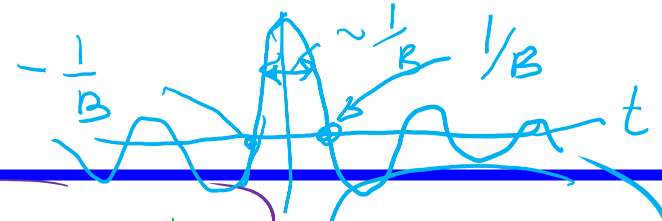
$B > \frac{1}{\tau}$

$|s(t)| = A$

$E = |s(t)|^2 \cdot \tau$

?

Sistemi Radar



$s(t) \rightarrow \boxed{FA} \rightarrow y(t) = R_{ss}(t) = \text{sinc}\,\pi t B$

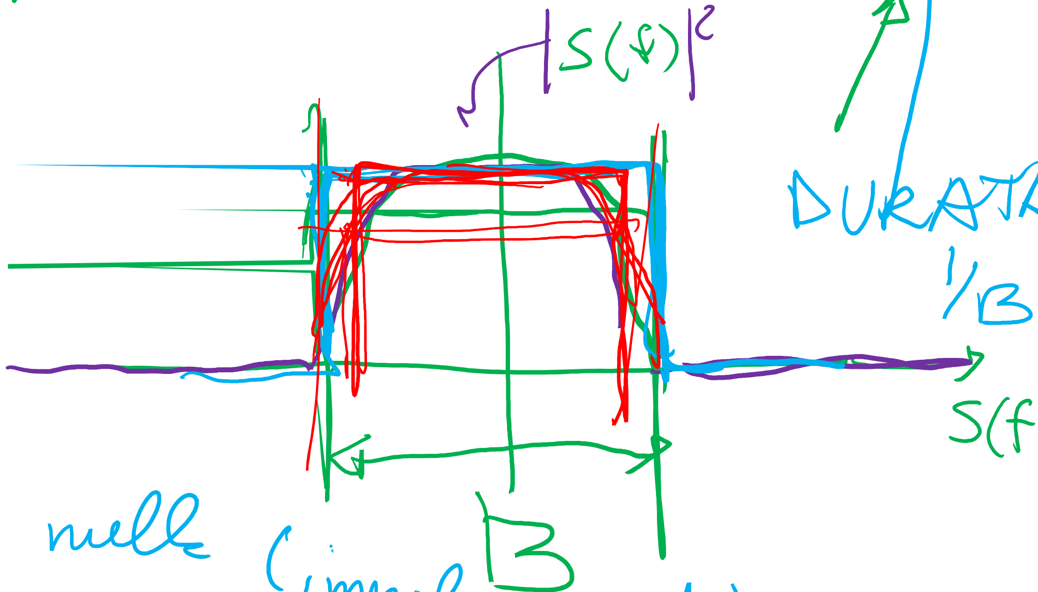
$S(f)$

durata τ

$H(f) = S^*(f)$

$Y(f) = S(f) \cdot S^*(f) = |S(f)|^2$

DURATA $1/B$



Se $B = \frac{1}{\tau}$ non cambia nulla (impulso rect)

Se $B > \frac{1}{\tau}$ Compressione $= \frac{\tau}{1/B} = \tau B \gg 1$

Sistemi Radar

$\frac{c}{2}\tau$

$\frac{1}{B} = 1\mu s$

$\cancel{P_t \cdot \tau = 87 \cdot 10^3 \cdot 10^{-6}}$

ris 150 m

$\tau \simeq 1\mu s$

$\cancel{P_t = 87kW}$

$\div 100$

150km con imp rect

$\tau \to 100\mu s$

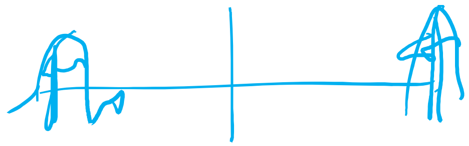
870kW

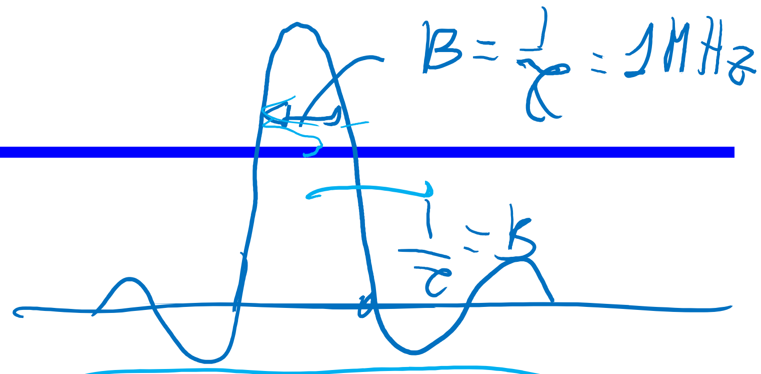
dopo compressione ris in dista $= \frac{c}{2} \cdot \frac{1}{B} = 150m$

$\tau = \frac{1}{1MHz} = 1\mu s$

$\frac{c}{2B} = 150m \to B = \frac{3 \cdot 10^8 \, m/s}{2 \cdot 150 \, m} = 10^6 \, 1/s = 1MHz$

Sistemi Radar





$B = \frac{1}{\tau} = 1\,MHz$

$\frac{1}{\tau} = B$

$\tau = 1\,\mu s$

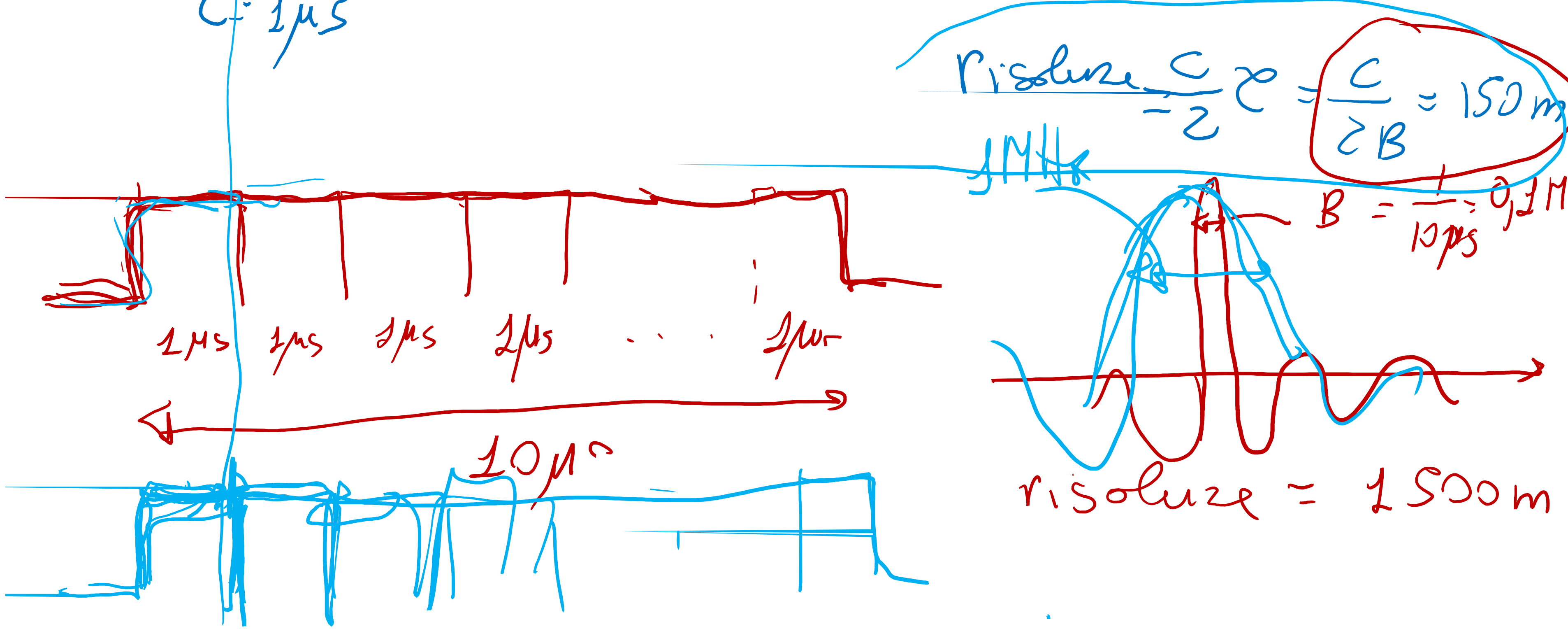
risoluz $= \frac{c}{2}\tau = \frac{c}{2B} = 150\,m$

$1\,MHz$

$B = \frac{1}{10\,\mu s} = 0{,}1\,MHz$

1μs 1μs 1μs 1μs . . . 1μs

10μs



risoluz $= 1500\,m$