
Equazione Radar e filtro adattato

Pierfrancesco Lombardo

Matched filtering (I)

- If the radar transmission is coherent, it is possible to transmit a long duration pulse (with the advantage of reduced peak power) and compress the echoes received to resemble the echoes from a very short pulse (with the advantages of good range resolution).

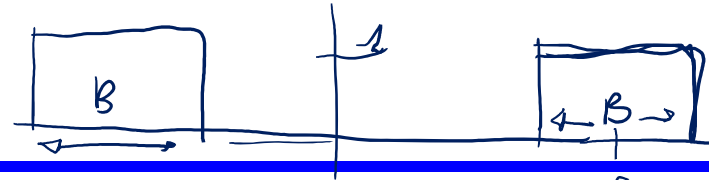
$$\underline{|S(t)|^2}$$

Handwritten note: "out" written below the expression.

Matched filter characteristics

- 1) A matched filter gives the best possible Signal/Noise power ratio (S/N) at Receiver output. This S/N ratio is proportional to the energy contained in the signal waveform.
- 2) A receiver can be matched to a transmitted waveform by making its impulse response the time reverse of that waveform.
ie. $y(t)_{\text{FILTER}} = s^*(-t)_{\text{SIGNAL}}$ it follows that: $Y(\omega)_{\text{FILTER}} = S^*(\omega)_{\text{SIGNAL}}$
(Receiver frequency response is complex conjugate of transmit signal spectrum)
- 3) The output waveform of a matched filter receiver is the delayed autocorrelation of the transmitted signal waveform.

Matched filtering (II)



$s(t)$ Input signal
 $y(t)$ Output signal

$h(t)$ Impulse response
 t_1 time of maximum

$B_n = B$

Signal output power at time t_1

$$\rightarrow P_{S_{out}} = |y(t_1)|^2 = \left| \int_{-\infty}^{+\infty} Y(f) e^{j2\pi f t_1} df \right|^2 = \left| \int_{-\infty}^{+\infty} S(f) H(f) e^{j2\pi f t_1} df \right|^2$$

Noise output power

$$\rightarrow N_{out} = \int_{-\infty}^{+\infty} N(f) |H(f)|^2 df = \left(\frac{N_0}{2} \right) \int_{-\infty}^{+\infty} |H(f)|^2 df$$

With noise power spectral density $N(f)$

$$P_n = \frac{kT_0}{2} \cdot 2 \cdot B_n$$

Matched filtering (III)

$$\frac{S}{N} \Big|_{out} = \frac{\left| \int_{-\infty}^{+\infty} S(f)H(f)e^{j2\pi ft_1} df \right|^2}{\int_{-\infty}^{+\infty} N(f)|H(f)|^2 df}$$



$$N(f) = \frac{N_0}{2}$$

$$\frac{S}{N} \Big|_{out} = \frac{2 \left| \int_{-\infty}^{+\infty} S(f)H(f)e^{j2\pi ft_1} df \right|^2}{N_0 \int_{-\infty}^{+\infty} |H(f)|^2 df}$$

Schwartz inequality:

$$\left| \int f \cdot g dt \right|^2 \leq \int |f|^2 dt \cdot \int |g|^2 dt$$

$$\frac{S}{N} \Big|_{out} \leq \frac{\left(\int_{-\infty}^{+\infty} |S(f)|^2 df \right) \left(\int_{-\infty}^{+\infty} |H(f)|^2 df \right)}{N_0 \int_{-\infty}^{+\infty} |H(f)|^2 df}$$

$$\frac{2E}{N_0}$$

$f = g$

$$\left| \int |f|^2 dt \right|^2 = \int |f|^2 dt \cdot \int |f|^2 dt$$

Matched filtering (IV)

$$\left. \frac{S}{N} \right|_{out} = \frac{\left| \int_{-\infty}^{+\infty} S(f)H(f)e^{j2\pi ft_1} df \right|^2}{N_0 \int_{-\infty}^{+\infty} |H(f)|^2 df} = \frac{E}{N_0}$$

Obtained for

$$H(f) = G \cdot S^*(f) e^{-j2\pi ft_1}$$

$$h(t) = G \cdot s^*(t_1 - t)$$

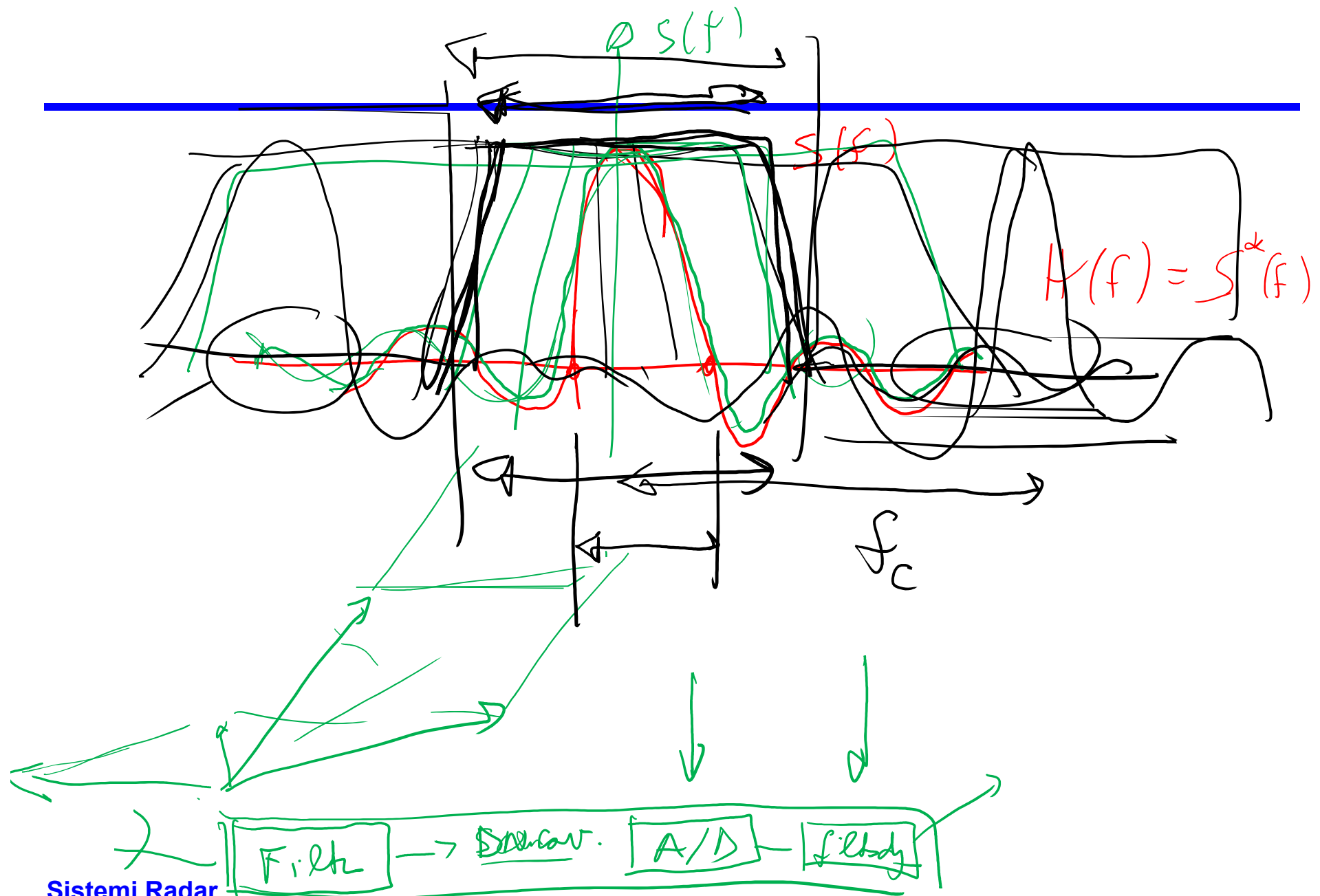
$$N_{out} = N_0 \int_{-\infty}^{+\infty} |H(f)|^2 df = G^2 N_0 \int_{-\infty}^{+\infty} |S(f)|^2 df = G^2 E_s N_0$$

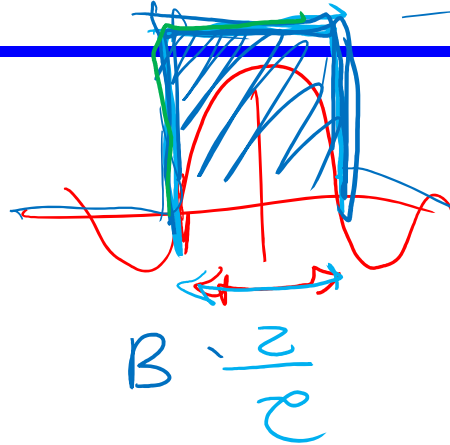
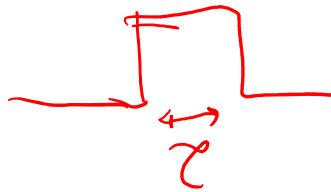
$$|y(t_1)|^2 = \left| \int_{-\infty}^{+\infty} S(f)H(f)e^{j2\pi ft_1} df \right|^2 = G^2 \left| \int_{-\infty}^{+\infty} |S(f)|^2 df \right|^2 = G^2 |E_s|^2$$

To maintain output noise power level equal to input use
 - especially important in digital matched filter
 implementation to use bit dynamics

$$G^2 E_s = 1$$

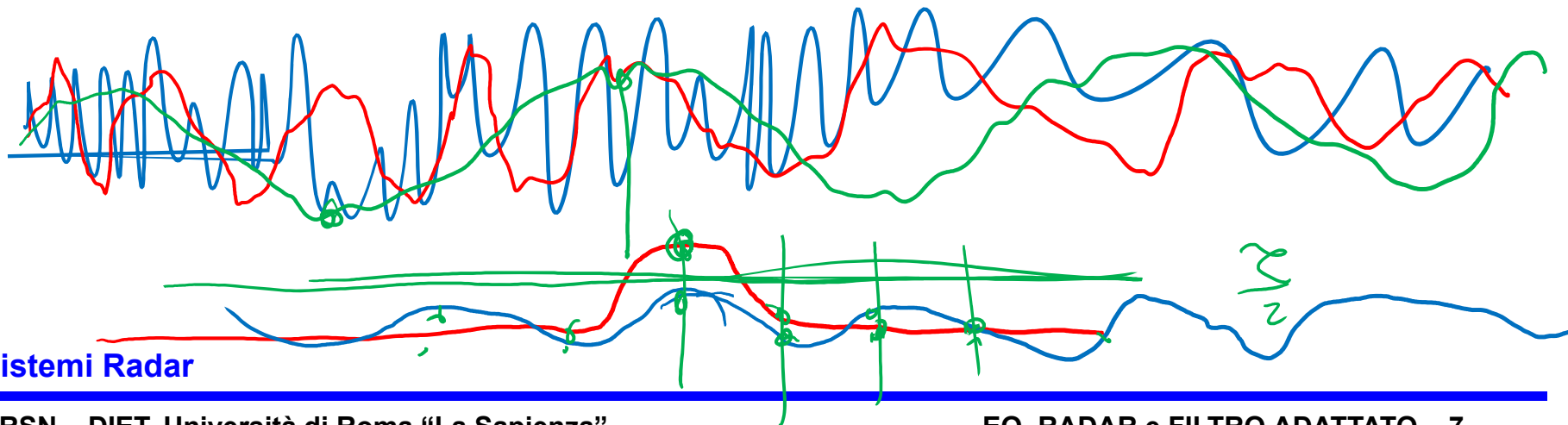
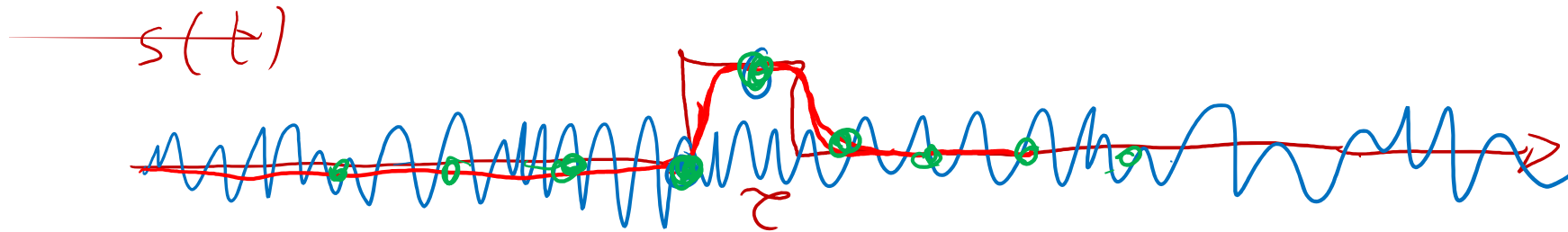
$$\Rightarrow G = \sqrt{E_s}$$





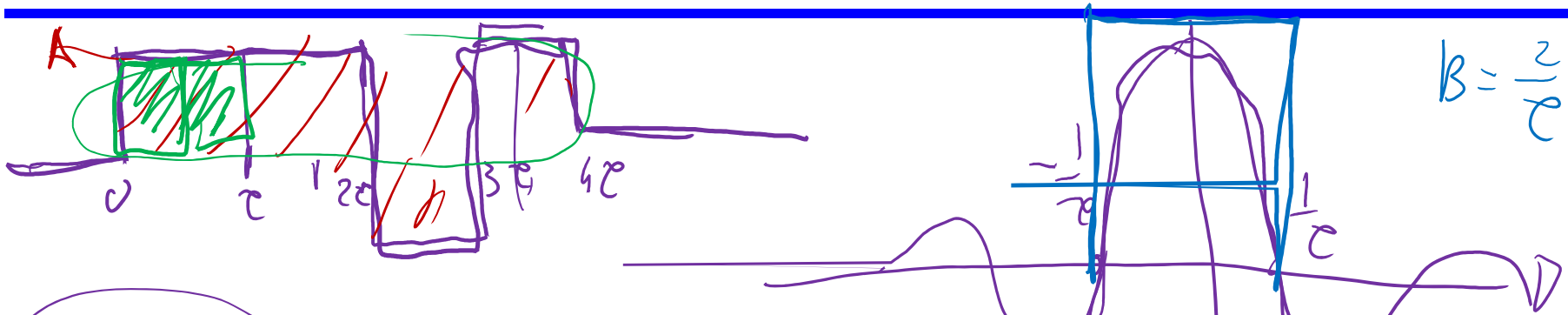
$$t = \frac{1}{B}$$

$$\underline{\underline{\text{SinCTT } Bt}}$$



$s(t)$

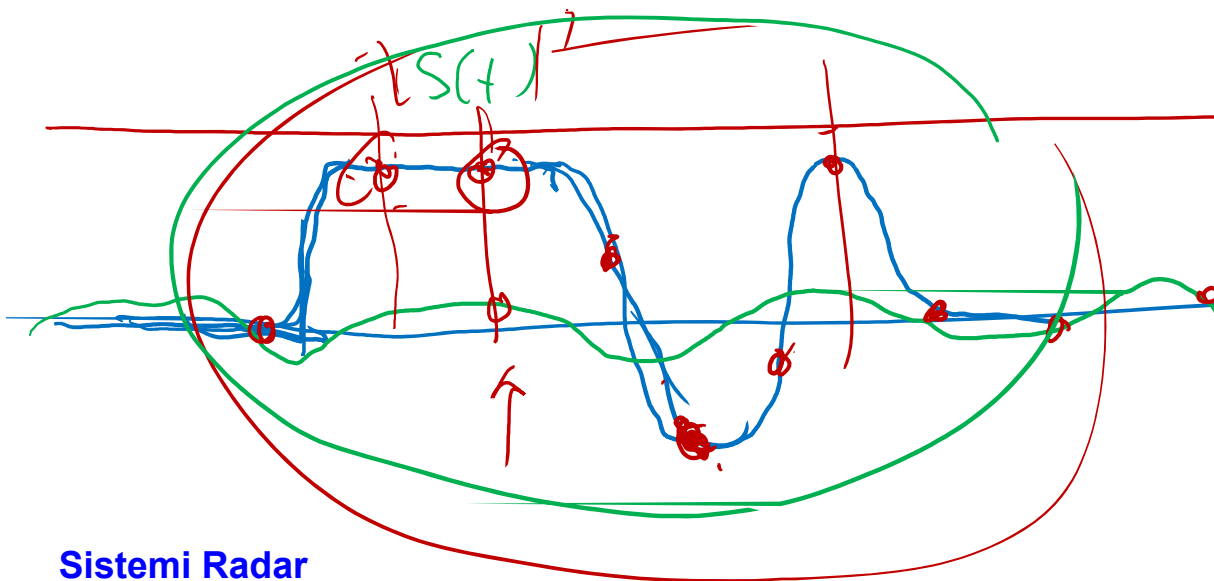
$S(f)$



$$B = \frac{2}{\tau}$$

$$\text{Sinc}(\pi f \tau) = \left[e^{-j2\pi \frac{\tau}{2}} + e^{-j2\pi \frac{3\tau}{2}} - e^{-j2\pi \frac{5\tau}{2}} + e^{-j2\pi \frac{7\tau}{2}} \right]$$

$$A = \sqrt{P_r}$$



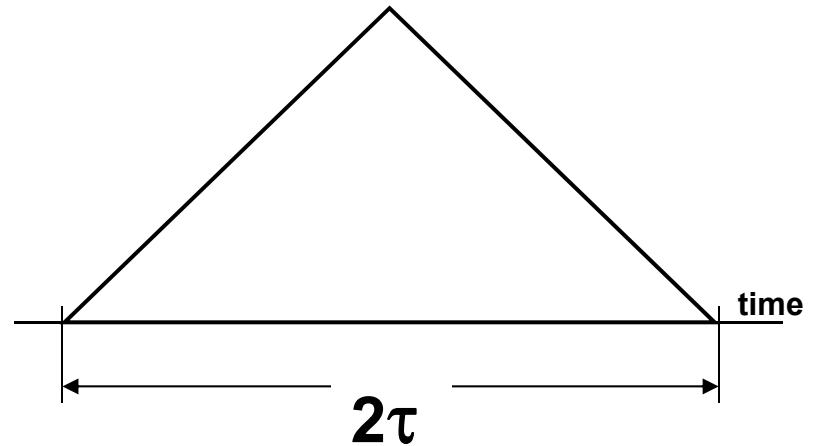
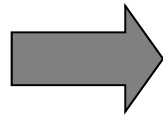
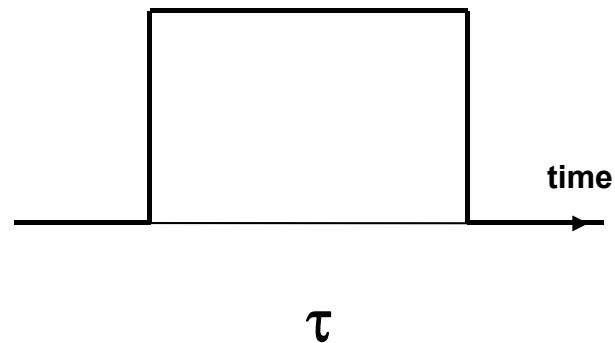
$$SNR = \frac{P_{ist}}{P_{nois}} = \frac{2A^2}{N_0 B}$$

$$SNR = \frac{P_{ist}}{P_{nois}} = \frac{2A^2 \tau}{N_0 \tau}$$

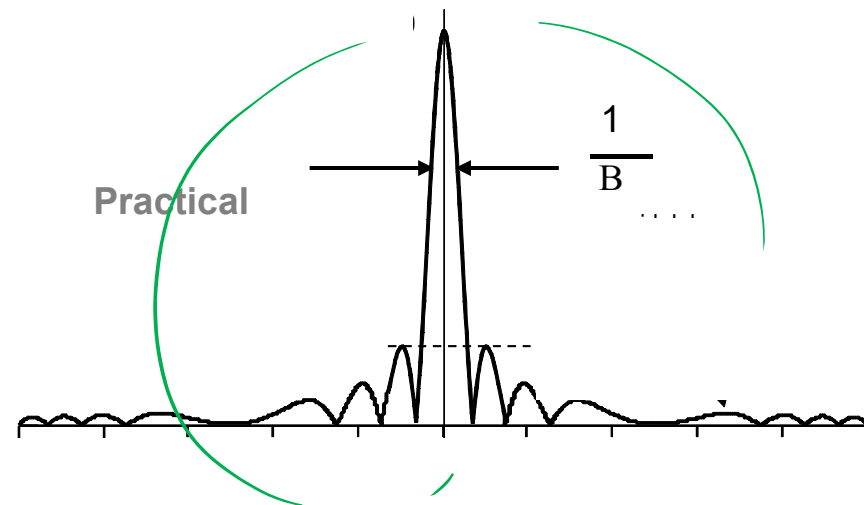
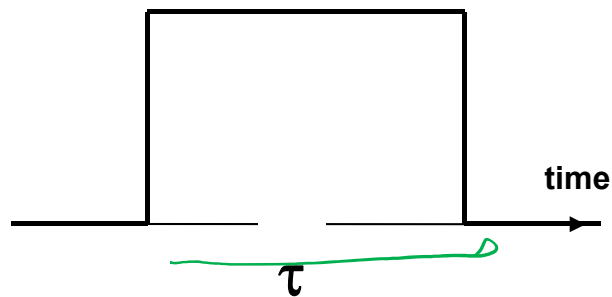
Sistemi Radar

Matched filtering (V)

Non-modulated Rectangular pulse:

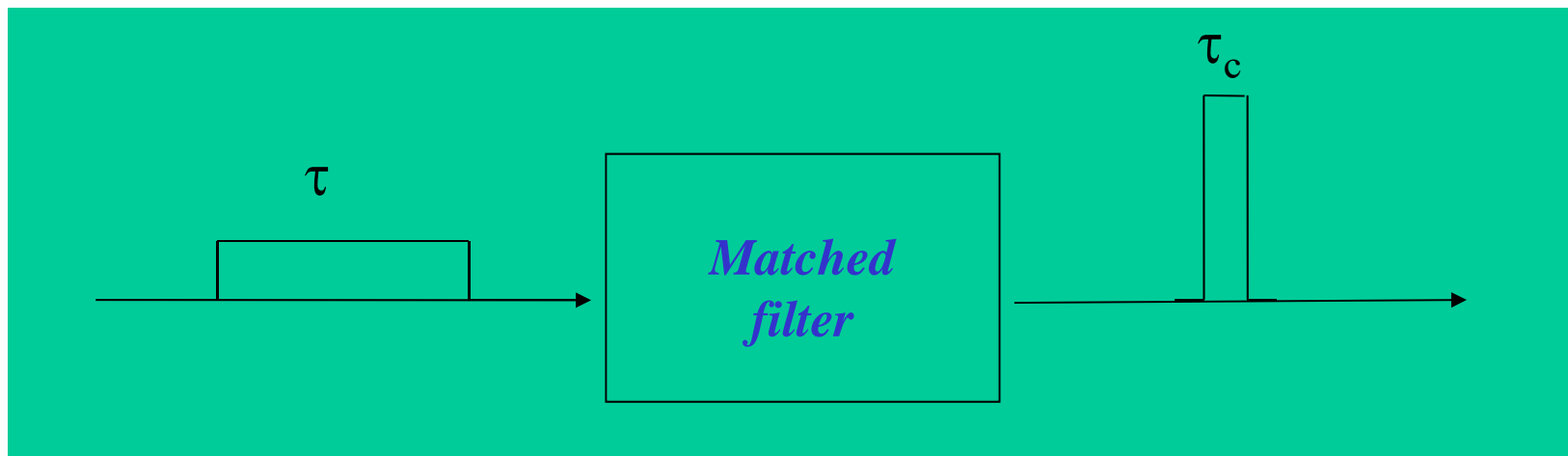


Phase-modulated rectangular pulse:
With overall bandwidth B



Matched filtering (VI)

- We want to transmit a long pulse (length τ) to get energy on target
- But we want a short compressed output (length $\tau_c \approx 1/B$) and larger amplitude:



Passive Filter \longrightarrow principle of energy conservation:

$$E_o = E_i \quad \longrightarrow \quad P_{po} \tau_c = P_{pi} \tau$$

Matched filtering (VII)

Passive Filter

principle of energy conservation:

$$E_o = E_i \quad \longrightarrow \quad P_{po} \tau_c = P_{pi} \tau$$

$$P_{po} = P_{pi} \quad \tau / \tau_c = P_{pi} B \tau$$

Noise power level does not change

Signal power improved with respect to noise power of

$$\tau / \tau_c = P_{pi} B \tau$$

COMPRESSION RATIO

Equazione Radar con compressione

Varie forme dell'Equazione Radar

L'equazione radar, e di conseguenza l'espressione della portata, può essere particolarizzata in dipendenza delle applicazioni

$$\left(\frac{S}{N}\right)_r = \frac{P_r}{P_n} = \frac{P_t G A_e \sigma}{(4\pi R^2)^2 k T_0 B F L} B \tau = \frac{P_t G A_e \sigma \tau}{(4\pi R^2)^2 k T_0 F L}$$

- 1 Si suppose di aver fissato il massimo valore del guadagno d'antenna G (vincolo la larghezza del fascio e quindi la risoluzione angolare): utilizzando $G=4\pi A_e/\lambda^2$

$$\left(\frac{S}{N}\right)_r = \frac{P_t G^2 \lambda^2 \sigma \tau}{(4\pi)^3 R^4 k T_0 F L} \quad \Rightarrow \quad R_{\max} = \left[\frac{P_t G^2 \lambda^2 \sigma \tau}{(4\pi)^3 k T_0 B F L (S/N)_{\min}} \right]^{1/4} \quad \text{Preferibili le basse frequenze}$$

- 2 Si suppose di aver fissato il massimo valore dell'area geometrica e quindi efficace d'antenna A_e : utilizzando $A_e = \lambda^2 G / 4\pi$

$$\left(\frac{S}{N}\right)_r = \frac{P_t A_e^2 \sigma \tau}{4\pi R^4 \lambda^2 k T_0 F L} \quad \Rightarrow \quad R_{\max} = \left[\frac{P_t A_e^2 \sigma \tau}{4\pi \lambda^2 k T_0 F L (S/N)_{\min}} \right]^{1/4} \quad \text{Preferibili le alte frequenze}$$

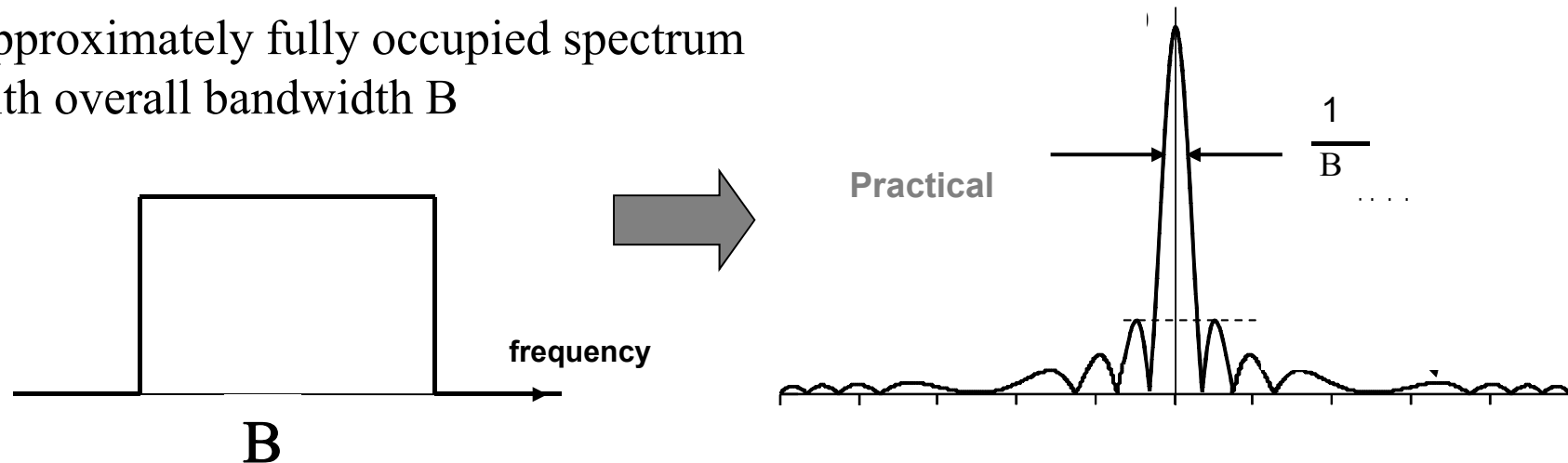
Sistemi Radar

Pulse compression yields Sidelobes

Output is signal autocorrelation:

$$|y(t)|^2 = \left| \int_{-\infty}^{+\infty} S(f)H(f)e^{j2\pi ft} df \right|^2 = G^2 \left| \int_{-\infty}^{+\infty} |S(f)|^2 e^{j2\pi f(t-t_1)} df \right|^2 = G^2 |R_{ss}(t-t_1)|^2$$

Approximately fully occupied spectrum
with overall bandwidth B



E' necessario un controllo dei lobi laterali: pesatura?