# A simple experiment for measuring the adiabatic coefficient of air

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Measurement of the ratio of specific heats (adiabatic coefficient), $\gamma$, of air is a classical experiment in an introductory laboratory of thermodynamics. Several methods (Clement and Desormes, Rüchhardt/Rinkel, Clark and Katz, Kundt's tube)[1,2] exist for determining $\gamma$ of air, most of them having been more recently improved.[3–6] The oldest of these methods is the Clement and Desormes experiment, in which a small mass of air first undergoes an adiabatic expansion, followed by an isochoric process to reach the initial temperature. Although this method has some pedagogical interest concerning the illustration of adiabatic processes, it has been abandoned by most modern thermodynamics laboratories. Two reasons for this abandonment are: first, a relatively large volume of confined air at a pressure higher than atmospheric is required, and second, it is a rather inaccurate method because of the difficulties of measuring the pressure change of the air in the isochoric process. Recently, Mottman[6] has presented an experiment which differs from that of Clement and Desormes in that instead of venting out air from a receptacle, it takes the air of a plastic bottle through a complete Otto cycle.

Here, we present an alternative, simple and accurate experiment to determine the adiabatic coefficient, $\gamma$, of air, within the experimental style of the Clement and Desormes and Mottman experiments. The experiment has a moderate cost since only a large glass syringe, a glass bottle with two holes and two rubber stoppers, and a common digital blood pressure monitor are required. The method allows one to measure experimentally the isothermal compressibility, $k_T$, and the adiabatic (or isoentropic) compressibility, $k_S$, of air, and it uses the Reech relation[1]

$$\gamma = \frac{c_p}{c_v} = \frac{k_T}{k_S}, \tag{1}$$

where

$$k_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T, \quad k_S = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_S. \tag{2}$$

If the changes in volume ($\Delta V$) and pressure ($\Delta P$) in isothermal and adiabatic processes are small compared with the initial volume ($V$) and pressure ($P$), we can write

$$k_T = -\frac{1}{V}\left(\frac{\Delta V}{\Delta P}\right)_T, \quad k_S = -\frac{1}{V}\left(\frac{\Delta V}{\Delta P}\right)_S. \tag{3}$$

Then, measuring $(-\Delta V/\Delta P)_T$ and $(-\Delta V/\Delta P)_S$, the adiabatic coefficient $\gamma$ can be obtained from Eq. (1).

The experimental setup is shown in Fig. 1. The volume changes were measured using a 100-ml glass syringe with 1-ml scale divisions, with an uncertainty of $\pm 0.5$ ml. The pressure changes were measured using a common digital blood pressure monitor (for example, model DS-115, A. T. Digital) with a gauge pressure range of 0–300 mm Hg. Atmospheric pressure ($P = 694 \pm 0.5$ mm Hg) was obtained from a standard Hg barometer. The initial air volume ($V = 1198 \pm 1$ ml) was obtained by filling the system, syringe plus bottle, with water and then measuring the volume of this water.

The syringe was mounted horizontally with the output end connected to the upper hole of the glass bottle. The lower hole of the bottle was connected to the air connector of the pressure monitor using a short tube plug. A rubber band of $\sim 25$ cm length and $\sim 1$ cm width was fixed around the piston of the syringe to control the volume change. The experiment was performed as follows. The piston was placed in the *initial* position (maximum scale reading) and the pressure monitor was turned on. After a 0 appeared on the display, the piston was pushed by hand quickly but continuously until it was stopped by the rubber band bumping against the syringe input end. This is the adiabatic compression stage. The pressure increased, reached a maximum value, and then started to decrease. The reading corresponding to the maximum value of the pressure seen on the display was recorded: this is $(\Delta P)_S$. After waiting about 3 min the pressure stabilized because the internal temperature had again come into equilibrium with the laboratory temperature. The new pressure value read on the display was recorded: this is $(\Delta P)_T$. Last, the volume change, $-\Delta V$, was read directly on the scale of the syringe. Finally, in order to prepare the apparatus for a new measurement, the pressure monitor was turned off, the rubber stopper was removed from the upper hole of the bottle, the piston was moved back to the *initial* position, and the upper rubber stopper was replaced.

While measuring a single value of $-\Delta V$ will suffice to provide a reasonable estimate of $k_T$, $k_S$, and $\gamma$, considerable improvement in the final results is obtained by making several measurements of $(\Delta P)_T$ and $(\Delta P)_S$ for different values

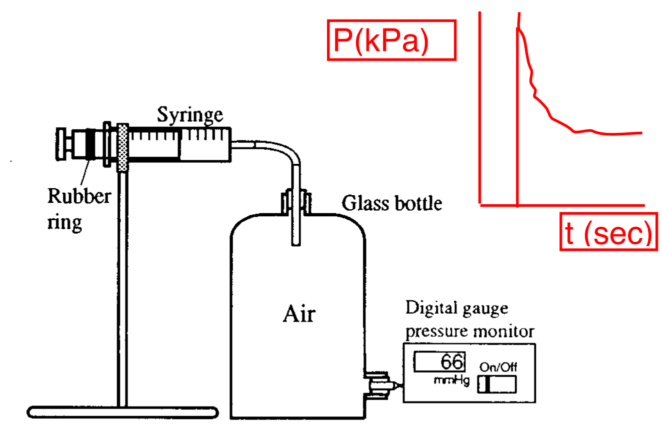


Fig. 1. Experimental setup.

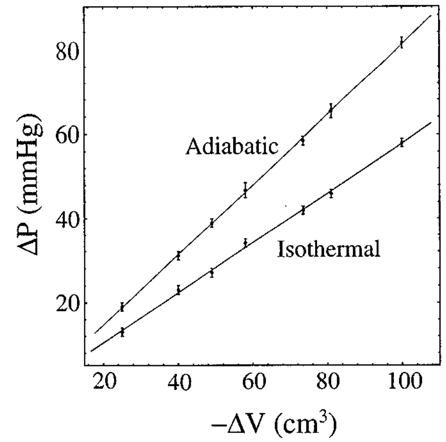


Fig. 2. Pressure increase ($\Delta P$) vs volume decrease ($-\Delta V$) for adiabatic and isothermal compression stages. The dots indicate the mean values and the bars indicate the standard deviations of the means. The lines are least-squares fits to the data.

of $-\Delta V$. We have made ten measurements of $(\Delta P)_T$ and $(\Delta P)_S$ for each of seven different values of $-\Delta V$ (between 25 and 100 ml). Plots of the average values of $(\Delta P)_T$ and $(\Delta P)_S$ vs $-\Delta V$ are shown in Fig. 2 as well as the corresponding linear regressions. Using Eqs. (2) with $V = 1198$ ml, from the slope of each linear regression we obtained:[7] $k_T = 0.001\,41 \pm 0.000\,03$ mm Hg$^{-1}$ and $k_S = 0.001\,00 \pm 0.000\,02$ mm Hg$^{-1}$ [for a diatomic ideal gas at $P = 694$ mm Hg, one has $k_T = 1/P = 0.001\,44$ mm Hg$^{-1}$ and $k_S = 1/(\gamma P) = 0.001\,03$ mm Hg$^{-1}$ with $\gamma = 1.4$]. From Eq. (1) we obtained $\gamma = 1.41 \pm 0.05$.

We notice that measuring $(\Delta P)_T$ and $(\Delta P)_S$ for a given volume change $-\Delta V$, from Eqs. (1) and (3) one has

$$\gamma = \frac{(\Delta P)_S}{(\Delta P)_T}. \tag{4}$$

Then, one can obtain $\gamma$ in a direct way by plotting $(\Delta P)_S$ vs $(\Delta P)_T$ for different values of $-\Delta V$. Moreover, since knowledge of the values of $-\Delta V$ is not necessary, one can obtain $\gamma$ by using a syringe without scale divisions.

Finally, we should remark that the simple experiment proposed here can also be used for demonstration of Boyle's law and can serve as a simple illustration of both isotherm (very slow motion of the piston) and adiabatic (quick motion of the piston) processes. Thus, it can be suitable not only as a laboratory experiment but also as a demonstration in class lectures.

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[7] We used mm Hg as pressure unit since the reading from the digital pressure monitor is given in this unit. Notice that it is not necessary to convert pressure units to obtain $\gamma$. Uncertainties for $k_T$ and $k_S$ are propagated through the linear least-squares fit of the data in Fig. 2.

## SCIENCE IS EFFECTIVE, BUT...

Science is effective, but what does it tell us about ourselves and how we must live? The brief answer to this is: nothing. Science has always worked assiduously to avoid being a religion, faith or morality. It does not tell us why we should do things or how we should live; it offers, instead, solutions. Life is a series of separate problems with separate answers. It is not an issue in itself so much as a container of issues.

Bryan Appleyard, *Understanding the Present—Science and the Soul of Modern Man* (Pan Books, London, 1992), p. 9.

## SURVIVAL SKILLS

Since science survival skills are rarely taught in a direct way, most young scientists need a mentor. Some will find one in graduate school, or as a postdoctoral researcher, or perhaps as an assistant professor. Those who do not, to paraphrase Mencken, have an excellent chance of moving from graduate study to scientific retirement without passing through a career.

Peter J. Feibelman, *A Ph.D. Is Not Enough—A Guide to Survival in Science* (Addison–Wesley, Reading, MA, 1993), p. ix.

Corso di Termodinamica e Laboratorio

## ESPERIENZA 4: misura del coefficiente adiabatico

Obiettivi dell'esperienza: misura del coefficiente o indice adiabatico dell'aria mediante compressibilità isoentropica e isoterma

Strumenti a disposizione:
- bottiglia
- misuratore di bassa pressione
- siringa
- distanziatori ed elastico di fissaggio

## Premessa

L'esperienza si basa sulla uguaglianza del coefficiente adiabatico $\gamma$ al rapporto tra la compressibilità isoterma $\beta_T$ e quella adiabadica $\beta_S$, ovvero

$$\gamma = \frac{c_p}{c_v} = \frac{\beta_T}{\beta_S}$$

dove le compressibilità sono definite da $\beta_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$ e $\beta_S = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_S$

Notare che nell'esperienza non è possibile effettuare variazioni di volume infinitesimali e quindi si impiegano variazioni di volume $\Delta V$ piccole ma finite, a cui corrispondano piccole variazioni di pressione $\Delta P$. Con questa premessa, si suppone la seguente approssimazione

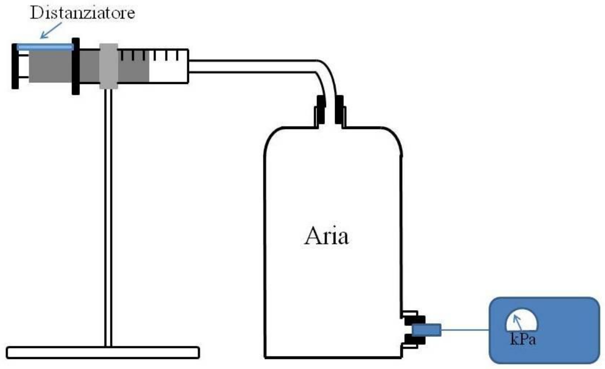
$$\beta_T \cong -\frac{1}{V}\left(\frac{\Delta V}{\Delta P}\right)_T \quad \text{e} \quad \beta_S \cong -\frac{1}{V}\left(\frac{\Delta V}{\Delta P}\right)_S$$

che consente di ottenere il coefficiente adiabatico mediante il seguente rapporto

$$\gamma = \frac{\left(\frac{\Delta V}{\Delta P}\right)_T}{\left(\frac{\Delta V}{\Delta P}\right)_S}$$

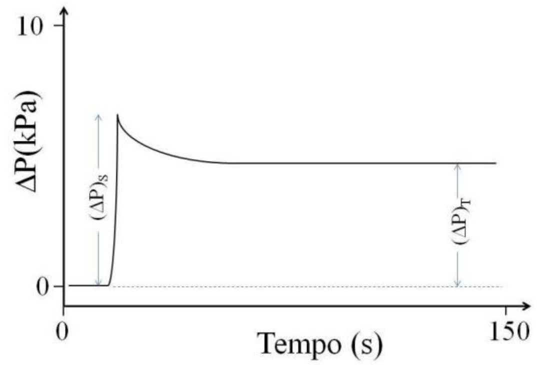
## Disposizione sperimentale

La disposizione sperimentale per la misura è come in figura. I distanziatori sono appoggiati sul pistone della siringa e tenuti in posizione fissa mediante un elastico. Il volume di aria da considerare, incluso dei tubi di collegamento con la siringa e con il misuratore di bassa pressione, è di 1111 mL (con siringa a fine corsa)

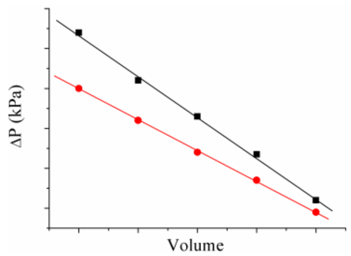


# Misura del coefficiente adiabatico

**a**) selezionare una serie di distanziatori necessari a fornire le variazioni di volume (si consiglia distanziatori che consentano di effettuare variazioni tra 10 e 50 mL) e determinare la posizione di riferimento del pistone della siringa a cui corrisponde il volume di riferimento $V_0$ (da ristabilire ad ogni fine misura e prima della successiva)
**b**) per ogni distanziatore, assicurarsi che il fissaggio sul pistone della siringa sia stabile (usare l'elastico per legare il distanziatore al pistone)
**c**) calibrare il misuratore di bassa pressione
**d**) generare una rapida pressione sul pistone della siringa in maniera tale che si misuri (mediante DataStudio) una curva di pressione in funzione del tempo come quella riportata in figura (scala temporale complessiva della misura dell'ordine di 100-120 s). Attenzione a non rilasciare il pistone durante la misura, ma mantenerlo a fine corsa senza forzare




**e)** verificare che il salto di pressione rapido, effettuato con la spinta sul pistone della siringa, avvenga in tempi rapidissimi, tra i 100 e 300 ms a seconda delle variazioni di volume da imprimere
**f)** determinare dal grafico precedente i salti di pressione isoentropico (spinta veloce) e quello isotermo (equilibrio finale)
**g)** riportare in tabella i valori di $\Delta V$ , $(\Delta P)_S$ e $(\Delta P)_T$
**h)** con i dati ottenuti, costruire un grafico nel piano di Clapeyron in cui le ordinate riportano le variazioni di pressione, mentre in ascissa si riporta il volume totale $V_0 - \Delta V$ (vedi figura)




**i)** dai due fit lineari, estrarre i coefficienti angolari $m_S$ (retta superiore) e $m_T$ (retta inferiore) e valutare il coefficiente adiabatico da $\gamma = \left(\frac{\Delta V}{\Delta P}\right)_T \Big/ \left(\frac{\Delta V}{\Delta P}\right)_S = \frac{m_S}{m_T}$

**j)** calcolare le compressibilità da $\beta_T \cong -\frac{1}{V}\left(\frac{\Delta V}{\Delta P}\right)_T = -\frac{1}{V_0 m_T}$ e $\beta_S \cong -\frac{1}{V}\left(\frac{\Delta V}{\Delta P}\right)_S = -\frac{1}{V_0 m_S}$

## *Approfondimenti*

- ricavare la relazione tra coefficiente adiabatico e compressibilità
- spiegare perché le differenze di pressione iniziale e finale corrispondono rispettivamente ai salti di pressione adiabatici e isotermi