Synthetic Aperture Radar

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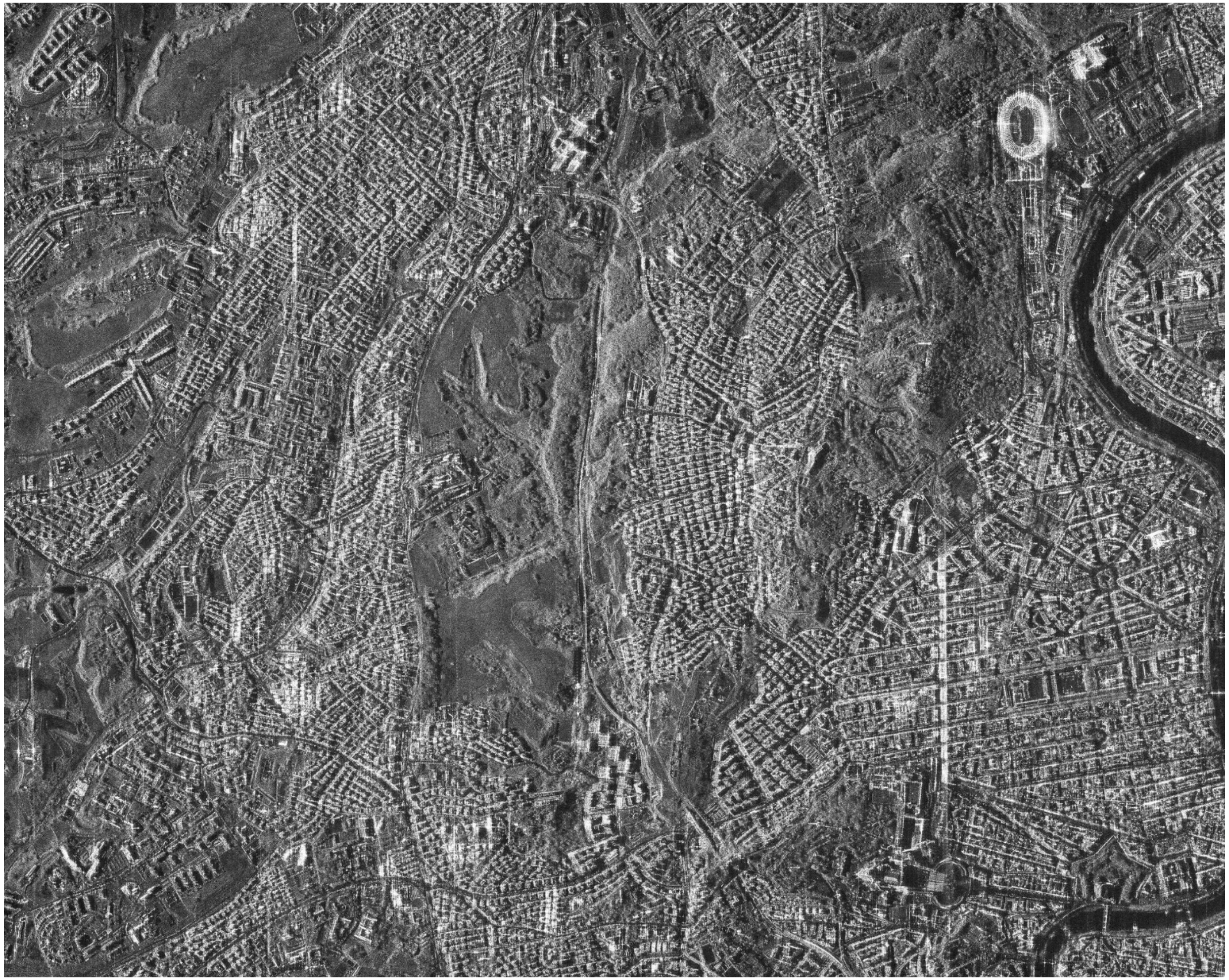
Outline

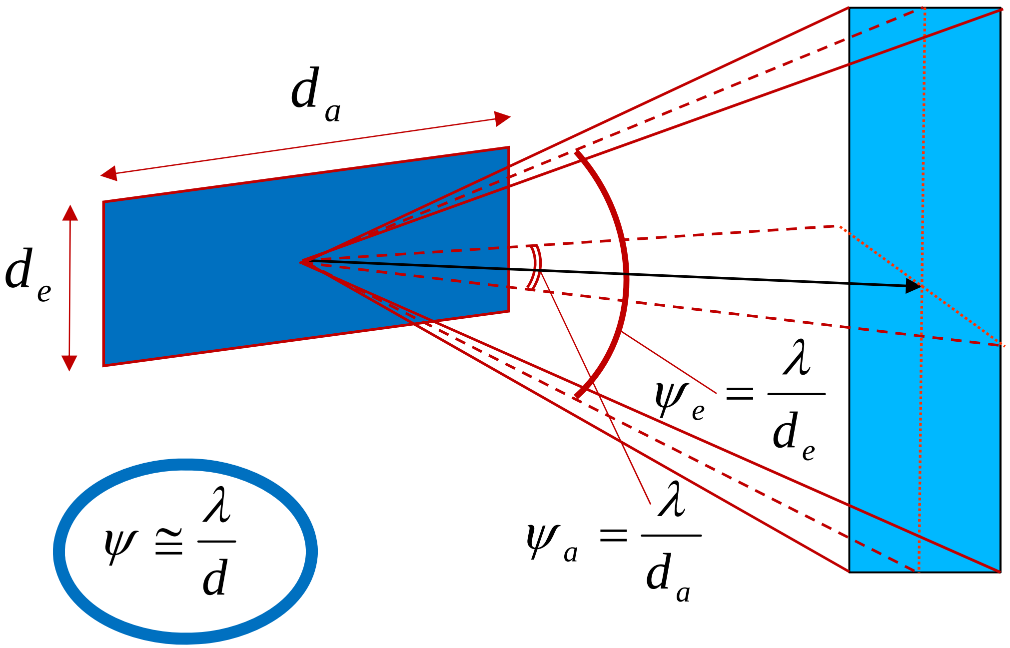
- **SAR Basics**
  - SAR System parameters, range resolution and swaths
  - Real Aperture Radar (RAR)
  - Doppler Frequency approach to SAR
  - Synthetic antenna approach to SAR
- **SAR Focusing algorithms**
  - Range Cell Migration and focus parameter variation
  - Range-Doppler Algorithm
  - Chirp Scaling Algorithm
  - Range Migration Algorithm
- **SAR imaging modes**
  - Fundamental limitation of SAR
  - Squinted SAR
  - Spotlight SAR Inverse SAR
- **Examples of advanced SAR applications**
  - Coherent Multichannel SAR/ISAR using multiple platforms
  - Passive SAR and ISAR

Principles of SAR Image Formation



Sample image from ASI - Italian Space Agency (www.asi.it)

Radar Antenna Beam



Example airborne SAR	
Wavelength (λ)	3.1 cm (X band)
Antenna ($d_a \times d_e$)	1.8 m × 0.18 m
Altitude	10 km
Off-nadir angle (α_0)	Adjustable 15° - 60°

airborne case

$$\psi_e = \frac{\lambda}{d_e} = \frac{0.031}{0.18} = 0.1722\,(rad) \rightarrow 9.87°$$

$$\psi_a = \frac{\lambda}{d_a} = \frac{0.031}{1.8} = 0.01722\,(rad) \rightarrow 0.987°$$

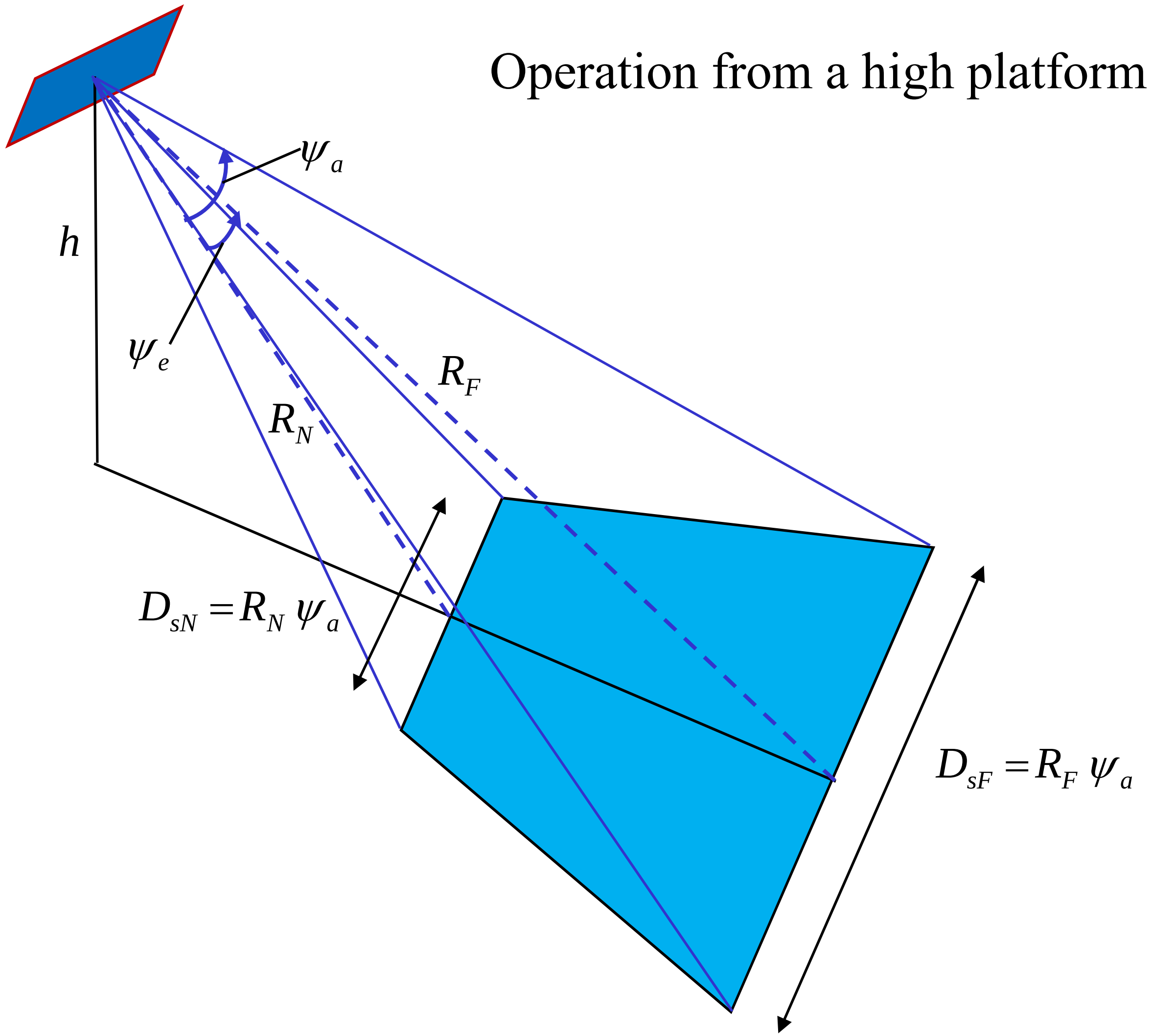
spaceborne case

$$\psi_e = \frac{\lambda}{d_e} = \frac{0.0567}{1} = 0.0567\,(rad) \rightarrow 3.2487°$$

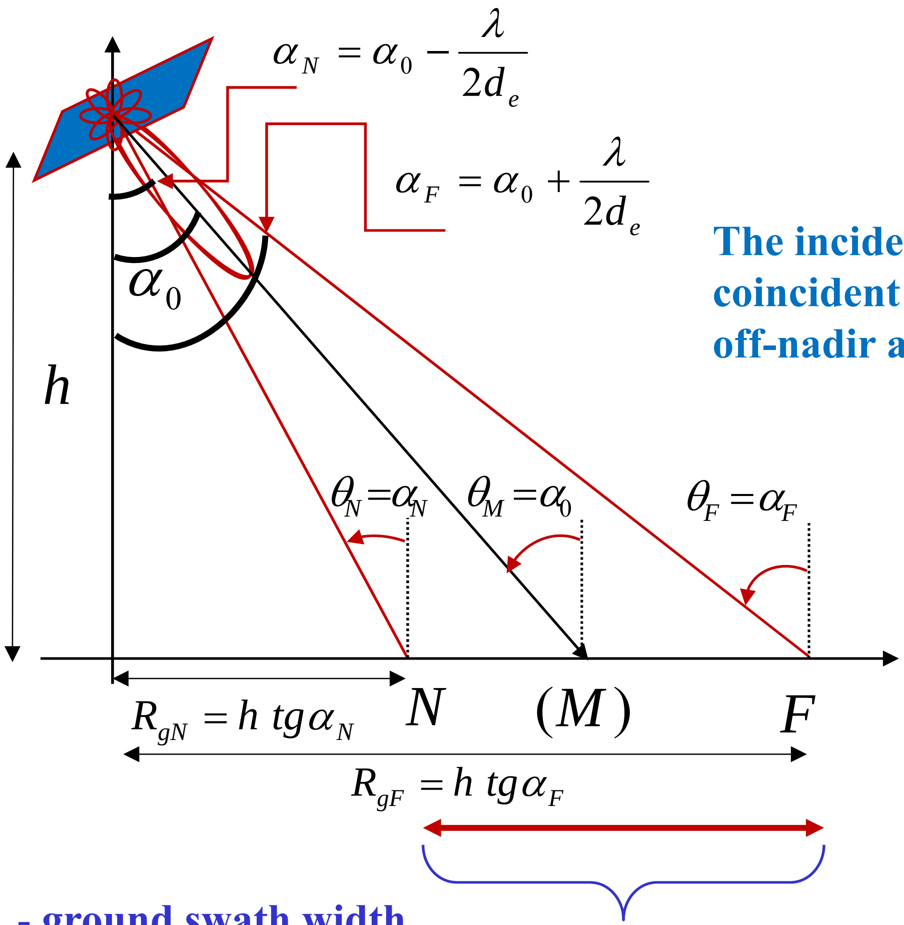
$$\psi_a = \frac{\lambda}{d_a} = \frac{0.0567}{10} = 0.00567\,(rad) \rightarrow 0.32487°$$

Radar Antenna Footprint

Operation from a high platform



Air-borne SAR: ground range swath



The incidence angle θ is coincident with the local off-nadir angle α

Example airborne SAR	
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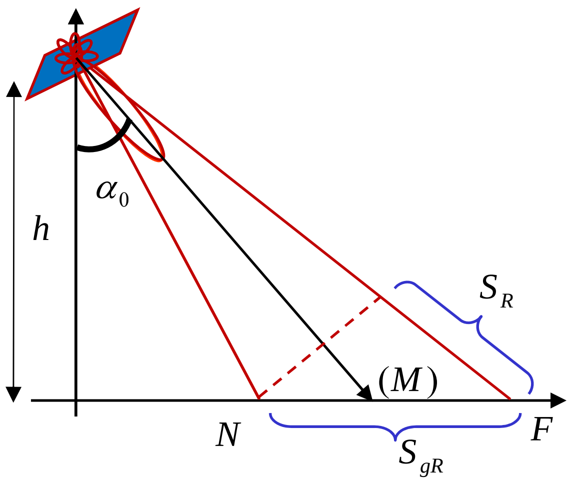


- ground swath width

$$S_{gR} = h\ tg(\alpha_0 + \frac{\lambda}{2d_e}) - h\ tg(\alpha_0 - \frac{\lambda}{2d_e})$$

$\alpha_0 = 15°$ $\quad S_{gR} = 1.85\ km$

$\alpha_0 = 60°$ $\quad S_{gR} = 7.06\ km$

- **Swath**
- **appro**

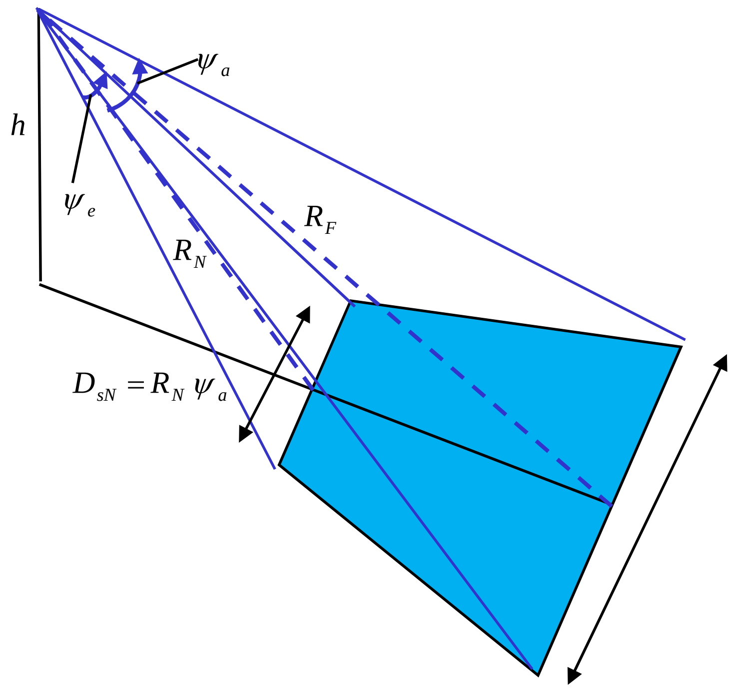
x:

$$S_{gR} \cong \frac{\lambda}{d_e}\frac{R_0}{\cos\alpha_0} = \frac{\lambda}{d_e}\frac{h}{\cos^2\alpha_0}$$

$$S_{gR} \cong 0.17\frac{10}{0.93} = 1.85\ km$$

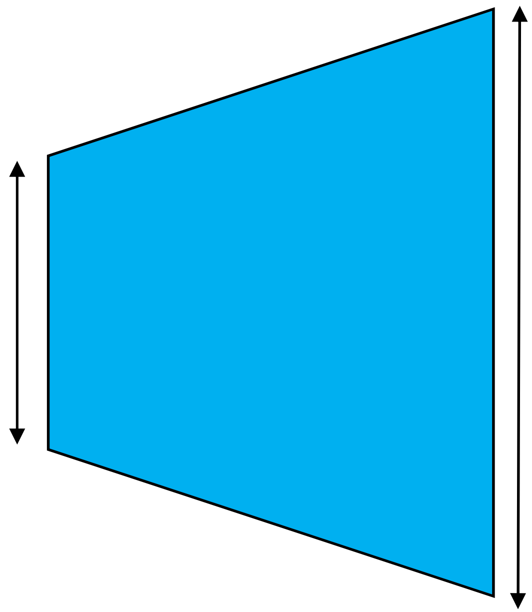
$$S_{gR} \cong 0.17\frac{10}{0.25} = 6.89\ km$$

Azimuth antenna footprint



$$R_N = \frac{h}{\cos\alpha_N}$$

$$D_{sN} = R_N\,\psi_a$$



$$R_F = \frac{h}{\cos\alpha_F}$$

$$D_{sF} = R_F\,\psi_a$$

$\alpha_0 = 15°$

$$R_N = \frac{10}{\cos(\pi/12 - 0.17)} = 10.156\,km \qquad R_0 = \frac{10}{\cos(\pi/12)} = 10.353\ km \qquad R_F = \frac{10}{\cos(\pi/12 + 0.17)} = 10.637\ km$$

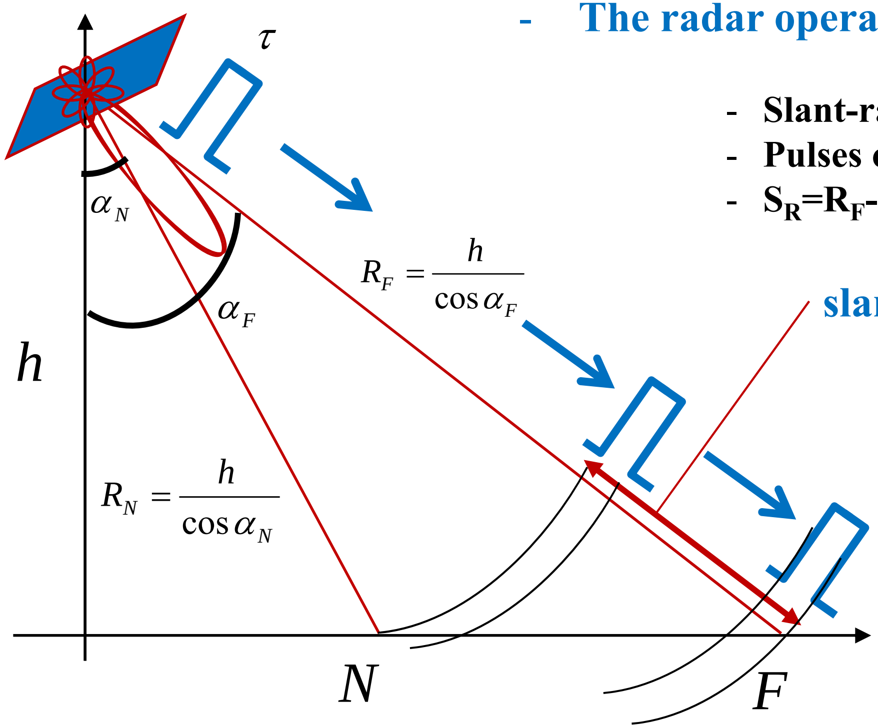
$$D_{sN} = \frac{\lambda}{d_e}R_N = 0.017\cdot 10.156 = 0.175\,km \qquad D_{s0} = \frac{\lambda}{d_e}R_N = 0.017\cdot 10.353 = 0.178\,km \qquad D_{sF} = \frac{\lambda}{d_e}R_N = 0.017\cdot 10.637 = 0.183\,km$$

$\alpha_0 = 60°$

$$R_N = \frac{10}{\cos(\pi/12 - 0.17)} = 17.463\,km \qquad R_0 = \frac{10}{\cos(\pi/12)} = 20.000\ km \qquad R_F = \frac{10}{\cos(\pi/12 + 0.17)} = 23.604\ km$$

$$D_{sN} = \frac{\lambda}{d_e}R_N = 0.017\cdot 10.156 = 0.301\,km \qquad D_{s0} = \frac{\lambda}{d_e}R_N = 0.017\cdot 10.353 = 0.344\,km \qquad D_{sF} = \frac{\lambda}{d_e}R_N = 0.017\cdot 10.637 = 0.407\,km$$

Radar pulses & range resolution



- **The radar operates in time domain ("fast time") by sending RF pulses:**
  - **Slant-range is direct transposition of time to space (scale factor c/2)**
  - **Pulses come back from distances going from R_N to R_F**
  - **$S_R=R_F-R_N$ is the slant-range swath corresponding to S_{gR}**

slant range swath

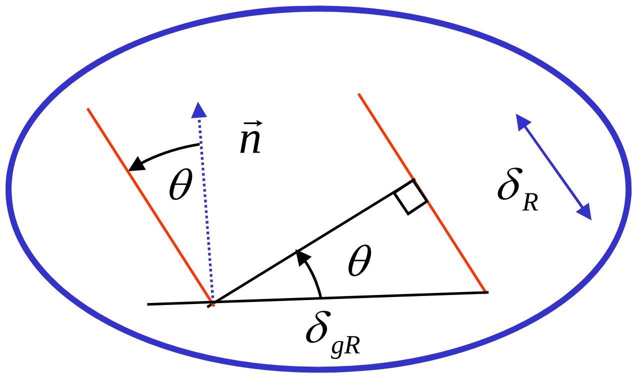
$$S_R = R_F - R_N = \frac{h}{\cos(\alpha_F)} - \frac{h}{\cos(\alpha_N)}$$

- **Slant range resolution: $\delta_R = c\tau/2$ (τ = pulse length)**

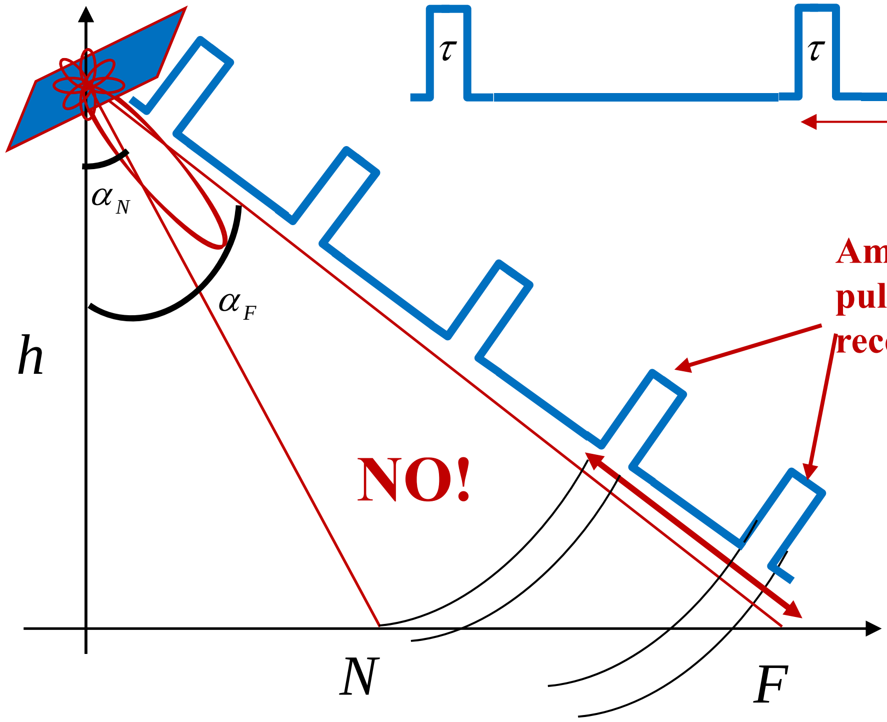
all scatterers with echo from a distance inside slant range resolution cannot be resolved

- **Ground range resolution: $\delta_{gR} = c\tau/(2\sin\theta)$**

all scatterers with ground range inside groundrange resolution cannot be resolved



Range ambiguities



Ambiguities: echoes to subsequent pulses from these ranges reach the receiver at the same time

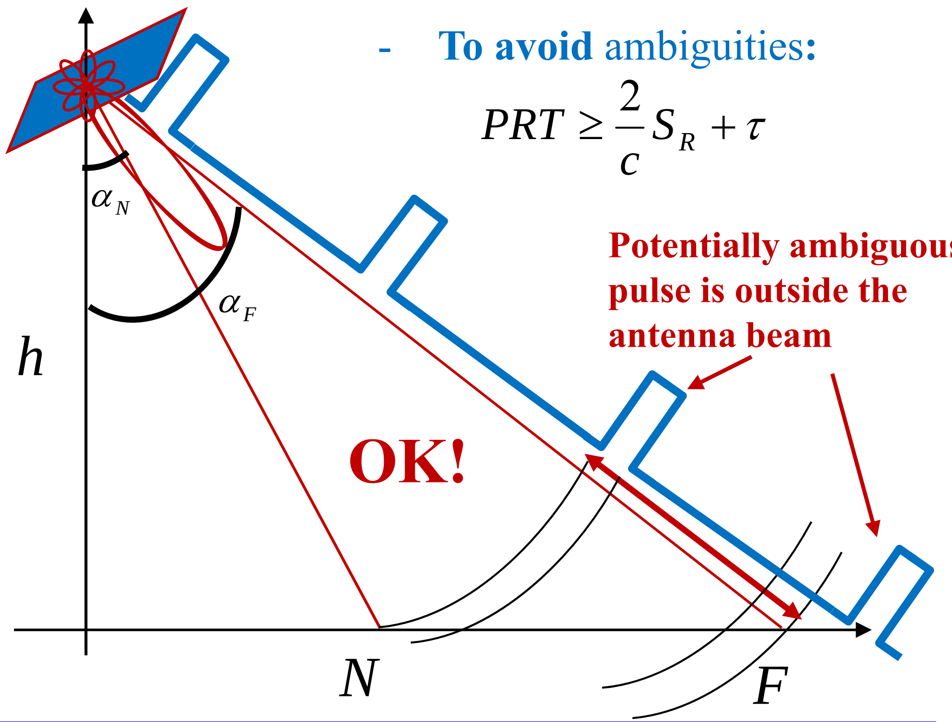
- **Pulse Repetition Frequency (PRF)**

  (assume $\tau << 2S_R/c$ negligible)

$$PRF = \frac{1}{PRT} < \frac{c}{2S_R}$$

- **To avoid** ambiguities:

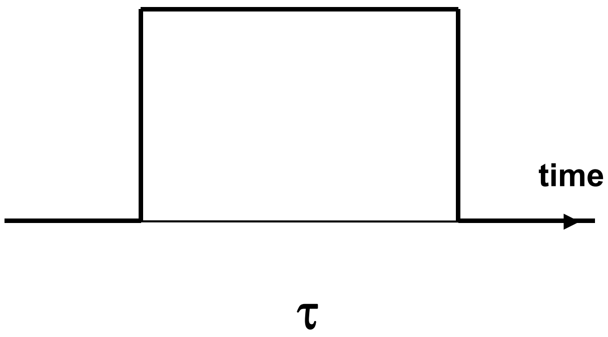
$$PRT \geq \frac{2}{c} S_R + \tau$$

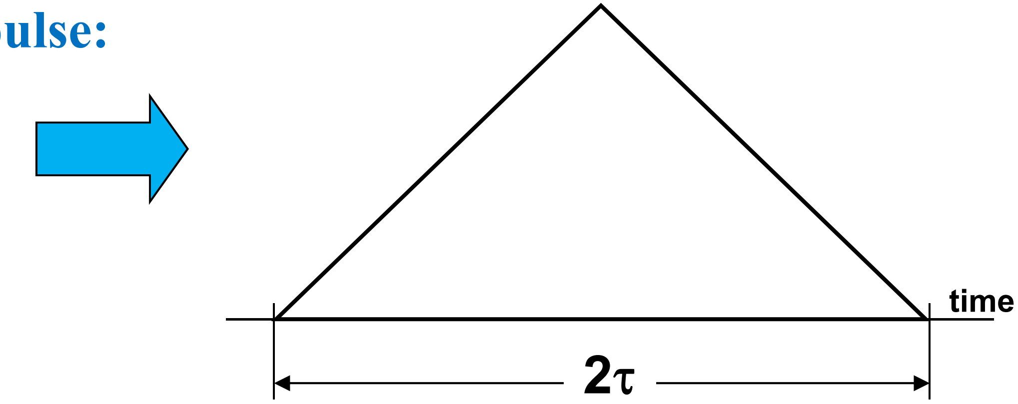


Potentially ambiguous pulse is outside the antenna beam

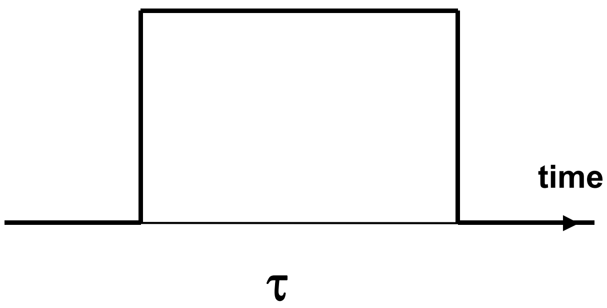
Pulse compression and range resolution

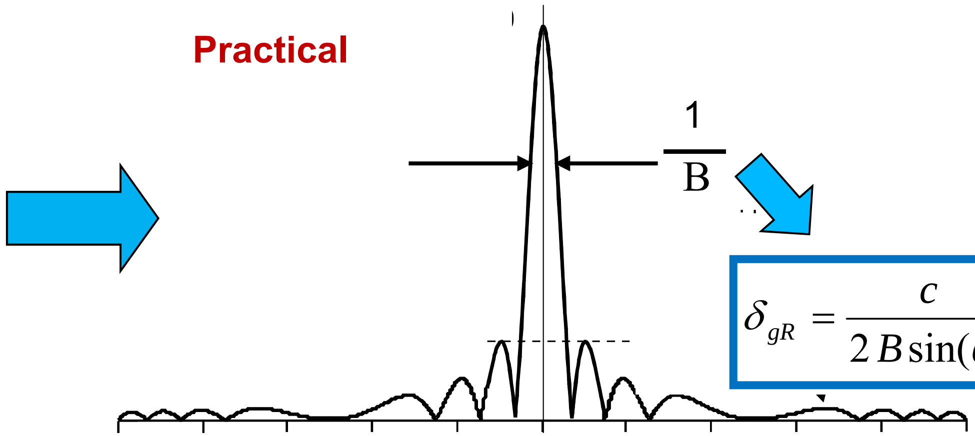
Non- modulated Rectangular pulse:





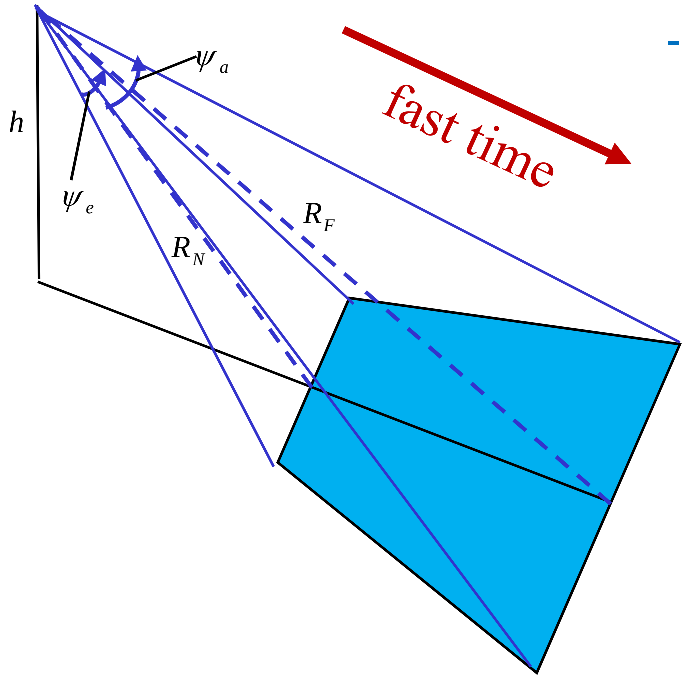
Phase-modulated rectangular pulse with overall bandwidth B





$$\delta_{gR} = \frac{c}{2B\sin(\theta)}$$

Single pulse radar echo



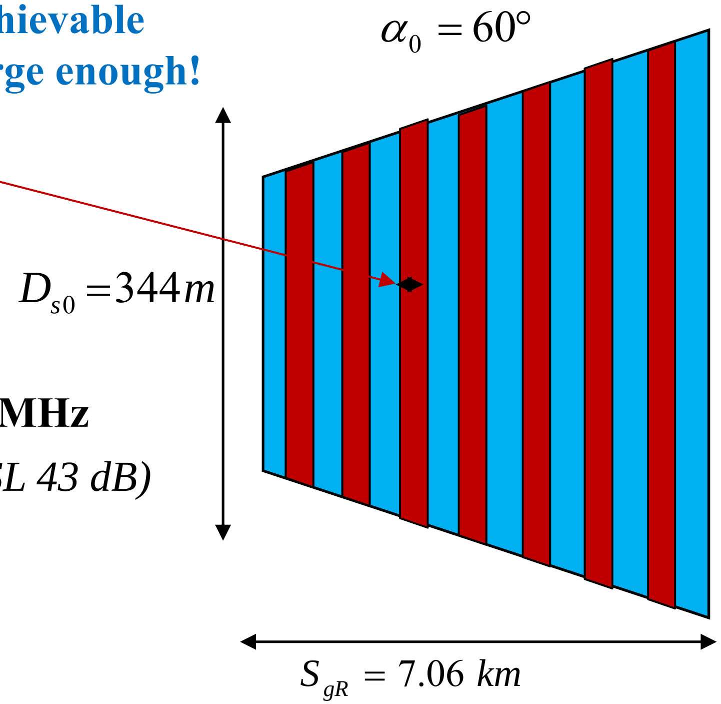
- **Any desired value is achievable using a pulse with B large enough!**

$$\delta_{gR} = \frac{k}{\sqrt{3}} \frac{c}{B}$$

Example: B = 450 MHz

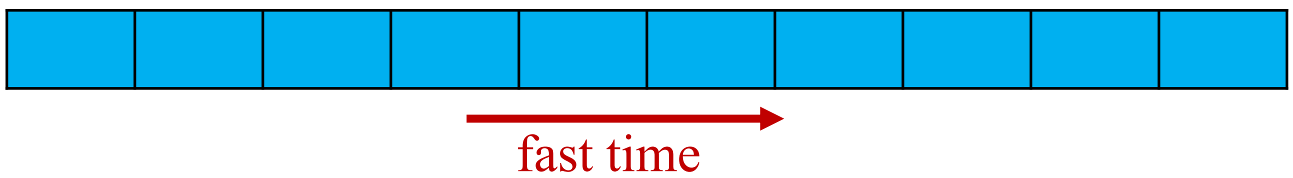
k= 1.3 Hamming (PSL 43 dB)

$\boldsymbol{\delta_{gR}}$**=0.5 m**

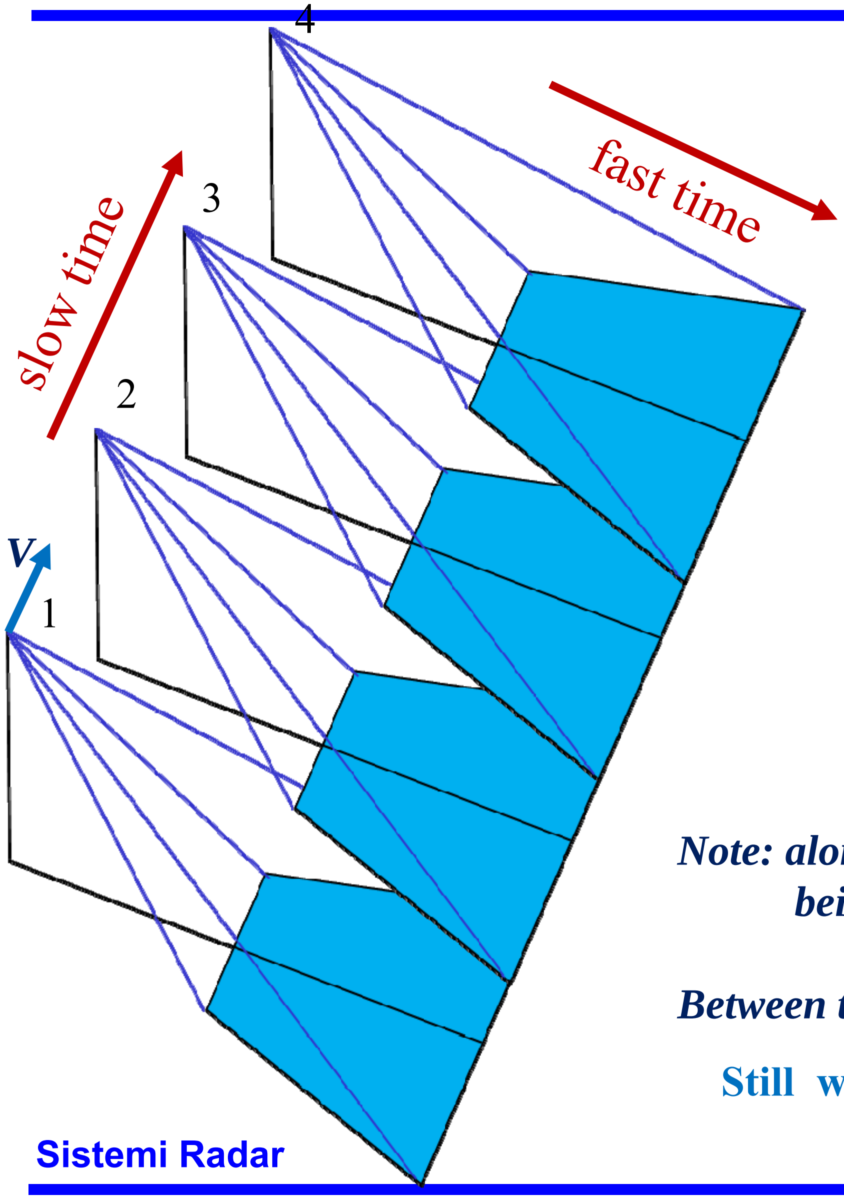


Two problems:

- **1 pixel represents a ground patch of: 0.5 m × 344 m !!!!**
- **Vector collecting "fast time" samples: not a matrix – not an image !!!!**



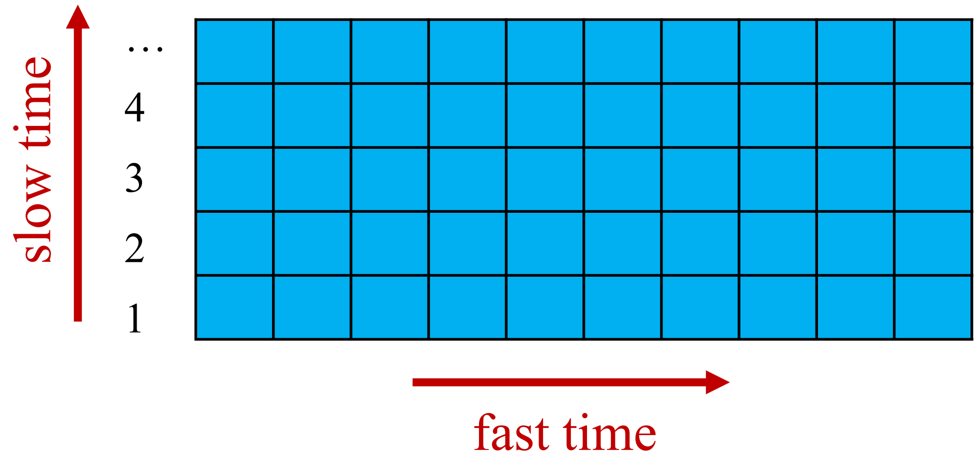
Real Aperture Radar



Exploit platform motion in "slow time"

→ **pulses collected from different positions**

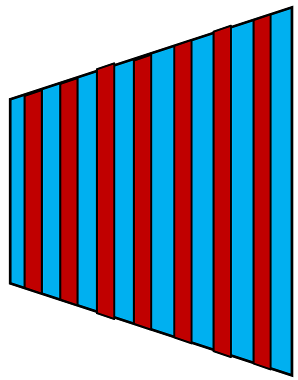
→ **a matrix → an image !!!!**



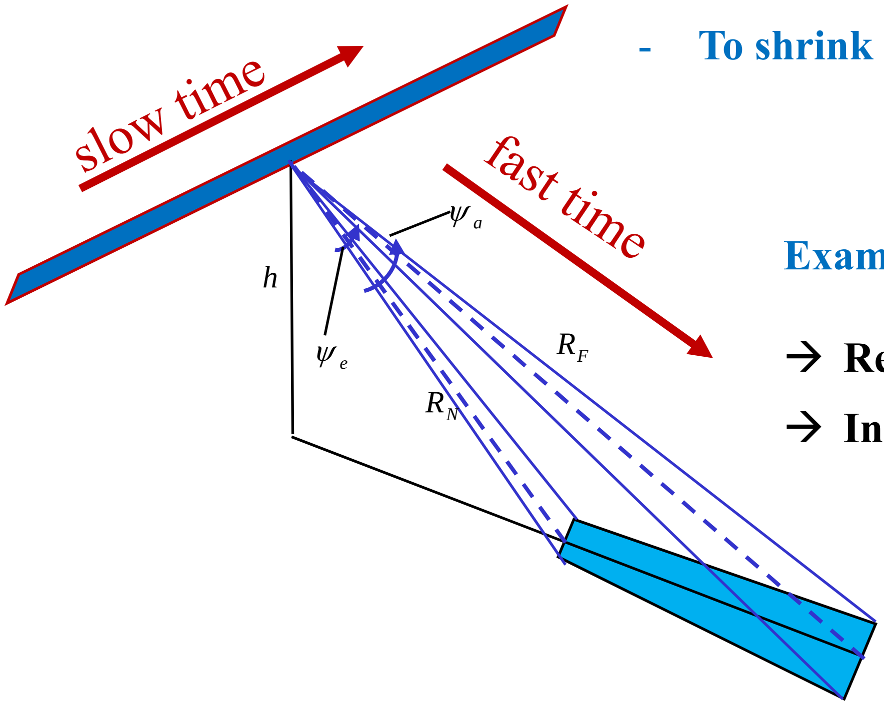
Note: along slow time … space = V * time
being V the platform velocity

Between two pulses displacement of V PRT

Still wide pixel along slow time !!!

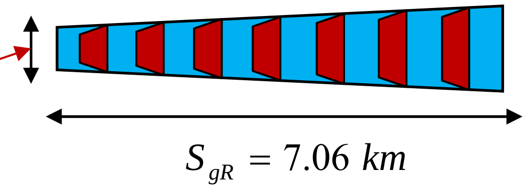


Real Aperture Radar (II)



- To shrink resolution cell → increase antenna length d_a

Example: to achieve D_{s0}=0.5 m



→ **Reduce beamwidth 344/0.5=688 times!**

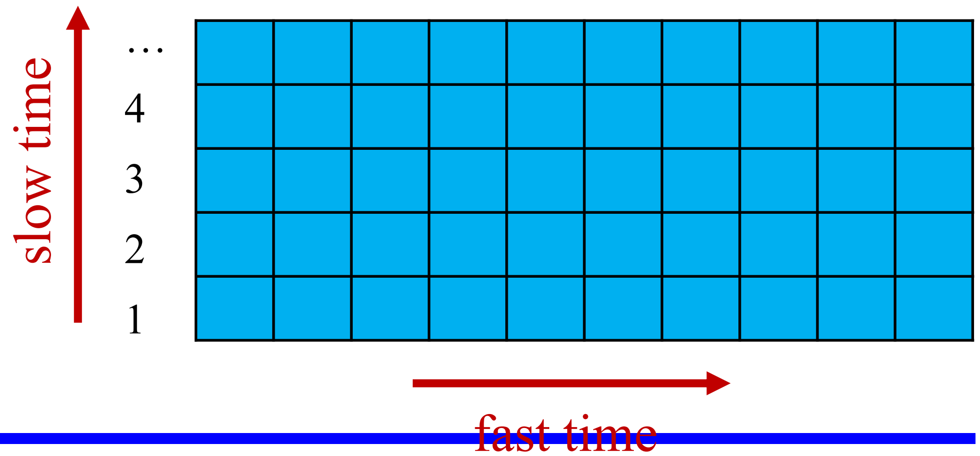
→ **Increase antenna length da 688 times:**

d_a=1.8*688= 1238.4 dm !!!!!!!! IMPOSSIBLE!!!

Using platform motion:

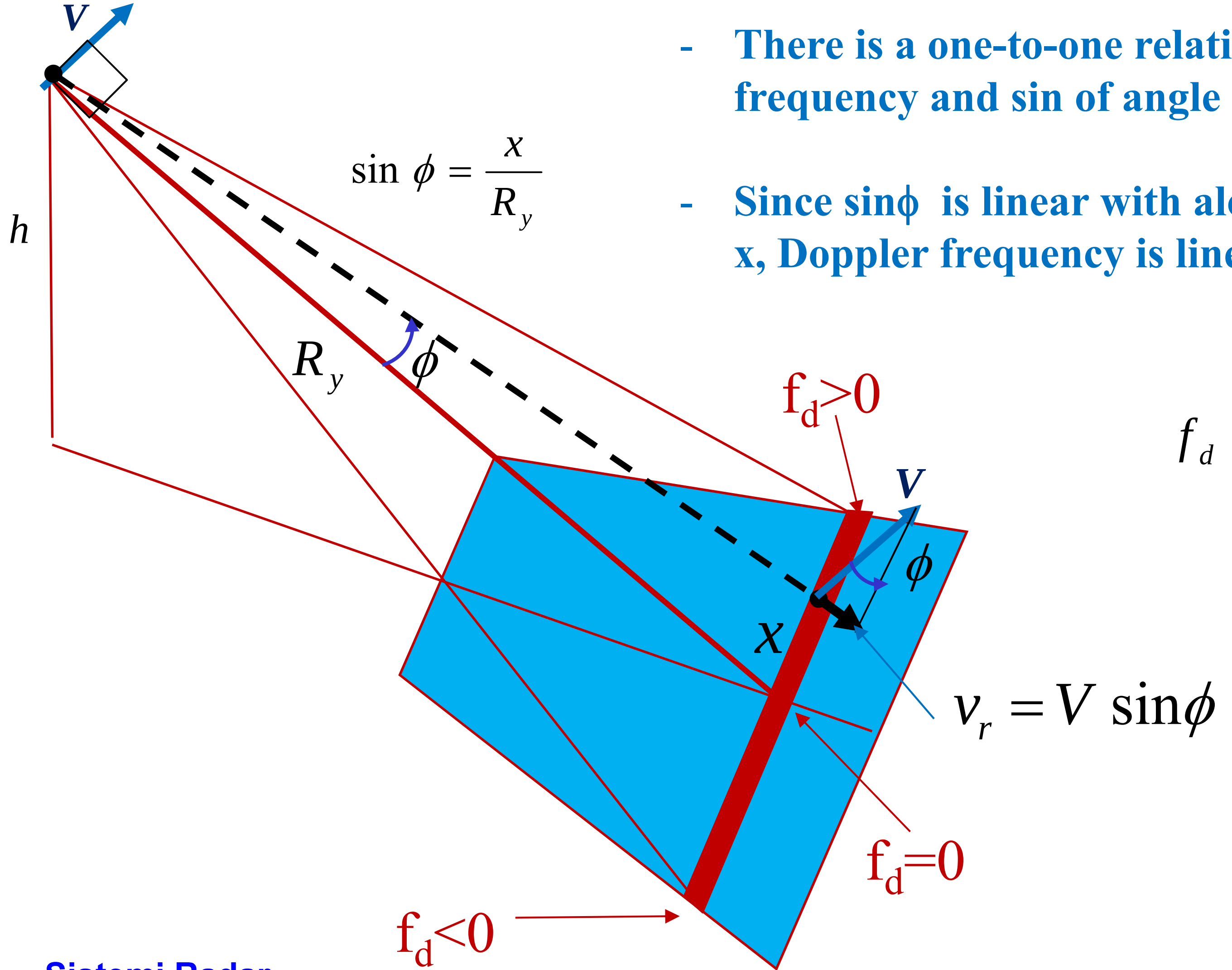
→ **a matrix → an image !!!!**

with 0.5 m × 0.5 m ground resolution



Angle-Doppler frequency relationship

- There is a one-to-one relationship between Doppler frequency and sin of angle ϕ
- Since $\sin\phi$ is linear with along-track displacement x, Doppler frequency is linear with x

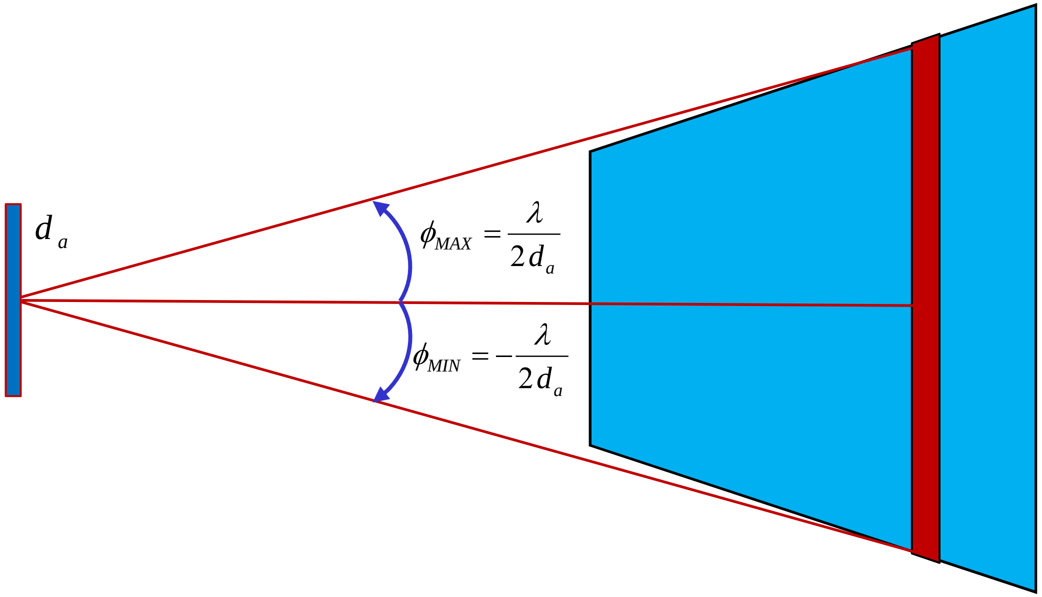


$$f_d = \frac{2}{\lambda} v_r = \frac{2}{\lambda} V \sin\phi$$

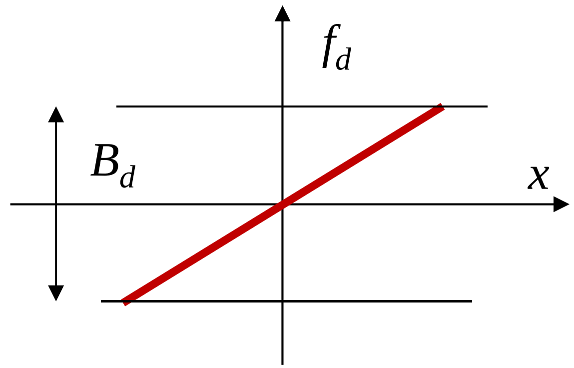
$$f_d \cong \frac{2V}{\lambda R_y} x$$

Doppler frequency bandwidth

$$f_{d\,MAX} = \frac{2}{\lambda} V \sin\phi_{MAX} = \frac{2}{\lambda} V \sin\left(\frac{\lambda}{2d_a}\right) \cong \frac{2}{\lambda} V \frac{\lambda}{2d_a} = \frac{V}{d_a}$$



$$f_{d\,MIN} = \frac{2}{\lambda} V \sin\phi_{MIN} \cong -\frac{V}{d_a}$$



Doppler frequency bandwidth:

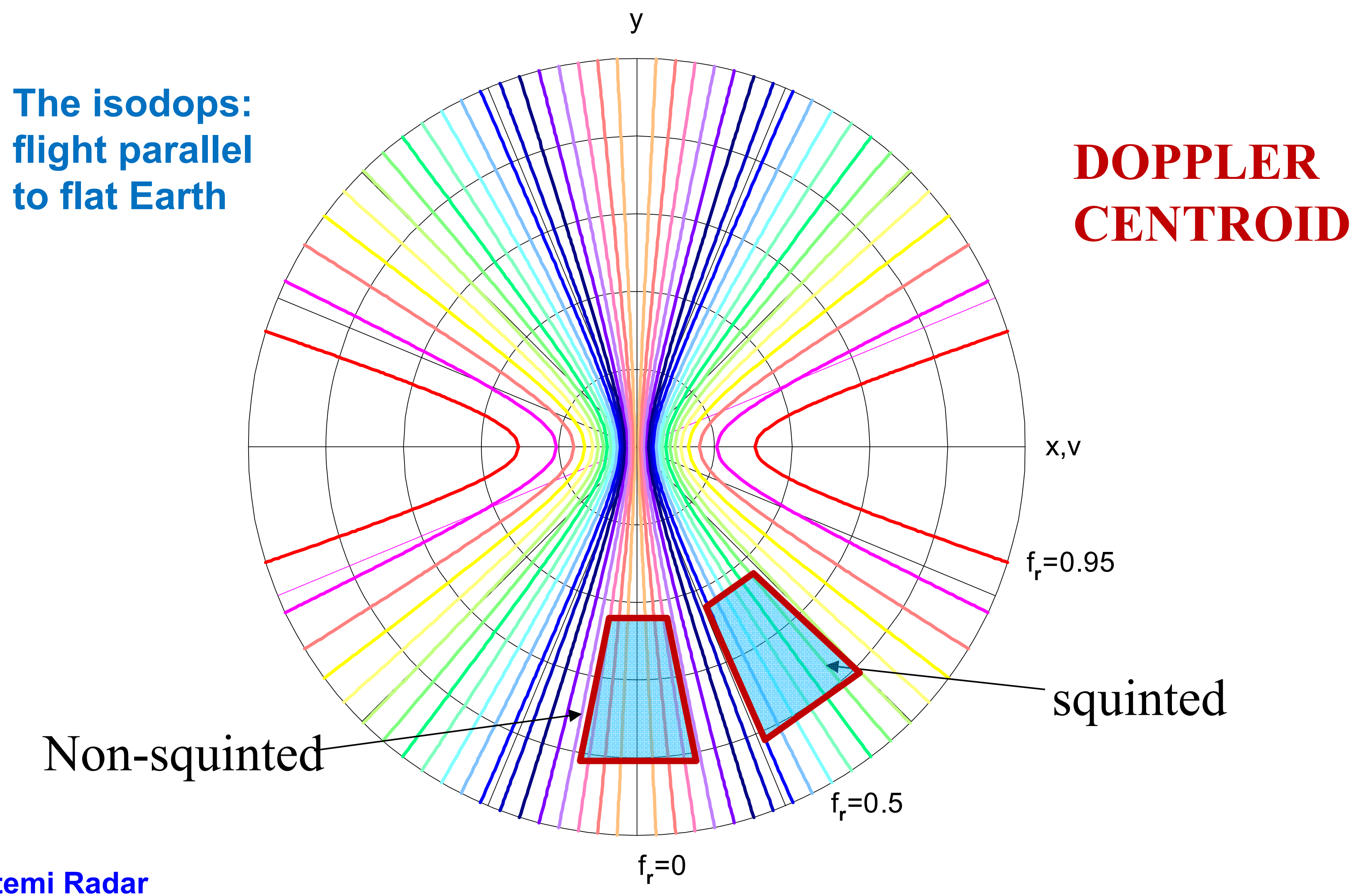
$$B_d = f_{d\,MAX} - f_{d\,MIN} = \frac{2V}{d_a}$$

⇓

- **Minimum PRF**

$$PRF \geq B_d = \frac{2V}{d_a}$$

Frequency approach to SAR



Along-track resolution by Doppler

- **Doppler frequency resolution *(Fourier Transform)***

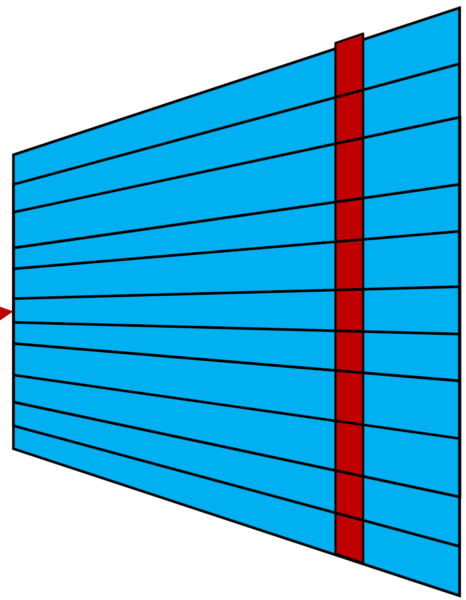
$$\Delta f_d = \frac{1}{T_{oss}} = \frac{1}{N \cdot PRT} = \frac{PRF}{N}$$

$$\begin{cases} \Delta f_d = \dfrac{1}{T_{oss}} = \dfrac{2}{\lambda} V \ \delta \sin\phi \\ \Delta f_d = \dfrac{1}{T_{oss}} = \dfrac{2V}{\lambda R_y} \ \delta x \end{cases}$$

$$\begin{cases} \delta \sin\phi = \dfrac{\lambda}{2V} \Delta f_d = \dfrac{\lambda}{2V} \dfrac{1}{T_{oss}} = \dfrac{\lambda}{2V} \dfrac{PRF}{N} \\ \delta x = \dfrac{\lambda R_y}{2V} \Delta f_d = \dfrac{\lambda R_y}{2V} \dfrac{1}{T_{oss}} = \dfrac{\lambda R_y}{2V} \dfrac{PRF}{N} \end{cases}$$

- **N pulses at min PRF: FFT provides N Doppler filters**

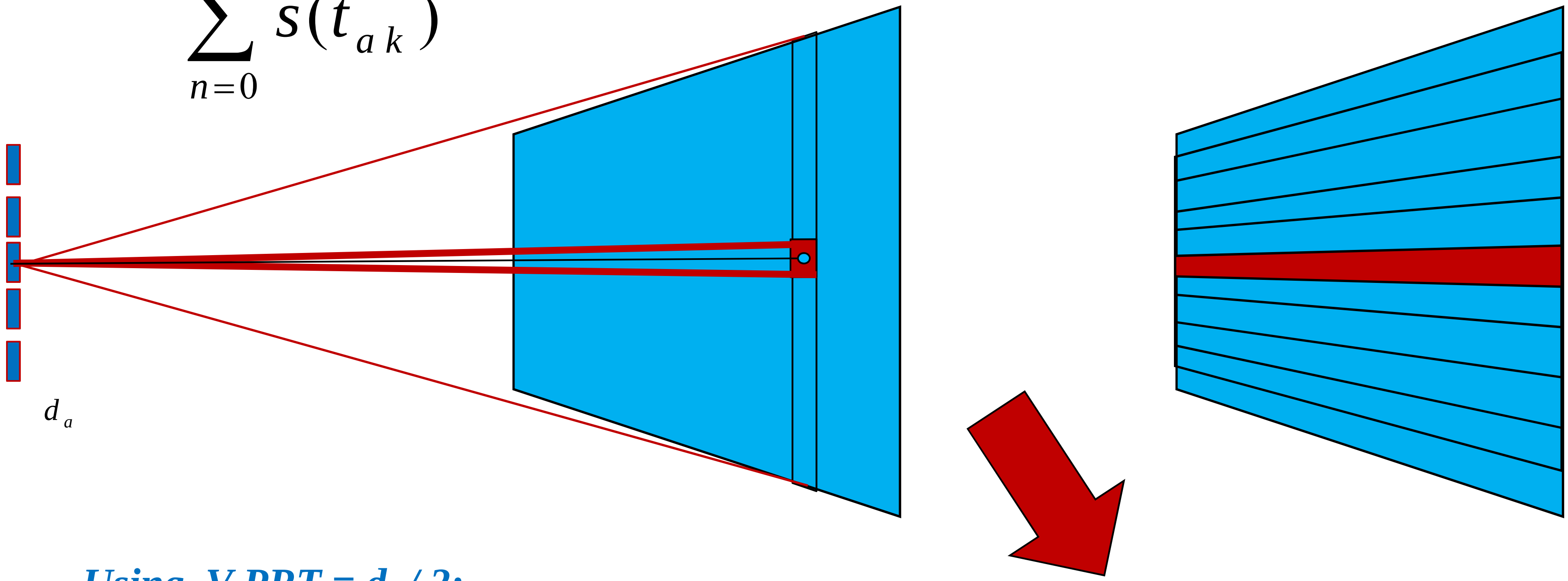
$$\begin{cases} \delta \sin\phi \geq \dfrac{\lambda}{2V} \dfrac{2V}{N d_a} = \dfrac{1}{N} \dfrac{\lambda}{d_a} = \dfrac{\psi_a}{N} \\ \delta x \geq \dfrac{\lambda R_y}{2V} \dfrac{2V}{N d_a} = \dfrac{1}{N} \dfrac{\lambda}{d_a} R_y = \dfrac{D_{sy}}{N} \end{cases}$$



Synthetic antenna principle

- **By exploiting platform motion emulate “synthetic antenna array”**

$$\sum_{n=0}^{N-1} s(t_{ak})$$



-Using $V\,PRT = d_a/2$:

$$\frac{2V\,PRT}{\lambda}\,\delta \sin\phi = \frac{1}{N} \quad \rightarrow \delta \sin\phi = \frac{\lambda}{2V \cdot PRT}\frac{1}{N} = \frac{\lambda}{d_a}\frac{1}{N} = 2\frac{\psi_a}{N}$$

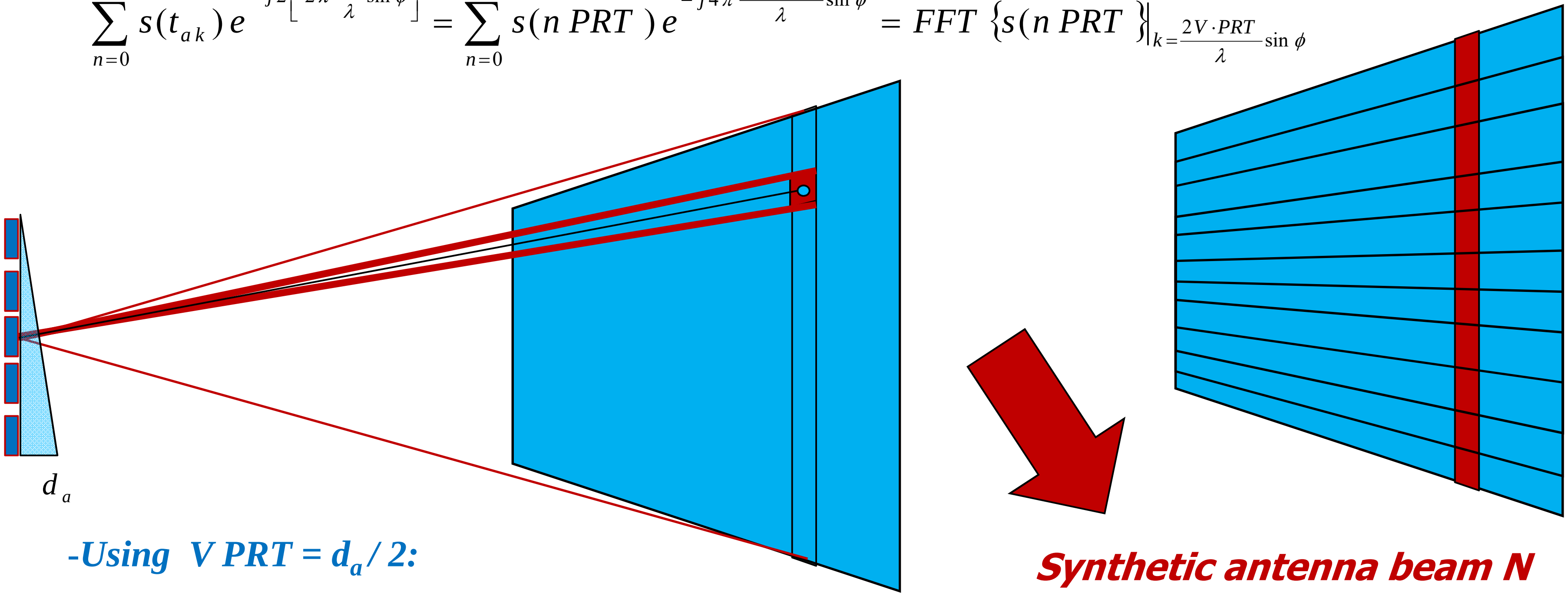
Synthetic antenna beam N times narrower than real antenna beam

Synthetic antenna principle (II)

- **By exploiting platform motion emulate "synthetic antenna array"**

-To steer in direction ϕ, add all returns of the N pulses after compensating a linear phase term $\Delta\phi = 2\pi k \frac{d}{\lambda} \sin\phi$ ***both*** ***in TX and in RX (twice as in standard array):***

$$\sum_{n=0}^{N-1} s(t_{ak})\, e^{-j2\left[2\pi \frac{nd}{\lambda}\sin\phi\right]} = \sum_{n=0}^{N-1} s(n\,PRT)\, e^{-j4\pi \frac{nV\cdot PRT}{\lambda}\sin\phi} = FFT\left\{s(n\,PRT)\right\}\Big|_{k=\frac{2V\cdot PRT}{\lambda}\sin\phi}$$

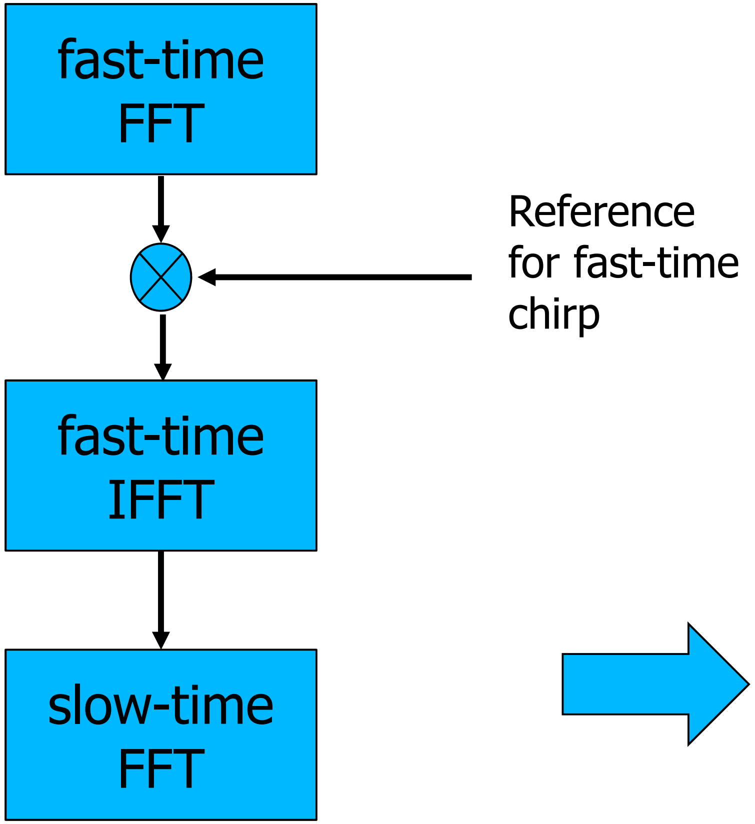


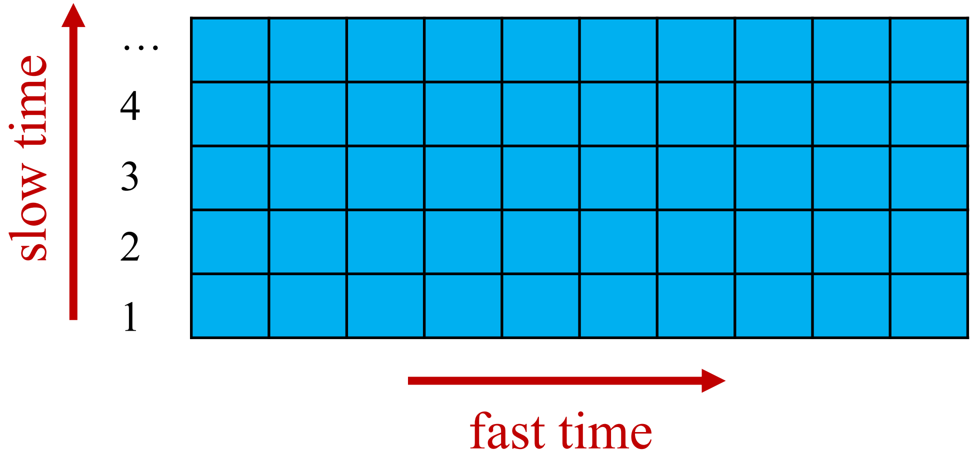
-Using $V\,PRT = d_a/2$:

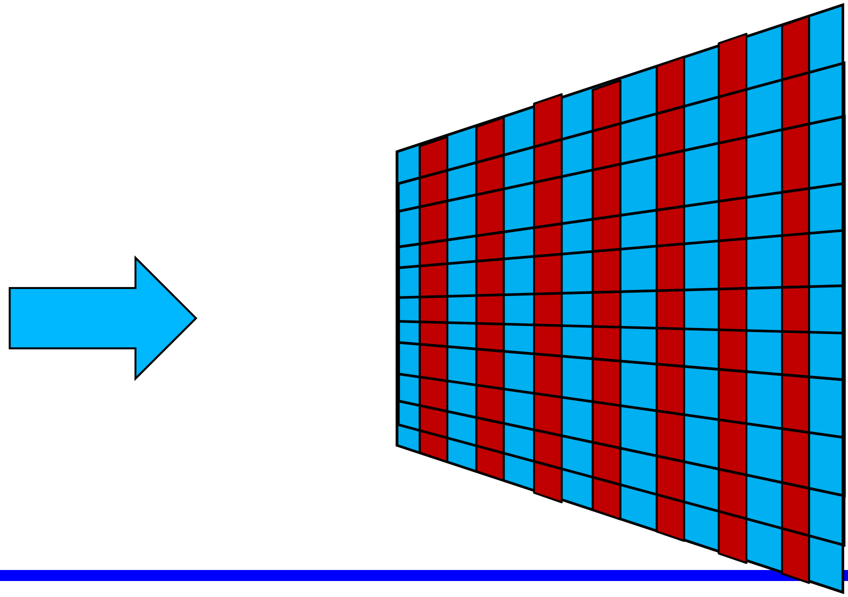
$$\frac{2V\,PRT}{\lambda}\,\delta\sin\phi = \frac{1}{N} \quad \rightarrow \delta\sin\phi = \frac{\lambda}{2V\cdot PRT}\frac{1}{N} = \frac{\lambda}{d_a}\frac{1}{N} = 2\frac{\psi_a}{N}$$

Synthetic antenna beam N times narrower than real antenna beam

Unfocused SAR Processing scheme







Limit to Doppler frequency resolution

Longer T_{oss} = longer pulse sequence → Higher Doppler frequency resolution

$$\Delta f_d = \frac{1}{T_{oss}} = \frac{PRF}{N}$$

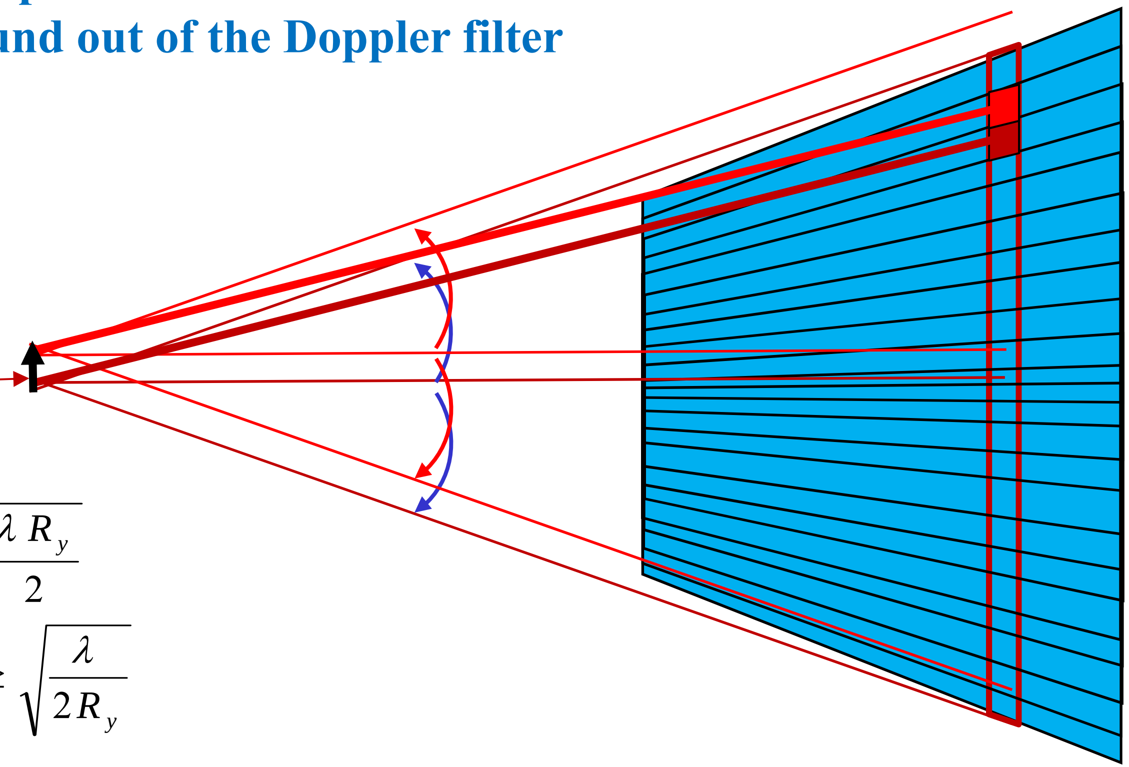
$$\begin{cases} \delta \sin\phi = \dfrac{\lambda}{2V}\dfrac{1}{T_{oss}} \\ \delta x = \dfrac{\lambda R_y}{2V}\dfrac{1}{T_{oss}} \end{cases}$$

- **This all applies as long as the platform motion does not force motion of point on ground out of the Doppler filter**

$$V\,T_{oss} \le \delta x = \frac{\lambda R_y}{2V}\frac{1}{T_{oss}}$$

$$(V\,T_{oss})^2 \le \frac{\lambda R_y}{2}$$

$$V\,T_{oss} \le \sqrt{\frac{\lambda R_y}{2}}$$



Maximum resolution:

$$\delta x \ge \sqrt{\frac{\lambda R_y}{2}}$$

$$\delta \sin\phi \ge \sqrt{\frac{\lambda}{2R_y}}$$

Max unfocused SAR resolution

Longer T_{oss} = longer pulse sequence → Higher Doppler frequency resolution

$$\Delta f_d = \frac{1}{T_{obs}} = \frac{PRF}{N}$$

$$N = T_{obs}\, PRF = T_{obs}\, B_d = T_{obs} \frac{2V}{d_a} \le \frac{2}{d_a}\sqrt{\frac{\lambda R_y}{2}}$$

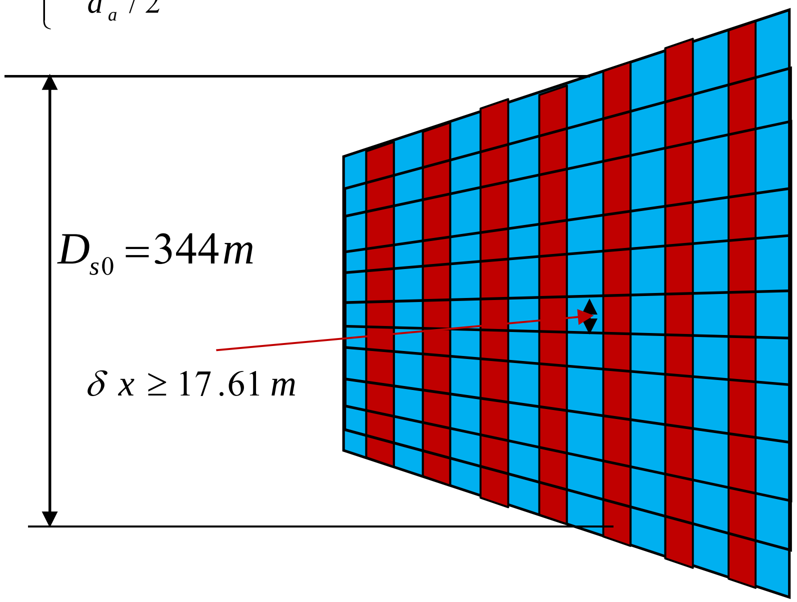
$$V\, T_{oss} \le \sqrt{\frac{\lambda R_y}{2}} = \begin{cases} \sqrt{\lambda R_N / 2} = 16.45\, m \\ \sqrt{\lambda R_0 / 2} = 17.61\, m \\ \sqrt{\lambda R_F / 2} = 19.13\, m \end{cases}$$

$$N \le \begin{cases} \frac{\sqrt{\lambda R_N / 2}}{d_a / 2} = 18.28 \\ \frac{\sqrt{\lambda R_0 / 2}}{d_a / 2} = 19.56 \\ \frac{\sqrt{\lambda R_F / 2}}{d_a / 2} = 21.25 \end{cases}$$

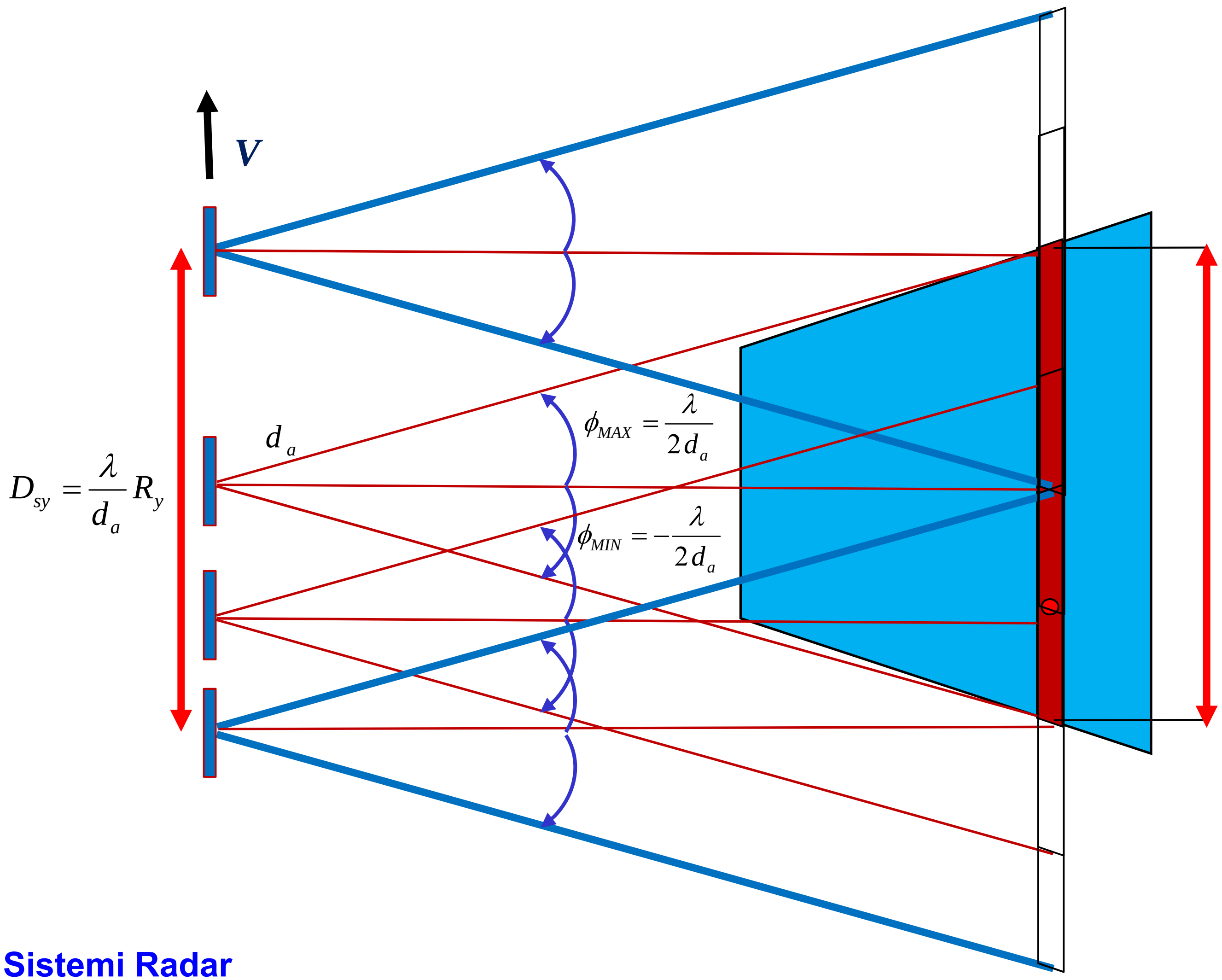
Maximum resolution:

$$\delta x \ge \sqrt{\frac{\lambda R_y}{2}} = \begin{cases} \sqrt{\lambda R_N / 2} = 16.45\, m \\ \sqrt{\lambda R_0 / 2} = 17.61\, m \\ \sqrt{\lambda R_F / 2} = 19.13\, m \end{cases}$$

$$\delta \sin\phi \ge \sqrt{\frac{\lambda}{2 R_y}} = \begin{cases} \sqrt{\frac{\lambda}{2 R_N}} \to \phi \cong 0.0540\,° \\ \sqrt{\frac{\lambda}{2 R_F}} \to \phi \cong 0.0464\,° \end{cases}$$



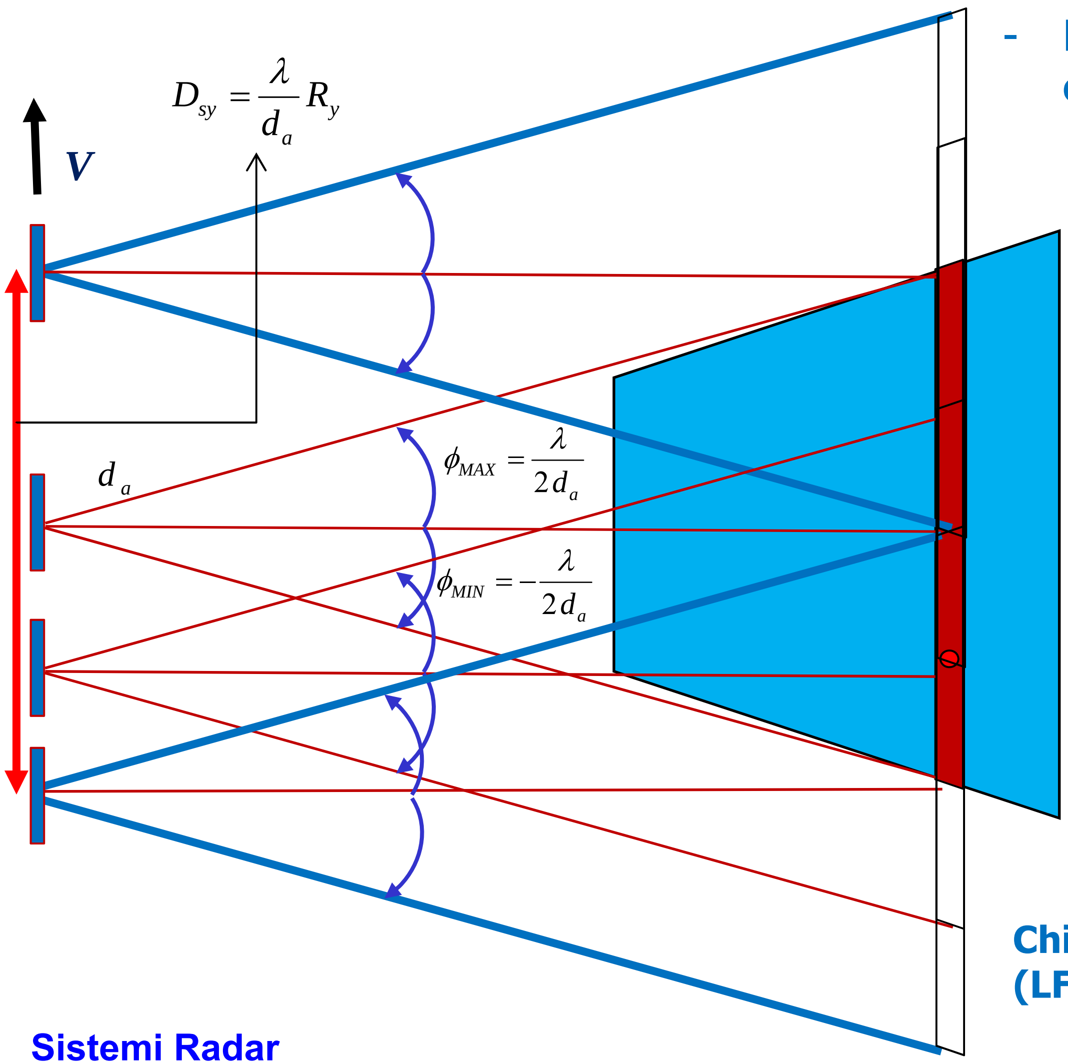
Maximum observation time for point target



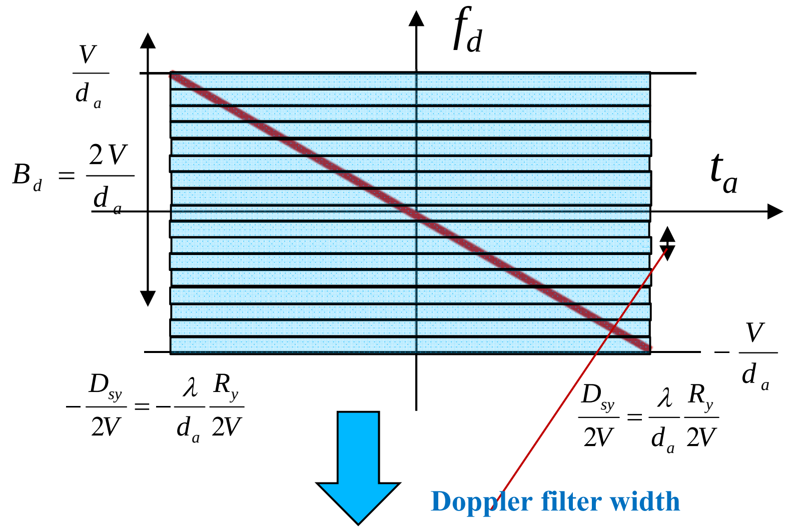
- **Maximum T_{obs} for the echo to be received inside the antenna beam**

$$T_{obs} = \frac{D_{sy}}{V} = \frac{\lambda}{d_a} \frac{R_y}{V}$$

Slow-time Chirp signal from point target



- **For long T_{oss} the geometry induces a chirp signal in the slow time t_a**



Received echo from point target migrates through Doppler frequency filters in the slow time t_a

Chirp slope: (LFM rate)

$$\beta_{t_a} = \frac{B_d}{T_{obs}} = \frac{2V}{d_a}\frac{1}{\frac{\lambda}{d_a}\frac{R_y}{V}} = \frac{2V^2}{\lambda R_y}$$

Slow-time Chirp signal from point target (II)

- **Chirp signal in the slow time t_a**

$$s(t_a) = rect_{T_{obs}}(t_a)\, e^{-j\pi \beta_{t_a} t_a^2}$$

$$T_{obs} = \frac{\lambda}{d_a}\frac{R_y}{V} \qquad \beta_{t_a} = \frac{2V^2}{\lambda R_y}$$

- **Chirp signal in the along-track space domain *x*** $\quad x = V\, t_a$

$$s(x) = rect_{D_{sy}}(x)\, e^{-j\pi \beta x^2}$$

$$D_{sy} = \frac{\lambda}{d_a} R_y \qquad \beta = \frac{2}{\lambda R_y}$$

Focused SAR

To exploit long T_{oss} we can think in terms of:

- **Compress the chirp signal in the slow time t_a domain**

  → **Resolution in slow time** $$\delta t_a = \frac{1}{B_d} = \frac{d_a}{2V}$$

  → **Resolution in along-track range** $$\delta x = V\,\delta t_a = \frac{d_a}{2}$$

- **Compensate for the liner frequency modulation + narrow Doppler filter at zero Doppler using the whole T_{oss}**

$$T_{oss} = \frac{D_{sy}}{V} = \frac{\lambda}{d_a}\frac{R_y}{V}$$

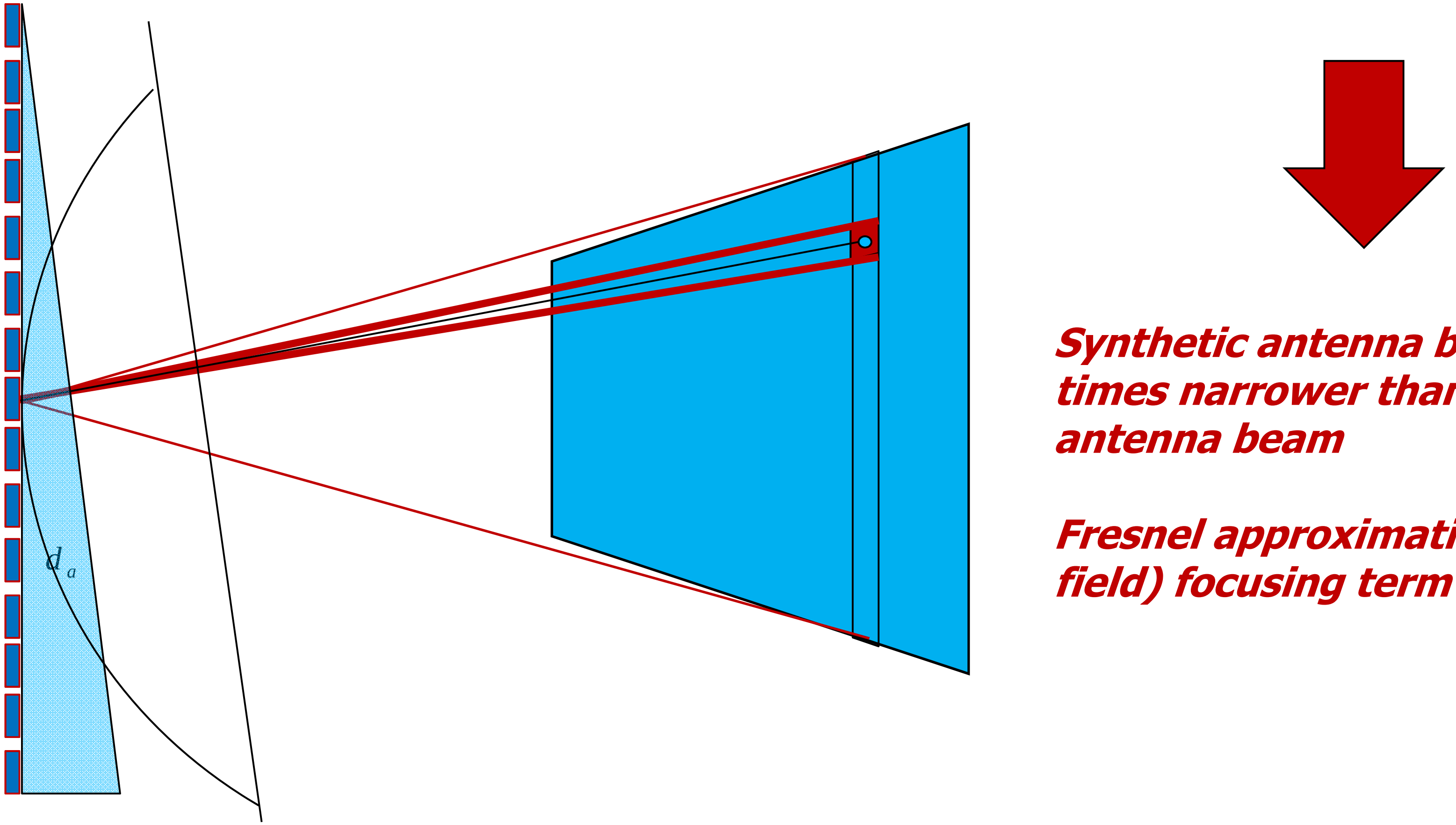
$$\delta x = \frac{\lambda R_y}{2V}\Delta f_d = \frac{\lambda R_y}{2V}\frac{1}{T_{oss}} = \frac{\lambda R_y}{2V}\frac{1}{\frac{D_{sy}}{V}} = \frac{\lambda R_y}{2V}\frac{1}{\frac{\lambda}{d_a}\frac{R_y}{V}} = \frac{d_a}{2}$$

- To achieve high resolution -→ Small-sized ANTENNA appears better !

Synthetic antenna principle (II)

- **By exploiting platform motion emulate "synthetic antenna array"**

-For long sequence of pulses, to steer in direction ϕ, compensating a linear phase term is not enough: SECOND ORDER TERM is needed → Quadratic phase of the Fresnel area

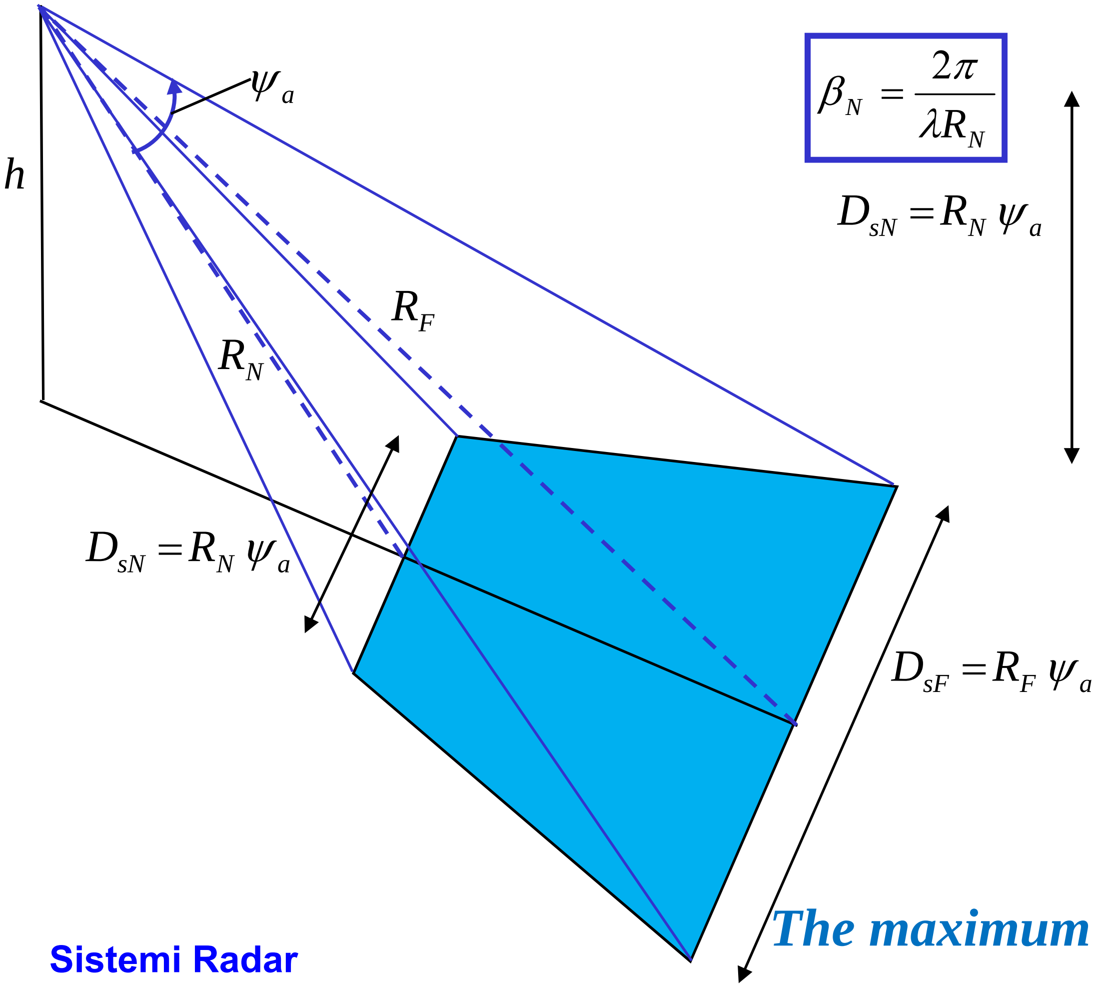


Synthetic antenna beam N times narrower than real antenna beam

Fresnel approximation (near-field) focusing term is required

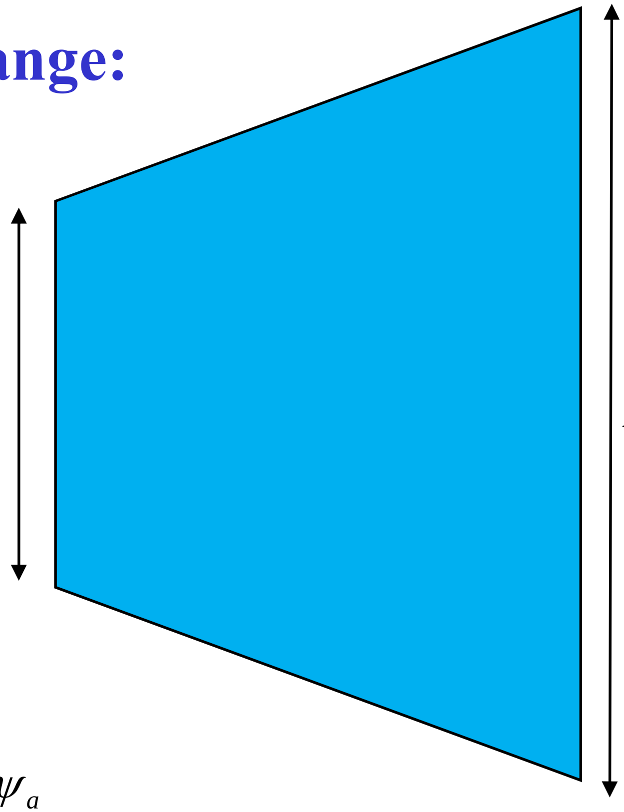
Range variation of aperture and slope (I)

D_{sy} and β vary with range:



$$\beta_N = \frac{2\pi}{\lambda R_N}$$

$$D_{sN} = R_N \, \psi_a$$



$$\beta_F = \frac{2\pi}{\lambda R_F}$$

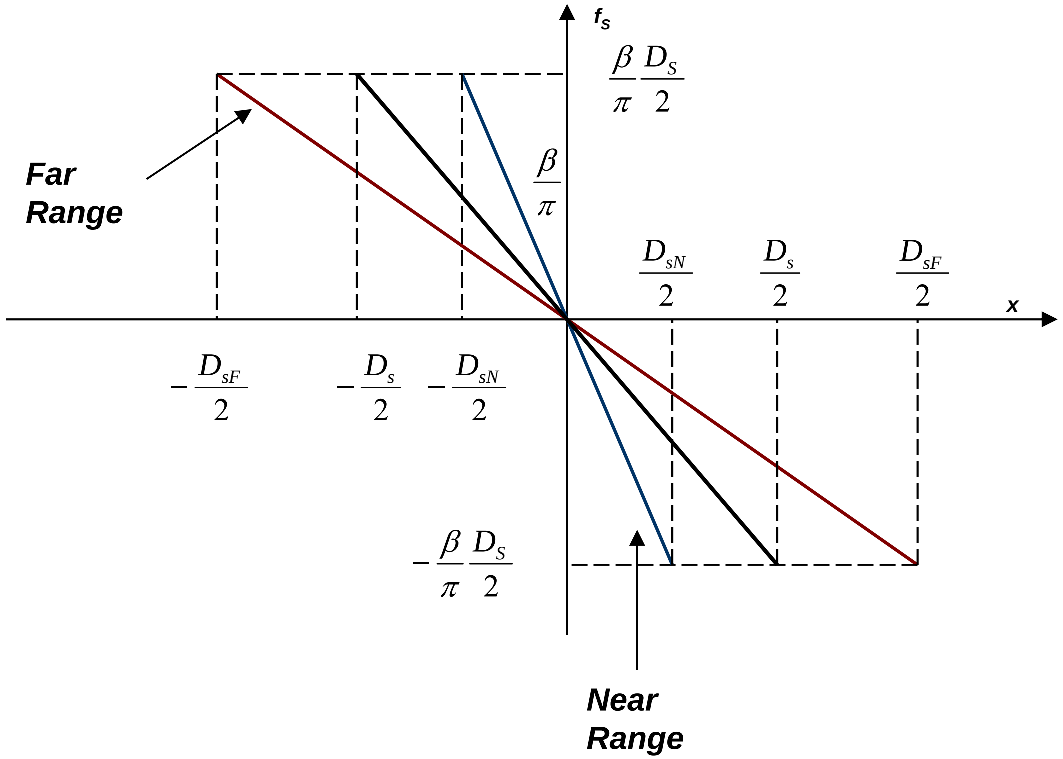
$$D_{sF} = R_F \, \psi_a$$

$$\frac{1}{r_A} = \frac{\beta_N}{\pi} \times D_{sN} = \frac{\beta_F}{\pi} \times D_{sF} = \frac{2}{d_a}$$

The maximum resolution does not vary with range

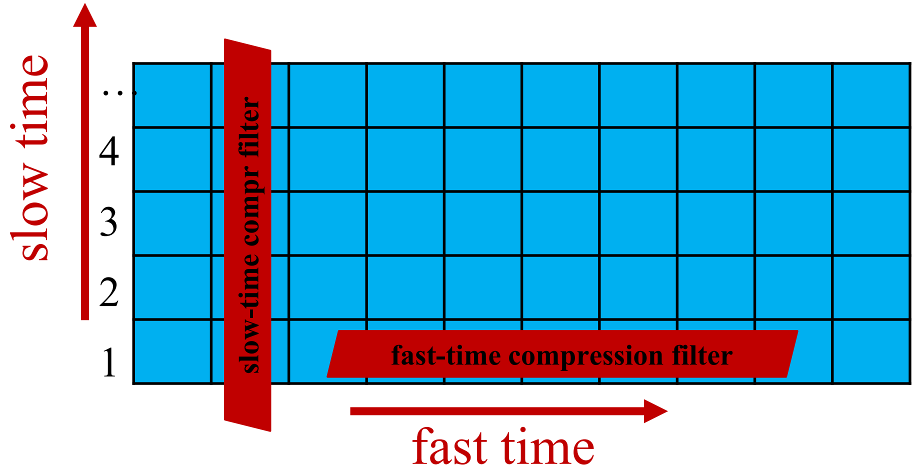
Range variation of aperture and slope (II)

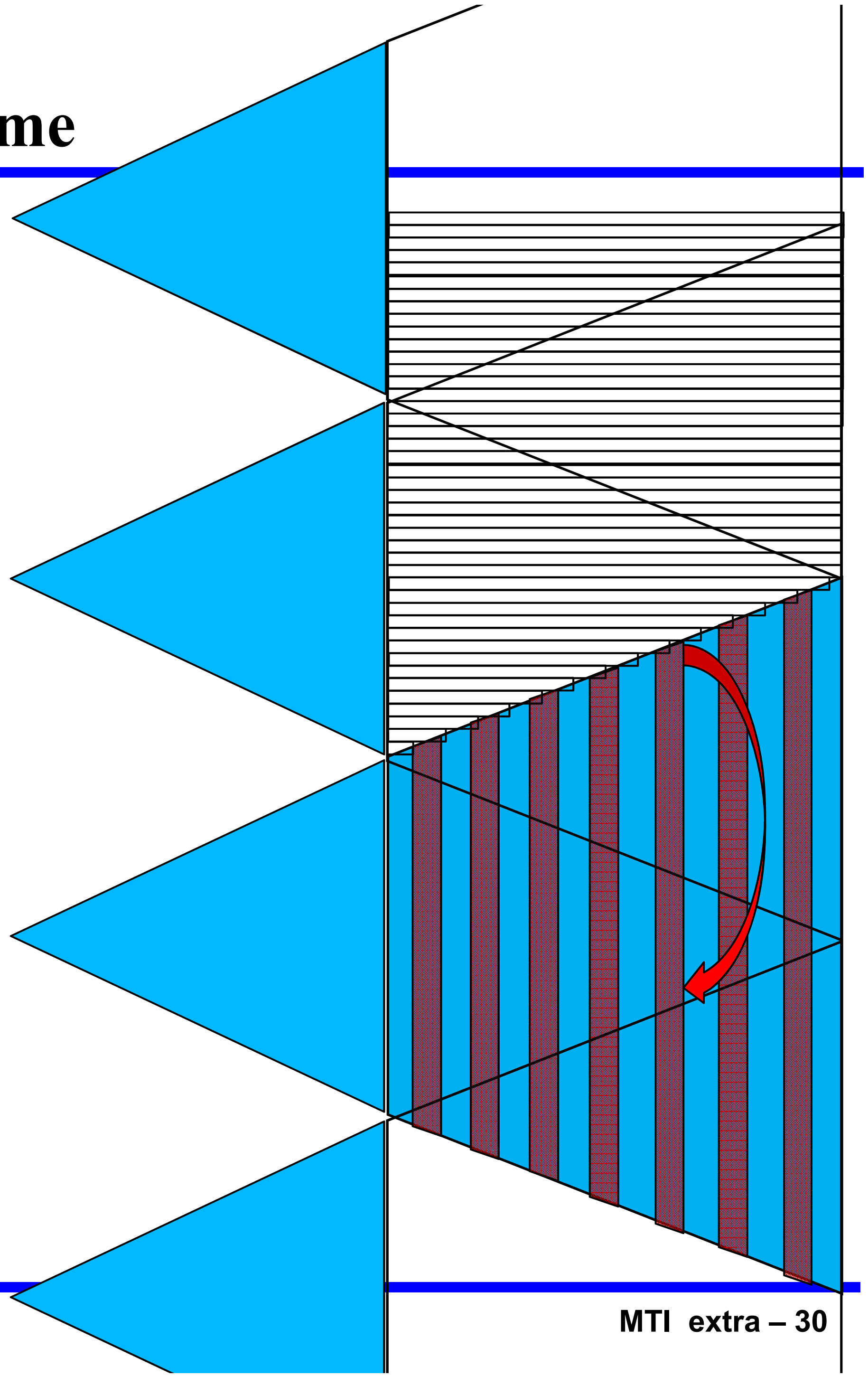
The maximum resolution does not vary with range



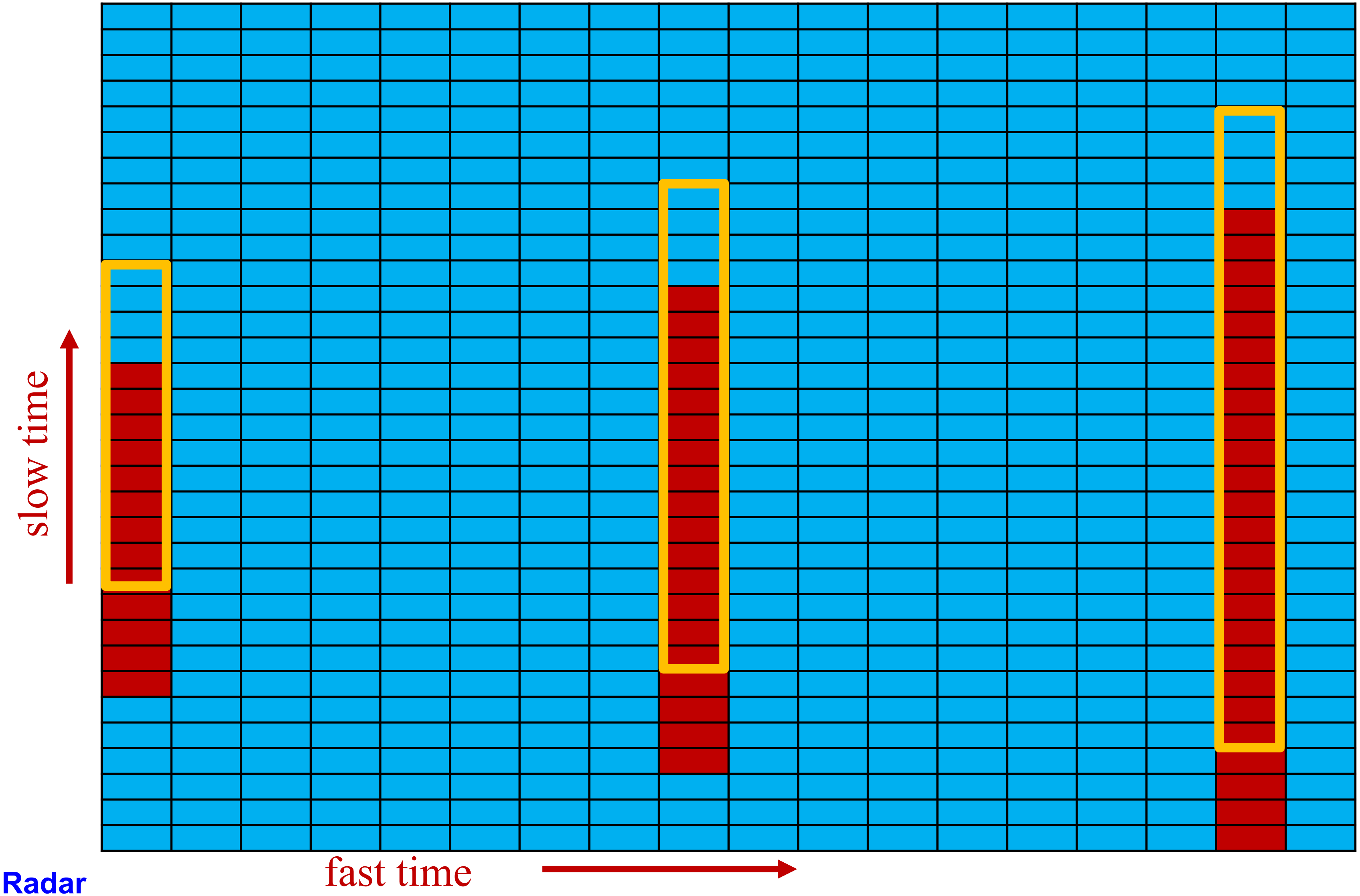
- **Note: compression filter length and filter parameter (beta) vary from N to F**
  → a different slow-time filter must be applied for every fast-time sample

Focused SAR processing scheme

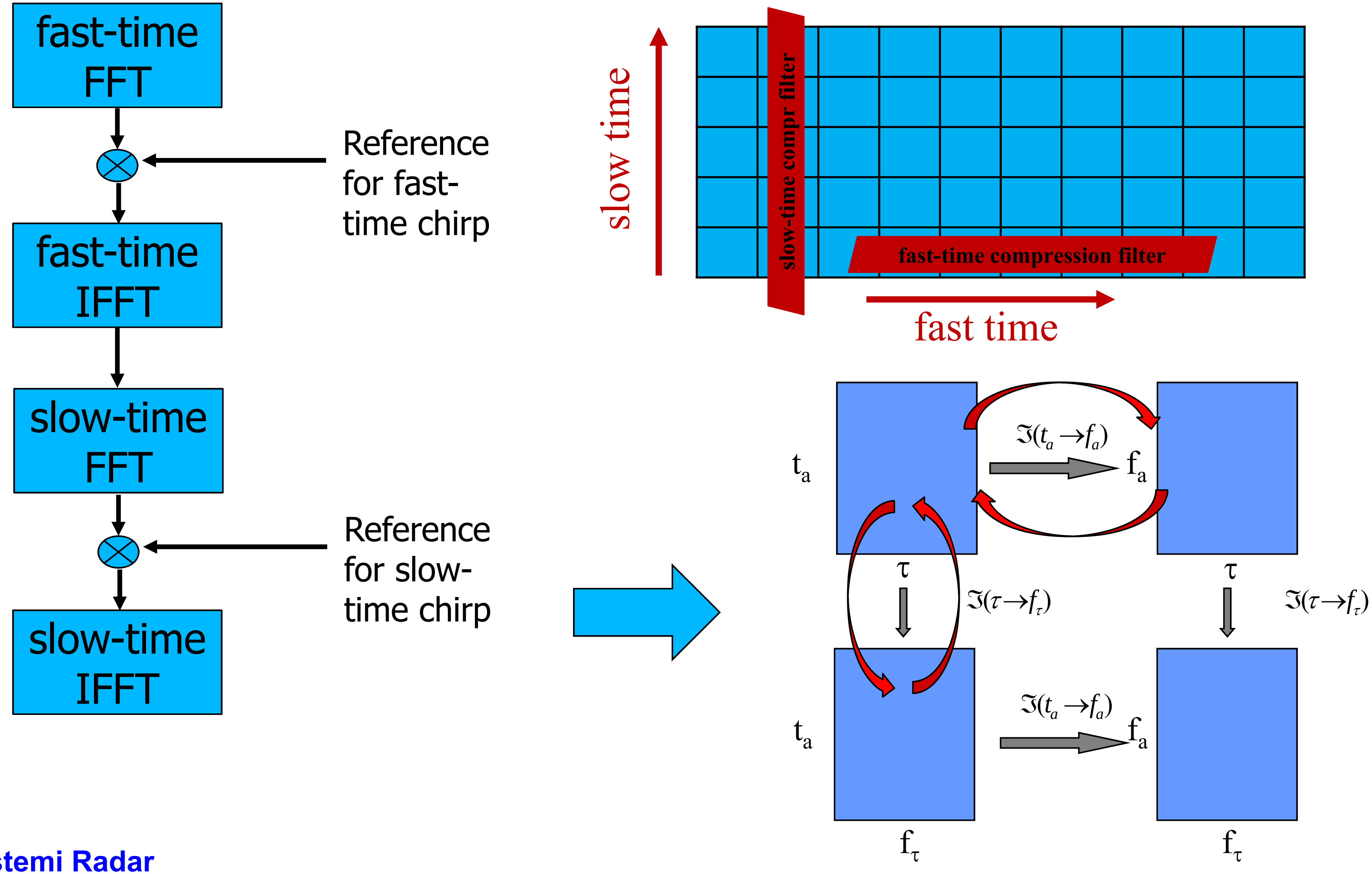




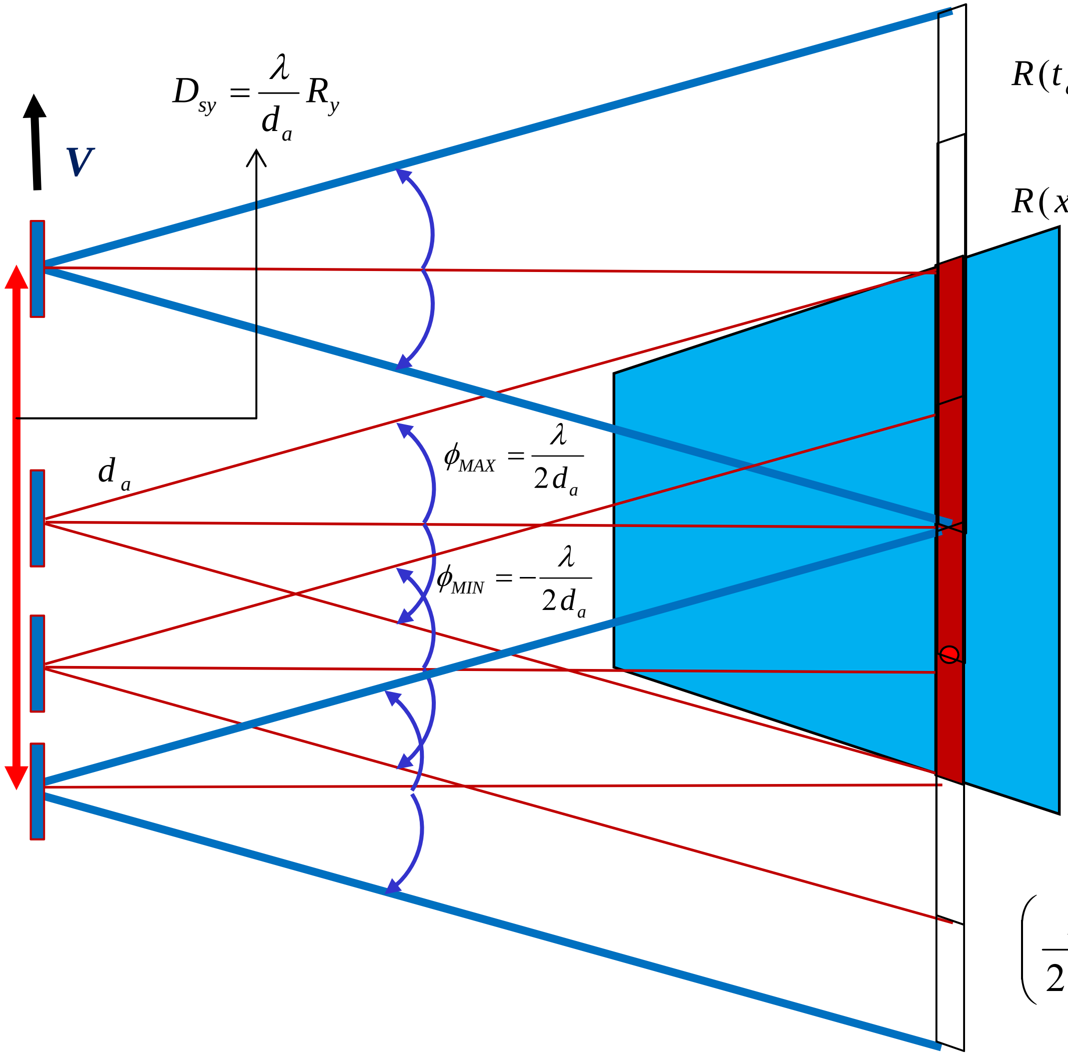
focused SAR Processing scheme (II)



frequency domain SAR processing scheme



Radar-point target range varies with slow-time



$$R(t_a) = \sqrt{R_y^2 + V^2 t_a^2} = R_y \sqrt{1 + \frac{V^2 t_a^2}{R_y^2}} \cong R_y + \frac{V^2 t_a^2}{2R_y} + \cdots$$

$$R(x) = \sqrt{R_y^2 + x^2} = R_y \sqrt{1 + \frac{x^2}{R_y^2}} \cong R_y + \frac{x^2}{2R_y} + \cdots$$

Note: the quadratic term in x (t_a) is the same term responsible for the slow-time chirp:

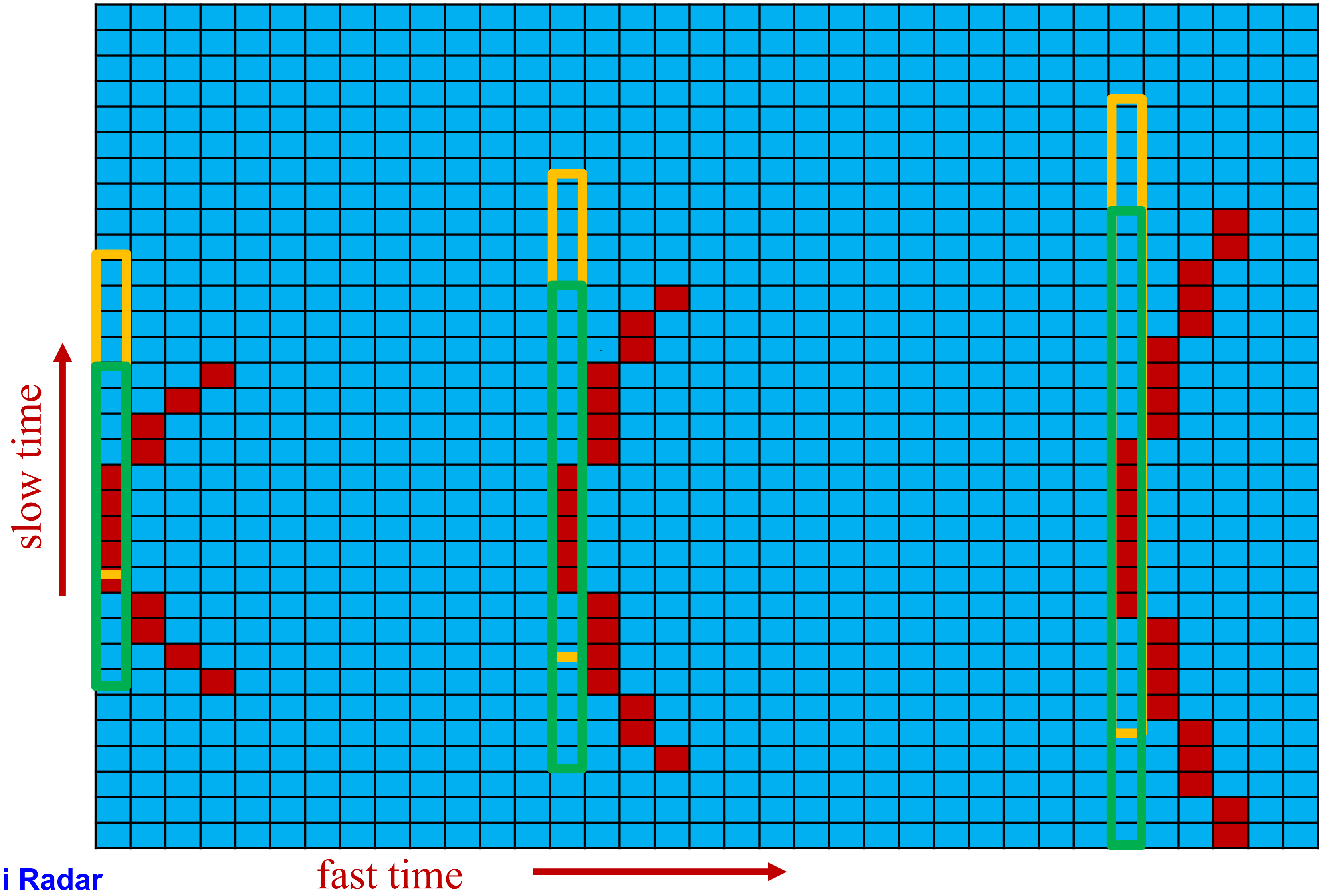
- **quadratic phase term**
- **linear frequency modulation**

if $\frac{x^2}{2R_y} > \delta_R$ **the echo from the point target migrates through range bins**

$$\left(\frac{\lambda}{2d_a} R_y\right)^2 \frac{1}{2R_y} < \delta_R \quad \Rightarrow \quad \delta_R > \frac{\lambda^2 R_y}{8 d_a^2}$$

Sistemi Radar

Range cell migration (RCM)



RCM compensation

Hyperbolic shaped (approx. quadratic) range cell migration appears unless range resolution is coarse enough

For the sample airborne SAR case (using worst case Far range distance)

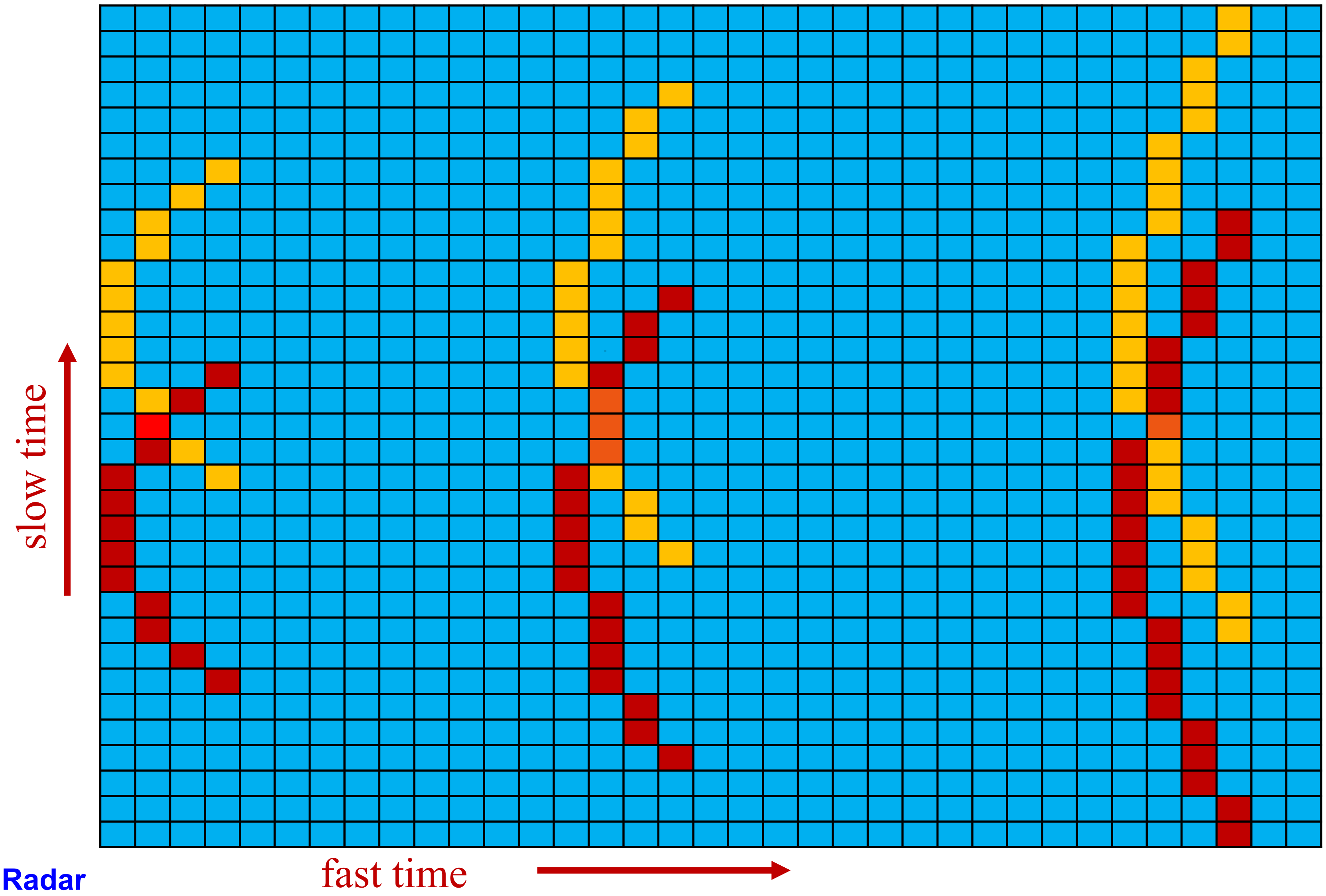
$$\delta_R > \frac{\lambda^2 R_y}{8 d_a^2} = 3.5 \ m$$

If higher rage resolution is required, it is necessary to compensate the point target migration through range bins

Note:

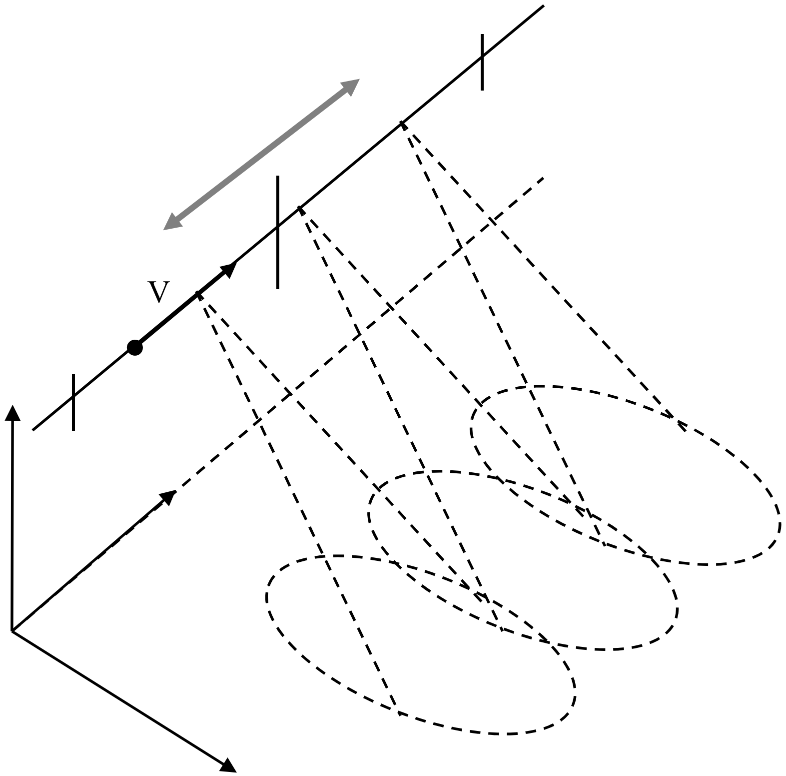
1) **Range Cell Migration shape is range dependent ! →different compensation fron N to F**
2) **For targets at same range and different along-track displacement RCM compensation is different → Compensation in time domain must be repeated continuously in slow-time**

RCM compensation (II)

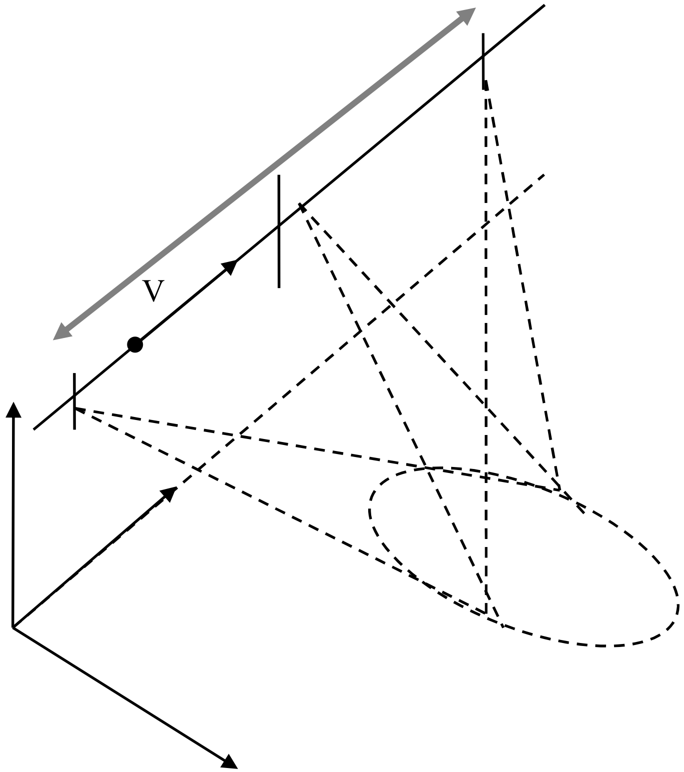


Spotlight Mode SAR

Spotlight Mode SAR steers the real antenna toward the scene center to exceed the limit on the synthetic aperture of the stripmap mode



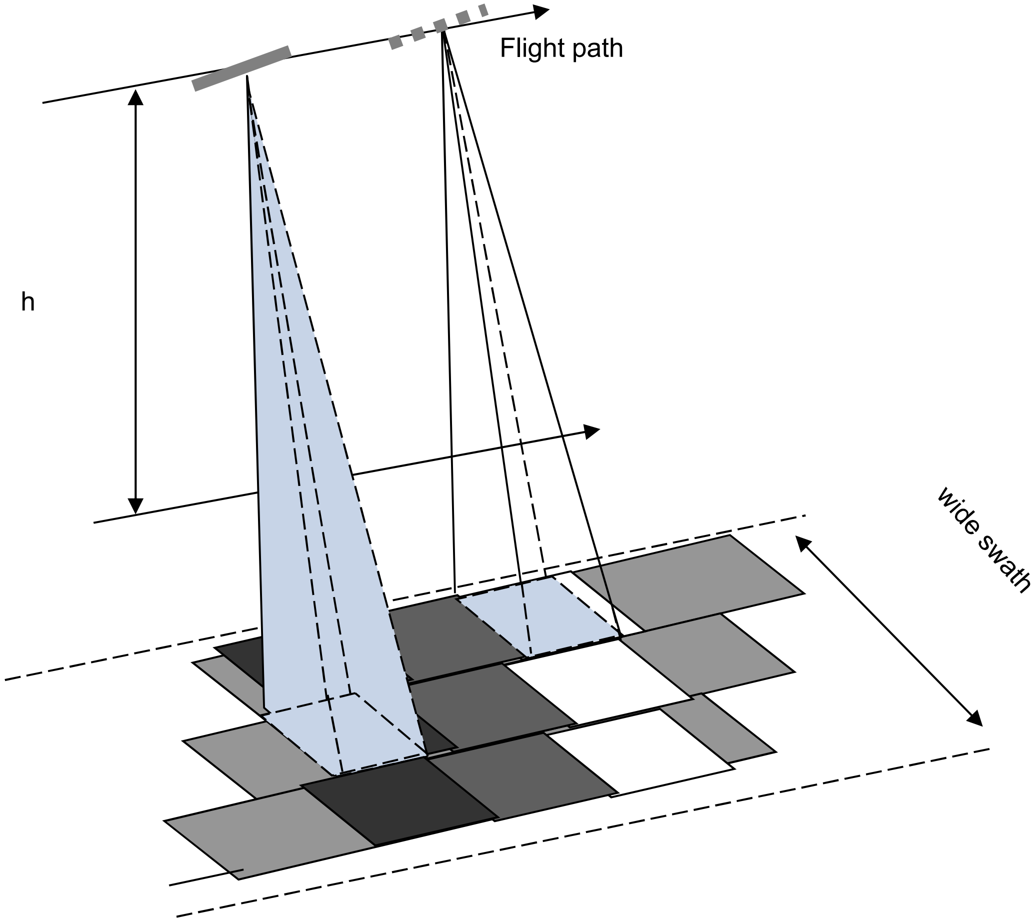
STRIPMAP Mode



SPOTLIGHT Mode

ScanSAR Mode

ScanSAR Mode acquisition are performed by using the same azimuth antenna steering of the stripmap mode, but switching the beam in elevation after each burst to cover a wider swath



Fundamental limitation of SAR

Avoidance of Range Ambiguities: $1/PRF > 2\,S_R/c$

Avoidance of Azimuth Ambiguities: $PRF > 2v/\lambda$ *Antenna beamwidth AZ

Range Swath: $S_R = \psi_e R_o / \cos\alpha = \lambda/d_e\, R_o / \cos\alpha$

Antenna beamwidth AZ $\psi_a = \lambda/d_a$

$$\frac{2v}{\lambda}\frac{\lambda}{d_a} < PRF < \frac{c}{2}\frac{d_e \cos\alpha}{\lambda R_o} \Rightarrow \frac{2v}{\lambda}\frac{\lambda}{d_a} < \frac{c}{2}\frac{d_e \cos\alpha}{\lambda R_o}$$

$$\frac{S_R}{d_a/2} < \frac{c}{2v}$$

$$d_e d_a > \frac{4v}{c}\frac{\lambda R_o}{\cos\alpha}$$