Integrazione Coerente e Filtraggio Ottimo

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Discriminazione in base alla Doppler (III)

S_i- C_i- N_i : potenza di segnale utile, clutter e noise in ingresso al dispositivo anti-clutter;

S_o- C_o- N_o : potenza di segnale utile, clutter e noise in uscita al dispositivo anti-clutter;

$$S_0 = \left| H(f_d) \right|^2 S_i$$

$$C_0 = \int_{-\infty}^{\infty} \left| H(f) \right|^2 S_i(f) \; df$$

$$C_i = \int_{-\infty}^{\infty} S_i(f) \; df$$

$$IF = \frac{S_o / C_o}{S_i / C_i} = \frac{S_o}{S_i} \cdot \frac{C_i}{C_o} = G \cdot CA$$

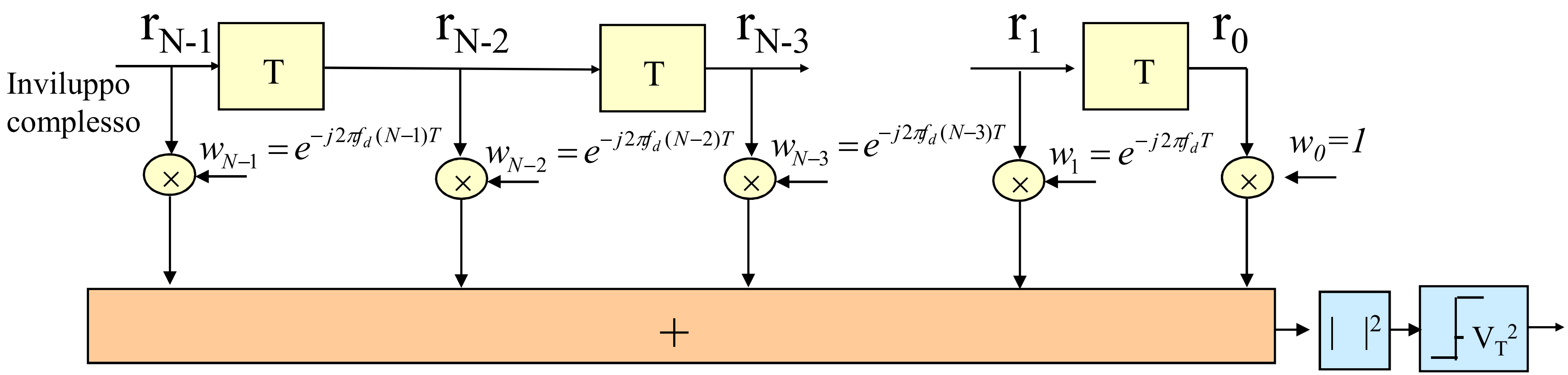
Clutter attenuation

Guadagno sul segnale

$$IF = \frac{\left| H(f_d) \right|^2 \int_{-\infty}^{\infty} S_i(f) \; df}{\int_{-\infty}^{\infty} \left| H(f) \right|^2 S_i(f) \; df}$$

Integrazione: notazione vettoriale

Filtro adattato = Riallineamento delle fasi = uscita FFT



Vettori di ritorni agli N impulsi e pesi

$$\mathbf{r} = \begin{bmatrix} r_0 \\ r_1 \\ \\ r_{N-3} \\ r_{N-2} \\ r_{N-1} \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} w_0 \\ w_1 \\ \\ w_{N-3} \\ w_{N-2} \\ w_{N-1} \end{bmatrix} \qquad \mathbf{s}_0 = \begin{bmatrix} 1 \\ e^{j2\pi f_d T} \\ \\ e^{j2\pi f_d (N-3)T} \\ e^{j2\pi f_d (N-2)T} \\ e^{j2\pi f_d (N-1)T} \end{bmatrix}$$

Modello di segnale e clutter

Eco dal bersaglio, con $S(f_d T) = e^{j2\pi f_d T}$ o con f_d=k/(NT): $S_k = e^{j2\pi k/N}$ $\mathbf{S}_k = diag\{\mathbf{s}_0(k)\}$

$$A\mathbf{s}_0(k) = A\begin{bmatrix} 1 \\ S_k \\ S_k^2 \\ S_k^3 \\ \\ S_k^{N-2} \\ S_k^{N-1} \end{bmatrix} = A\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & S_k & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & S_k^2 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & S_k^3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & S_k^{N-2} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & S_k^{N-1} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ \\ 1 \\ 1 \end{bmatrix} = A\,\mathbf{S}_k\,\mathbf{u}$$

Matrice di covarianza del clutter $P_C \cdot \mathbf{R}_0$ $\mathbf{R}_0$ Matrice dei coefficienti di correlazione

Improvement factor complessivo

Potenza di segnale in ingresso $P_{Si} = |A|^2$

Potenza di clutter in ingresso $P_{Ci} = P_C$

Rapporto segnale a clutter in ingresso $SCR_i = \dfrac{P_{Si}}{P_{Ci}} = \dfrac{|A|^2}{P_C}$

Segnale in uscita $A\,\mathbf{w}\,\mathbf{s}_0(k)$

Potenza di segnale in uscita $P_{So} = |A|^2 \left|\mathbf{w}\,\mathbf{s}_0(k)\right|^2$

Guadagno sul segnale $G_S = \dfrac{P_{So}}{P_{Si}} = \dfrac{|A|^2 \left|\mathbf{w}\,\mathbf{s}_0(k)\right|^2}{|A|^2} = \left|\mathbf{w}\,\mathbf{s}_0(k)\right|^2$

Potenza di clutter in uscita $P_{Co} = P_C\,\mathbf{w}\,\mathbf{R}_0\mathbf{w}^H$

Clutter attenuation $CA = \dfrac{P_{Ci}}{P_{Co}} = \dfrac{P_C}{P_C\,\mathbf{w}\,\mathbf{R}_0\mathbf{w}^H} = \dfrac{1}{\mathbf{w}\,\mathbf{R}_0\mathbf{w}^H}$

$$IF(k) = \frac{SCR_0(k)}{SCR_i} = \frac{P_{So}/P_{Co}}{P_{Si}/P_{Ci}} = \frac{P_{So}}{P_{Si}}\frac{P_{Ci}}{P_{Co}} = G_S \cdot CA = \frac{\left|\mathbf{w}\,\mathbf{s}_0(k)\right|^2}{\mathbf{w}\,\mathbf{R}_0\mathbf{w}^H}$$

Guadagno sul segnale – Integrazione coerente

$$\mathbf{w}_{\text{int}} = \mathbf{s}_0^H(k) \qquad \mathbf{w}_{\text{int}} = \mathbf{u}^H diag\{\mathbf{s}_0^H(k)\} = \mathbf{u}^H \mathbf{S}_k^H$$

$$\mathbf{w}_{opt} = \mathbf{s}_0^H(k)\mathbf{R}_0^{-1} \qquad \mathbf{w}_{opt} = \mathbf{s}_0^H(k)\mathbf{R}_0^{-1} = \mathbf{u}^H diag\{\mathbf{s}_0^H(k)\}\mathbf{R}_0^{-1} = \mathbf{u}^H \mathbf{S}_k^H \mathbf{R}_0^{-1}$$

$$\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix} \qquad \mathbf{a} = \begin{bmatrix} a_0 \\ a_1 \\ a_3 \\ a_4 \\ \\ a_{N-2} \\ a_{N-1} \end{bmatrix}$$

$$G_{S\_\text{int}} = \left|\mathbf{w}\,\mathbf{s}_0(k)\right|^2 = \left|\mathbf{u}^H \mathbf{S}_k^H\, \mathbf{s}_0(k)\right|^2 = \left|\mathbf{u}^H\, \mathbf{u}\right|^2 = N^2$$

$$G_{S\_opt} = \left|\mathbf{w}\,\mathbf{s}_0(k)\right|^2 = \left|\mathbf{u}^H \mathbf{S}_k^H\, \mathbf{R}_0^{-1}\, \mathbf{s}_0(k)\right|^2 = \left|\mathbf{u}^H \mathbf{S}_k^H\, \mathbf{R}_0^{-1}\, \mathbf{S}_k \mathbf{u})\right|^2$$

Clutter Attenuation – Integrazione coerente

$$CA_{\text{int}}^{-1} = \mathbf{w}\,\mathbf{R}_0\mathbf{w}^H = \mathbf{u}^H\mathbf{S}_k^H\mathbf{R}_0\mathbf{S}_k\,\mathbf{u}$$

$$\mathbf{S}_k^H\mathbf{R}_0\mathbf{S}_k = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & S_k^{\;*} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & S_k^{\;*2} & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & S_k^{\;*3} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & S_k^{\;*(N-2)} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & S_k^{\;*(N-1)} \end{bmatrix} \mathbf{R}_0 \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & S_k & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & S_k^{\;2} & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & S_k^{\;3} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & S_k^{\;(N-2)} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & S_k^{\;(N-1)} \end{bmatrix}$$

$$\left[\mathbf{S}_k^H\mathbf{R}_0\mathbf{S}_k\right]_{i,j} = \left[\mathbf{R}_0\right]_{i,j}\cdot S_k^{\;(j-i)} = \rho\left[(j-i)T\right]\cdot S_k^{\;(j-i)}$$

$$CA_{\text{int}}^{-1}(k) = \mathbf{u}^H\mathbf{S}_k^H\mathbf{R}_0\mathbf{S}_k\,\mathbf{u} = \sum_{i,j=0}^{N-1}\left[\mathbf{S}_k^H\mathbf{R}_0\mathbf{S}_k\right]_{i,j} = \sum_{i,j=0}^{N-1}\left[\mathbf{R}_0\right]_{i,j}\cdot S_k^{\;(j-i)} = \sum_{i,j=0}^{N-1}\rho\left[(j-i)T\right]\cdot S_k^{\;(j-i)} =$$

$$= \sum_{r=-(N-1)}^{N-1}\;\sum_{i=1}^{N-|r|}\rho\left[rT\right]\cdot S_k^{\;r} = \sum_{r=-(N-1)}^{N-1}\left(N-|r|\right)\rho\left[rT\right]\cdot S_k^{\;r} = N + \sum_{r=-(N-1)}^{1}\left(N-|r|\right)\rho\left[rT\right]\cdot S_k^{\;r} + \sum_{r=1}^{N-1}\left(N-|r|\right)\rho\left[rT\right]\cdot S_k^{\;r} =$$

$$= N + \sum_{n=1}^{N-1}\left(N-n\right)\rho\left[-nT\right]\cdot S_k^{\;-n} + \sum_{r=1}^{N-1}\left(N-r\right)\rho\left[rT\right]\cdot S_k^{\;r} = N + 2\,\mathrm{Re}\left\{\sum_{r=1}^{N-1}\left(N-r\right)\rho\left[rT\right]\cdot S_k^{\;r}\right\}$$

IF – Integrazione coerente

$$IF_{\text{int}}(k) = \frac{N^2}{N + 2\,\text{Re}\left\{\sum_{r=1}^{N-1} (N-r)\ \rho[r\,T] \cdot S_k^{\ r}\right\}}$$

$$IF_{\text{int}}(k) = \begin{cases} \dfrac{N^2}{N} = N & \rho = 0 \\ \dfrac{N^2}{N + 2\,\text{Re}\left\{\sum_{r=1}^{N-1} (N-r)\ S_k^{\ r}\right\}} & \end{cases}$$

Spettro Esponenziale – Integrazione coerente

$$\mathbf{R}_0=\begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 & \cdots & \rho^{(N-2)} & \rho^{(N-1)} \\ \rho^* & 1 & \rho & \rho^2 & \cdots & \rho^{(N-3)} & \rho^{(N-2)} \\ \rho^{*2} & \rho^* & 1 & \rho & \cdots & \rho^{(N-4)} & \rho^{(N-3)} \\ \rho^{*3} & \rho^{*2} & \rho^* & 1 & \cdots & \rho^{(N-5)} & \rho^{(N-4)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \rho^{*(N-2)} & \rho^{*(N-3)} & \rho^{*(N-4)} & \rho^{*(N-5)} & \cdots & 1 & \rho \\ \rho^{*(N-1)} & \rho^{*(N-2)} & \rho^{*(N-3)} & \rho^{*(N-4)} & \cdots & \rho^* & 1 \end{bmatrix}$$

$$\mathbf{S}_k^H\mathbf{R}_0\mathbf{S}_k=\begin{bmatrix} 1 & \rho S_k & \rho^2 S_k^2 & \rho^3 S_k^3 & \cdots & (\rho S_k)^{(N-2)} & (\rho S_k)^{(N-1)} \\ \rho^* S_k^* & 1 & \rho S_k & \rho^2 S_k^2 & \cdots & (\rho S_k)^{(N-3)} & (\rho S_k)^{(N-2)} \\ (\rho^* S_k^*)^2 & \rho^* S_k^* & 1 & \rho S_k & \cdots & (\rho S_k)^{(N-4)} & (\rho S_k)^{(N-3)} \\ (\rho^* S_k^*)^3 & (\rho^* S_k^*)^2 & \rho^* S_k^* & 1 & \cdots & (\rho S_k)^{(N-5)} & (\rho S_k)^{(N-4)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ (\rho^* S_k^*)^{(N-2)} & (\rho^* S_k^*)^{(N-3)} & (\rho^* S_k^*)^{(N-4)} & (\rho^* S_k^*)^{(N-5)} & \cdots & 1 & \rho S_k \\ (\rho^* S_k^*)^{(N-1)} & (\rho^* S_k^*)^{(N-2)} & (\rho^* S_k^*)^{(N-3)} & (\rho^* S_k^*)^{(N-4)} & \cdots & \rho^* S_k^* & 1 \end{bmatrix}$$

$$\left[\mathbf{S}_k^H\mathbf{R}_0\mathbf{S}_k\right]_{i,j}=\left[\mathbf{R}_0\right]_{i,j}\cdot S_k^{\,(j-i)}=\rho^{|j-i|}\cdot S_k^{\,(j-i)}$$

$$CA_{\text{int}}^{-1}(k)=N+2\,\mathrm{Re}\left\{\sum_{r=1}^{N-1}(N-r)\,(\rho\cdot S_k)^r\right\}=N+2\,\mathrm{Re}\left[N\sum_{r=1}^{N-1}(\rho S_k)^r-\sum_{r=1}^{N-1}r\,(\rho S_k)^r\right]=$$

$$=N+2\,\mathrm{Re}\left[N\sum_{r=1}^{N-1}(\rho S_k)^r-\rho S_k\left[\frac{\partial}{\partial\alpha}\sum_{r=1}^{N-1}\alpha^r\right]_{\alpha=\rho S_k}\right]=N+2\,\mathrm{Re}\left[N\left(\frac{1-(\rho S_k)^N}{1-(\rho S_k)}-1\right)-\rho S_k\left[\frac{\partial}{\partial\alpha}\left(\frac{1-\alpha^N}{1-\alpha}-1\right)\right]_{\alpha=\rho S_k}\right]=$$

$$=N+2\,\mathrm{Re}\left[N\left(\frac{1-(\rho S_k)^N}{1-(\rho S_k)}-1\right)-\rho S_k\left[\frac{-N(\rho S_k)^{N-1}}{1-(\rho S_k)}+\frac{1-(\rho S_k)^N}{(1-(\rho S_k))^2}\right]\right]=N+2\,\mathrm{Re}\left[N\left(\frac{1}{1-(\rho S_k)}-1\right)-\rho S_k\left[\frac{1-(\rho S_k)^N}{(1-(\rho S_k))^2}\right]\right]=$$

$$=N+2\,\mathrm{Re}\left[N\,\frac{\rho S_k}{1-(\rho S_k)}-\rho S_k\left[\frac{1-(\rho S_k)^N}{(1-(\rho S_k))^2}\right]\right]=N+2\,\mathrm{Re}\left[\frac{\rho S_k}{1-(\rho S_k)}\left[N-\frac{1-(\rho S_k)^N}{1-(\rho S_k)}\right]\right]$$

IF Spettro esponenziale – Integrazione coerente

$$IF_{\text{int}}(k) = \frac{N^2}{N + 2\,\text{Re}\left\{\sum_{r=1}^{N-1} (N - r)\, \rho[rT] \cdot S_k^{\,r}\right\}}$$

$$IF_{\text{int}}(k) = \frac{N^2}{N + 2\,\text{Re}\left[\frac{\rho S_k}{1 - (\rho S_k)}\left[N - \frac{1 - (\rho S_k)^N}{1 - (\rho S_k)}\right]\right]}$$

Clutter Attenuation ed IF – Integrazione ottima

$$CA_{\text{int}}^{-1} = \mathbf{w}\,\mathbf{R}_0^{-1}\mathbf{w}^H = \mathbf{u}^H\mathbf{S}_k^H\mathbf{R}_0^{-1}\mathbf{S}_k\,\mathbf{u}$$

$$G_{S\_opt} = \left|\mathbf{w}\,\mathbf{s}_0(k)\right|^2 = \left|\mathbf{u}^H\mathbf{S}_k^H\,\mathbf{R}_0^{-1}\,\mathbf{s}_0(k)\right|^2 = \left|\mathbf{u}^H\mathbf{S}_k^H\,\mathbf{R}_0^{-1}\,\mathbf{S}_k\mathbf{u})\right|^2$$

$$IF_{opt} = \frac{\left|\mathbf{w}\,\mathbf{s}_0(k)\right|^2}{\mathbf{w}\,\mathbf{R}_0^{-1}\mathbf{w}^H} = \frac{\left|\mathbf{u}^H\mathbf{S}_k^H\,\mathbf{R}_0^{-1}\,\mathbf{S}_k\mathbf{u})\right|^2}{\mathbf{u}^H\mathbf{S}_k^H\mathbf{R}_0^{-1}\mathbf{S}_k\,\mathbf{u}} = \mathbf{u}^H\mathbf{S}_k^H\mathbf{R}_0^{-1}\mathbf{S}_k\,\mathbf{u}$$

IF Spettro Esponenziale – Integrazione ottima

$$\mathbf{R}_0^{-1} = \frac{1}{1-|\rho|^2}\begin{bmatrix} 1 & -\rho & 0 & 0 & \cdots & 0 & 0 \\ -\rho^* & 1+|\rho|^2 & -\rho & 0 & \cdots & 0 & 0 \\ 0 & -\rho^* & 1+|\rho|^2 & \rho & \cdots & 0 & 0 \\ 0 & 0 & -\rho^* & 1+|\rho|^2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1+|\rho|^2 & \rho \\ 0 & 0 & 0 & 0 & \cdots & -\rho^* & 1 \end{bmatrix}$$

$$\mathbf{S}_k^H \mathbf{R}_0^{-1} \mathbf{S}_k = \frac{1}{1-|\rho|^2}\begin{bmatrix} 1 & -\rho S_k & 0 & 0 & \cdots & 0 & 0 \\ -\rho^* S_k^* & 1+|\rho|^2 & -\rho S_k & 0 & \cdots & 0 & 0 \\ 0 & -\rho^* S_k^* & 1+|\rho|^2 & \rho S_k & \cdots & 0 & 0 \\ 0 & 0 & -\rho^* S_k^* & 1+|\rho|^2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1+|\rho|^2 & \rho S_k \\ 0 & 0 & 0 & 0 & \cdots & -\rho^* S_k^* & 1 \end{bmatrix}$$

$$IF_{opt} = \mathbf{u}^H \mathbf{S}_k^H \mathbf{R}_0^{-1} \mathbf{S}_k \, \mathbf{u} =$$

$$= \frac{1}{1-|\rho|^2}\left\{ N(1+|\rho|^2) - 2|\rho|^2 - 2(N-1)\,\mathrm{Re}[\rho S_k] \right\} =$$

$$= \frac{N}{1-|\rho|^2}\left\{ 1+|\rho|^2 - \frac{2}{N}|\rho|^2 - 2\,\mathrm{Re}[\rho S_k](1-\frac{1}{N}) \right\}$$

Cancellatore-Integratore ottimo (I)

The characteristics of the clutter are characterized by the covariance matrix Φ_C of the N clutter returns. If the power spectrum of the clutter is denoted $S_C(f)$ and the corresponding autocorrelation function is $R_C(t_i - t_j)$, then the elements of Φ_C are given by

$$\Phi_{ij} = R_C(t_i - t_j) \tag{15.15}$$

where t_i is the transmission time of the ith pulse. For example, for a gaussian-shaped clutter spectrum we have

$$S_C(f) = P_C \frac{1}{\sqrt{2\pi}\,\sigma_f} \exp\left[-\frac{(f - f_d)^2}{2\sigma_f^2}\right] \tag{15.16}$$

where P_C is the total clutter power, σ_f is the standard deviation of the clutter spectral width, and f_d is the average doppler shift of the clutter.

Cancellatore-Integratore ottimo (II)

The correspond ing autocorrelation function is

$$R_C(\tau) = P_C \exp(-4\pi\sigma_f^2\tau^2) \exp(-j2\pi f_d\tau) \quad (15.17)$$

For two pulses separated in time by the interpulse period T the complex correlation coefficient between two clutter returns is

$$\rho_T = \exp(-4\pi\sigma_f^2 T^2) \exp(-j2\pi f_d T) \quad (15.18)$$

The second factor in this expression represents the phase shift caused by the doppler shift of the clutter returns.

For a known target doppler shift the received target return can be represented by an N-dimensional vector:

$$s = P_S \mathbf{f} \quad (15.19)$$

where the elements of the vector **f** are $f_i = \exp[j2\pi f_s t_i]$. On the basis of this de-

Cancellatore-Integratore ottimo (III)

scription of signal and clutter it has been shown[12] that the optimum doppler filter will have weights given by

$$w_{opt} = \Phi_C^{-1} s \tag{15.20}$$

and the corresponding signal-to-clutter improvement is

$$I_{SCR} = \frac{\mathbf{w}_{opt}{}^T \mathbf{s} \cdot \mathbf{s}^{T*} \mathbf{w}_{opt}{}^*}{\mathbf{w}_{opt}{}^T \, \Phi_C \mathbf{w}_{opt}{}^*} \tag{15.21}$$

where the asterisk denotes complex conjugation and superscript T is the transposition operator. An example where the optimum performance is determined for the case of clutter at zero doppler having a wide gaussian-shaped spectrum and a normalized width of $\sigma_f T = 0.1$ is shown in Fig. 15.18. In this case a coherent processing interval of CPI = nine pulses was assumed, and the limitation due to thermal noise was ignored by setting the clutter level at 100 dB above noise.

It should be kept in mind that Eq. (15.21) for the optimum weights will yield a different result for each different target doppler shift, so that a large number of

Cancellatore-Integratore ottimo (IV)

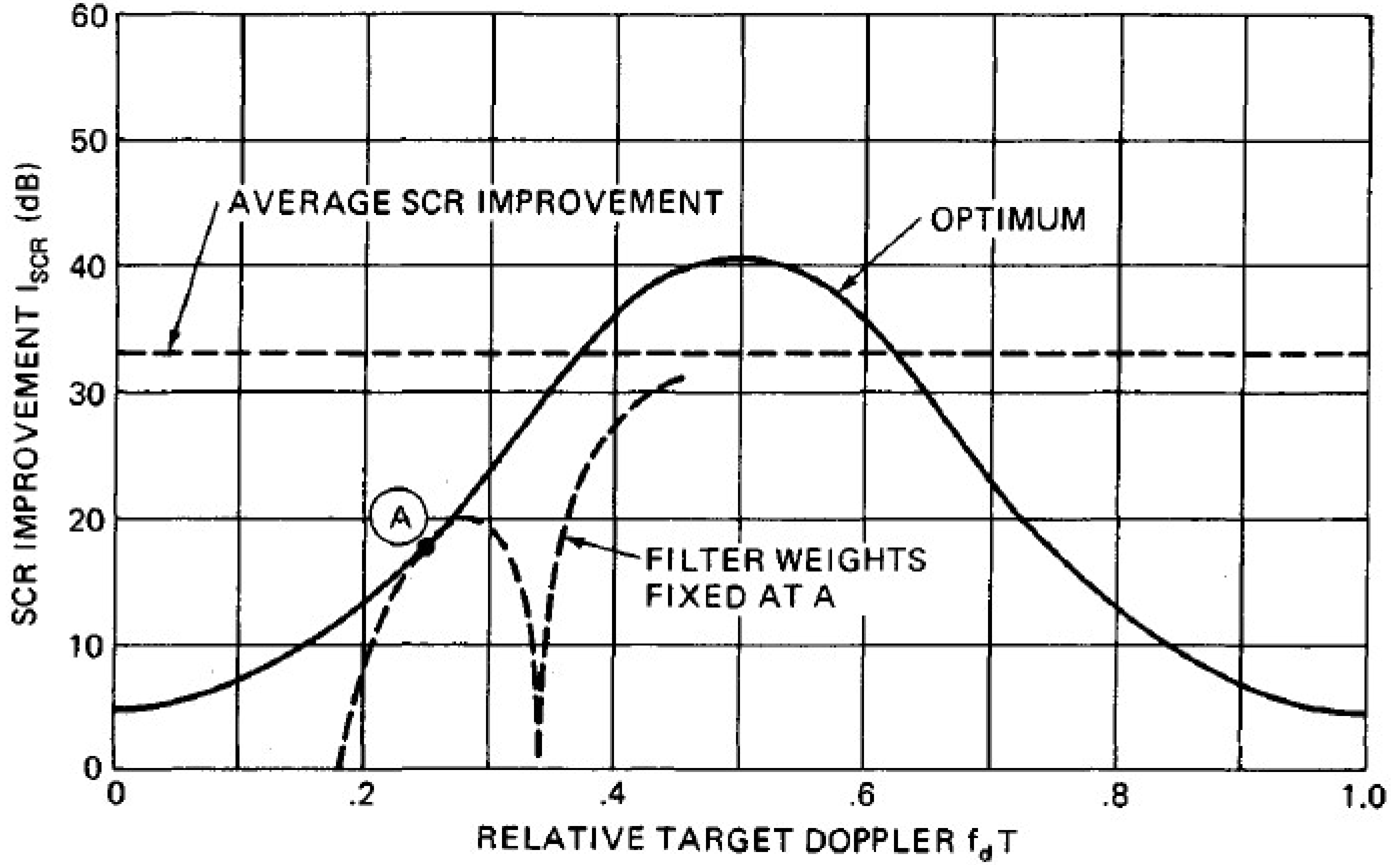


FIG. 15.18 Optimum signal-to-clutter ratio improvement (I_{SCR}) for gaussian-shaped clutter spectrum and a CPI of nine pulses; clutter-to-noise ratio, 100 dB.

Shrader & Gregers-Hansen "MTI Radar"
in M. Skolnik – Radar Handbook 2° Ed.

Cancellatore-Integratore ottimo (V)

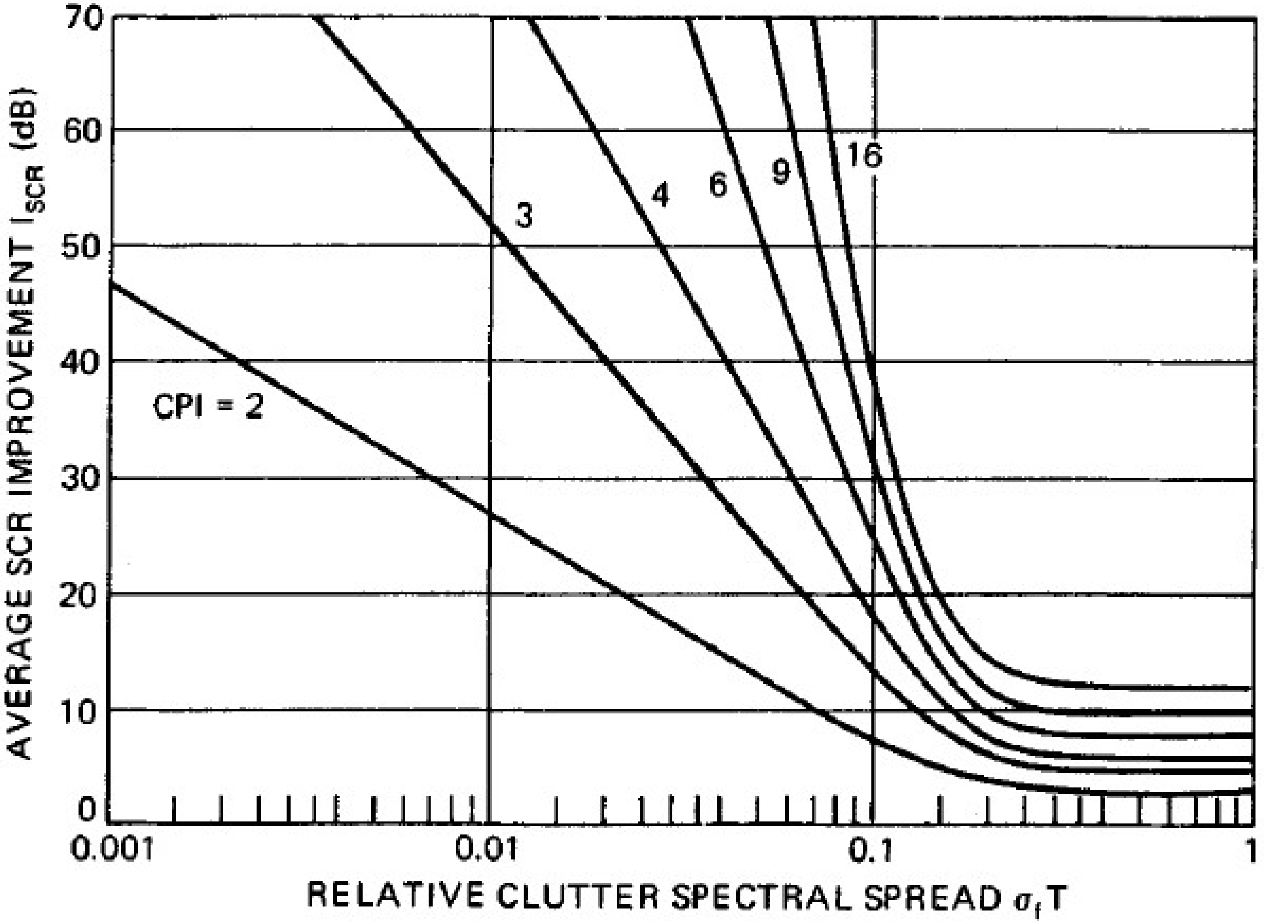


FIG. 15.19 Reference curve of optimum average SCR improvement for a gaussian-shaped clutter spectrum.

Shrader & Gregers-Hansen "MTI Radar"
in M. Skolnik – Radar Handbook 2° Ed.

Ottimo vs. Banco di Filtri (I)

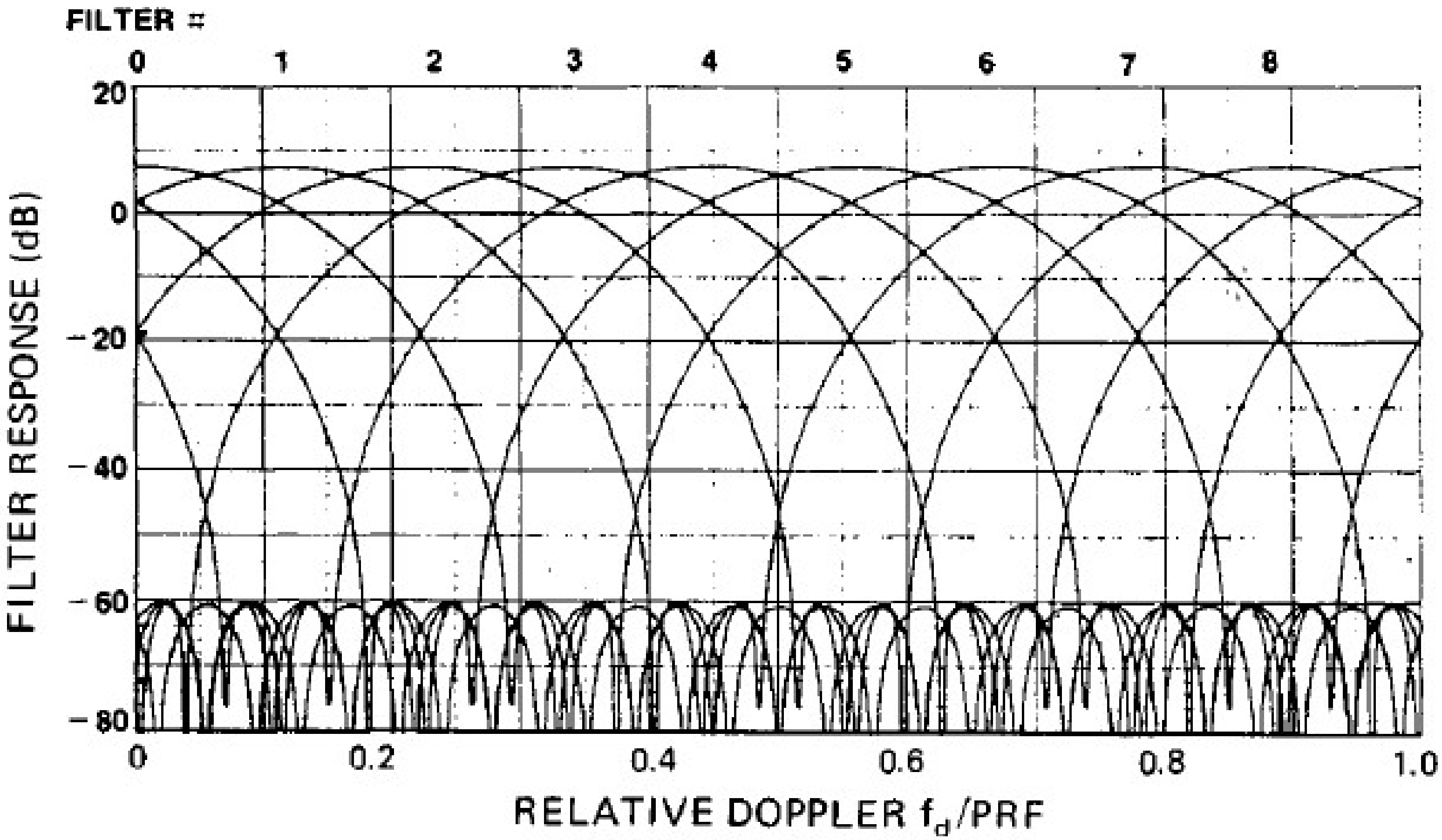


FIG. 15.28 Doppler filter bank of 68 dB Chebyshev filters. CPI = nine pulses.

Ottimo vs. Banco di Filtri (II)

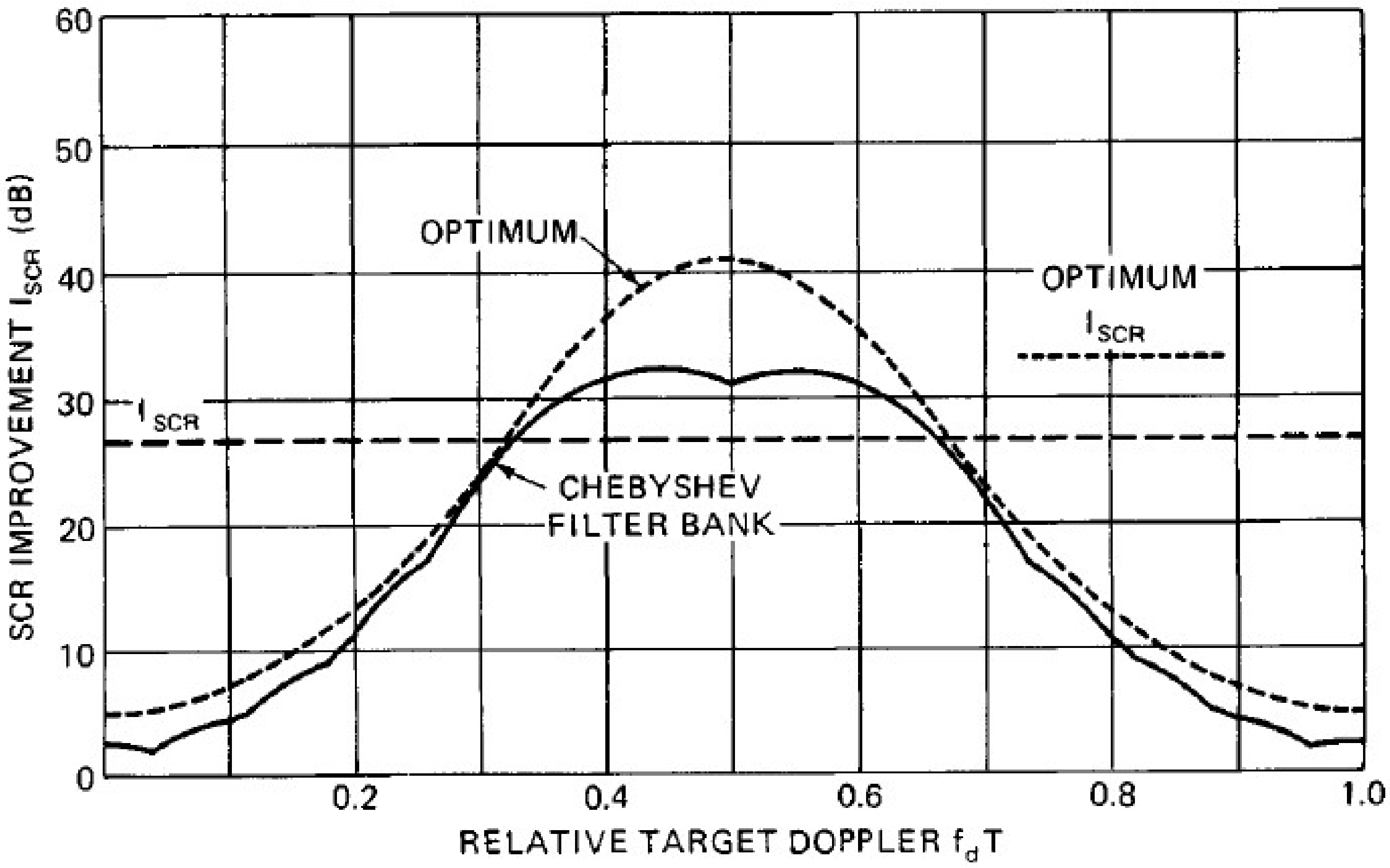


FIG. 15.29 SCR improvement of 68 dB Chebyshev doppler filter bank compared with the optimum.

Ottimo vs. Banco di Filtri (III)

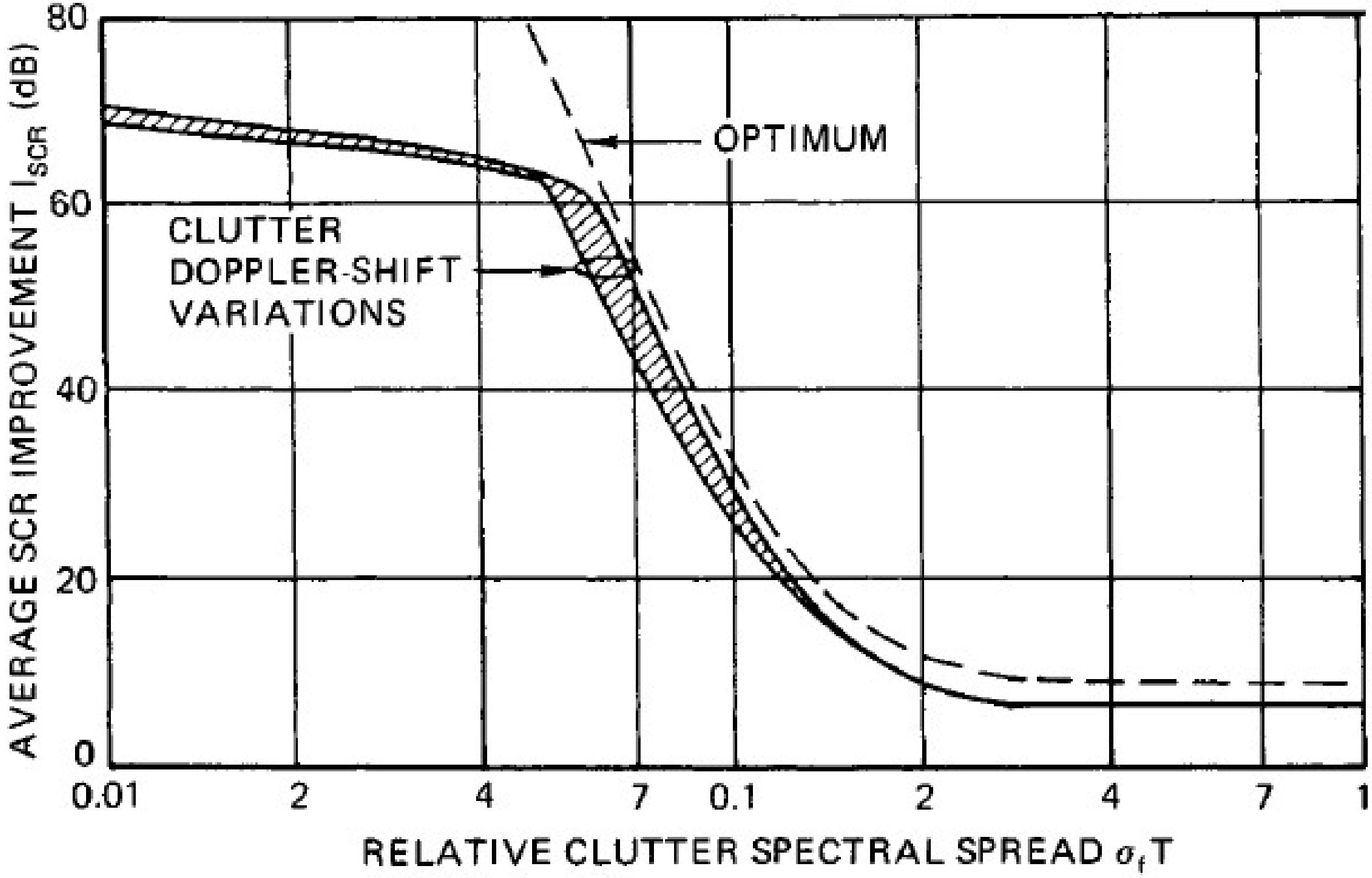
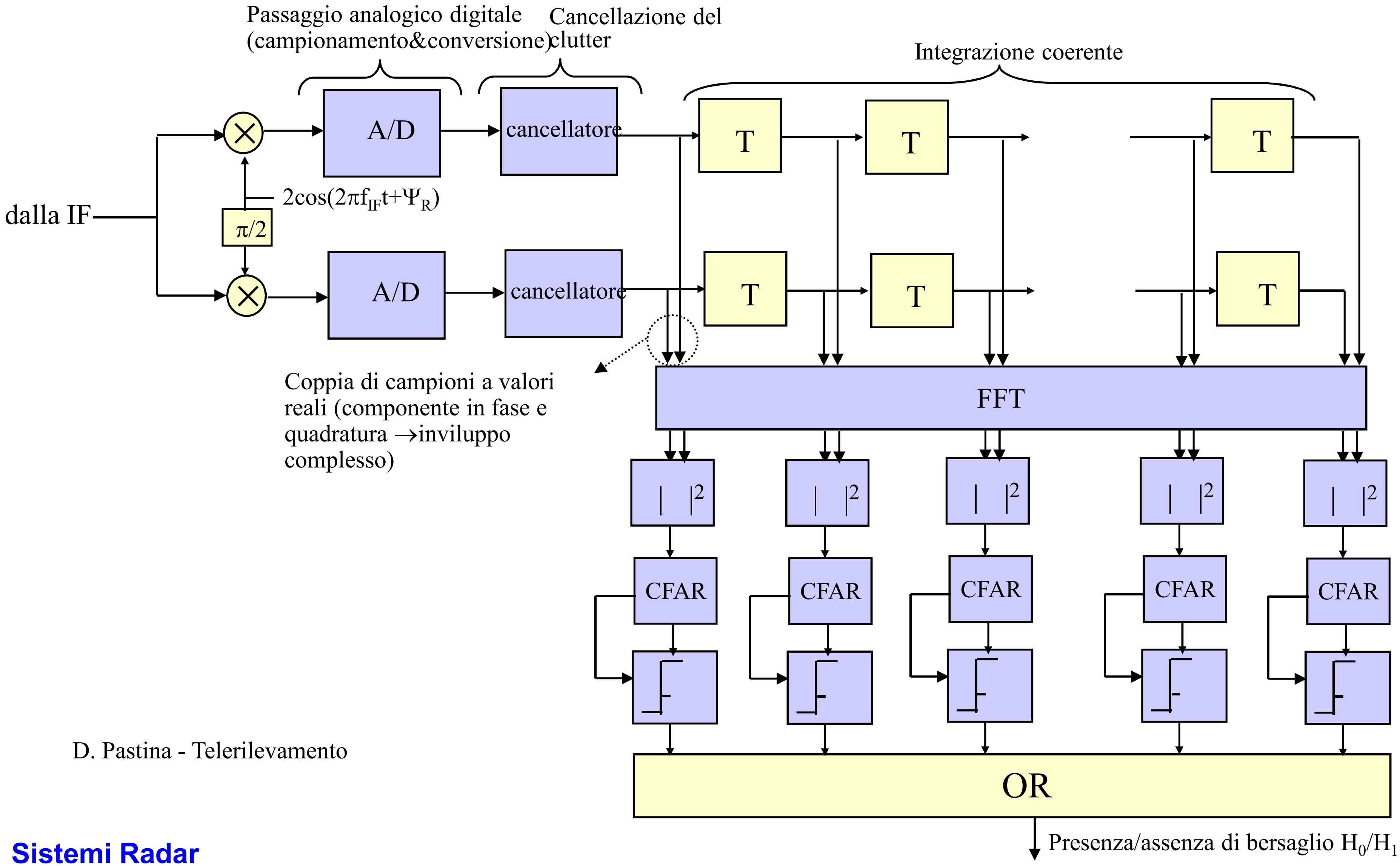


FIG. 15.30 Average SCR improvement for the 68 dB Chebyshev filter bank shown in Fig. 15.28. CPI = nine pulses. Optimum is from Fig. 15.19.

MTD- Moving Target Detector



D. Pastina - Telerilevamento

Filtri vs. clutter di pioggia

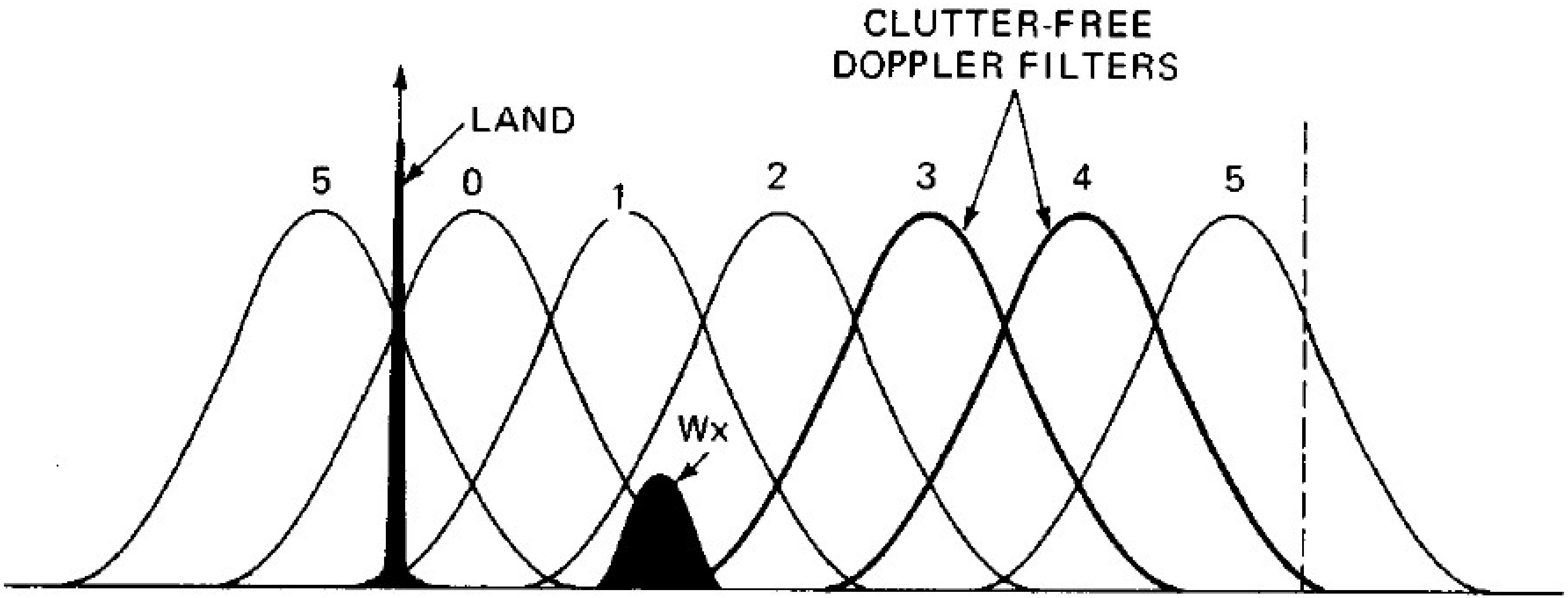


FIG. 15.7 Suppression of multiple clutter sources by using a doppler filter bank.