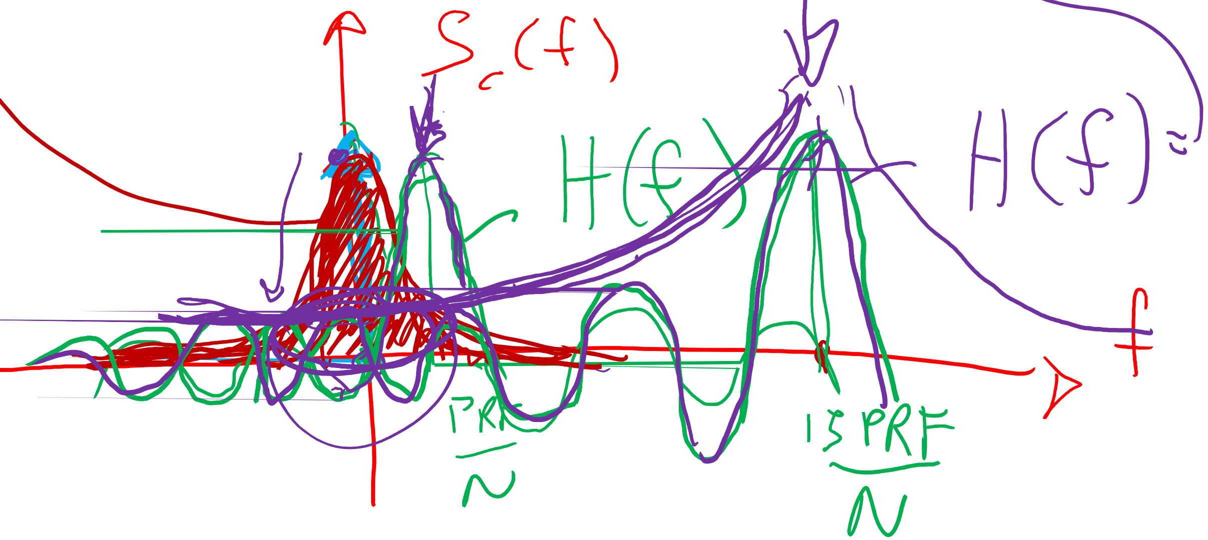
$$P_{c\,out} = \int_{-\infty}^{+\infty} P_c(f)\,|H(f)|^2\,df$$

Densità spettrale di clutter

$$P_{c\,in} = \int_{-\infty}^{+\infty} P_c(f)\,df$$

Pierfrancesco Lombardo



$$CA = \frac{P_{c\,in}}{P_{c\,out}}$$

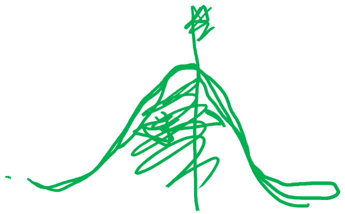
Power Spectral Density (PSD) models

- Correct spectral shape impacts clutter cancellation and target detection performance
- The clutter spectrum is not concentrated at zero Doppler only, but spreads at higher frequencies
- There are several reasons for the clutter spreading:
  - Wind-induced variations of the clutter reflectivity (sea waves, windblown vegetations, etc.)
  - Amplitude modulation by the mechanically scanning antenna beam
  - Pulse-to-pulse instabilities of the radar system components
  - Transmitted frequency drift
- The pulse-to-pulse fluctuation is generally referred to as **internal clutter motion (ICM)**

Power Spectral Density (PSD) models

A general analytic representation for the (normalized) total PSD:

$$P_c(f) = \frac{r}{1+r}\delta(f) + \frac{1}{1+r}P_{ac}(f), \; -\infty < f < \infty$$



r = ratio of dc power to ac power in the spectrum ($r \neq 0$ only for ground clutter)

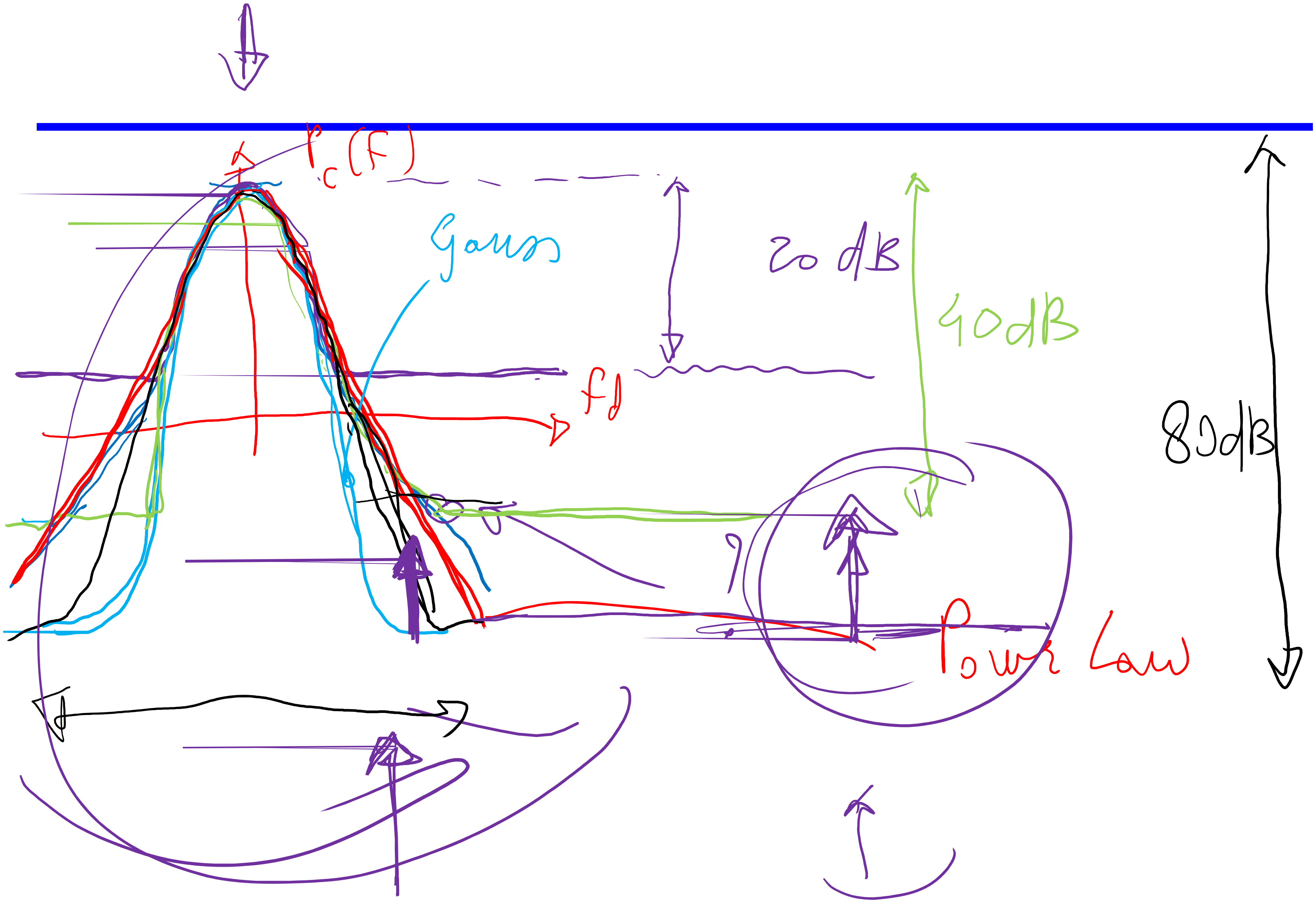
shape of the ac component of the spectrum

$$\int_{-\infty}^{\infty} P_{ac}(f)df = 1$$

It is customary to model the effect of amplitude modulation by a **Gaussian-shaped PSD:**

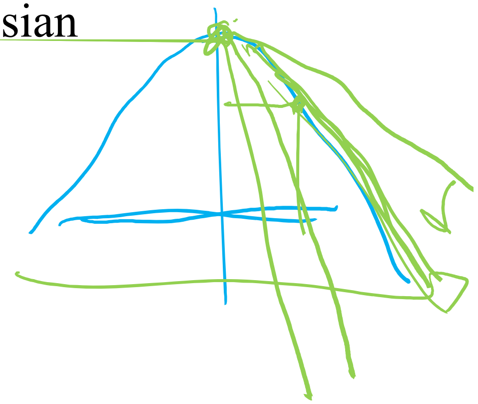
$$P_{ac}(f) = \frac{1}{\sqrt{2\pi\sigma}}\exp\left(-\frac{f^2}{2\sigma^2}\right)$$

Barlow proposed this model for **windblown clutter spectra**, for noncoherent radar systems and over limited spectral dynamic ranges (up to a level 20 dB below the peak level and to a maximum Doppler velocity of 0.67 m/s)


Gauss
20 dB
40dB
80dB
Power Law

Windblown ground clutter spectra

- Essentially all modern measurements of ground clutter spectra, with increased sensitivity compared to those of Barlow, without exception show spectral shapes wider than Barlow's Gaussian in their tails
- It had become theoretically well understood from 1965-67 on, that branch motion in windblown vegetation generates spectra wider than Gaussian
- Fishbein introduced the **power-law** clutter spectral shape:



$$P_{ac}(f) = \frac{n\sin(\pi/n)}{2\pi f_c}\,\frac{1}{1+(f/f_c)^n}$$

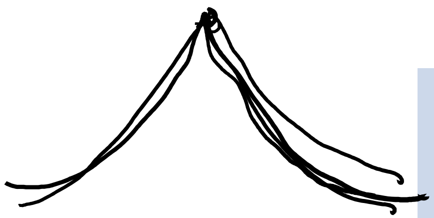
n is the shape parameter

break-point Doppler frequency where the shape function is 3 dB below its peak zero-Doppler level

Fishbein *et al.* indicated that measured clutter rejection ratios up to 40 dB were matched under the assumption of a theoretical power-law spectral shape with $n = 3$

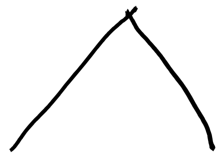
Windblown ground clutter spectra

- Many measurements followed those of Fishbein *et al.* in which spectral tails wider than Gaussian were observed and modeled as power-law over the spectral ranges of power then available (typically 35 to 40 dB below the zero-Doppler peak)
- Values of power-law exponent *n* used in these representations were usually on the order of 3 or 4, but sometimes greater
- The evidence that clutter spectra have power-law shapes over spectral dynamic ranges reaching 30 to 40 dB below zero-Doppler peaks is essentially empirical, not theoretical
- However, there is no simple physical model or fundamental underlying reason requiring clutter spectral shapes to be power law
- Recently, **Billingsley** showed that measurements at MIT-LL of windblown ground clutter power spectra to levels substantially lower than most earlier measurements (i.e., 60 to 80 dB below zero-Doppler peaks) indicate spectral shapes that fall off much more rapidly than constant power-law at the lower levels, at rates of decay approaching **exponential**:



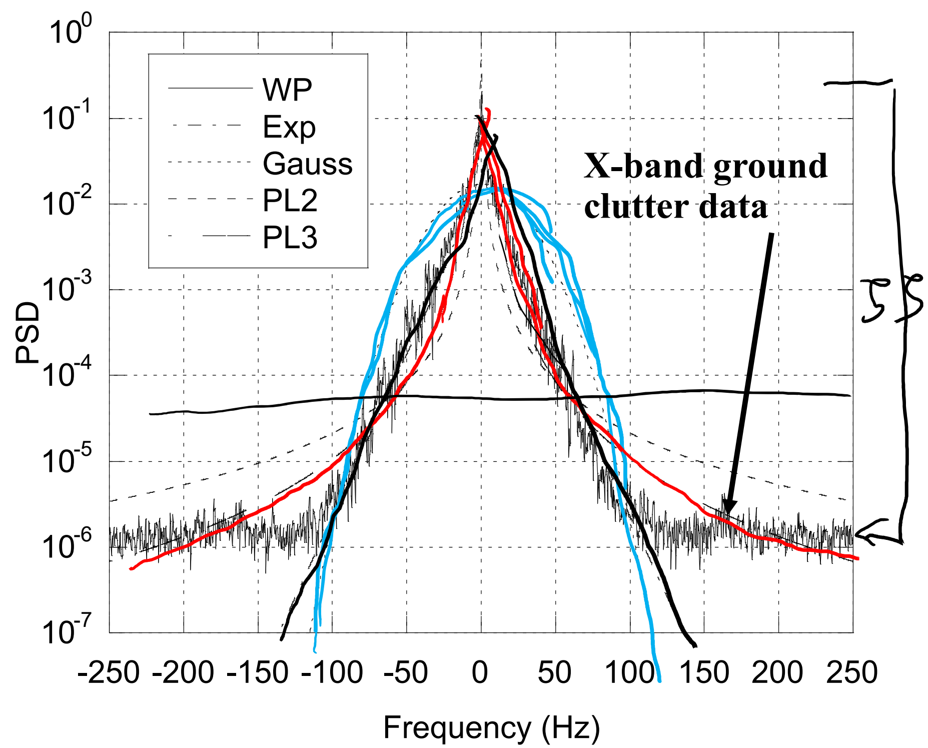
$$P_{ac}(f) = \frac{\lambda \beta_e}{4} \exp\left(-\frac{\lambda \beta_e}{2} |f| \right)$$

λ is the radar transmission wavelength and β_e is the exponential shape parameter



Windblown ground clutter spectra

Source: MIT-LL, courtesy of Mr. J. B. Billingsley



At low enough power level the power-law spectral tail always becomes much wider than Gaussian or exponential

Like the Gaussian function, the exponential function is simple and analytically tractable

As for the power-law for spectral dynamic ranges 30 to 40 dB down, the evidence of exponential PSD for spectral dynamic ranges reaching 60 to 80 dB down is essentially empirical

At high wind speeds, most of the foliage is in motion, and the ratio *r* of dc to ac power in the clutter PSD is relatively low. In such circumstances, and in the higher microwave bands where little foliage penetration occurs, the dc component can become vanishing small (*r~0*)

Ground clutter spectra

Recent studies have demonstrated that the ground clutter spectrum consists of three components:

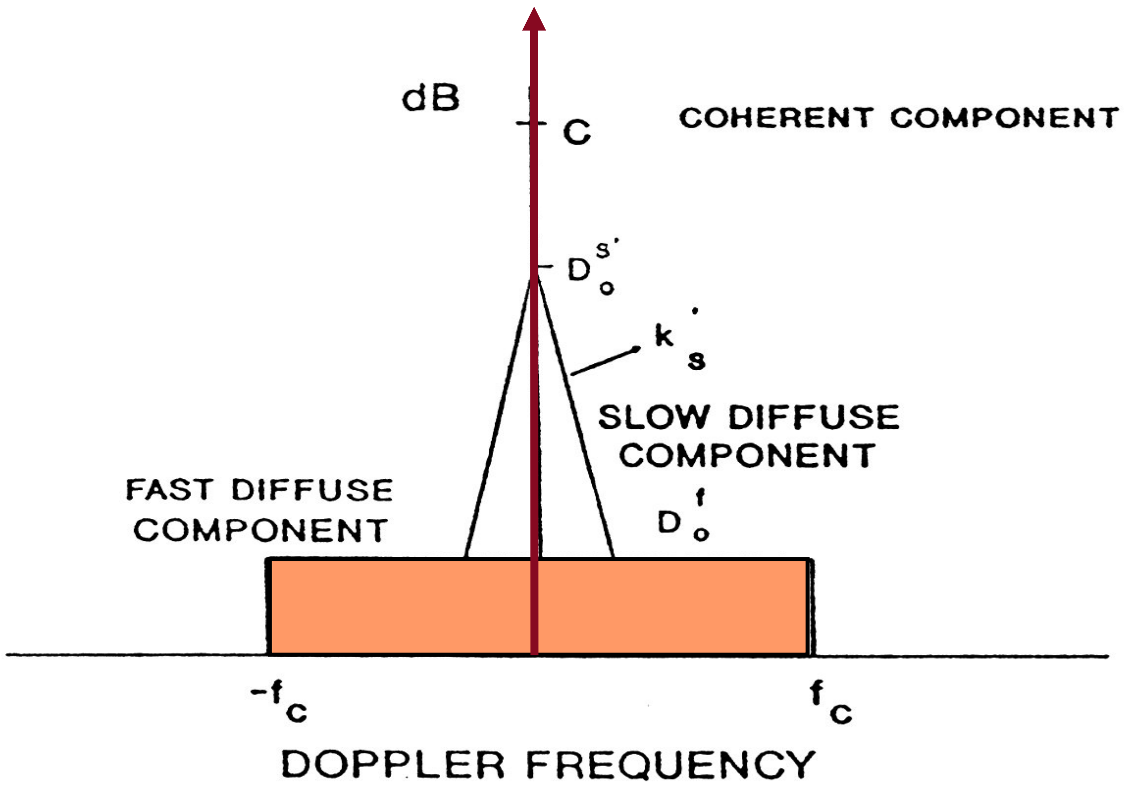
- coherent component
- slow diffuse component
- fast-diffuse component

The *coherent component* was the results of radar returns from steady objects such as buildings, highways and from movable objects at rest.
The coherent component is at zero Doppler.

Slow-diffuse & fast-diffuse components

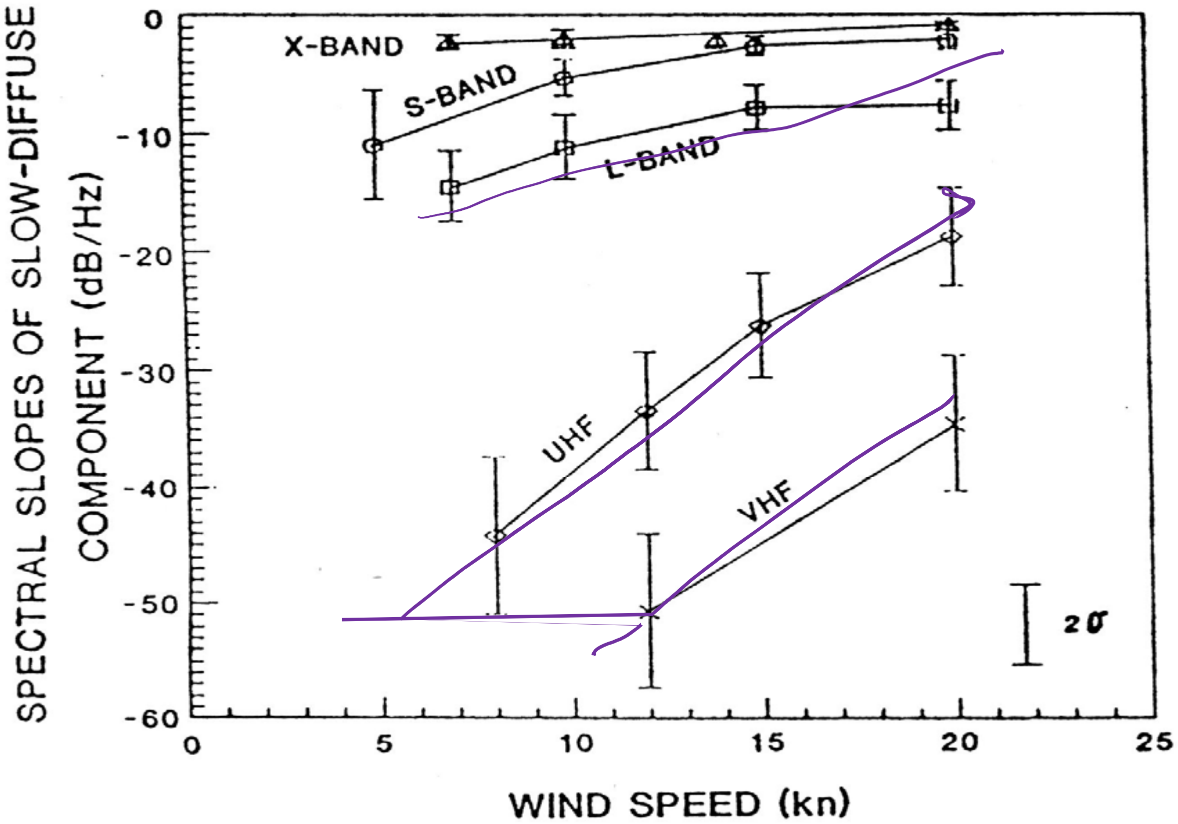
The *slow-diffuse component* is the consequence of motions of objects with moderate inertia (tree branches).
The slow-diffuse component occupies a relative narrow region around zero Doppler.
The spectrum is approximately symmetrical and its spectral density in Db scale decreases linearly with increasing absolute values of Doppler frequency.
The *fast-diffuse component* is the result of movements of light objects such as a tree leaves. This component has a spectral density similar to a band-limited noise. Its magnitude is usually compared to other components.
The spectral extent is of the same order as the Doppler shifts that corresponds to the wind speed.

Ground clutter spectra



Symbolic diagram of a ground-clutter spectral model.

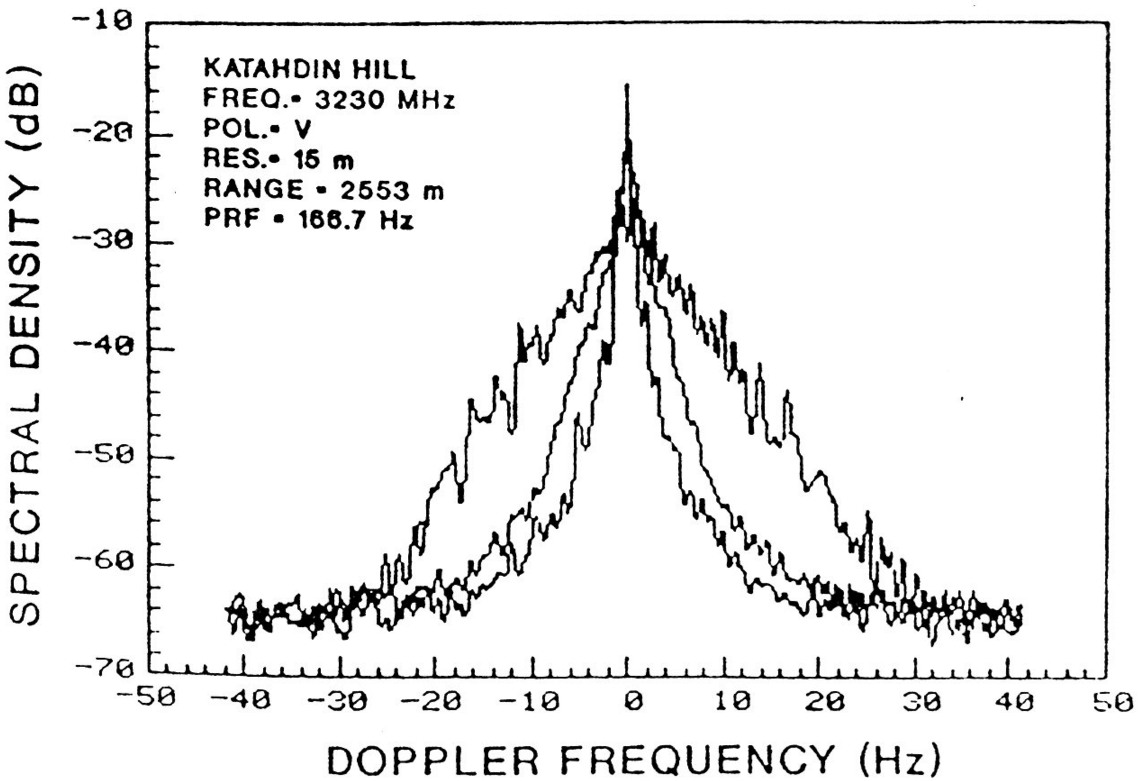
Spectral slope of slow-diffuse components



Spectral slopes of the slow-diffuse component at various frequency bands - from a small forested patch at Katahdin Hill.

Variation of the spectral slope diffuse components

S-band



Variation of the spectral slope of the slow diffuse component over an observation interval.

Sea clutter spectra

• Capillary waves with wavelengths on the order of centimeters or less. Generated by turbulent gusts of near surface wind; their restoring force is the surface tension.
• Longer gravity waves (sea or swell) with wavelengths ranging from a few hundred meters to less than a meter. Swells are produced by stable winds and their restoring force is the force of gravity.

The relative motion of the sea surface with respect to the radar causes an intrinsic Doppler shift of the return from individual scatterers.

Because the motion of the scattering elements have varying directions and speeds the total echo contains a spectrum of Doppler frequencies.

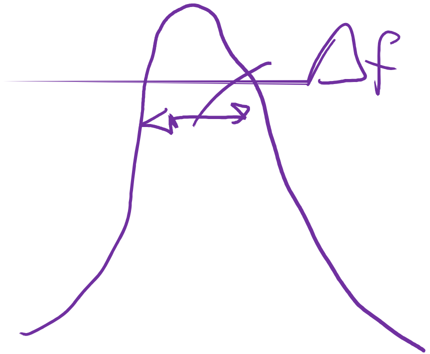
Two effects are of interest:
• the spectral shape and width
• the mean Doppler shift of the entire spectrum.

• Bragg scattering, present in both HH and VV data, but stronger in VV data.
• Whitecaps scattering; the amplitude of both like-polarizations are roughly equal and are noticeably stronger than the background scatter, particularly in HH, in which the Bragg scattering is often weak.
• Spikes; absent in VV data and strong only in up-wind HH data

Sea clutter spectra

The spectrum of sea clutter is generally assumed to have Gaussian shape. An approximate relationship between the -3 dB bandwidth Δf of the spectrum and sea state S (Douglas scale) has been derived by Nathanson:

$$\Delta f = 3.6 f_{0(GH_Z)} S$$



The standard deviation of the Gaussian spectrum is related to Δf by the expression:

$$\sigma_f = 0.42 \Delta f$$

Sea clutter spectra

• In the literature, it has been often assumed that the sea clutter has **Lorentzian** spectrum (i.e., power-law with $n=2$).

•More recently, Lee *et al.* showed that the spectral lineshapes can be decomposed into three **basis functions** which are **Lorentzian**, **Gaussian**, and **Voigtian** (convolution of the Gaussian and Lorentzian):

$$P_L(f)=\frac{\Gamma/2\pi^2}{(f-f_L)^2+(\Gamma/2\pi)^2}$$

peak of the Lorentzian function

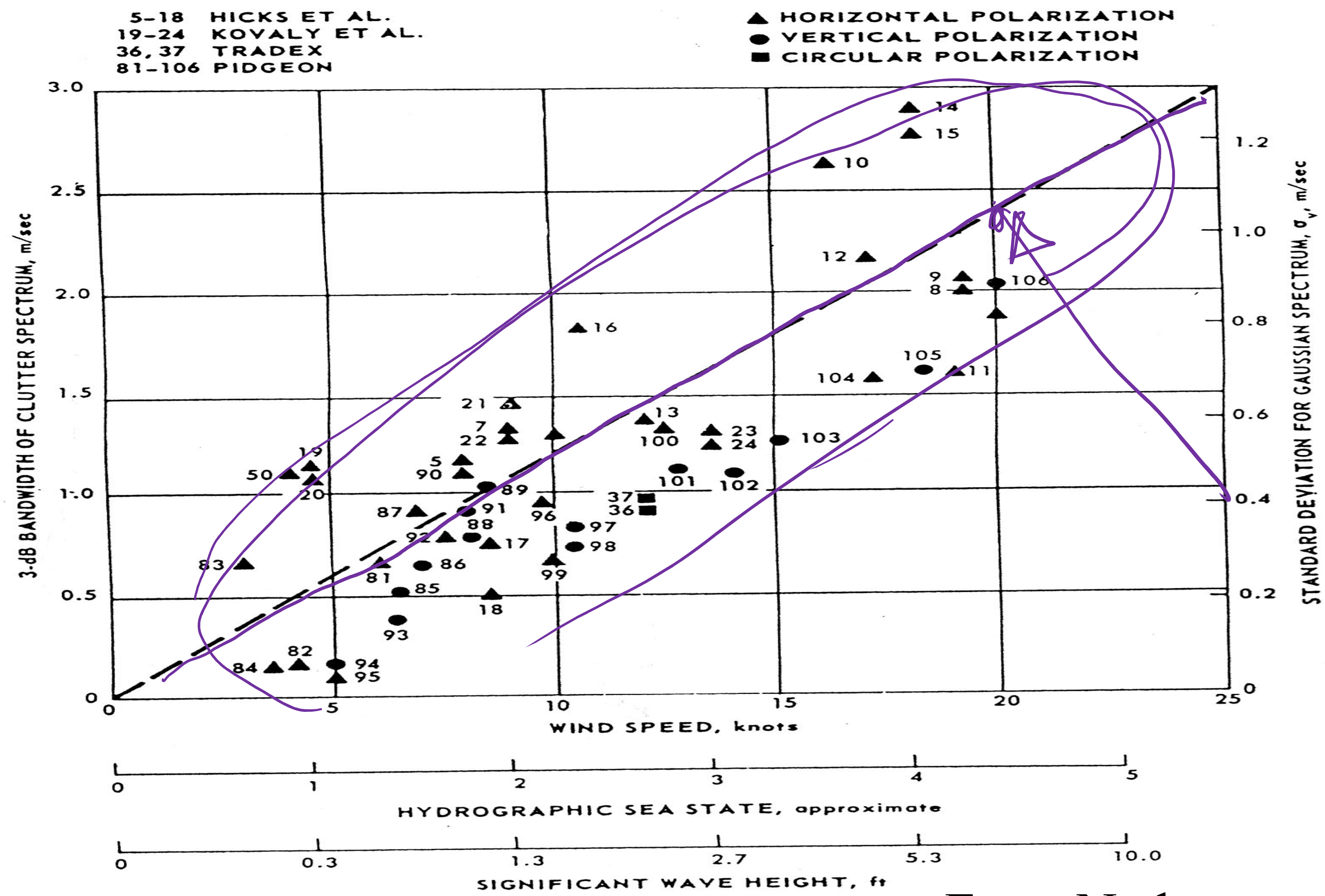
Γ^{-1} = characteristic scatterer lifetime

$$P_V(f;a)=\frac{a}{\pi}\int_{-\infty}^{\infty}\frac{\exp(-x^2)}{\left(\frac{f-f_V}{f_{Ve}}-x\right)^2+a^2}dx$$

$a=\Gamma/2\pi f_{Ve}$ shape parameter

f_V = centre of the Voigt function

Bandwidth of clutter spectrum



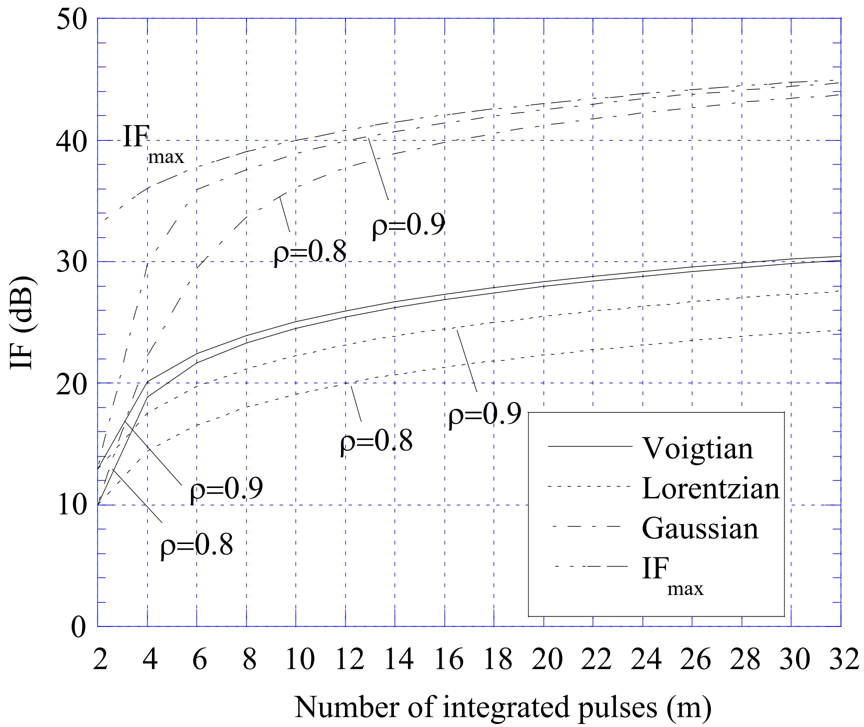
From Nathanson, 1990

Impact of spectral model on performance prediction (I)

Swerling-I target signal:

$$P_D = (P_{FA})^{\frac{1}{1+SIR \cdot \mathbf{p}^H \mathbf{M}^{-1} \mathbf{p}}}$$

Receiver Operating Characteristic (ROC)



P_D is a monotonic increasing function of the receiver **improvement factor (IF)**:

$$IF = \frac{SIR_{out}}{SIR} = \mathbf{p}^H \mathbf{M}^{-1} \mathbf{p}$$

SIR_{out} is the signal-to-interference ratio at the output of the WMF

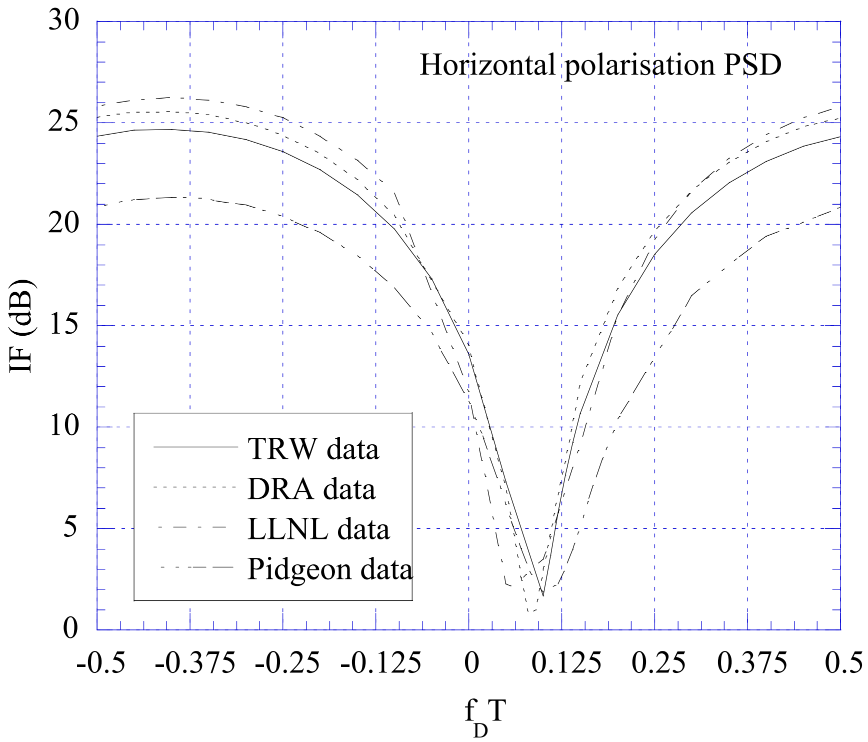
When operating in Gaussian disturbance, the performance of the Optimum Detector can be described equivalently in terms of ROCs or in terms of the IF

The maximum value of the Improvement Factor is equal to **IF_{max}=m(CNR+1)** where **m** is the number of integrated pulses.

ρ = clutter one-lag correlation coeff.

Impact of spectral model on performance prediction (II)

Real sea clutter data suggest that several types of scattering processes from ocean waves are present; as a consequence the spectral lineshapes should be better modeled as a combination of:
Gaussian, **Lorentzian** and **Voigtian** basis functions



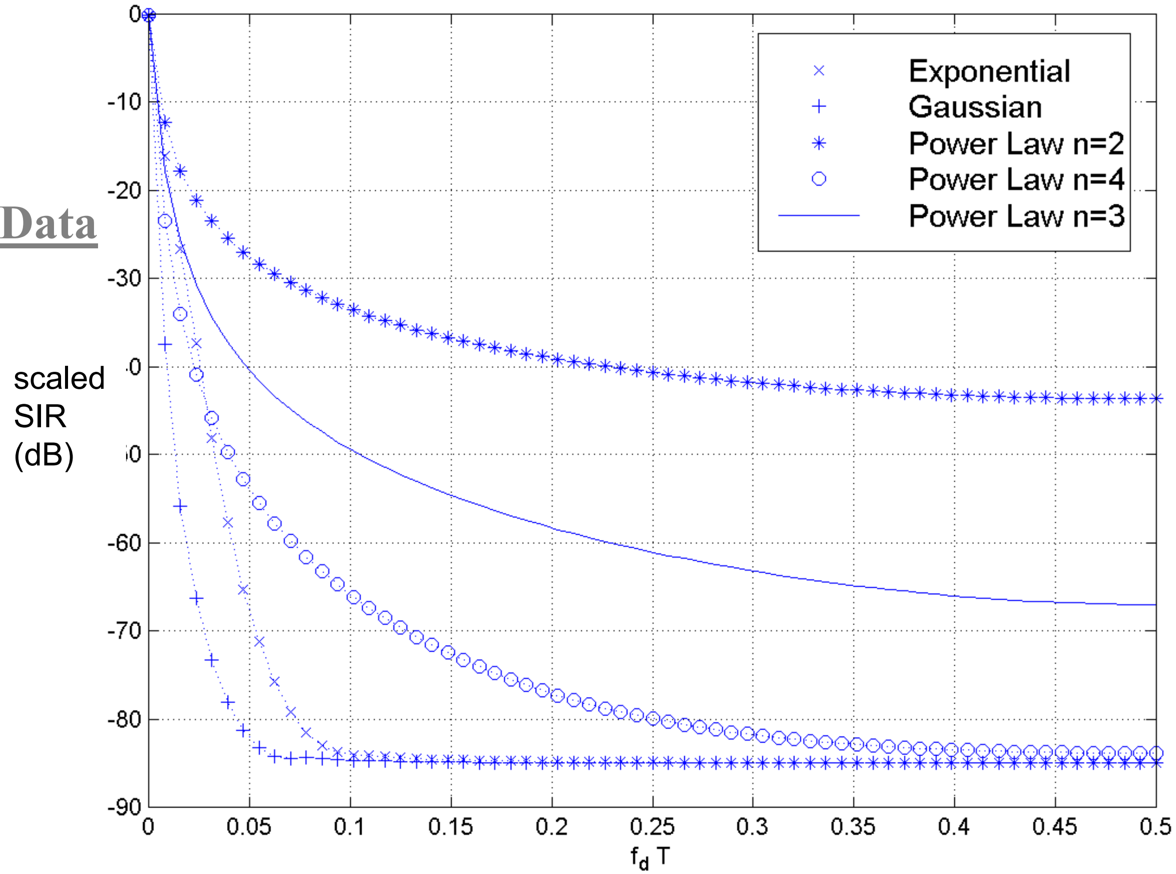
Four sets of data, labeled as TRW data, DRA data, LLNL data and Pidgeon data, obtained by different experimenters at different times and locations

The PSD parameters have been obtained by nonlinear least squares (NLLS) fitting

Impact of spectral model on performance prediction (III)

Predicted Signal to Interference Ratio (SIR) using different spectral models

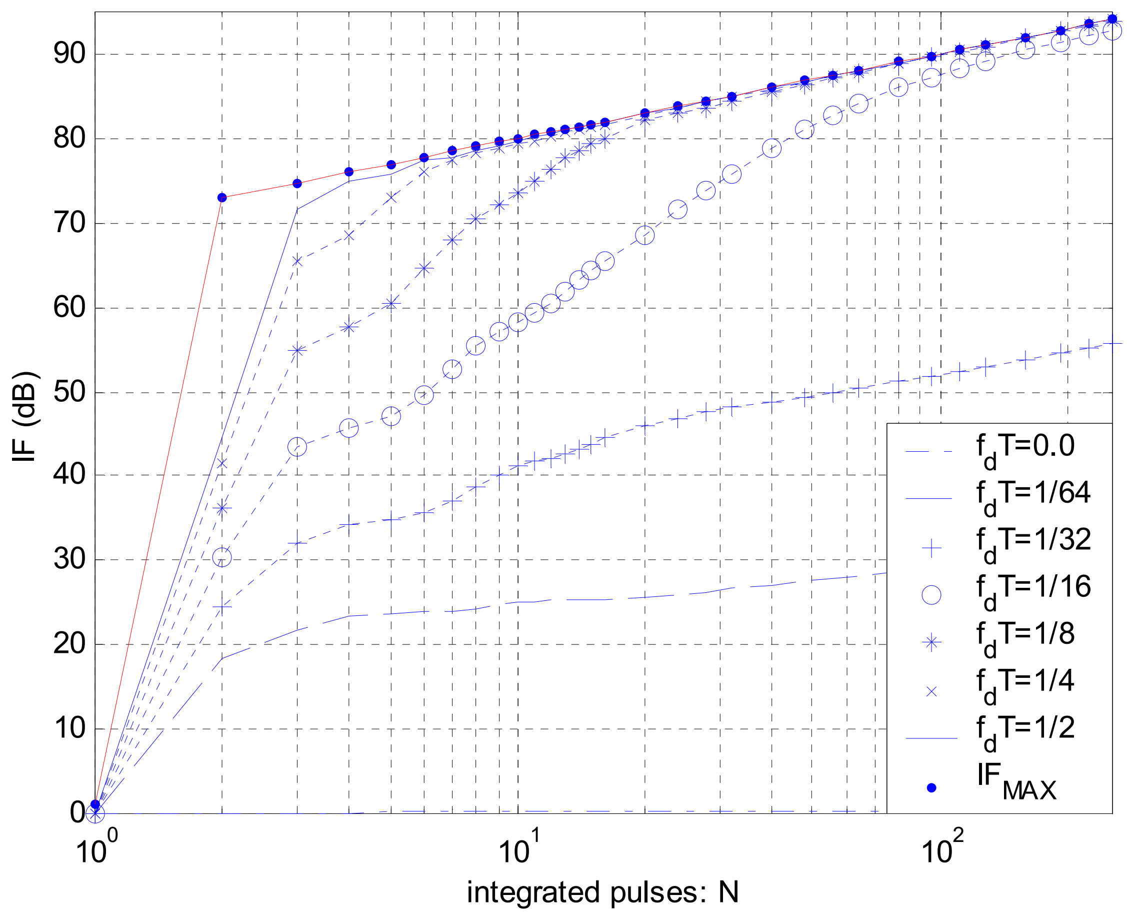
Land Clutter Data



Visibility curve for ground-based surveillance radar: L-band; N=32 coherent pulses;; windy conditions (wind speed 30 mph, β=5.7 s/m).

Impact of spectral model on performance prediction (IV)

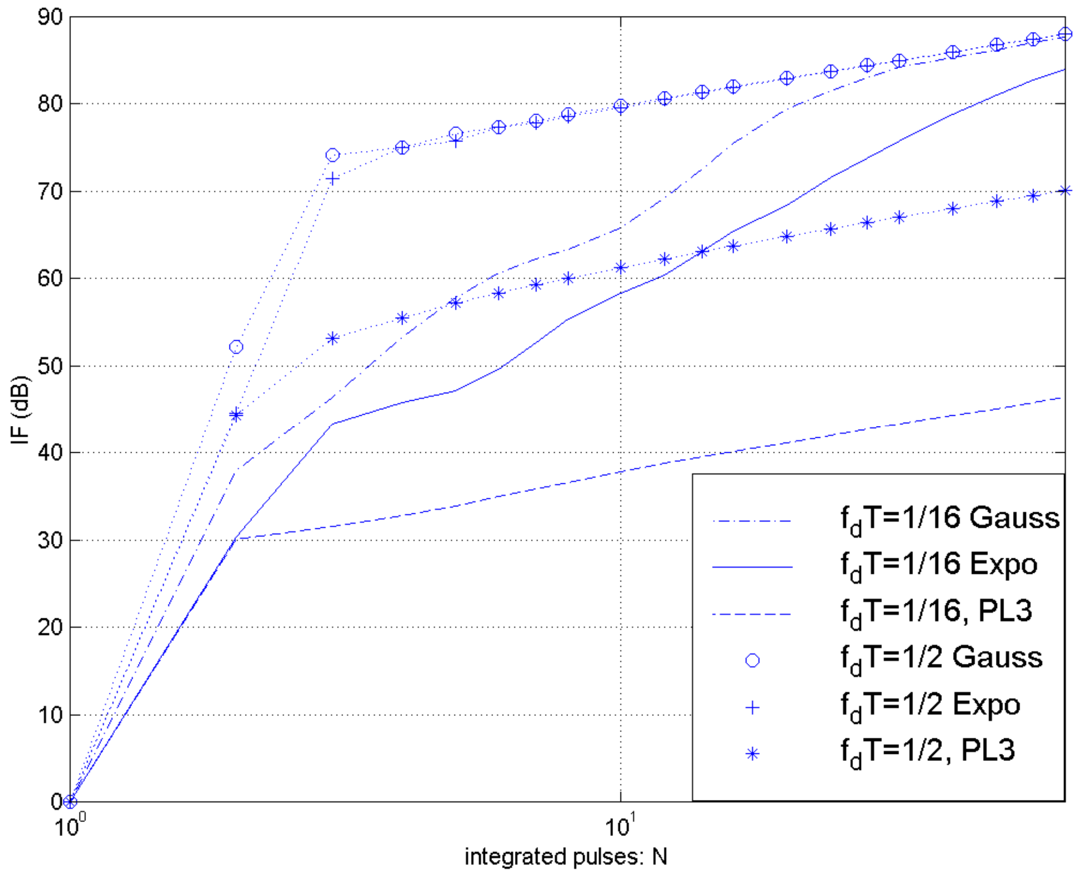
IF of the optimum filter vs number N of PRIs - Exponential Spectrum



Ground-based surveillance radar: L-band; windy conditions (wind speed 30 mph, β=5.7 s/m).

Impact of spectral model on performance prediction (V)

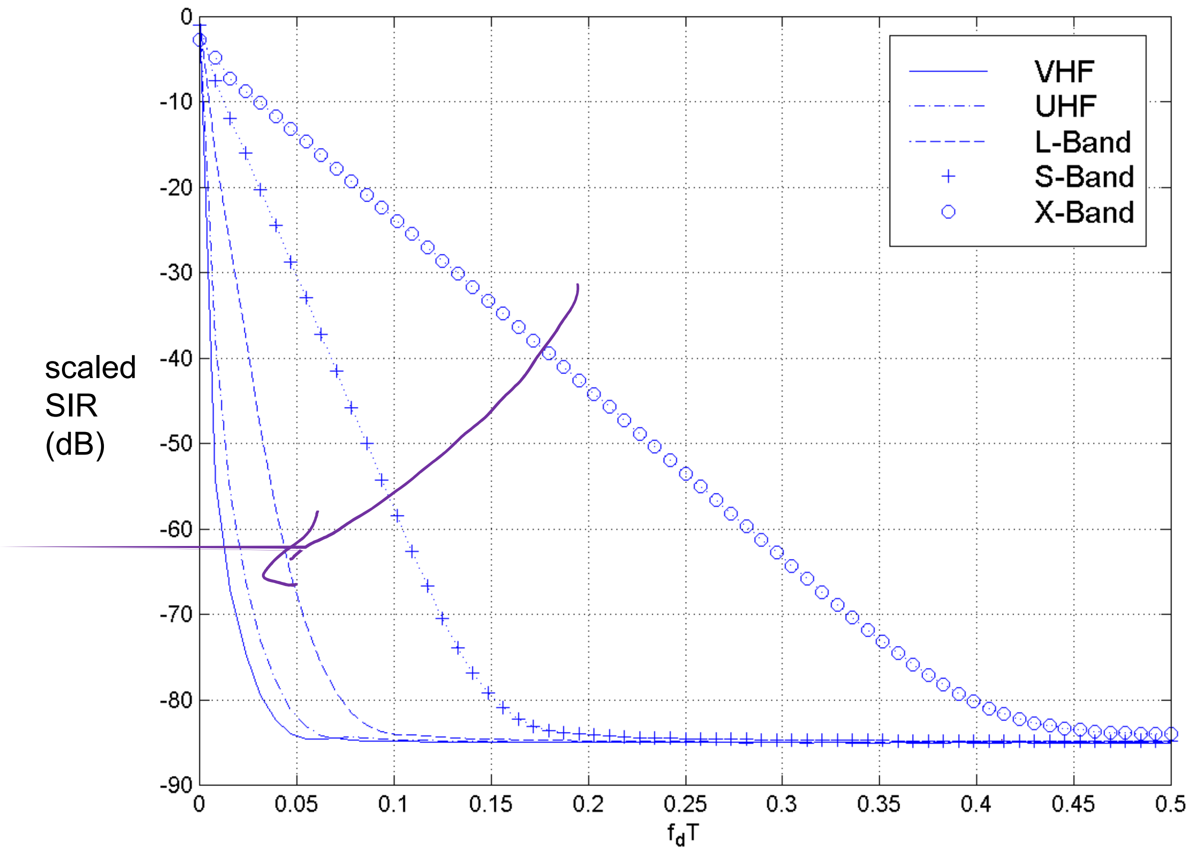
Comparison of IF for the optimum filter assuming different spectral models



IF plot vs. number of pulses with $f_d T$ as parameter for ground-based surveillance radar: L-band; Exponential, Gaussian, Power-Law; windy conditions (wind speed 30 mph, β=5.7 s/m).

Impact of spectral model on performance prediction (VI)

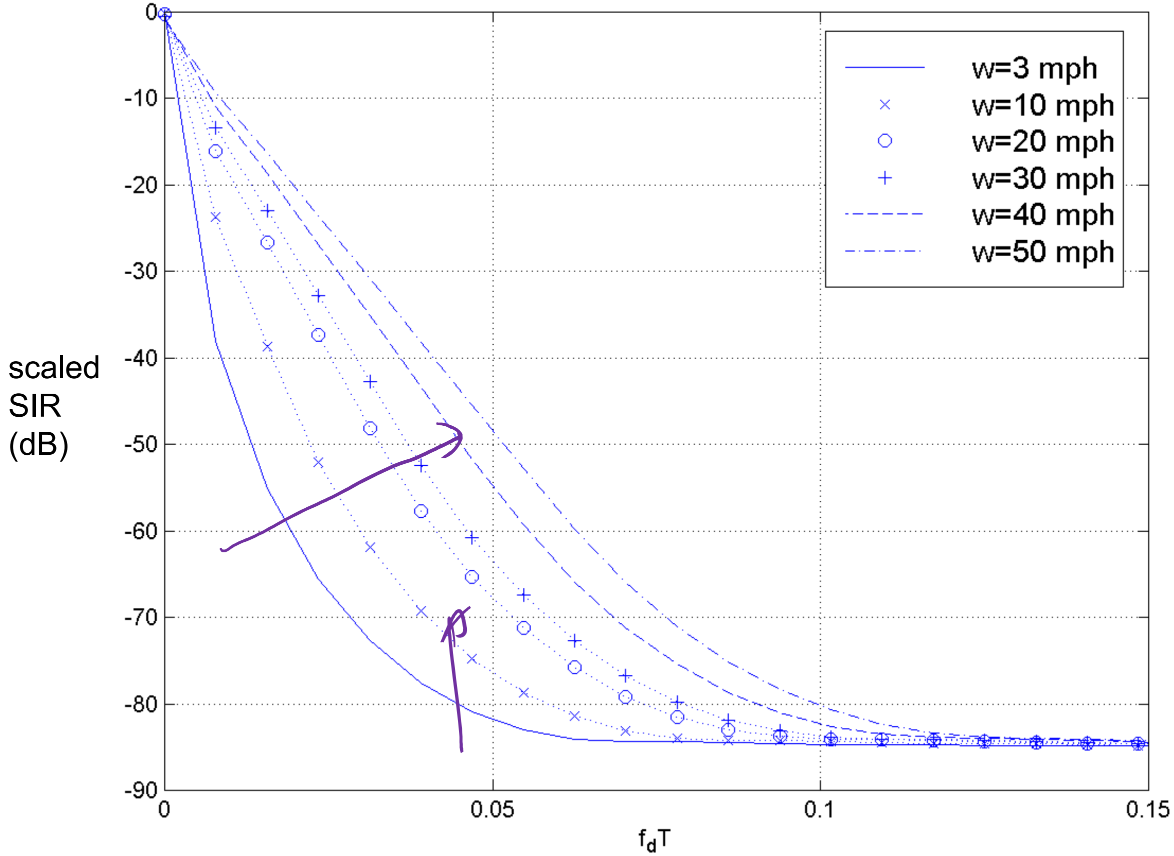
Predicted SIR at different frequencies – Exponential Spectrum



Visibility curve for ground-based surveillance radar : windy conditions (wind speed 30 mph, β=5.7 s/m); N=32 coherent pulses;

Impact of spectral model on performance prediction (VII)

Predicted SIR at different wind speeds– Exponential Spectrum



Visibility curve for ground-based surveillance radar : L band; N=32 coherent pulses.