Radar di Tracking

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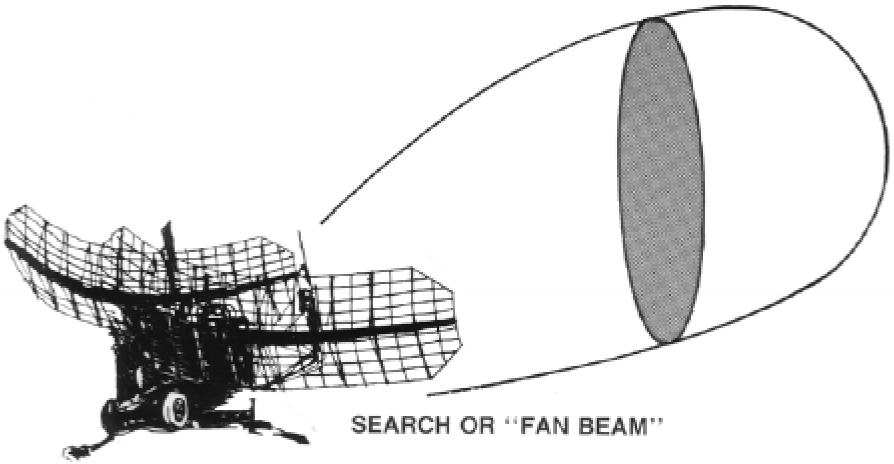
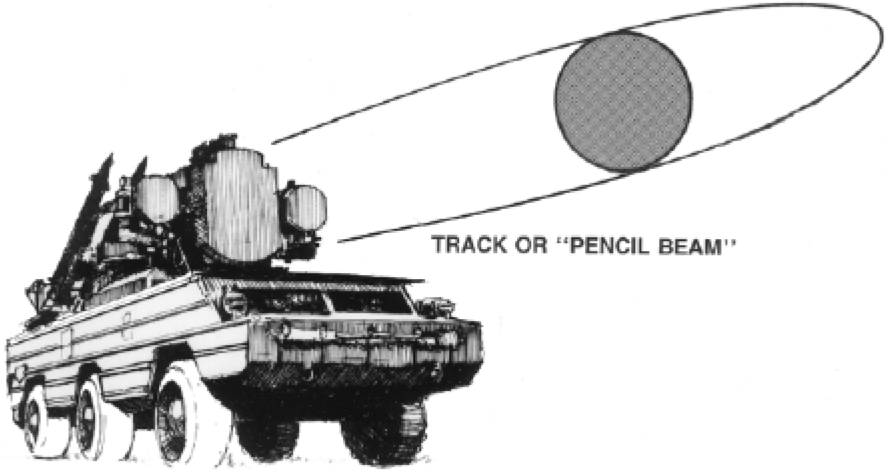
Antenna patterns



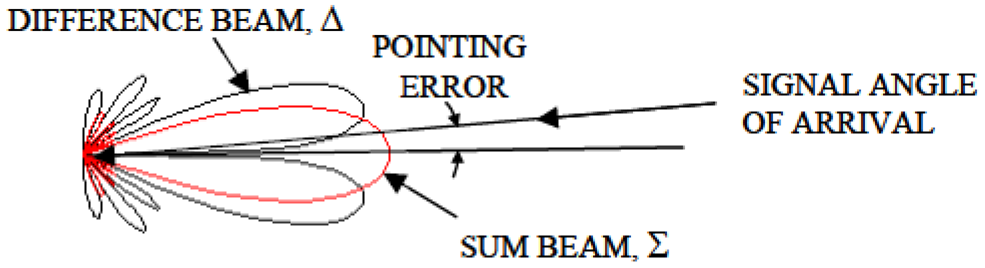
- Fan beam for 2-d search



- Pencil beam for tracking for 3-d search

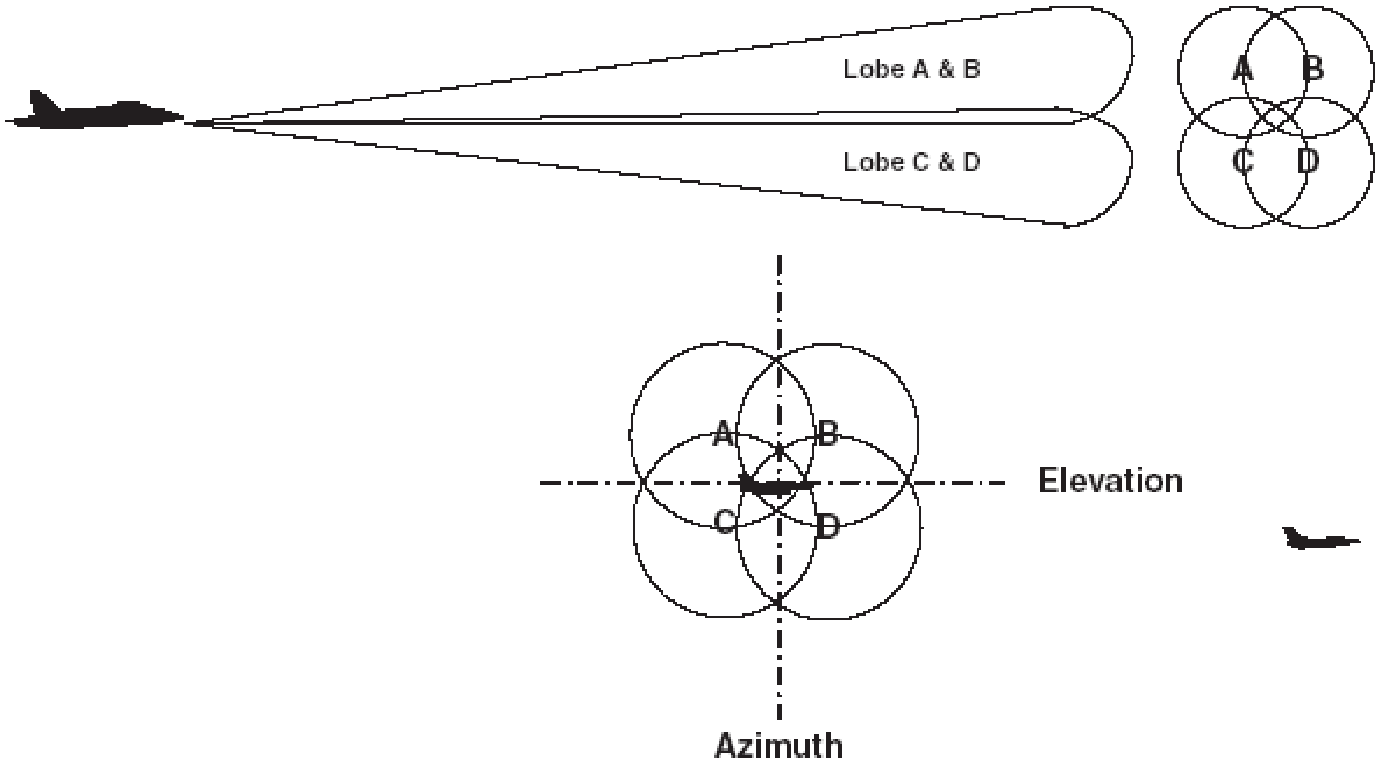
Track versus Search

- Search radars
  - > Long, medium, short ranges (20 km to 2000 km)
  - > High power density on the target: high peak power, long pulses, long pulse trains, high antenna gain
  - > Low PRFs, large range bins
  - > Search options: rapid search rate with narrow beams or slower search rate with wide beams
- Tracking radar
  - > Accurate angle and range measurement required
  - > Minimize time on target for rapid processing
  - > Special tracking techniques: monopulse, conical scan, beam switching



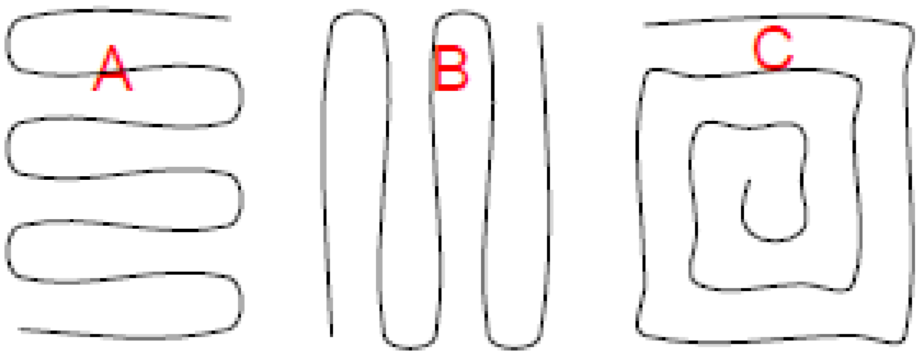
Monopulse Technique

Angle tracking – monopulse



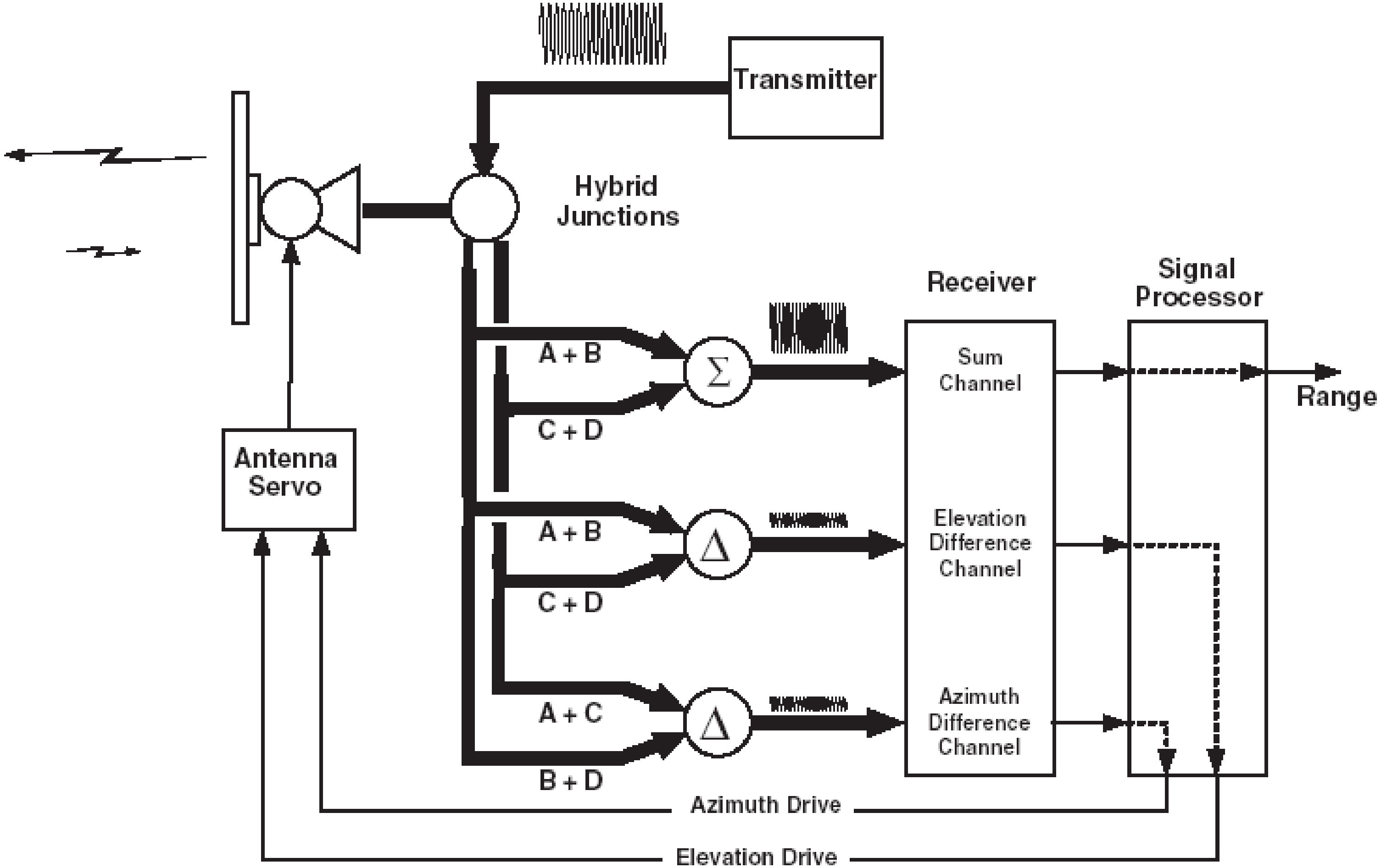
Fase di ricerca

Pattern di ricerca in angolo partendo da cueing:



Zona di range definita

Angle tracking loop – monopulse



Stima di distanza: Cramer-Rao Bound (I)

- **Modello del segnale ricevuto** $r(t)$ sotto l'ipotesi H_1:

$$r(t) = d(t) + a\ s_0(t - t_d)\ e^{j2\pi f_d t} \qquad p(\mathbf{r}|H_1) = \frac{1}{\pi^N \sigma_n^{2N}} \exp\left\{-\frac{1}{\sigma_n^2}\left|\mathbf{r} - a\,\mathbf{s}_0[t_0, f_0]\right|^2\right\}$$

$$\frac{\partial}{\partial t_0}\ln\left[p(\mathbf{r}|H_1)\right] = -\frac{1}{\sigma_n^2}\frac{\partial}{\partial t_0}\left|\mathbf{r} - a\,\mathbf{s}_0[t_0, f_0]\right|^2 = -\frac{1}{T_s\,\sigma_n^2}\frac{\partial}{\partial t_0}\int\left|r(t) - a\,s_0(t-t_0)e^{j2\pi f_0 t}\right|^2 dt =$$

$$= -\frac{1}{T_s\,\sigma_n^2}\int\frac{\partial}{\partial t_0}\left[\left(r^*(t) - a^*\,s_0^*(t-t_0)e^{-j2\pi f_0 t}\right)\left(r(t) - a\,s_0(t-t_0)e^{j2\pi f_0 t}\right)\right]dt =$$

$$= -\frac{1}{T_s\,\sigma_n^2}\int\left[a^*\,\dot{s}_0^*(t-t_0)e^{-j2\pi f_0 t}\left(r(t) - a\,s_0(t-t_0)e^{j2\pi f_0 t}\right) + \left(r^*(t) - a^*\,s_0^*(t-t_0)e^{-j2\pi f_0 t}\right)a\,\dot{s}_0(t-t_0)e^{j2\pi f_0 t}\right]dt =$$

$$= -\frac{2}{T_s\,\sigma_n^2}\int \mathrm{Re}\left[a^*\,\dot{s}_0^*(t-t_0)e^{-j2\pi f_0 t}\left(r(t) - a\,s_0(t-t_0)e^{j2\pi f_0 t}\right)\right]dt =$$

$$= -\frac{2}{T_s\,\sigma_n^2}\int \mathrm{Re}\left[a^*\,r(t)\dot{s}_0^*(t-t_0)e^{-j2\pi f_0 t} - |a|^2\,\dot{s}_0^*(t-t_0)\,s_0(t-t_0)\right]dt =$$

Stima di distanza: Cramer-Rao Bound (II)

$$\frac{\partial}{\partial t_0}\ln\left[p(\mathbf{r}|H_1)\right]=-\frac{2}{T_s\,\sigma_n^2}\int \mathrm{Re}\left[a^*\,r(t)\dot{s}_0^*(t-t_0)e^{-j2\pi f_0 t}-|a|^2\,\dot{s}_0^*(t-t_0)\,s_0(t-t_0)\right]dt=$$

$$=-\frac{2}{T_s\,\sigma_n^2}\int \mathrm{Re}\left[a^*\,\dot{s}_0^*(t-t_0)e^{-j2\pi f_0 t}\left[a\;s_0(t-t_0)e^{j2\pi f_0 t}+n(t)\right]-|a|^2\,\dot{s}_0^*(t-t_0)\,s_0(t-t_0)\right]dt=$$

$$=-\frac{2}{T_s\,\sigma_n^2}\int \mathrm{Re}\left[|a|^2\,\dot{s}_0^*(t-t_0)s_0(t-t_0)+a^*\,\dot{s}_0^*(t-t_0)e^{-j2\pi f_0 t}\,n(t)-|a|^2\,\dot{s}_0^*(t-t_0)\,s_0(t-t_0)\right]dt=$$

$$=-\frac{2}{T_s\,\sigma_n^2}\int \mathrm{Re}\left[a^*\,\dot{s}_0^*(t-t_0)e^{-j2\pi f_0 t}\,n(t)\right]dt$$

Stima di distanza: Cramer-Rao Bound (III)

$$E\left\{\left[\frac{\partial}{\partial t_0}\ln\left[p(\mathbf{r}|H_1)\right]\right]^2\right\}=\left(\frac{2}{T_s\,\sigma_n^2}\right)^2 E\left\{\int \mathrm{Re}\left[a^*\,\dot{s}_0^*(t-t_0)e^{-j2\pi f_0 t}\,n(t)\right]dt\int \mathrm{Re}\left[a^*\,\dot{s}_0^*(t'-t_0)e^{-j2\pi f_0 t'}\,n(t')\right]dt'\right\}=$$

$$=\left(\frac{1}{T_s\,\sigma_n^2}\right)^2 E\left\{\int\int\left[a^*\,\dot{s}_0^*(t-t_0)e^{-j2\pi f_0 t}\,n(t)+a\,\dot{s}_0(t-t_0)e^{j2\pi f_0 t}\,n^*(t)\right]\left[a^*\,\dot{s}_0^*(t'-t_0)e^{-j2\pi f_0 t'}\,n(t')+a\,\dot{s}_0(t'-t_0)e^{j2\pi f_0 t'}\,n^*(t')\right]dt\,dt'\right\}=$$

$$=\left(\frac{1}{T_s\,\sigma_n^2}\right)^2 E\left\{\int\int\left[a^*\,\dot{s}_0^*(t-t_0)e^{-j2\pi f_0 t}\,n(t)a\,\dot{s}_0(t'-t_0)e^{j2\pi f_0 t'}\,n^*(t')+a\,\dot{s}_0(t-t_0)e^{j2\pi f_0 t}\,n^*(t)a^*\,\dot{s}_0^*(t'-t_0)e^{-j2\pi f_0 t'}\,n(t')\right]dt\,dt'\right\}=$$

$$=\left(\frac{1}{T_s\,\sigma_n^2}\right)^2 E\left\{2\int|a|^2\left|\dot{s}_0(t-t_0)\right|^2|n(t)|^2dt\right\}=\left(\frac{1}{T_s\,\sigma_n^2}\right)^2 2|a|^2\sigma_n^2\int\left|\dot{s}_0(t-t_0)\right|^2dt=\frac{2|a|^2}{T_s^2\,\sigma_n^2}\int\left|\dot{s}_0(t-t_0)\right|^2dt$$

$$\sigma_{t_0}^2\geq E\left\{\left[\frac{\partial}{\partial t_0}\ln\left[p(\mathbf{r}|H_1)\right]\right]^2\right\}^{-1}=\frac{1}{\dfrac{2|a|^2}{T_s^2\,\sigma_n^2}\displaystyle\int\left|\dot{s}_0(t-t_0)\right|^2dt}$$

Stima di distanza: Cramer-Rao Bound (IV)

$$\sigma_{t_0}^2 \ge E\left\{\left[\frac{\partial}{\partial t_0}\ln\left[p(\mathbf{r}|H_1)\right]\right]^2\right\}^{-1} = \frac{1}{\frac{2|a|^2}{T_s^2\,\sigma_n^2}\int \left|\dot{s}_0(t-t_0)\right|^2 dt} = \frac{1}{2\,SNR\cdot\beta^2}$$

$$S = |a|^2 \left|\mathbf{s}_0(t-t_0)\right|^2 = |a|^2 \frac{1}{T_s}\int \left|s_0(t-t_0)\right|^2 dt$$

$$SNR = \frac{|a|^2}{\sigma_n^2}\frac{1}{T_s}\int \left|s_0(t-t_0)\right|^2 dt$$

$$\beta^2 = \frac{\int \left|\dot{s}_0(t-t_0)\right|^2 dt}{\int \left|s_0(t-t_0)\right|^2 dt} = \frac{\int (2\pi f)^2 \left|S_0(f)\right|^2 df}{\int \left|S_0(f)\right|^2 df}$$

$$\int \left|s_0(t-t_0)\right|^2 dt = \int \left|S_0(f)\right|^2 df$$

$$\int \left|\dot{s}_0(t-t_0)\right|^2 dt = \int \left|j2\pi f\, S_0(f)\right|^2 df = \int (2\pi f)^2 \left|S_0(f)\right|^2 df$$

Stima di frequenza: Cramer-Rao Bound (I)

- **Modello del segnale ricevuto** $r(t)$ sotto l'ipotesi H_1:

$$r(t) = d(t) + a\, s_0(t - t_d)\, e^{j2\pi f_d t} \qquad p(\mathbf{r}|H_1) = \frac{1}{\pi^N \sigma_n^{2N}} \exp\left\{-\frac{1}{\sigma_n^2}\left|\mathbf{r} - a\,\mathbf{s}_0[t_0, f_0]\right|^2\right\}$$

$$\frac{\partial}{\partial f_0}\ln\left[p(\mathbf{r}|H_1)\right] = -\frac{1}{\sigma_n^2}\frac{\partial}{\partial f_0}\left|\mathbf{r} - a\,\mathbf{s}_0[t_0, f_0]\right|^2 = -\frac{1}{T_s\,\sigma_n^2}\frac{\partial}{\partial f_0}\int\left|r(t) - a\,s_0(t-t_0)e^{j2\pi f_0 t}\right|^2 dt =$$

$$= -\frac{1}{T_s\,\sigma_n^2}\int\frac{\partial}{\partial f_0}\left[\left(r^*(t) - a^*\,s_0^*(t-t_0)e^{-j2\pi f_0 t}\right)\left(r(t) - a\,s_0(t-t_0)e^{j2\pi f_0 t}\right)\right]dt =$$

$$= -\frac{1}{T_s\,\sigma_n^2}\int\left[j2\pi t\; a^*\,s_0^*(t-t_0)e^{-j2\pi f_0 t}\left(r(t) - a\,s_0(t-t_0)e^{j2\pi f_0 t}\right) - \left(r^*(t) - a^*\,s_0^*(t-t_0)e^{-j2\pi f_0 t}\right)j2\pi t\; a\,\dot{s}_0(t-t_0)e^{j2\pi f_0 t}\right]dt =$$

$$= -\frac{2}{T_s\,\sigma_n^2}\int\mathrm{Re}\left[j2\pi t\; a^*\,s_0^*(t-t_0)e^{-j2\pi f_0 t}\left(r(t) - a\,s_0(t-t_0)e^{j2\pi f_0 t}\right)\right]dt =$$

$$= -\frac{2}{T_s\,\sigma_n^2}\int\mathrm{Re}\left[j2\pi t\; a^*\,s_0^*(t-t_0)e^{-j2\pi f_0 t}\,r(t) - j2\pi t\,|a|^2\,|s_0(t-t_0)|^2\right]dt =$$

Stima di frequenza: Cramer-Rao Bound (II)

$$\frac{\partial}{\partial f_0}\ln\left[p(\mathbf{r}|H_1)\right]=-\frac{2}{T_s\,\sigma_n^2}\int \mathrm{Re}\left[j2\pi t\ a^*\,s_0^*(t-t_0)e^{-j2\pi f_0 t}\,r(t)-j2\pi t\left|a\right|^2\left|s_0(t-t_0)\right|^2\right]dt=$$

$$=-\frac{2}{T_s\,\sigma_n^2}\int \mathrm{Re}\left[j2\pi t\ a^*\,s_0^*(t-t_0)e^{-j2\pi f_0 t}\left[a\ s_0(t-t_0)e^{j2\pi f_0 t}+n(t)\right]-j2\pi t\left|a\right|^2\left|s_0(t-t_0)\right|^2\right]dt=$$

$$=-\frac{2}{T_s\,\sigma_n^2}\int \mathrm{Re}\left[j2\pi t\ \left|a\right|^2\left|s_0(t-t_0)\right|^2+j2\pi t\ a^*\,s_0^*(t-t_0)e^{-j2\pi f_0 t}\,n(t)-j2\pi t\left|a\right|^2\left|s_0(t-t_0)\right|^2\right]dt=$$

$$=-\frac{2}{T_s\,\sigma_n^2}\int \mathrm{Re}\left[j2\pi t\ a^*\,s_0^*(t-t_0)e^{-j2\pi f_0 t}\,n(t)\right]dt$$

Stima di frequenza: Cramer-Rao Bound (III)

$$E\left\{\left[\frac{\partial}{\partial f_0}\ln\left[p(\mathbf{r}|H_1)\right]\right]^2\right\}=\left(\frac{2}{T_s\sigma_n^2}\right)^2 E\left\{\int \mathrm{Re}\left[j2\pi t\, a^* s_0^*(t-t_0)e^{-j2\pi f_0 t}\, n(t)\right]dt \int \mathrm{Re}\left[j2\pi t'\, a^* s_0^*(t'-t_0)e^{-j2\pi f_0 t'}\, n(t')\right]dt'\right\}=$$

$$=\left(\frac{1}{T_s\sigma_n^2}\right)^2 E\left\{\int\int\left[j2\pi t\, a^* s_0^*(t-t_0)e^{-j2\pi f_0 t}\, n(t)-j2\pi t\, a s_0(t-t_0)e^{j2\pi f_0 t}\, n^*(t)\right]\left[j2\pi t'\, a^* s_0^*(t'-t_0)e^{-j2\pi f_0 t'}\, n(t')-j2\pi t'\, a s_0(t'-t_0)e^{j2\pi f_0 t'}\, n^*(t')\right]dt\,dt'\right\}=$$

$$=\left(\frac{1}{T_s\sigma_n^2}\right)^2 E\left\{\int\int\left[-j2\pi t\, a^* s_0^*(t-t_0)e^{-j2\pi f_0 t}\, n(t)\, j2\pi t'\, a s_0(t'-t_0)e^{j2\pi f_0 t'}\, n^*(t')-j2\pi t\, a s_0(t-t_0)e^{j2\pi f_0 t}\, n^*(t)\, j2\pi t'\, a^* s_0^*(t'-t_0)e^{-j2\pi f_0 t'}\, n(t')\right]dt\,dt'\right\}=$$

$$=\left(\frac{1}{T_s\sigma_n^2}\right)^2 E\left\{\int 2(2\pi t)^2\,|a|^2\left|s_0(t-t_0)\right|^2|n(t)|^2\,dt\right\}=\left(\frac{1}{T_s\sigma_n^2}\right)^2 2|a|^2\sigma_n^2\int(2\pi t)^2\left|s_0(t-t_0)\right|^2 dt=\frac{2|a|^2}{T_s^2\sigma_n^2}\int(2\pi t)^2\left|s_0(t-t_0)\right|^2 dt$$

$$\sigma_{f_0}^2 \ge E\left\{\left[\frac{\partial}{\partial f_0}\ln\left[p(\mathbf{r}|H_1)\right]\right]^2\right\}^{-1}=\frac{1}{\dfrac{2|a|^2}{T_s^2\sigma_n^2}\displaystyle\int(2\pi t)^2\left|s_0(t-t_0)\right|^2 dt}$$

Stima di frequenza: Cramer-Rao Bound (IV)

$$\sigma_{f_0}^2 \geq E\left\{\left[\frac{\partial}{\partial f_0}\ln\left[p(\mathbf{r}|H_1)\right]\right]^2\right\}^{-1} = \frac{1}{\dfrac{2|a|^2}{T_s^2\,\sigma_n^2}\displaystyle\int (2\pi t)^2 \left|s_0(t-t_0)\right|^2 dt} = \frac{1}{2SNR\cdot\alpha^2}$$

$$S = |a|^2 \left|\mathbf{s}_0(t-t_0)\right|^2 = |a|^2 \frac{1}{T_s}\int \left|s_0(t-t_0)\right|^2 dt$$

$$SNR = \frac{|a|^2}{\sigma_n^2}\frac{1}{T_s}\int \left|s_0(t-t_0)\right|^2 dt$$

$$\alpha^2 = \frac{\displaystyle\int (2\pi t)^2 \left|s_0(t-t_0)\right|^2 dt}{\displaystyle\int \left|s_0(t-t_0)\right|^2 dt} = \frac{\displaystyle\int \left|\dot{S}_0(f)\right|^2 df}{\displaystyle\int \left|S_0(f)\right|^2 df}$$

$$\int \left|s_0(t-t_0)\right|^2 dt = \int \left|S_0(f)\right|^2 df$$

$$\int (2\pi t)^2 \left|s_0(t-t_0)\right|^2 dt = \int \left|\dot{S}_0(f)\right|^2 df$$

Stima di distanza e frequenza: CRB (I)

$$E\left\{\frac{\partial}{\partial t_0}\ln\left[p(\mathbf{r}|H_1)\right]\frac{\partial}{\partial f_0}\ln\left[p(\mathbf{r}|H_1)\right]\right\}=\left(\frac{2}{T_s\sigma_n^2}\right)^2 E\left\{\int \mathrm{Re}\left[a^*\dot{s}_0^*(t-t_0)e^{-j2\pi f_0 t}\,n(t)\right]dt\int \mathrm{Re}\left[j2\pi t'\,a^*s_0^*(t'-t_0)e^{-j2\pi f_0 t'}\,n(t')\right]dt'\right\}=$$

$$=\left(\frac{1}{T_s\sigma_n^2}\right)^2 E\left\{\int\int\left[a^*\dot{s}_0^*(t-t_0)e^{-j2\pi f_0 t}\,n(t)+a\dot{s}_0(t-t_0)e^{j2\pi f_0 t}\,n^*(t)\right]\left[j2\pi t'\,a^*s_0^*(t'-t_0)e^{-j2\pi f_0 t'}\,n(t')-j2\pi t'\,as_0(t'-t_0)e^{j2\pi f_0 t'}\,n^*(t')\right]dt\,dt'\right\}=$$

$$=\left(\frac{1}{T_s\sigma_n^2}\right)^2 E\left\{\int\int\left[-a^*\dot{s}_0^*(t-t_0)e^{-j2\pi f_0 t}\,n(t)\,j2\pi t'\,as_0(t'-t_0)e^{j2\pi f_0 t'}\,n^*(t')+a\dot{s}_0(t-t_0)e^{j2\pi f_0 t}\,n^*(t)\,j2\pi t'\,a^*s_0^*(t'-t_0)e^{-j2\pi f_0 t'}\,n(t')\right]dt\,dt'\right\}=$$

$$=\left(\frac{1}{T_s\sigma_n^2}\right)^2 E\left\{\int j2\pi t|a|^2\left[-\dot{s}_0^*(t-t_0)\ \ s_0(t-t_0)+\dot{s}_0(t-t_0)\ \ s_0^*(t'-t_0)\right]|n(t)|^2dt\right\}=$$

$$=-\left(\frac{1}{T_s\sigma_n^2}\right)^2|a|^2\sigma_n^2\int 4\pi t\ \ \mathrm{Im}\left[\dot{s}_0(t-t_0)s_0^*(t-t_0)\right]dt$$

$$Cov\left(t_0,f_0\right)\geq J^{-1}$$

Stima di distanza: MLE (I)

- **Modello del segnale ricevuto** $r(t)$ sotto l'ipotesi H_1:

$$r(t) = d(t) + a\ s_0(t - t_d)\ e^{j2\pi f_d t}$$

$$p(\mathbf{r}|H_1) = \frac{1}{\pi^N \sigma_n^{2N}} \exp\left\{-\frac{1}{\sigma_n^2}\left|\mathbf{r} - a\,\mathbf{s}_0[t_0, f_0]\right|^2\right\}$$

- **Stimatore ML**

$$\frac{\partial}{\partial t_0}\ln\left[p(\mathbf{r}|H_1)\right] = 0$$

$$\frac{\partial}{\partial t_0}\ln\left[p(\mathbf{r}|H_1)\right] = -\frac{2}{T_s\,\sigma_n^2}\int \mathrm{Re}\left[a^*\,r(t)\dot{s}_0^*(t-t_0)e^{-j2\pi f_0 t} - |a|^2\,\dot{s}_0^*(t-t_0)\,s_0(t-t_0)\right]dt = 0$$

Stima di distanza: MLE (II)

- **MLE:**

$$\frac{2\,|a|^2}{T_s\,\sigma_n^2}\int \mathrm{Re}\left[\dot{s}_0^*(t-t_0)\,s_0(t-t_0)\right]dt = \frac{2\,|a|^2}{T_s\,\sigma_n^2}\int \mathrm{Re}\left[\dot{s}_0^*(t-t_0)\,s_0(t-t_0)\right]dt =$$

$$= \frac{|a|^2}{T_s\,\sigma_n^2}\,\frac{\partial}{\partial t_0}\int s_0^*(t-t_0)\,s_0(t-t_0)\,dt = \frac{|a|^2}{T_s\,\sigma_n^2}\,\frac{\partial}{\partial t_0}\int \left|s_0(t-t_0)\right|^2 dt = \frac{|a|^2}{T_s\,\sigma_n^2}\,\frac{\partial}{\partial t_0}E = 0$$

$$\frac{\partial}{\partial t_0}\ln\left[p(\mathbf{r}\,|\,H_1)\right] = -\frac{2}{T_s\,\sigma_n^2}\int \mathrm{Re}\left[a^*\,r(t)\dot{s}_0^*(t-t_0)e^{-j2\pi f_0 t}\right]dt = 0$$

Dipende dalla fase di *a*

$$\frac{\partial}{\partial t_0}\ln\left[p(\mathbf{r}\,|\,H_1)\right] = -\frac{2}{T_s\,\sigma_n^2}\,\mathrm{Re}\left[a^*\int r(t)\dot{s}_0^*(t-t_0)e^{-j2\pi f_0 t}dt\right] = 0$$

Stima di ampiezza: MLE(I)

- **MLE per a**:

$$\frac{\partial}{\partial a_R}\ln\left[p(\mathbf{r}|H_1)\right]=-\frac{1}{\sigma_n^2}\frac{\partial}{\partial a_R}\left|\mathbf{r}-a\,\mathbf{s}_0[t_0,f_0]\right|^2=-\frac{1}{T_s\,\sigma_n^2}\frac{\partial}{\partial a_R}\int\left|r(t)-a\,s_0(t-t_0)e^{j2\pi f_0 t}\right|^2dt=$$

$$=-\frac{1}{T_s\,\sigma_n^2}\int\frac{\partial}{\partial a_R}\left[\left(r^*(t)-a^*\,s_0^*(t-t_0)e^{-j2\pi f_0 t}\right)\left(r(t)-a\,s_0(t-t_0)e^{j2\pi f_0 t}\right)\right]dt=$$

$$=-\frac{1}{T_s\,\sigma_n^2}\int\left[-s_0^*(t-t_0)e^{-j2\pi f_0 t}\left(r(t)-a\,s_0(t-t_0)e^{j2\pi f_0 t}\right)-\left(r^*(t)-a^*\,s_0^*(t-t_0)e^{-j2\pi f_0 t}\right)s_0(t-t_0)e^{j2\pi f_0 t}\right]dt=$$

$$=\frac{2}{T_s\,\sigma_n^2}\int\mathrm{Re}\left[s_0^*(t-t_0)e^{-j2\pi f_0 t}\left(r(t)-a\,s_0(t-t_0)e^{j2\pi f_0 t}\right)\right]dt=\frac{2}{T_s\,\sigma_n^2}\int\mathrm{Re}\left[r(t)s_0^*(t-t_0)e^{-j2\pi f_0 t}-a\left|s_0(t-t_0)\right|^2\right]dt=0$$

$$\frac{\partial}{\partial a_I}\ln\left[p(\mathbf{r}|H_1)\right]=-\frac{1}{T_s\,\sigma_n^2}\int\frac{\partial}{\partial a_I}\left[\left(r^*(t)-a^*\,s_0^*(t-t_0)e^{-j2\pi f_0 t}\right)\left(r(t)-a\,s_0(t-t_0)e^{j2\pi f_0 t}\right)\right]dt=$$

$$=-\frac{1}{T_s\,\sigma_n^2}\int\left[s_0^*(t-t_0)e^{-j2\pi f_0 t}\left(r(t)-a\,s_0(t-t_0)e^{j2\pi f_0 t}\right)-\left(r^*(t)-a^*\,s_0^*(t-t_0)e^{-j2\pi f_0 t}\right)s_0(t-t_0)e^{j2\pi f_0 t}\right]dt=$$

$$=-\frac{2}{T_s\,\sigma_n^2}\int\mathrm{Im}\left[s_0^*(t-t_0)e^{-j2\pi f_0 t}\left(r(t)-a\,s_0(t-t_0)e^{j2\pi f_0 t}\right)\right]dt=-\frac{2}{T_s\,\sigma_n^2}\int\mathrm{Im}\left[r(t)s_0^*(t-t_0)e^{-j2\pi f_0 t}-a\left|s_0(t-t_0)\right|^2\right]dt=0$$

Stima di ampiezza: MLE(II)

- **MLE per a**:

$$\begin{cases} \dfrac{2}{T_s\,\sigma_n^2} \displaystyle\int \mathrm{Re}\left[r(t)s_0^*(t-t_0)e^{-j2\pi f_0 t} - a\left|s_0(t-t_0)\right|^2 \right] dt = 0 \\ \dfrac{2}{T_s\,\sigma_n^2} \displaystyle\int \mathrm{Im}\left[r(t)s_0^*(t-t_0)e^{-j2\pi f_0 t} - a\left|s_0(t-t_0)\right|^2 \right] dt = 0 \end{cases}$$

$$\frac{2}{T_s\,\sigma_n^2} \int \left[r(t)s_0^*(t-t_0)e^{-j2\pi f_0 t} - a\left|s_0(t-t_0)\right|^2 \right] dt = 0$$

$$\int r(t)s_0^*(t-t_0)e^{-j2\pi f_0 t}\,dt = a \int \left|s_0(t-t_0)\right|^2 dt$$

$$a = \frac{\displaystyle\int r(t)s_0^*(t-t_0)e^{-j2\pi f_0 t}\,dt}{\displaystyle\int \left|s_0(t-t_0)\right|^2 dt}$$

Stima di distanza: MLE (III)

- **Sostituendo la stima MLE di a:**

$$\frac{\partial}{\partial t_0}\ln\left[p(\mathbf{r}|H_1)\right]=-\frac{2}{T_s\,\sigma_n^2}\;\mathrm{Re}\left[a^*\int r(t)\dot{s}_0^*(t-t_0)e^{-j2\pi f_0 t}dt\right]=0$$

$$\frac{\partial}{\partial t_0}\ln\left[p(\mathbf{r}|H_1)\right]=-\frac{2}{T_s\,\sigma_n^2}\;\mathrm{Re}\left[\frac{\left[\int r(t)s_0^*(t-t_0)e^{-j2\pi f_0 t}\,dt\right]^*}{\int\left|s_0(t-t_0)\right|^2dt}\int r(t)\dot{s}_0^*(t-t_0)e^{-j2\pi f_0 t}dt\right]=$$

$$=-\frac{2}{T_s\,\sigma_n^2}\;\frac{\left|\int r(t)\dot{s}_0^*(t-t_0)e^{-j2\pi f_0 t}dt\right|^2}{\int\left|s_0(t-t_0)\right|^2dt}=0$$

Stima di distanza: MLE (IV)

$$t_0 \quad tale \;\; che \quad \left| \int r(t)\; e^{-j2\pi f_0 t} \frac{\dot{s}_0^*(t-t_0)}{\int \left|s_0(t-t_0)\right|^2 dt}\, dt \right|^2 = 0$$

Compensazione Doppler

Derivata del segnale trasmesso normalizzata

Si trasla di t_0 fino a trovare il punto in cui l'integrale ha valore nullo

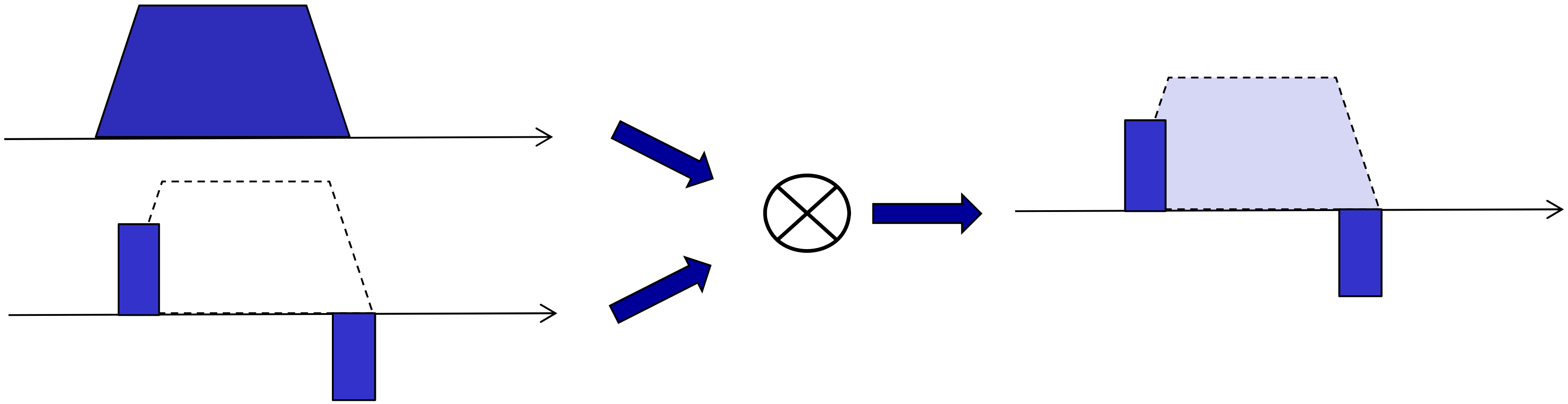
Nel seguito si assume forma d'onda ad energia unitaria per non preoccuparsi della normalizzazione:

$$\int \left|s_0(t-t_0)\right|^2 dt = 1$$

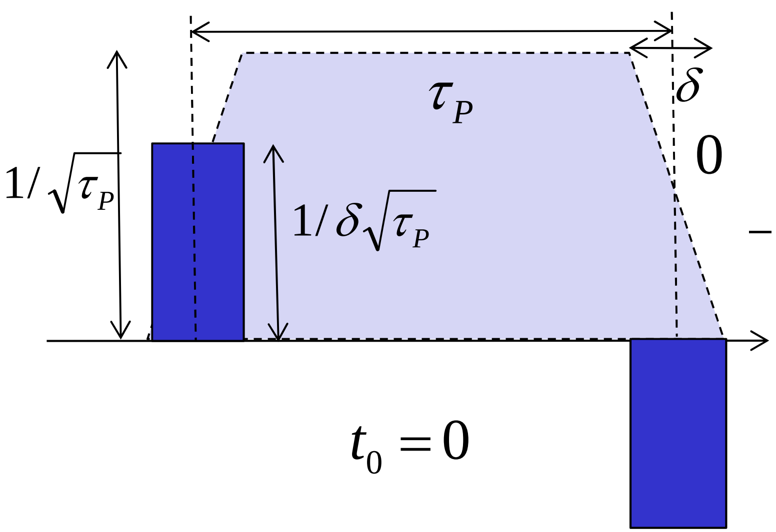
Stima di distanza: MLE (V)

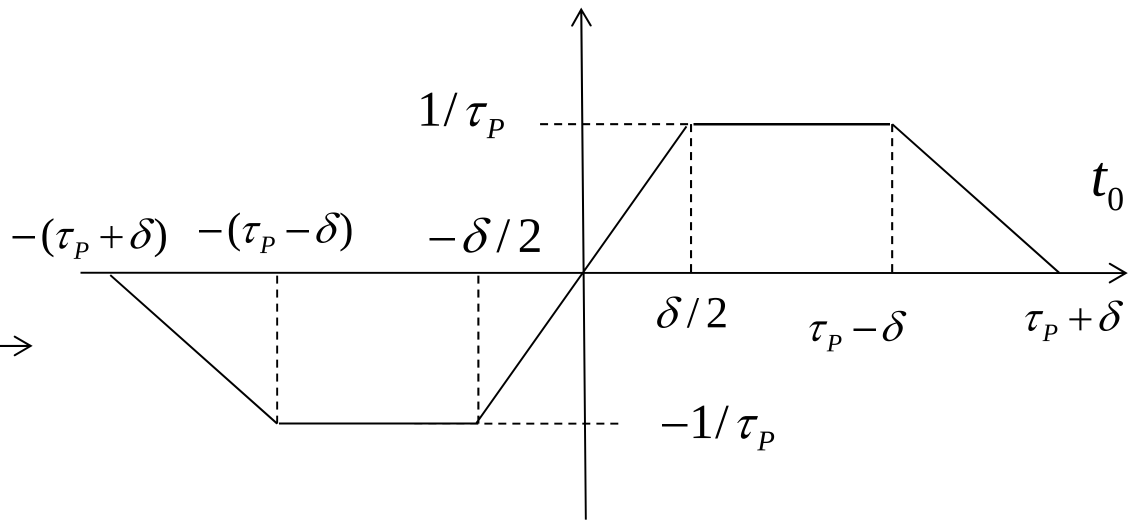
$$t_0 \quad tale \;\; che \quad \left| \int r(t)\; e^{-j2\pi f_0 t} \dot{s}_0^*(t-t_0)dt \right|^2 = 0$$

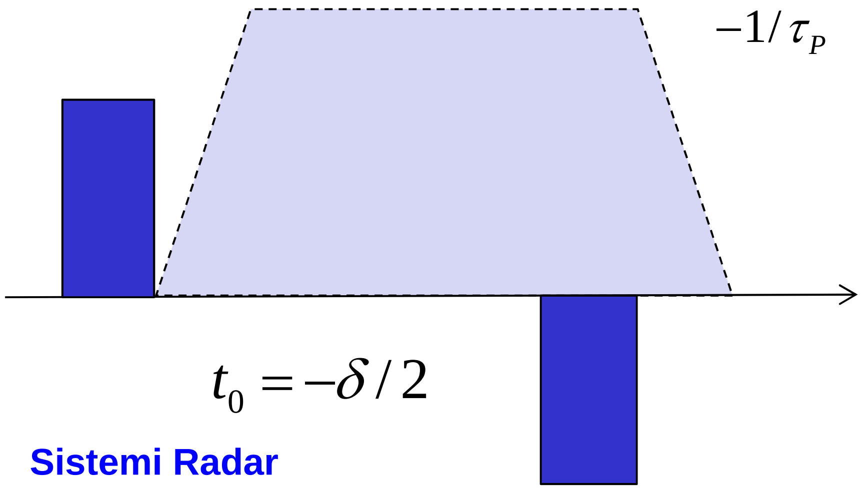
Si trasla di t_0 fino a trovare il punto in cui l'integrale ha valore nullo

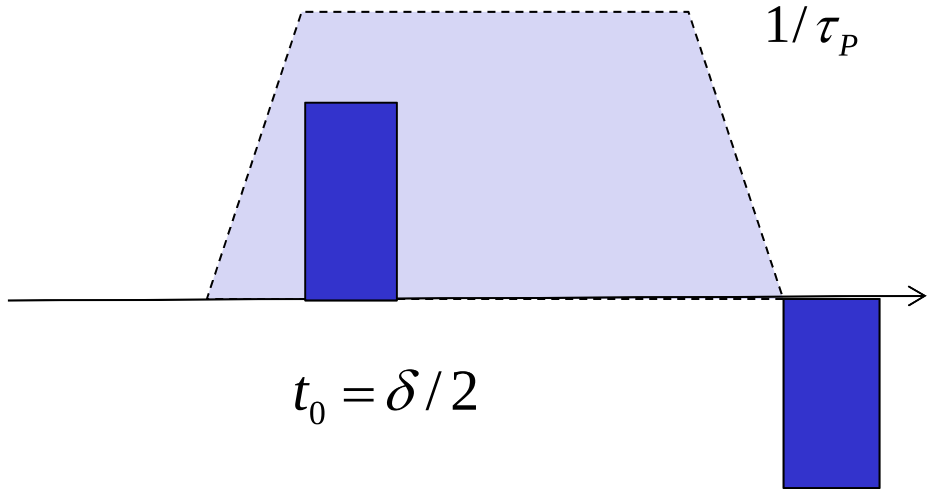


Stima di distanza: MLE (VI)

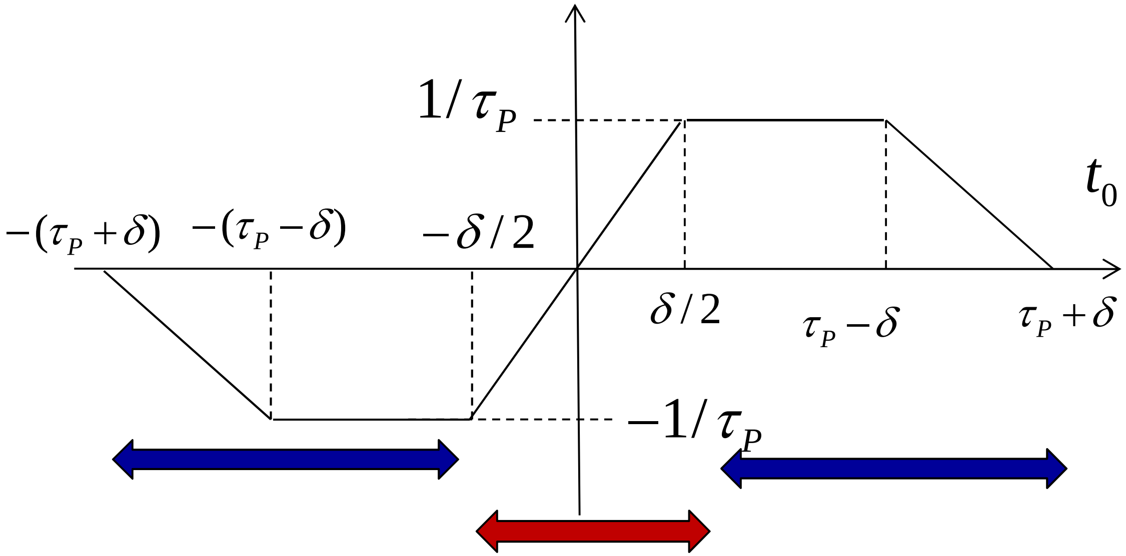








Stima di distanza: discriminatore (I)



Caratteristica del discriminatore di tempo:

- **Zona di lock-in:** caratteristica lineare (uscita direttamente proporzionale all'errore nella stima di tempo di arrivo)+

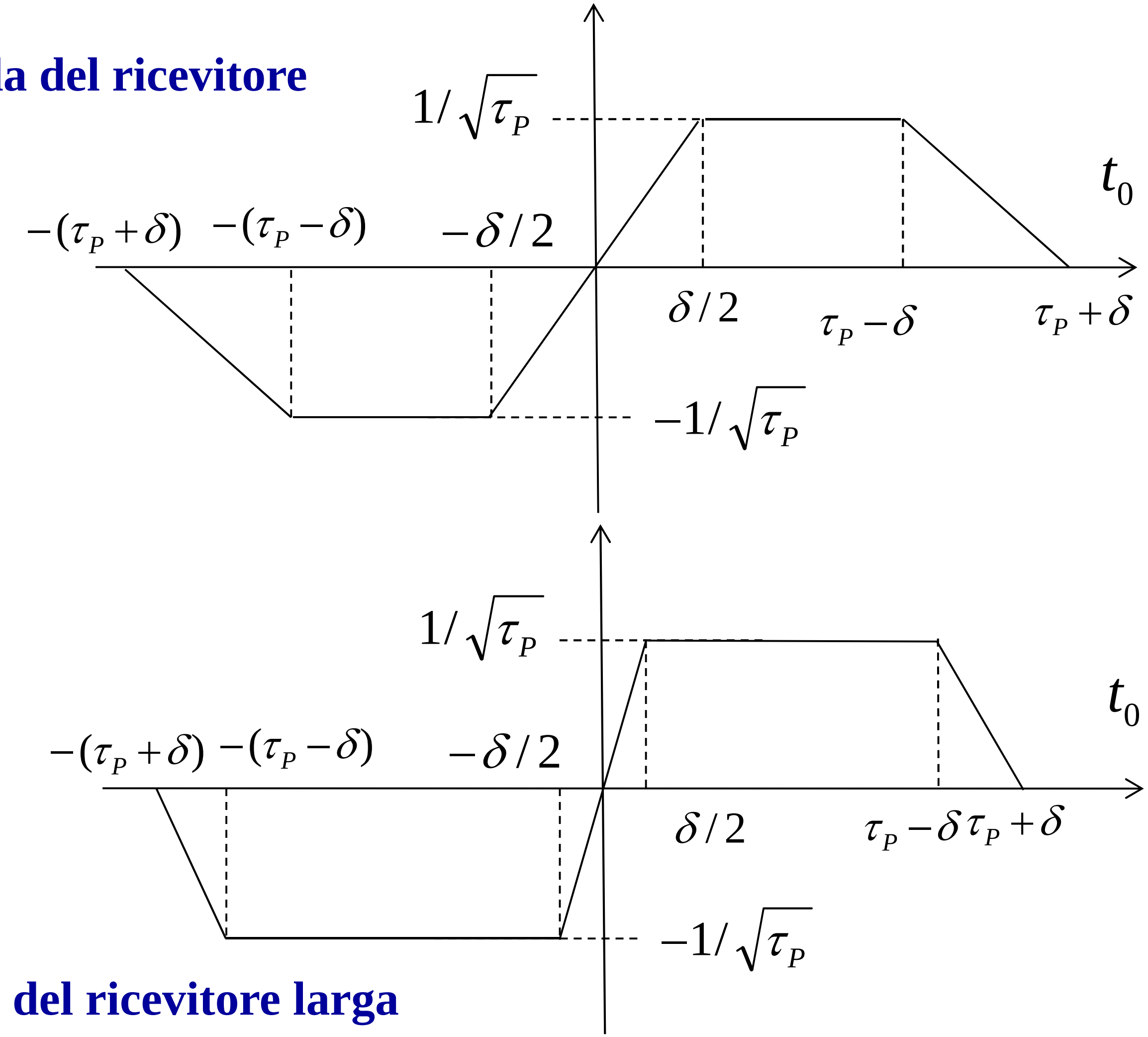
- **Zona di pull-in:** caratteristica non lineare (uscita non proporzionale all'errore nella stima di tempo di arrivo, ma con "segno" giusto)

Stima di distanza: CRB

$\delta \cong 1/B$ **Larghezza di banda del ricevitore**

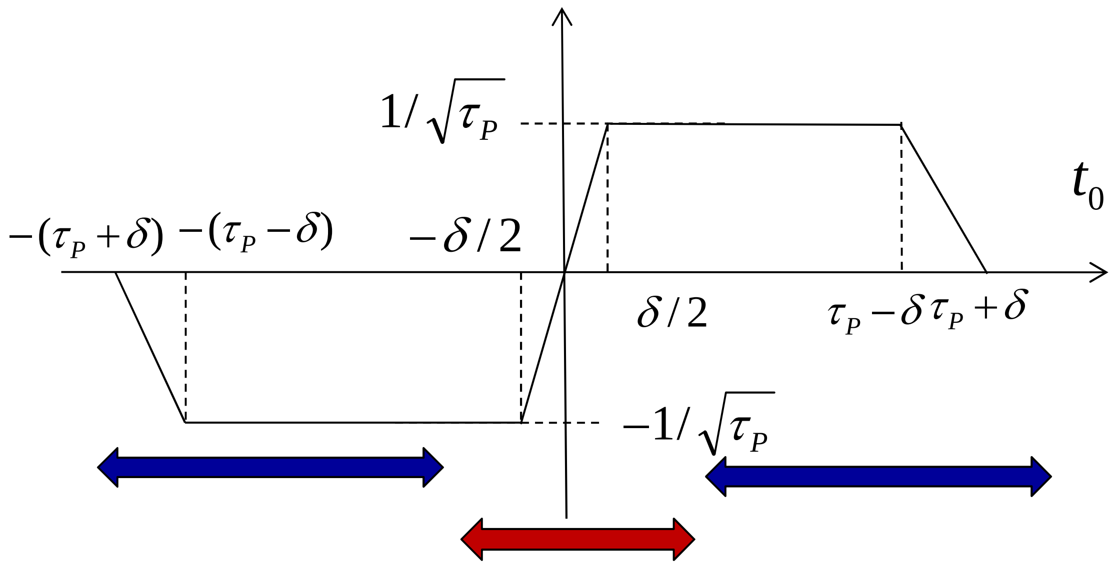
$$\beta^2 = \frac{\int \left|\dot{s}_0(t-t_0)\right|^2 dt}{\int \left|s_0(t-t_0)\right|^2 dt} = \frac{2(1/\delta\sqrt{\tau_P})^2\delta}{1} = \frac{2}{\delta\tau_P}$$

$$\sigma_{t_0}^2 \geq \frac{1}{2SNR\cdot\beta^2} = \frac{\delta\tau_P}{4SNR}$$



Accuratezza migliore per banda del ricevitore larga

Stima di distanza: discriminatore (II)



Caratteristica del discriminatore di tempo per banda larga:

- **Zona di lock-in:** caratteristica lineare estremamente piccola
  (= misuro ritardi in modo lineare solo in un range di ritardi molto piccolo)

- **Zona di pull-in:** caratteristica non lineare si accorcia leggermente, ma cambia poco

Stima di distanza: CRB & discriminatore

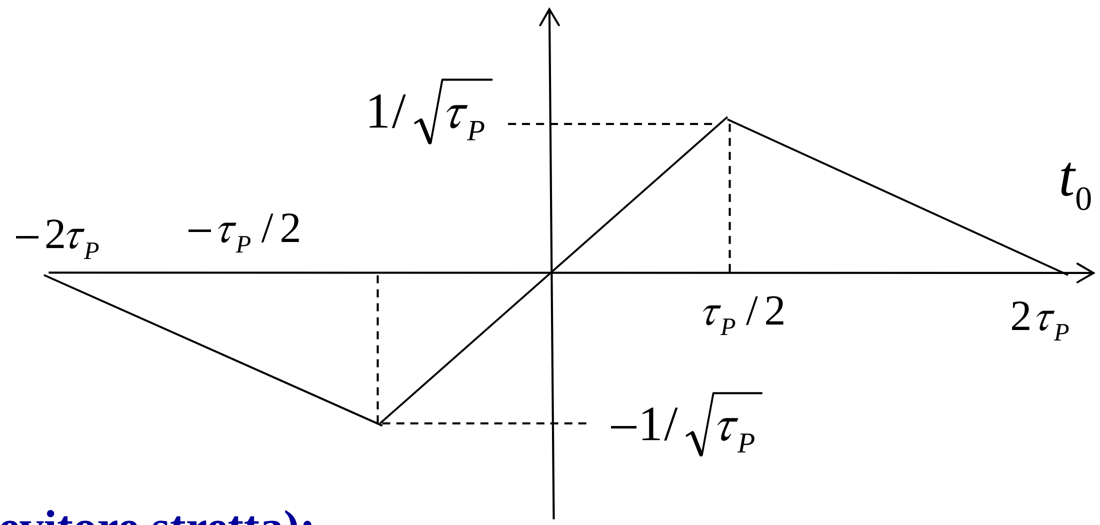
$$\delta \cong 1/B = \tau_P = 1/(1/\tau_P)$$

Larghezza di banda del ricevitore = banda dell'impulso rettangolare trasmesso

<u>Filtro di ricezione = Filtro adattato!</u>

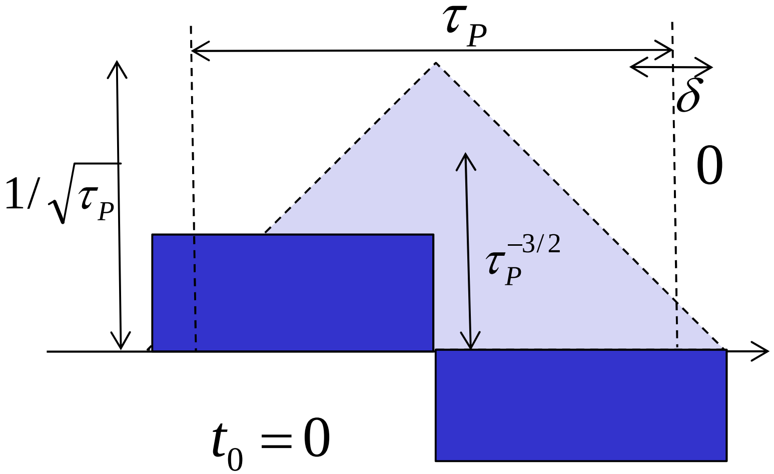
$$\beta^2 = \frac{2}{\delta \tau_P} = \frac{2}{\tau_P^2}$$

$$\sigma_{t_0}^2 \geq \frac{1}{2SNR \cdot \beta^2} = \frac{\tau_P^2}{4SNR}$$

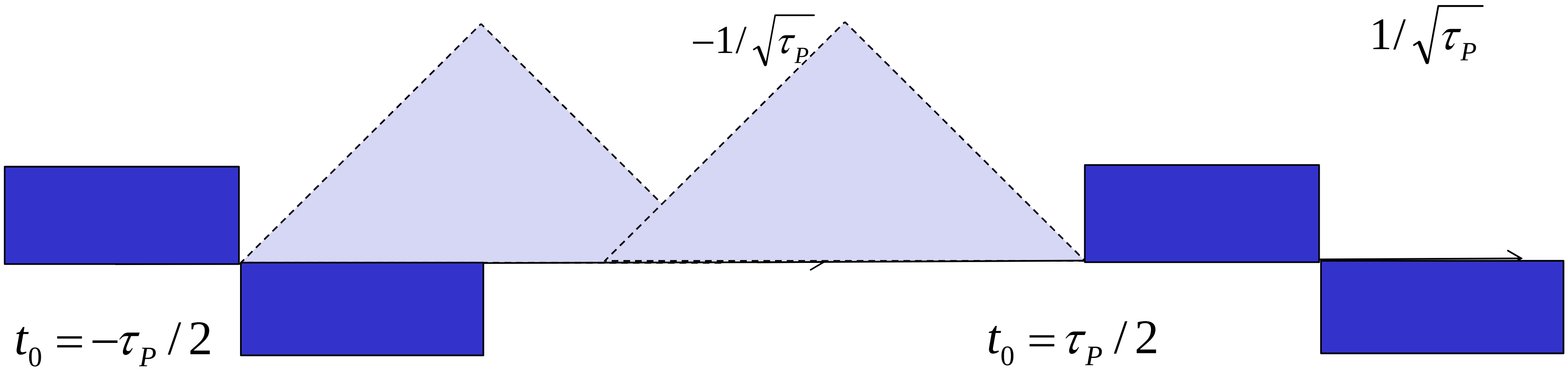


Con filtro adattato (banda del ricevitore stretta):
-Accuratezza peggiore
- zona di lock-in più larga
- anche zona di pull-in più larga

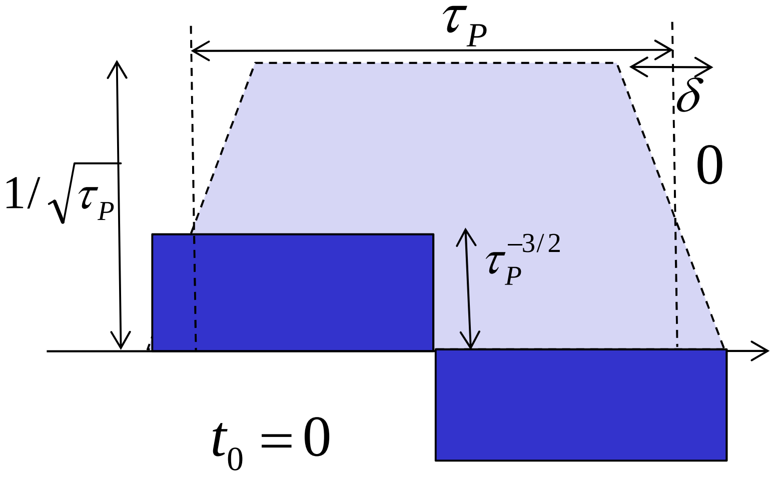
Stima di distanza: MLE & E-L gate



Early – Late Gate



Stima di distanza: Early-Late Gate



Early – Late Gate

Applicato a generico impulso trapezoidale

Early – Late Gate

Applicato a generico impulso trapezoidale

-Gates possono esser più larghi di durata, ma non più corti
- aggancio con gate larghi e inseguimento con gate stretti

Range tracking loop: Early-Late Gate

τ

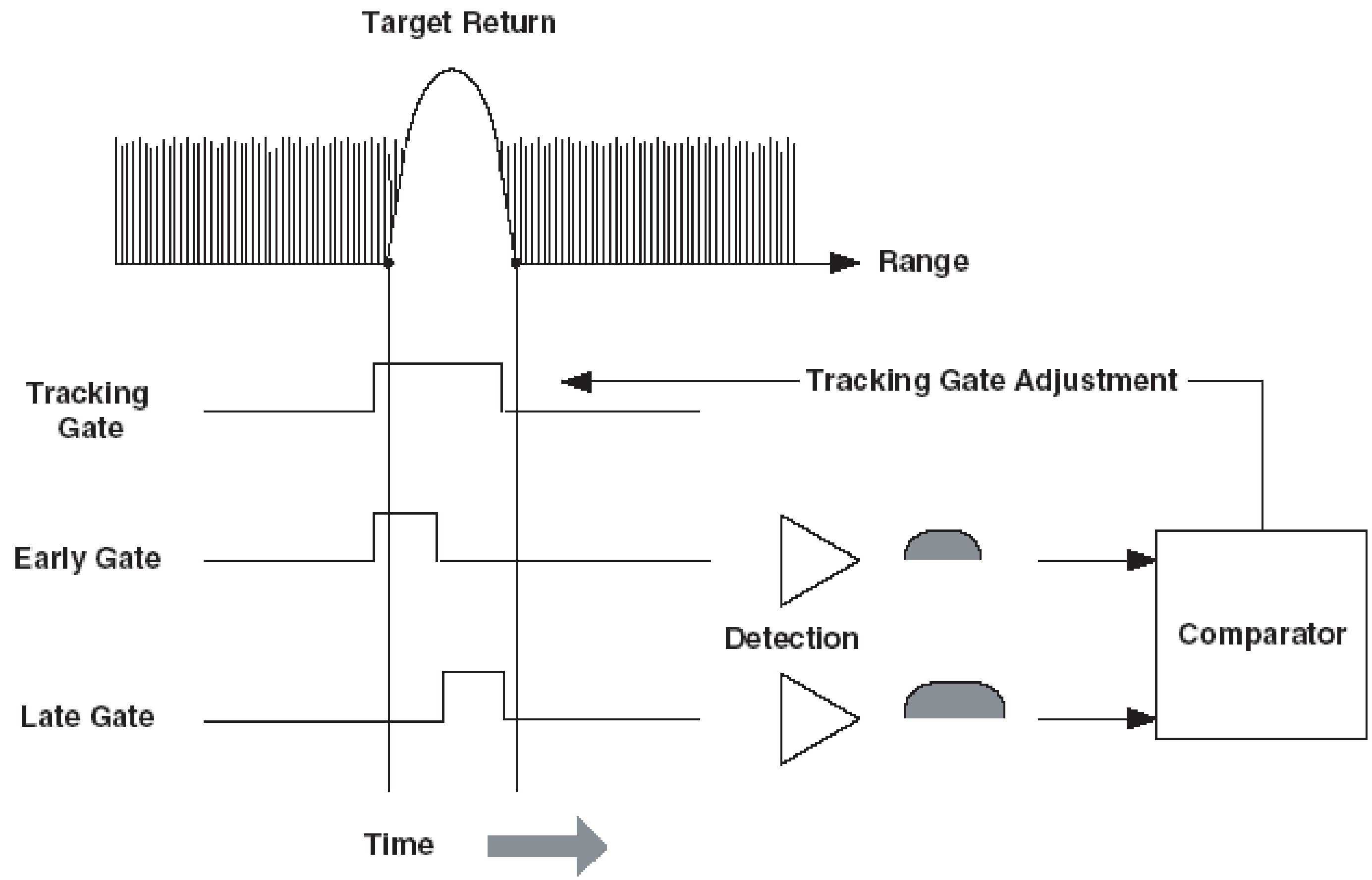
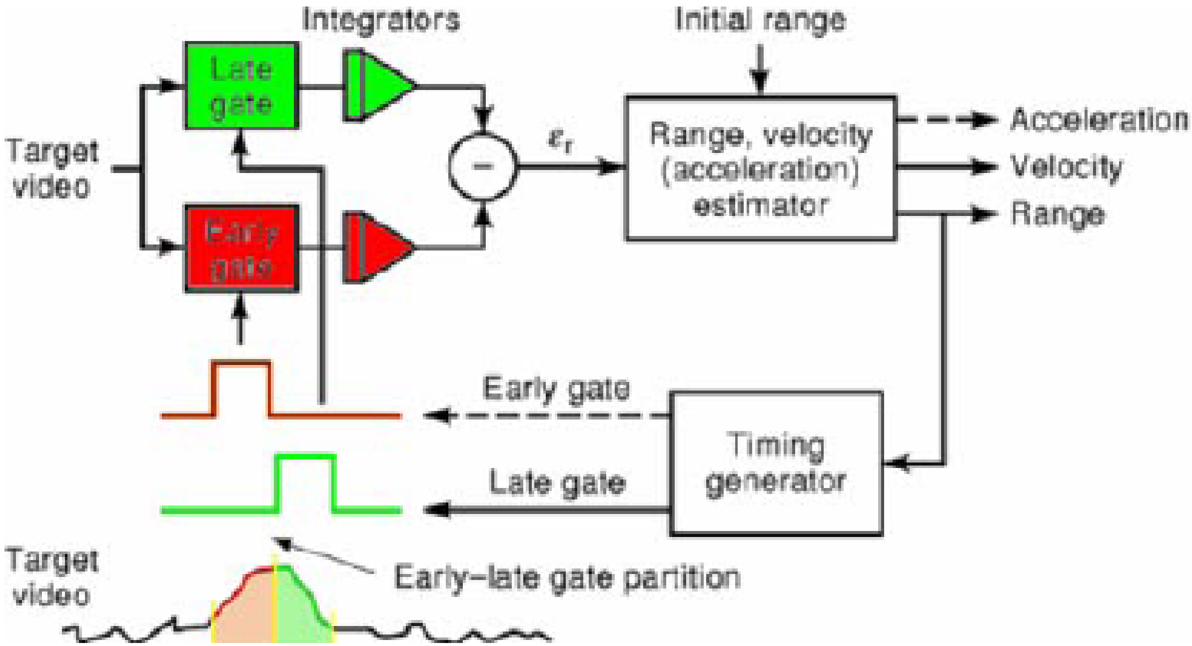


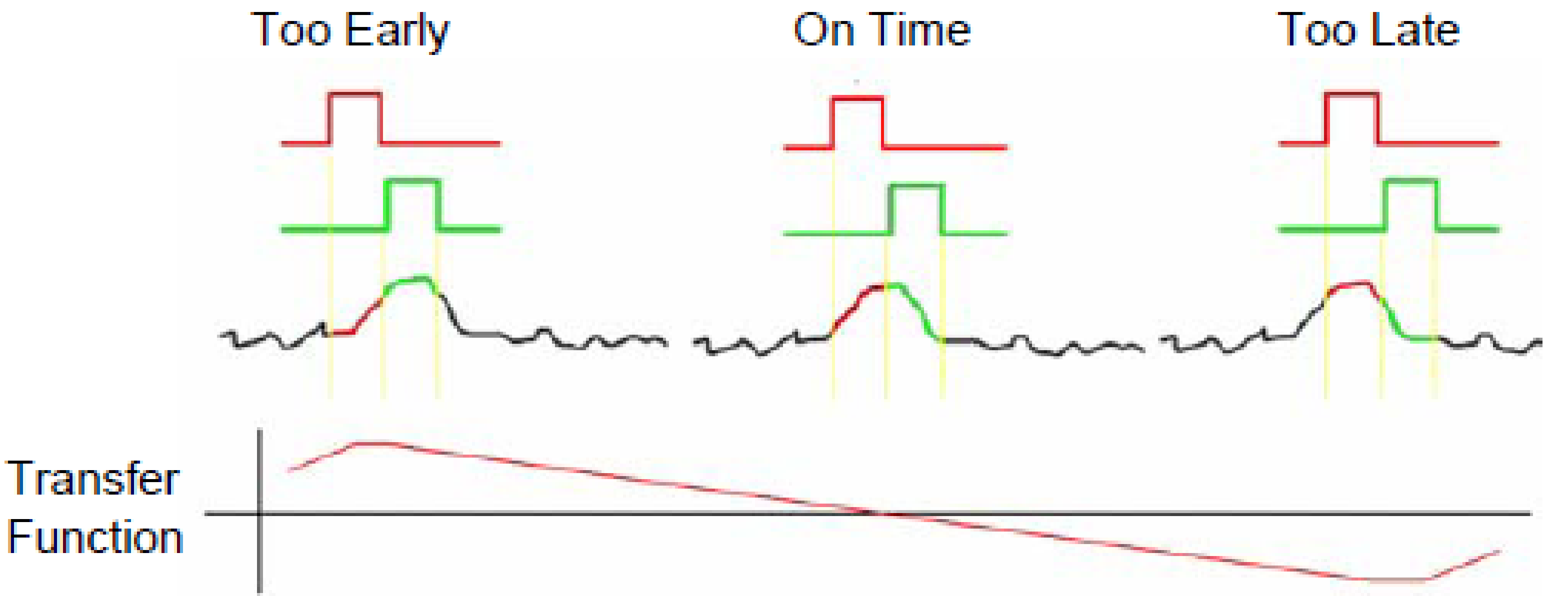
Figure 3.28 Range gate tracking.

Split Gate Tracker

- Consists of two sample-and-hold circuits triggered about one half of the pulse width apart. These are called the early and late gates
- The output of the S&H is the integral of the voltage in the echo pulse within each gate
- The difference between these voltages is equal to the range tracking error, and it is used to drive the tracking filter which in turn moves the gate timing to centre the gates ready for the next echo pulse



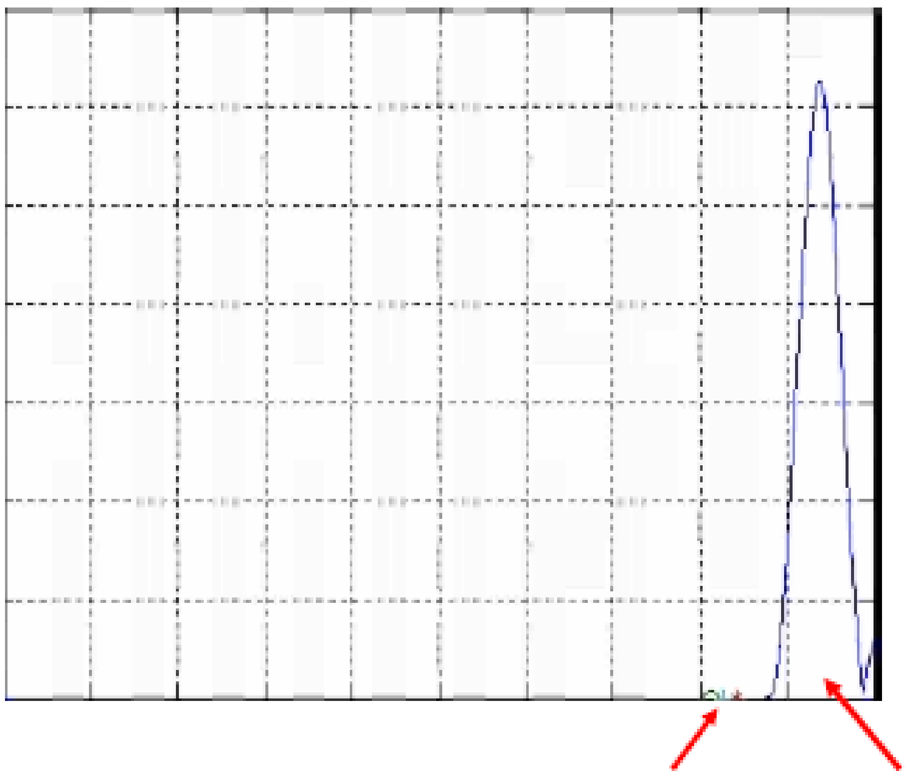
Split Gate Tracker: discriminatore (I)



- If the split gates are early with respect to the echo pulse, then a positive error is generated
- If they are aligned, then the error is zero
- If they are late, then the error is negative
- A continuous transfer function can be determined that maps the timing error into a voltage error

Split Gate Tracker: discriminatore (II)

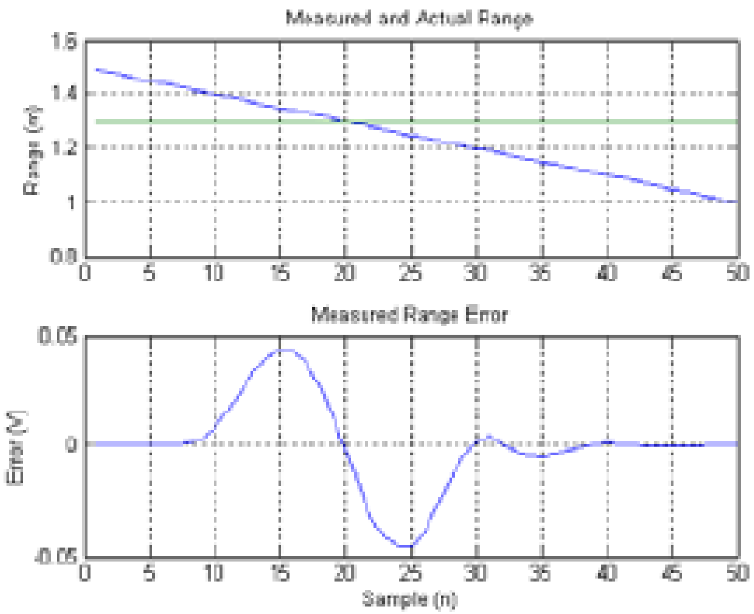
Echo Pulse Moving Through Gate



Split Gate Sampler

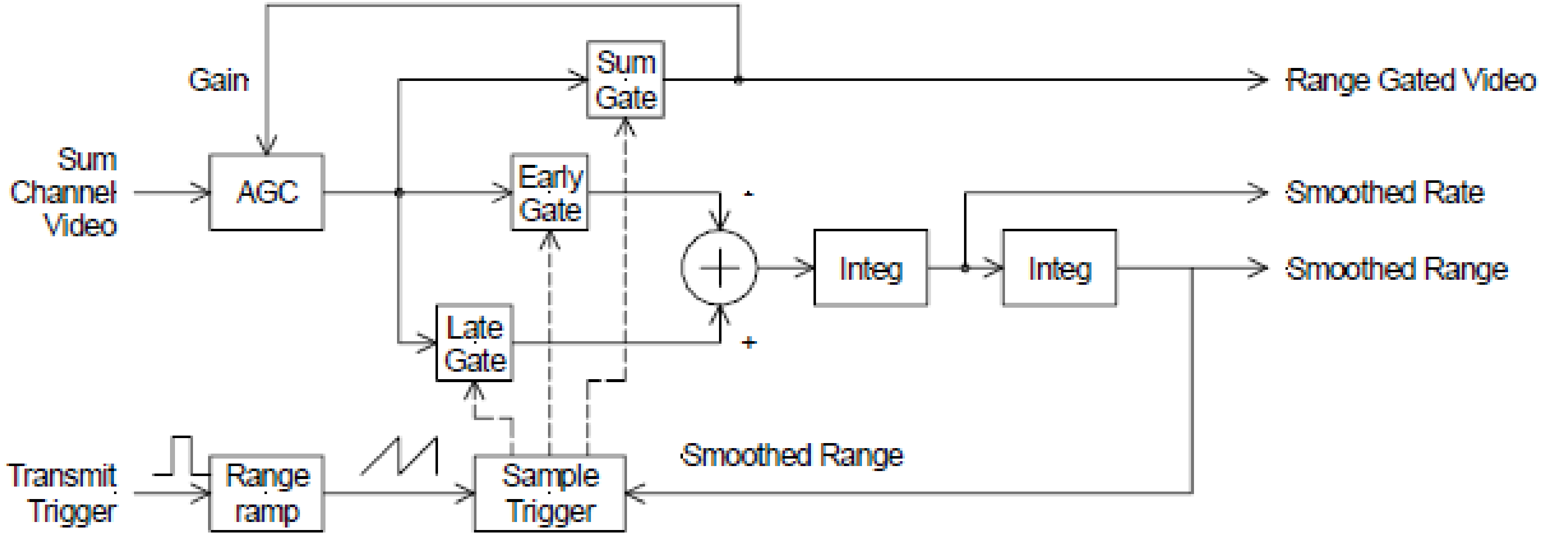
Echo Pulse

Measured Transfer Function

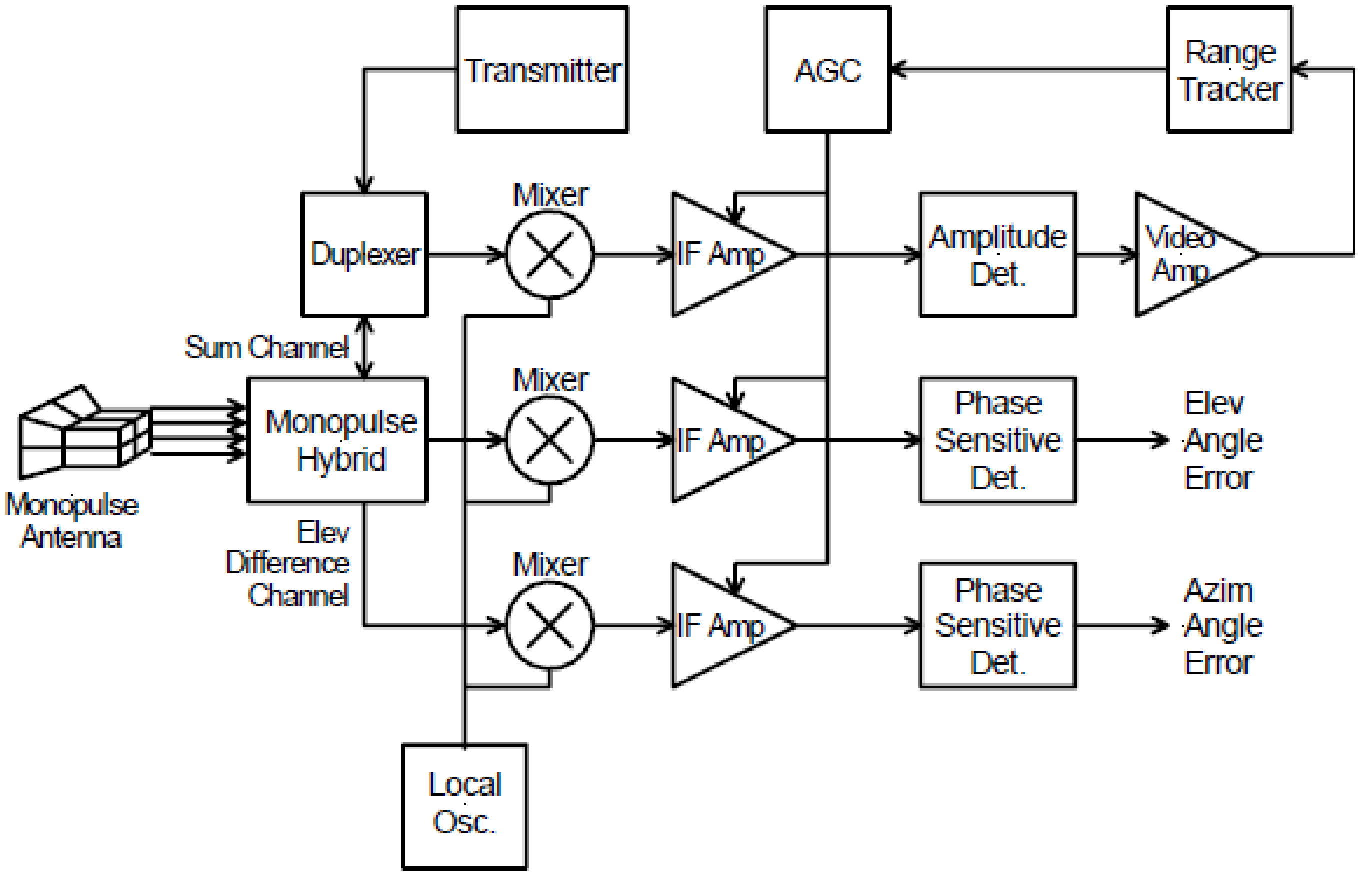


Tracking in analogico

- For constant loop bandwidth, the error signal V_L-V_E must be normalised otherwise the error will be a function of both the target RCS and the range
- Normalisation is achieved using an automatic gain control (AGC) loop that maintains a constant echo target echo amplitude
- The normalised range error drives a 2nd order tracker implemented by a pair of cascaded integrators

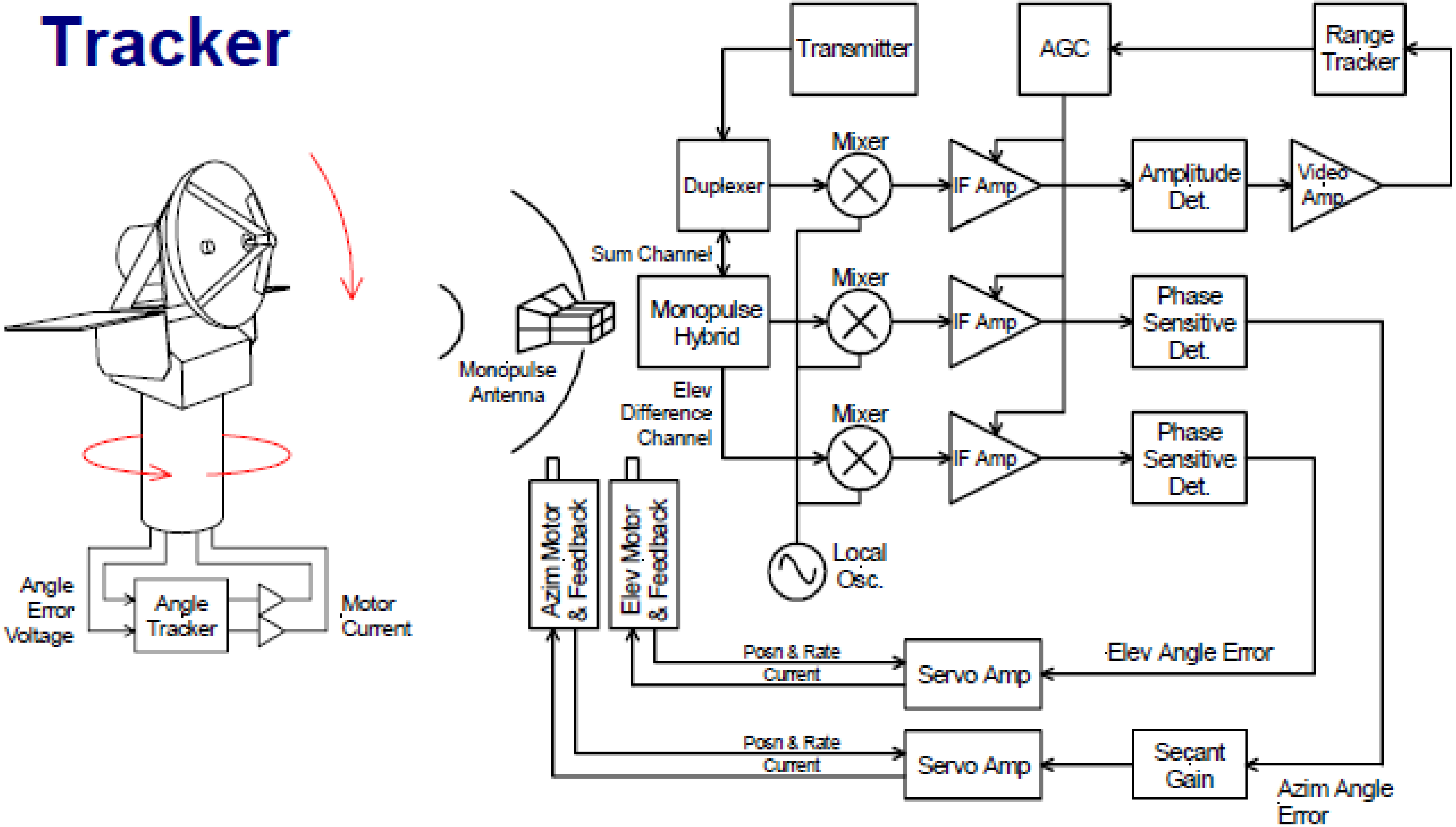


Tracking in analogico per angolo

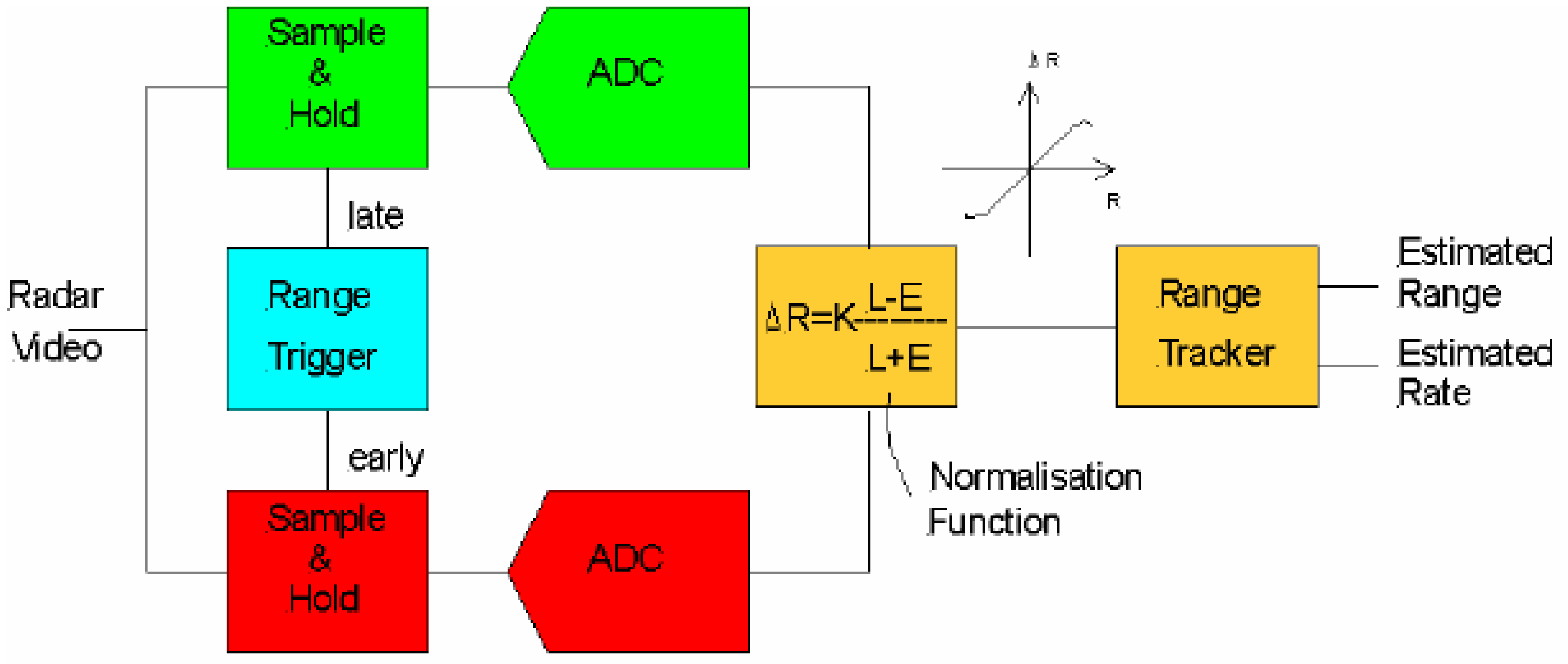


Tracking in analogico completo

Monopulse Tracker



Tracking in digitale



Il filtro di anello

- *Scopo del filtro di loop:*

  1. *ridurre il rumore, per produrre una stima accurata del parametro misurato*

     – *quindi si usa un filtro che effettui una media della quantità in ingresso: mantiene l'errore vero (valor medio), ma riduce la fluttuazione aleatoria di stima dovuta al rumore*
     – *filtro di tipo integratore/passabasso*

  2. *assicurare le risposte desiderate a segnali dinamici*

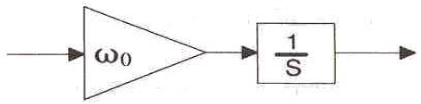
- *Parametri di progetto del loop:*

  1. *banda di rumore del sistema*
  2. *risposte al transitorio*
  3. *assicurare la stabilità*

Filtri di anello analogici

- *filtro analogico del I° ordine (integratore ideale)*

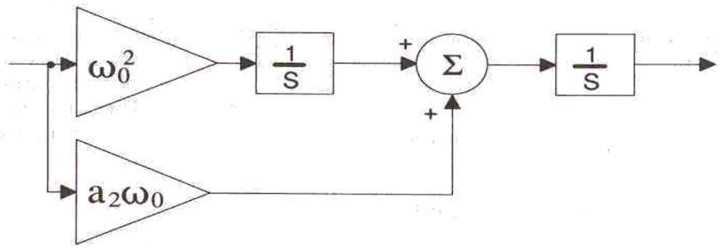
$$H(s) = \frac{\omega_0}{s}$$



(a)

- *filtro analogico del II° ordine*

$$H(s) = \frac{\omega_0^2}{s^2} + a_2 \frac{\omega_0}{s} = \omega_0 \frac{\omega_0 + a_2 s}{s^2}$$

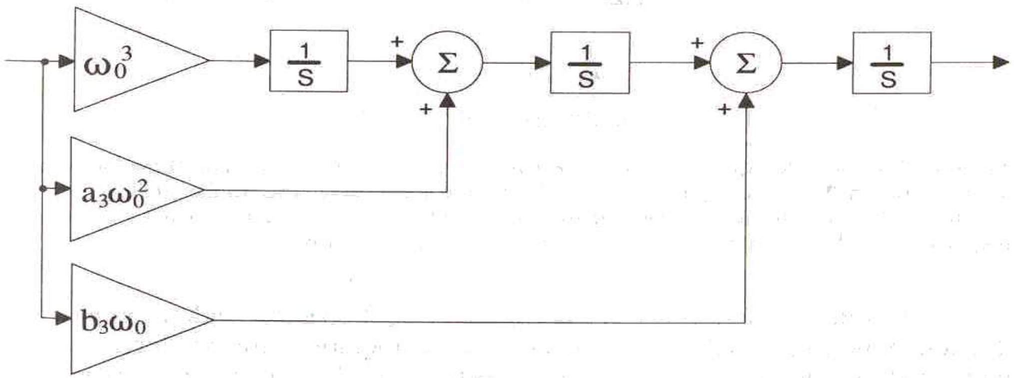


(b)

- *filtro analogico del III° ordine*

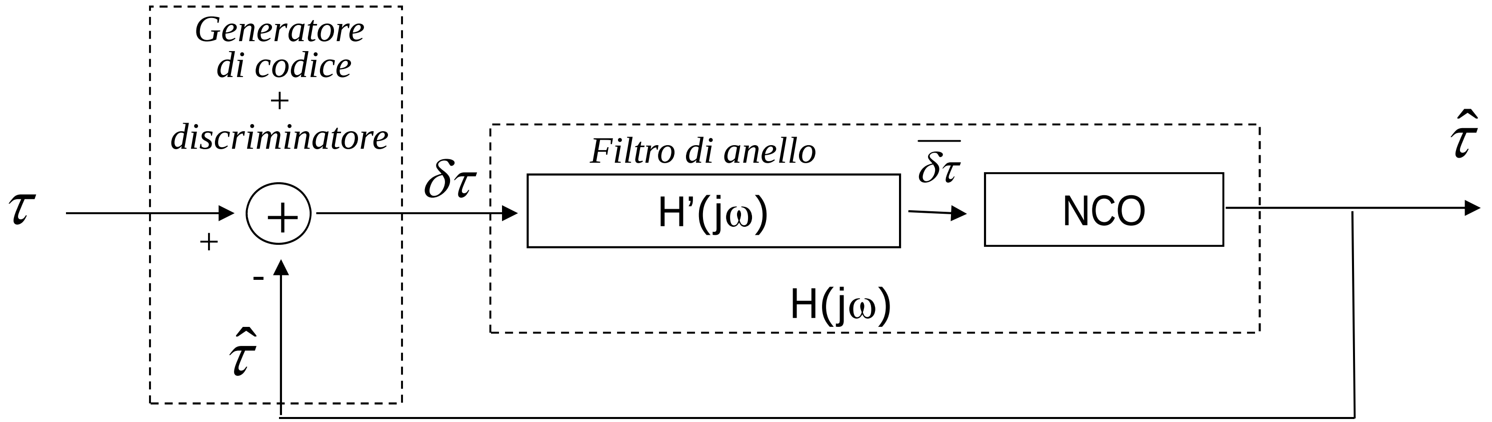
$$H(s) = \frac{\omega_0^3}{s^3} + a_3 \frac{\omega_0^2}{s^2} + b_3 \frac{\omega_0}{s} =$$

$$= \omega_0 \frac{\omega_0^2 + a_3\omega_0 s + b_3 s^2}{s^3}$$



(c)

Analisi del loop di codice (I)



Nota
- il NCO si comporta da integratore: trasforma il segnale di errore al suo ingresso in un effettivo ritardo

- quindi la funzione di trasferimento Dello NCO, può scriversi come

$$H_{NCO}(j\omega) = \frac{1}{j\omega}$$

Funzione di trasferimento del Loop

$$\hat{\mathrm{T}}(j\omega) = H(j\omega)\left[\mathrm{T}(j\omega) - \hat{\mathrm{T}}(j\omega)\right]$$

$$\hat{\mathrm{T}}(j\omega)\left[1 + H(j\omega)\right] = H(j\omega)\,\mathrm{T}(j\omega)$$

$$\hat{\mathrm{T}}(j\omega) = \frac{H(j\omega)}{1 + H(j\omega)}\mathrm{T}(j\omega)$$

Analisi del loop di codice (II)

Funzione di trasferimento dell'errore

$$\hat{T}(j\omega) = H(j\omega)\left[T(j\omega) - \hat{T}(j\omega)\right]$$

$$\hat{T}(j\omega)\left[1 + H(j\omega)\right] = H(j\omega)$$

$$E(j\omega) = T(j\omega) - \hat{T}(j\omega) = \left[1 - \frac{H(j\omega)}{1 + H(j\omega)}\right] T(j\omega)$$

$$\Rightarrow \quad H_e(j\omega) = \frac{E(j\omega)}{T(j\omega)} = \frac{1}{1 + H(j\omega)}$$

- Comportamento a regime: potenza di rumore termico in uscita dal loop (= varianza di errore nella stima del ritardo)

- Risposta al transitorio: uscita del loop per cambiamenti della distanza SV-RX

Comportamento a regime (I)

Varianza di errore $$\sigma_\tau^2 = P_e = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{N_0}{2}\left|\frac{H(j\omega)}{1+H(j\omega)}\right|^2 d\omega$$

- per loop del I° ordine:

$$\sigma_\tau^2 = \frac{1}{2\pi}\int_{-\infty}^{\aleph}\frac{N_0}{2}\left|\frac{\omega_0}{j\omega+\omega_0}\right|^2 d\omega = \frac{N_0}{4\pi}\int_{-\infty}^{\infty}\frac{\omega_0^2}{\omega^2+\omega_0^2}d\omega = \frac{\omega_0 N_0}{4\pi}\int_{-\infty}^{\infty}\frac{1}{1+x^2}dx =$$

$$= \frac{\omega_0 N_0}{4\pi}\int_{-\pi/2}^{\pi/2}\frac{1}{1+tg^2\phi}\frac{1}{\cos^2\phi}d\phi = \frac{\omega_0 N_0}{4\pi}\int_{-\pi/2}^{\pi/2}\frac{1}{1+tg^2\phi}\frac{1}{\cos^2\phi}d\phi = \frac{\omega_0 N_0}{4\pi}\int_{-\pi/2}^{\pi/2}\cos^2\phi\frac{1}{\cos^2\phi}d\phi = \frac{\omega_0 N_0}{4}$$

- Banda di rumore: $$B_n = \frac{\omega_0}{4}$$

Caratteristiche dei filtri di anello

Loop Order	Noise Bandwidth B_n (Hz)	Typical Filter Values	Steady-State Error	Characteristics
First	$\frac{\omega_0}{4}$	ω_0 $B_n = 0.25\omega_0$	$\frac{(dR/dt)}{\omega_0}$	Sensitive to velocity stress. Used in aided code loops and sometimes used in aided carrier loops. Unconditionally stable at all noise bandwidths.
Second	$\frac{\omega_0(1 + a_2^2)}{4a_2}$	ω_0^2 $a_2\omega_0 = 1.414\omega_0$ $B_n = 0.53\omega_0$	$\frac{(dR^2/dt^2)}{\omega_0^2}$	Sensitive to acceleration stress. Used in aided and unaided carrier loops. Unconditionally stable at all noise bandwidths.
Third	$\frac{\omega_0(a_3b_3^2 + a_3^2 - b_3)}{4(a_3b_3 - 1)}$	ω_0^3 $a_3\omega_0^2 = 1.1\omega_0^2$ $b_3\omega_0 = 2.4\omega_0$ $B_n = 0.7845\omega_0$	$\frac{(dR^3/dt^3)}{\omega_0^3}$	Sensitive to jerk stress. Used in unaided carrier loops. Remains stable at $B_n \leq 18$ Hz.

Accuratezza del tracking: range (I)

- For a matched system, the range noise variance on the output of a split gate tracker for $1<\beta\tau<2$ and $\tau<\tau_g<2\tau$ can be determined as follows

$$\sigma_r = \frac{\tau}{2.5\sqrt{2S/N}}$$

Where τ - pulse width (m)

τ_g – gate width (m)

S/N – signal to noise ratio

Accuratezza del tracking: range (II)

- Measurements made over *n* sample periods combine in the tracking filter to provide an output whose output RMS noise is reduced by $1/\sqrt{n}$
- In terms of the equivalent noise bandwidth β_n of filter

$$n = \frac{f_r}{2\beta_n} = \frac{1}{VRR}$$

where f_r – Pulse repetition frequency (Hz)

β_n - Bandwidth of tracking filter (Hz)

VRR – Variance reduction ratio

- For the split gate tracker, the RMS noise output after filtering will be

$$\sigma_r = \frac{\tau}{2.5\sqrt{(S/N)(f_r/\beta_n)}}$$

Accuratezza del tracking: angolo (I)

Singolo impulso

- Lobe Switching
- Conical Scan $k_{opt} = 1.4$
- Monopulse $1.5 < k < 2.3$

$$\sigma_t = \frac{\theta_{3dB}}{k\sqrt{2(S/N)}}$$

- Improvement in angle measurement accuracy in thermal noise is limited only by the measurement S/N

k dipende dalla derivata del fascio!
(analogamente a stima di distanza: derivata di forma d’onda)

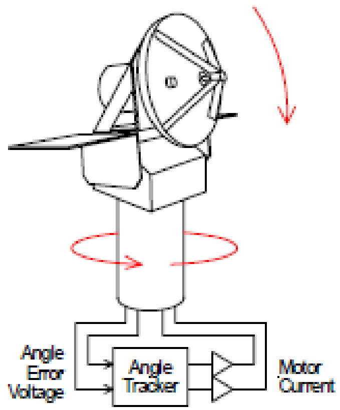
Accuratezza del tracking: angolo (II)

Molti impulsi, usati dal loop di tracking

- Extremely accurate angle measurements can be achieved by null steering the antenna
- For a point target, the theoretical improvement in accuracy in thermal noise is limited only by the signal to noise ratio of the measurement

$$\sigma_t = \frac{\theta_{3dB}}{k\sqrt{2(S/N)f_r/\beta_n}}$$

where: θ_{3dB} – Antenna Beamwidth (deg)
k – Constant dependant on the tracking type
S/N – Signal to noise ratio
f_r – Pulse repetition frequency (Hz)
β_n – Angle servo bandwidth (Hz)



N° impulsi equivalenti = f_r/β_n

Risposta al transitorio (I)

- variazioni della distanza (cioè del tempo di arrivo $\tau(t)$):
  - istantanee ⇒ gradino
  - lineari (allontanamento o avvicinamento a velocità costante) ⇒ rampa
  - quadratiche (allontanamento o avvicinamento a velocità costante) ⇒ rampa quadratica

- errore nella stima di distanza all'uscita del loop:
  - qual è l'errore nella stima $\hat{\tau}(t)$ del tempo di arrivo $\tau(t)$?
  - quanto la stima $\hat{\tau}(t)$ del loop è simile al tempo di arrivo vero $\tau(t)$?
  - cioè quanto è grande l'errore all'uscita del loop $e(t) = \hat{\tau}(t) - \tau(t)$?

Risposta al transitorio (II)

• variazioni della distanza dal satellite (cioè del tempo di arrivo $\tau(t)$):

- gradino $\Rightarrow$ $T^{(0)}(s) = \dfrac{R^{(0)}}{s}$ $\qquad R^{(0)} = R_0$
- rampa $\Rightarrow$ $T^{(1)}(s) = \dfrac{R^{(1)}}{s^2}$ $\qquad R^{(1)} = \dfrac{\partial R(t)}{\partial t} = V_0$
- rampa quadratica $\Rightarrow$ $T^{(2)}(s) = \dfrac{R^{(2)}}{s^3}$ $\qquad R^{(2)} = \dfrac{\partial^2 R(t)}{\partial t^2} = A_0$

errore nella stima di distanza all'uscita del loop:

$$E(s) = H_e(s)\,T(s) = \frac{1}{1+H(s)}\,T(s) \quad \Rightarrow \quad E^{(k)}(s) = \frac{1}{1+H(s)}\,\frac{R^{(k)}}{s^{k+1}}$$

$$E_{(n)}^{(k)}(s) = \frac{1}{1+\dfrac{\omega_0^n}{s^n}\left[1+N^{(n-1)}(s)\right]}\,\frac{R^{(k)}}{s^{k+1}} = R^{(k)}\,\frac{s^{n-k-1}}{\omega_0^n + \omega_0^n N^{(n-1)}(s) + s^n}$$

Risposta al transitorio (III)

- errore a regime nella stima di distanza all'uscita del loop:

$$\lim_{t\to\infty} e_{(n)}^{(k)}(t) = \left[s\,E_{(n)}^{(k)}(s)\right]_{s=0} = R^{(k)}\left[\frac{s^{n-k}}{\omega_0^n + \omega_0^n N^{(n-1)}(s) + s^n}\right]_{s=0} = \frac{R^{(k)}}{\omega_0^n}\left[s^{n-k}\right]_{s=0}$$

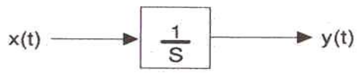
$$\lim_{t\to\infty} e_{(n)}^{(k)}(t) = \begin{cases} 0 & n > k \\ \dfrac{R^{(k)}}{\omega_0^n} & n = k \\ \infty & n < k \end{cases}$$

Caratteristiche dei filtri di anello

Loop Order	Noise Bandwidth B_n (Hz)	Typical Filter Values	Steady-State Error	Characteristics
First	$\frac{\omega_0}{4}$	ω_0 $B_n = 0.25\omega_0$	$\frac{(dR/dt)}{\omega_0}$	Sensitive to velocity stress. Used in aided code loops and sometimes used in aided carrier loops. Unconditionally stable at all noise bandwidths.
Second	$\frac{\omega_0(1 + a_2^2)}{4a_2}$	ω_0^2 $a_2\omega_0 = 1.414\omega_0$ $B_n = 0.53\omega_0$	$\frac{(dR^2/dt^2)}{\omega_0^2}$	Sensitive to acceleration stress. Used in aided and unaided carrier loops. Unconditionally stable at all noise bandwidths.
Third	$\frac{\omega_0(a_3b_3^2 + a_3^2 - b_3)}{4(a_3b_3 - 1)}$	ω_0^3 $a_3\omega_0^2 = 1.1\omega_0^2$ $b_3\omega_0 = 2.4\omega_0$ $B_n = 0.7845\omega_0$	$\frac{(dR^3/dt^3)}{\omega_0^3}$	Sensitive to jerk stress. Used in unaided carrier loops. Remains stable at $B_n \le 18$ Hz.

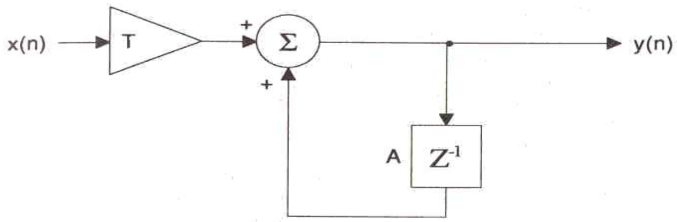
Conversione del filtro analogico in digitale

- *filtro analogico*



(a)

- *filtro digitale*



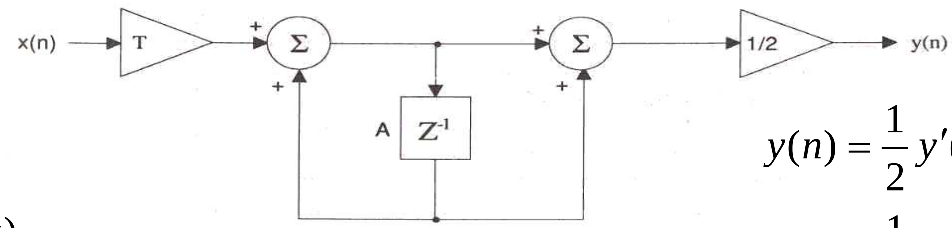
(b)

$$y(n) = T \cdot x(n) + y(n-1)$$

$$Y(z) = T \cdot X(z) + z^{-1}Y(z)$$

$$Y(z) = \frac{T}{1-z^{-1}} \cdot X(z)$$

- *filtro digitale con bilineare*



(c)

$$y'(n) = T \cdot x(n) + y(n-1)$$

$$Y'(z) = T \cdot X(z) + z^{-1}Y'(z)$$

$$Y'(z) = \frac{T}{1-z^{-1}} \cdot X(z)$$

$$y(n) = \frac{1}{2}y'(n) + \frac{1}{2}y'(n-1)$$

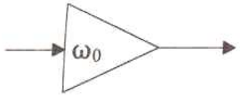
$$Y(z) = \frac{1}{2}Y'(z) + \frac{1}{2}z^{-1}Y'(z)$$

$$Y(z) = \frac{1}{2}(1+z^{-1})Y'(z)$$

$$Y(z) = \frac{T}{2}\frac{1+z^{-1}}{1-z^{-1}} \cdot X(z)$$

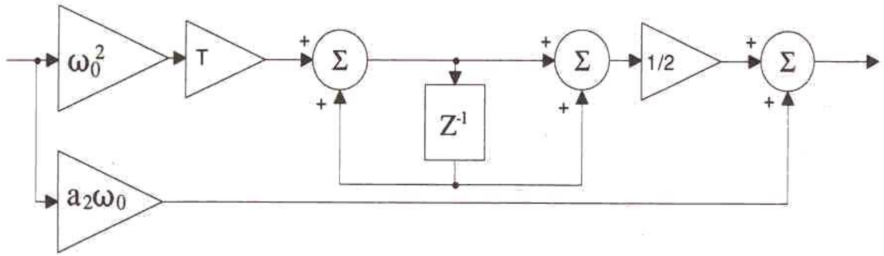
Filtri di anello digitali

(a)



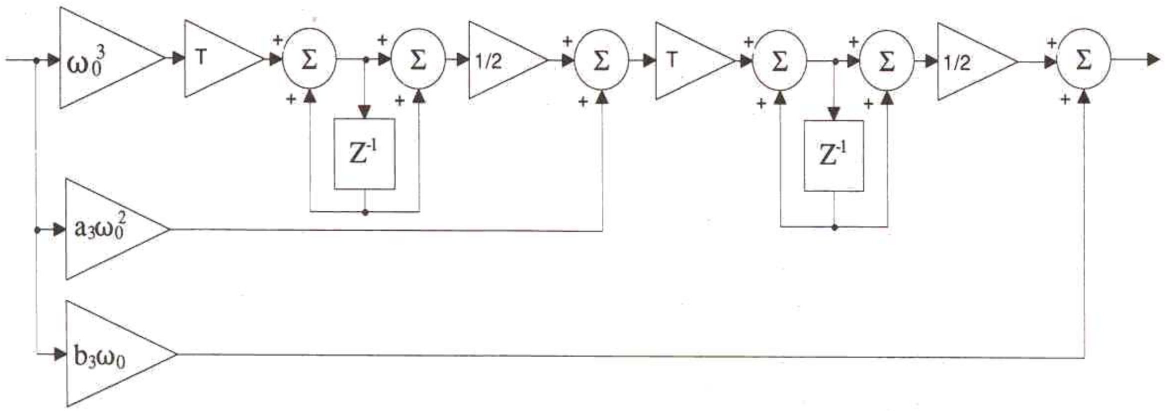
- *filtro digitale del I° ordine*

(b)



- *filtro digitale del II° ordine*

(c)



- *filtro digitale del III° ordine*

filtri di loop digitali*, escluso l'ultimo integratore (NCO)*