



# Prestazioni di rivelazione su impulso singolo Target fluttuante

$$A_0 e^{j\phi_0} e^{i \frac{2\pi}{\lambda} 2R_0} e^{j\phi_0} + A_1 e^{j\phi_0} e^{i \frac{2\pi}{\lambda} 2R_1} e^{j\phi_1}$$

$$\sum_n A_n e^{i\phi_0} \frac{e^{-j\frac{2\pi}{\lambda} 2R_n}}{e^{-j\frac{2\pi}{\lambda} 2R_0}} e^{j\phi_n}$$

$$R_n = R_0 + \Delta_n \quad \Delta_0 = 0$$

$$\sum_n A_n e^{i\phi_0} e^{j\frac{4\pi}{\lambda} R_0} e^{j\frac{4\pi}{\lambda} \Delta_n} e^{j\phi_n}$$

$$e^{j\phi_0} e^{j\frac{4\pi}{\lambda} R_0} \left( \sum_n A_n e^{j\phi_n} e^{j\frac{4\pi}{\lambda} \Delta_n} \right)$$

~~$\sum_n A_n e^{j\phi_n} e^{j\frac{4\pi}{\lambda} \Delta_n}$~~

$$\left| \sum_n A_n e^{j \rho_n} e^{j \frac{4\pi}{\lambda} \Delta_n} \right|$$

$$\underline{A_n = A_0}$$

$$\underline{\rho_n = \rho_0}$$

$$RCS = \underline{10 \text{ m}^2}$$

$$A_0 \cdot \left| \sum_n e^{j \frac{4\pi}{\lambda} \Delta_n} \right|$$

$$N=2 \quad (n=0, 1)$$


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$$A_0 \left( 1 + e^{j \frac{4\pi}{\lambda} \Delta_1} \right)$$

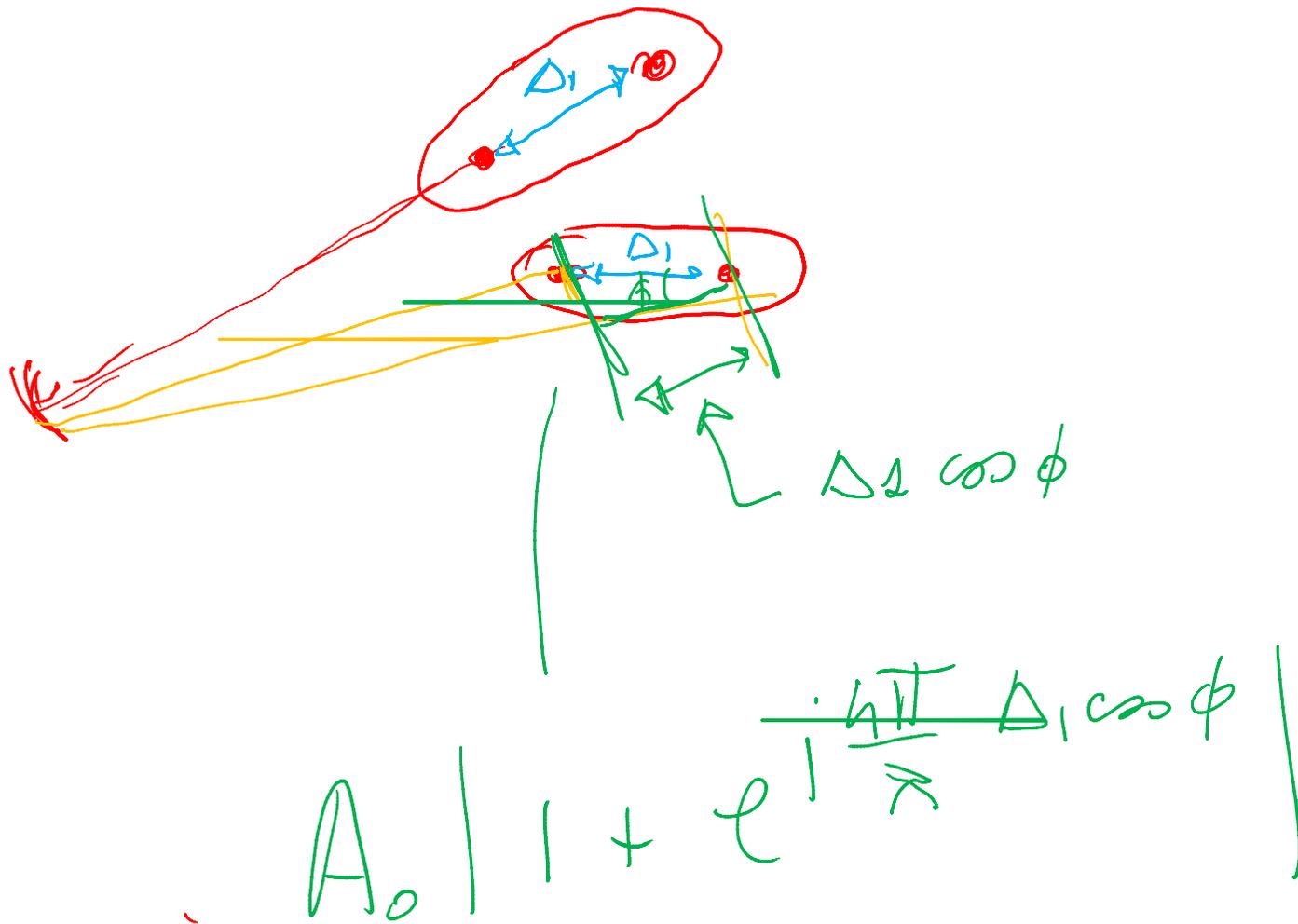
$$\left\{ \begin{array}{l} 2A_0 \\ \emptyset \\ \emptyset \\ 2A_0 \end{array} \right.$$

$$\Delta_1 = \frac{\lambda}{2}$$

$$\Delta_1 = \frac{\lambda}{4}$$

$$\Delta_1 = \frac{7}{4} \lambda$$

$$= 6 \lambda$$



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## **Sistemi Radar**

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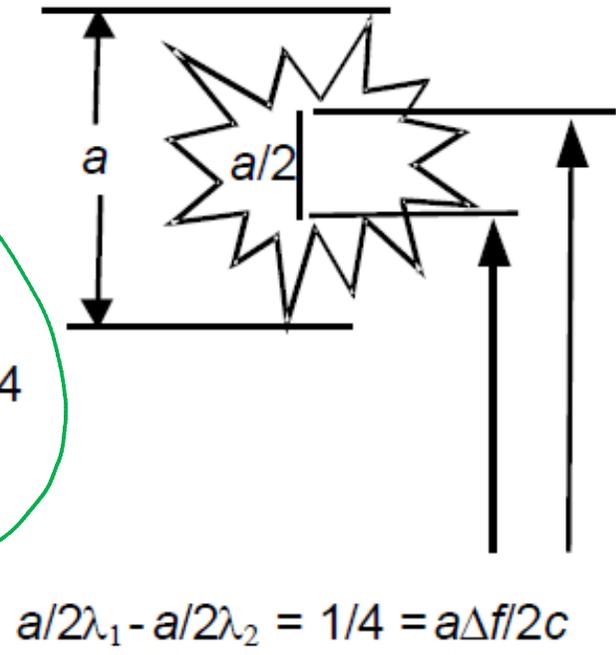
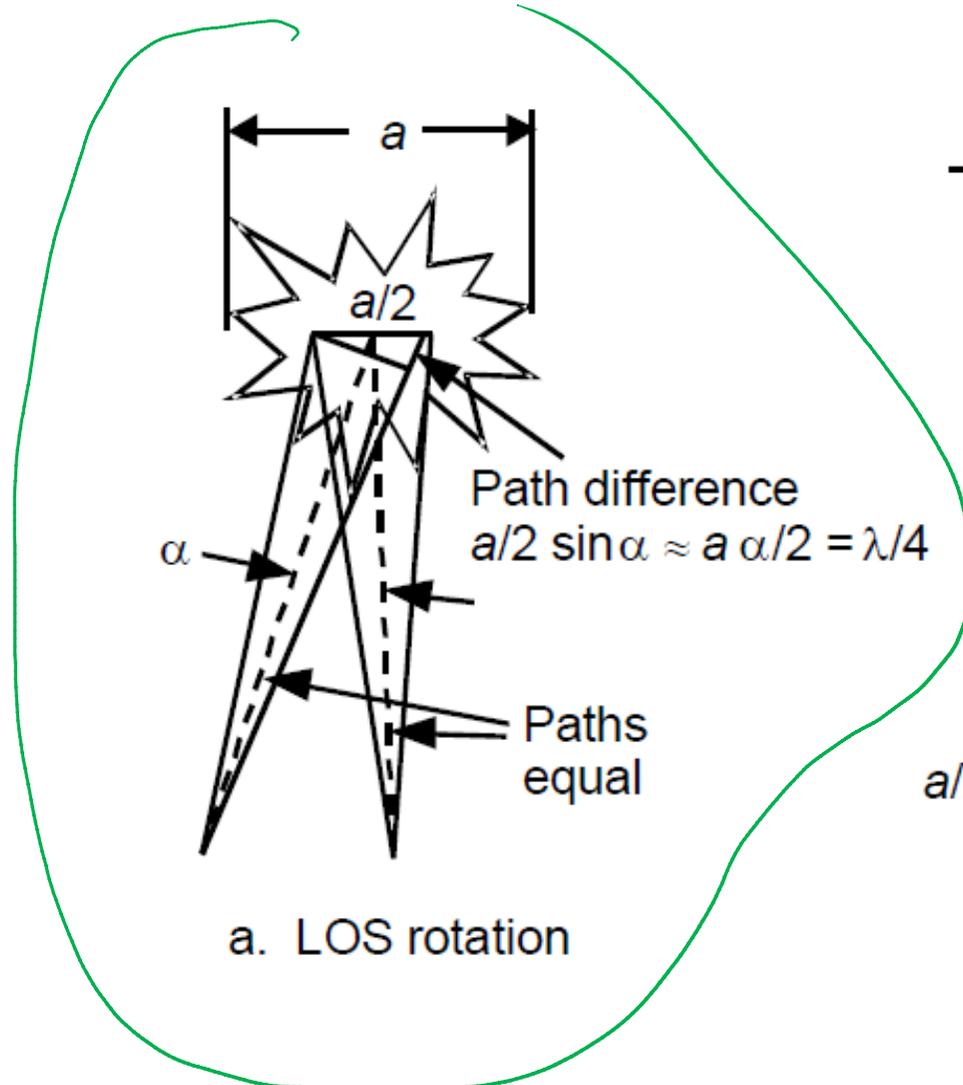
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## **Sistemi Radar**

# Bersagli fluttuanti (I)

- RCS Is the Ratio of Power Density Scattered by the Target in the Direction of the Radar, to the Power Density Incident on the Target
- RCS Generally Related To Target Size, but
  - Flat or Cylindrical Surfaces Produce Large Specular Returns
  - Corner Reflectors Produce Large Monostatic RCS
  - Stealth Techniques (Shaping, RAM, and Non-metallic Materials Can Reduce RCS)
- RCS of Large Complex Objects With Dimension  $a$  Can
  - Vary With Viewing Aspect (LOS)  $\geq \lambda / 2a$
  - Vary With Radar Frequency Changes  $\geq c / 2a$
- RCS of Targets Smaller Than  $\lambda / \pi$ 
  - Does Not Vary With LOS or Frequency Changes
  - Is Said to Be in the Raleigh Region
  - Varies With  $\lambda^{-4}$

# Bersagli fluttuanti (II)



# Bersagli fluttuanti (III)

## Requirements for RCS Decorrelation

Parameter	Value
Target Dimension (a)	2 m
Radar Frequency	9.5 GHz (X Band)
Wavelength ( $\lambda$ )	0.0316 m
LOS Rotation Required	8 mR ( $\approx 0.5$ deg)
Target Rotation Rate	5 deg/s
Decorrelation Time	100 ms
Frequency Change Required	75 MHz

## Rotating Radar Example

Parameter	Value
PRF	200 Hz
Time Between Pulses	5 ms
Antenna Rotation Period	10 s
Antenna Beamwidth	1 deg
Time on Target	30 ms

- For Target in Pervious Example
  - LOS Rotation Provides Dwell-to-dwell RCS Decorrelation (Swerling 1 and 3)
  - Frequency Agility Needed to Provide Pulse-to-pulse RCS Decorrelation (Swerling 2 and 4)

# Rivelazione per Bersagli fluttuanti (I)

Probabilità di rivelazione “istantanea” dipende dal valore di RCS “ $\sigma=|a|^2$ ” che si sperimenta in quello specifico momento. E’ di fatto una probabilità condizionata al valore di “ $\sigma=|a|^2$ ”:

$$P_d(\sigma) = \text{Prob}\{z > T | \sigma; H_1\} = \int_{T/\sigma_d}^{\infty} 2t \exp\left\{-t^2 - \frac{\sigma \cdot |\chi(\tau, \nu)|^2}{\sigma_d^2}\right\} \cdot I_0\left[\frac{2t \sqrt{\sigma} \cdot |\chi(\tau, \nu)|}{\sigma_d}\right] dt$$

Probabilità di rivelazione globale si ottiene rimuovendo il condizionamento per saturazione tramite la DDP della variabile aleatoria  $\sigma$ :

$$P_d = \int_0^{\infty} P_d(\sigma) p_{\sigma}(\sigma) d\sigma$$

In generale risulta  $P_d \neq P_d(\sigma_{av})$

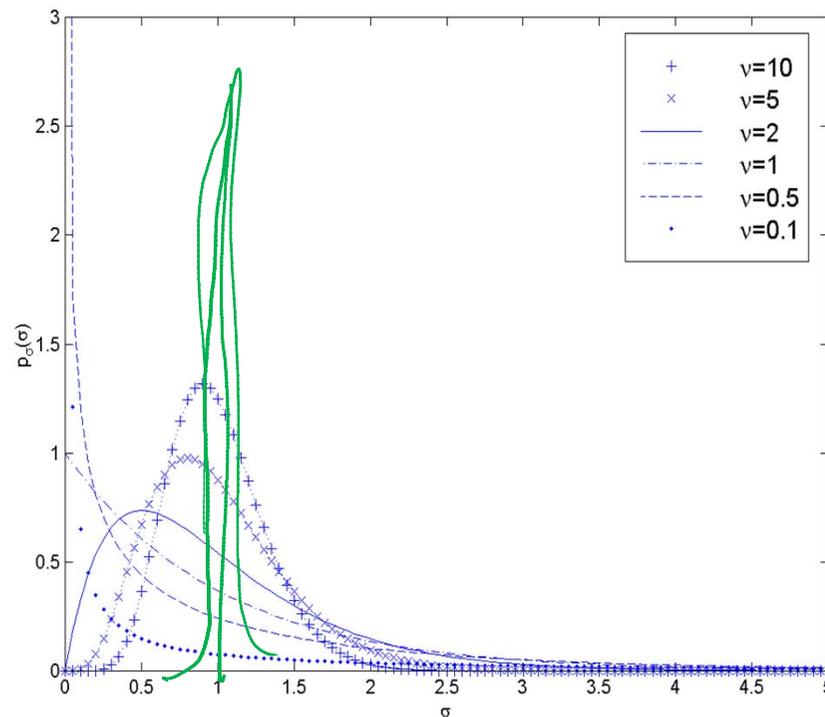
Solo se la DDP di  $\sigma$  è un Dirac (Bersaglio fisso) si ha:  $P_d = P_d(\sigma_{av})$

# Modello generale per la fluttuazione di RCS

Il modello di **RCS model** deve definire la DDP:

Un modello generale si ottiene usando la DDP gamma, con fattore di forma  $m$ :

$$p_{\sigma}(\sigma) = \frac{1}{(m-1)!} \left( \frac{m}{\sigma_{av}} \right)^m \sigma^{m-1} \exp \left[ -\frac{m}{\sigma_{av}} \sigma \right]$$



$m=1$  Swerling I-II

$m=2$  Swerling III-IV

$m \rightarrow \infty$  Swerling 0 (oppure 5)

$$\langle \sigma^n \rangle = \left( \frac{\sigma_{av}}{m} \right)^n \frac{(m+n-1)!}{(n-1)!}$$

$$\frac{\text{var } \sigma}{\sigma_{av}^2} = \frac{1}{m}$$

# Rivelazione per Bersagli fluttuanti (II)

$$P_d = \int_0^{\infty} P_d(\sigma) p_{\sigma}(\sigma) d\sigma = \int_0^{\infty} P_d(\sigma) \frac{1}{(m-1)! \left(\frac{\sigma}{\sigma_{av}}\right)^m} \sigma^{m-1} \exp\left[-\frac{m}{\sigma_{av}} \sigma\right] d\sigma$$

$$P_d = \frac{e^{-\frac{T^2/\sigma_d^2}{1+SNR/m}}}{(1+SNR/m)^{m-1}} \sum_{n=0}^{m-1} \binom{m-1}{n} \left(\frac{SNR}{m}\right)^n \sum_{k=0}^n \frac{1}{k!} \left(\frac{T^2/\sigma_d^2}{1+SNR/m}\right)^k$$

$m=1$ : Swerling I-II

$$P_d = e^{-\frac{T^2}{\sigma_d^2 (1+SNR)}} = P_{fa}^{1+SNR}$$

$$\left[ e^{-\frac{T^2}{\sigma_d^2}} \right]^{\frac{1}{1+SNR}}$$

$m=2$ : Swerling III-IV

$$P_d = e^{-\frac{T^2/\sigma_d^2}{1+SNR/2}} \left[ 1 + \frac{T^2}{2\sigma_d^2} \frac{SNR}{(1+SNR/2)^2} \right]$$

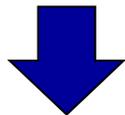
# Rivelazione per Bersagli fluttuanti (III)

Via analitica semplice

per  $m=1$ : Swerling I-II

$$z = |\tilde{z}| = \left| \underbrace{d_f(t)}_{\mu=1} + a \cdot \underbrace{\chi(\tau, \nu)}_{x+jy} \right| > T$$

Se “ $a$ ” ha ampiezza Rayleigh e fase uniforme, è una v.a. gaussiana complessa con valor medio nullo e varianza  $\sigma_{av}$



Anche  $\tilde{z}$  è una v.a. gaussiana complessa con valor medio nullo, e varianza  $\sigma_{av} + \sigma_d^2$

Dunque “ $z$ ” ha DDP Rayleigh, come il solo disturbo in  $H_p H_0$

$$P_d = \text{Prob}\{z > T \mid H_1\} = \int_T^\infty p_z(z \mid H_1) dz = e^{-\frac{T^2}{\sigma_d^2 + \sigma_{av}^2}} = e^{-\frac{T^2}{\sigma_d^2 (1 + \sigma_{av}^2 / \sigma_d^2)}} = e^{-\frac{T^2}{\sigma_d^2 (1 + SNR)}}$$

$$e^{-\frac{T^2}{\sigma_d^2}}$$

$$e^{-\frac{T^2}{\sigma_d^2 + \sigma_{av}^2}}$$

$$= e^{-\frac{T^2}{\sigma_d^2 (1 + \frac{\sigma_{av}^2}{\sigma_d^2})}}$$

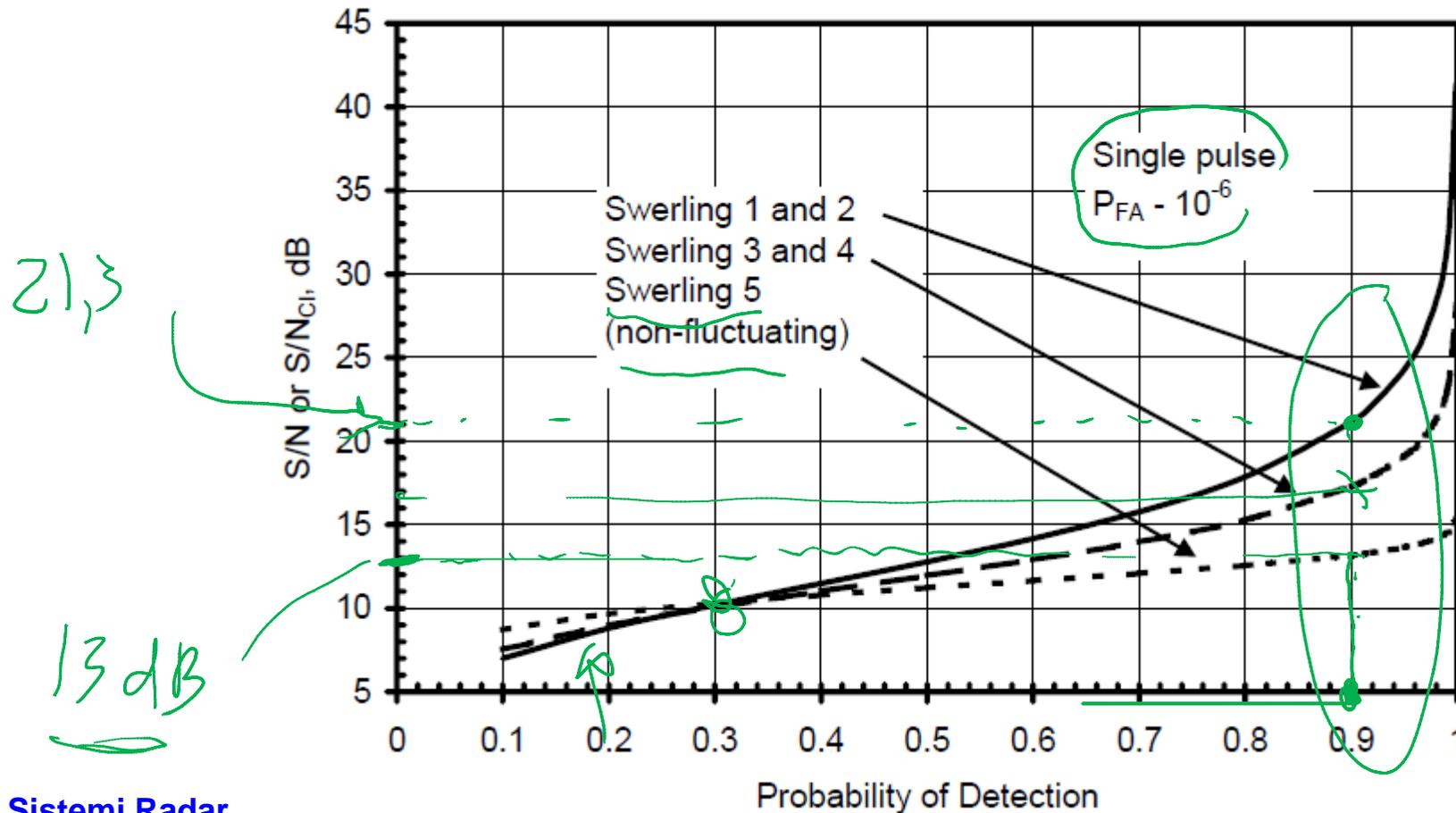
$$1 + SNR$$



$$P_d = e^{-\frac{T^2}{\sigma_d^2 (1 + SNR)}} = P_{fa}^{1 + SNR}$$

# Rivelazione per Bersagli fluttuanti (III)

## Plots of S/N vs. PD and Swerling Type



Sistemi Radar