Integrazione non coerente per bersagli fissi

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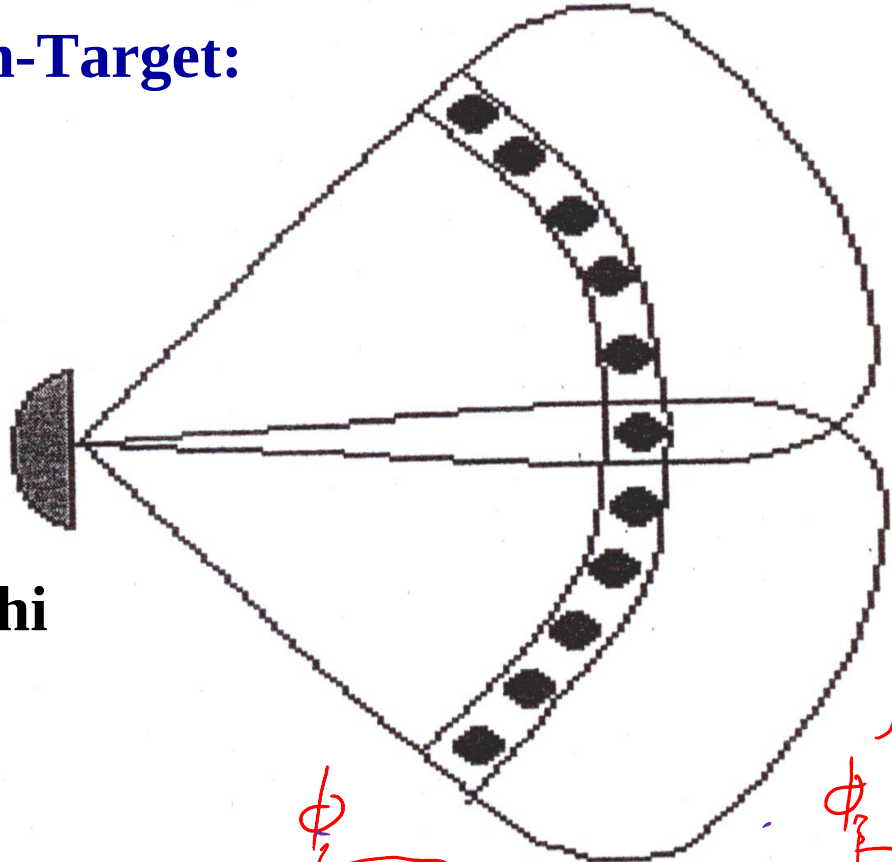
Integrazione di impulsi

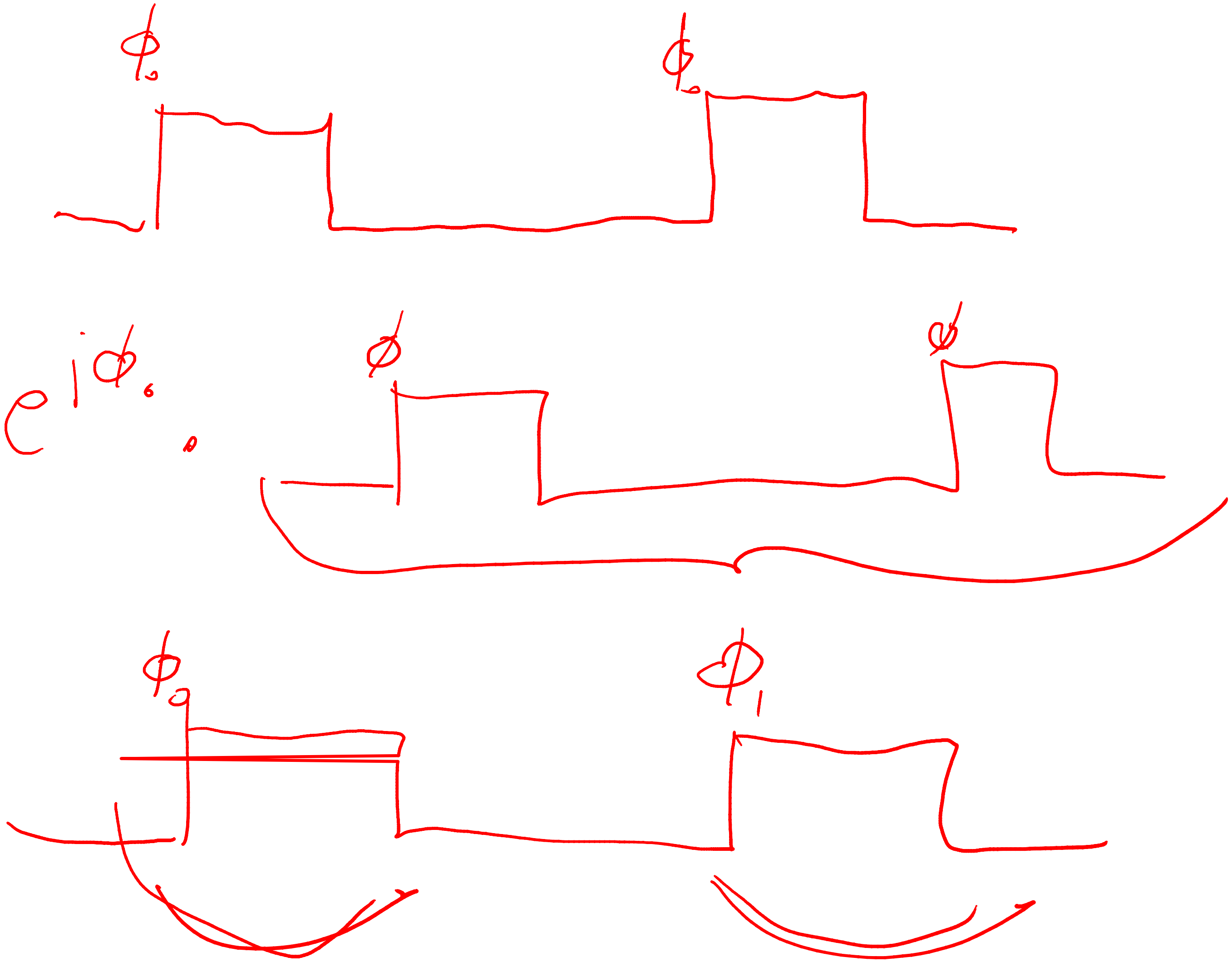
– **N impulsi nel Time-on-Target:**

$e^{j\phi_0} e^{j\phi(t)} \mathrm{rect}_{\tau_p}(t)$



Come sfruttare gli N echi per massimizzare la capacità di rivelazione?

ϕ_0 ϕ_1 ϕ_2 ϕ_3



$\phi_1 - \phi_0 = ??$

$$p(r/H_1) = \frac{1}{[\pi\sigma^2]^N} e^{-\frac{1}{\sigma^2}[\underline{r} - \alpha \underline{s}]^H \cdot [\underline{r} - \alpha \underline{s}]}$$

$$\underline{s} = \begin{bmatrix} s_0 \\ s_1 \\ \vdots \\ s_{M-1} \end{bmatrix}$$

$$\underline{r} = \begin{bmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \\ \vdots \end{bmatrix} = \alpha\, \underline{s}_0(t_0, f_0) + \underline{n}$$

$$p(\underline{r}/H_1) = \frac{1}{(\pi\sigma_n^2)^N} e^{-\frac{1}{\sigma_n^2}\|\underline{r} - a\underline{s}\|^2} \quad *$$

$$p(\underline{r}/H_0) = \frac{1}{(\pi\sigma_n^2)^N} e^{-\frac{1}{\sigma_n^2}\|\underline{r}\|^2} \quad *$$

$$\Lambda = \frac{p(\underline{r}/H_1)}{p(\underline{r}/H_0)} = e^{-\frac{1}{\sigma_n^2}\left\{\|\underline{r}-a\underline{s}\|^2-\|\underline{r}\|^2\right\}} \; * \; > \lambda_T$$

$$\lambda = \ln \Lambda = -\frac{1}{\sigma_n^2}\left\{\|\underline{r}-a\underline{s}\|^2-\|\underline{r}\|^2\right\} > T$$

$$[\underline{r}-a\underline{s}]^H[\underline{r}-a\underline{s}] = \underline{r}^H\underline{r} - a\,\underline{r}^H\underline{s} - a^*\underline{s}^H\underline{r} + \cancel{a^* a\,\underline{s}^H\underline{s}} =$$

$$= \|\underline{r}\|^2 - 2\,\mathrm{Re}\{a\,\underline{r}^H\underline{s}\} + |a|^2\|\underline{s}\|^2$$

$$* \quad \lambda = \cancel{-\frac{1}{\sigma_n^2}|a|^2\|\underline{s}\|^2} + \frac{2}{\sigma_n^2}\,\mathrm{Re}\{a\,\underline{r}^H\underline{s}\} \quad *$$

$$\boxed{\lambda = -\frac{|a|^2}{\sigma_n^2}\underbrace{\|\underline{s}\|^2}_{Q} + \frac{2}{\sigma_n^2}|a||\underline{r}^H\underline{s}|\cos\left(\angle a + \angle \underline{r}^H\underline{s}\right)}$$

$$a\,\underline{r}^H\underline{s} = |a|e^{j\angle a}\cdot|\underline{r}^H\underline{s}|\,e^{j\angle \underline{r}^H\underline{s}}$$

$$\mathrm{Re}\{a\underline{r}^H\underline{s}\} = |a||\underline{r}^H\underline{s}|\,\mathrm{Re}\left\{e^{j\left[\angle a + \angle \underline{r}^H\underline{s}\right]}\right\}$$

$$= |a|\,|\underline{r}^H\underline{s}|\cos\left(\angle a + \angle \underline{r}^H\underline{s}\right)$$

Rivelazione GLRT (I)

– **Rapporto di verosimiglianza :**

$r^H s$

$$\lambda = \exp\left\{\frac{2}{\sigma_n^2}\,|a|\,\left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right|\cos\left[-\angle a+\angle \mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right]\right\}\exp\left\{-\frac{|a|^2}{\sigma_n^2}\right\}$$

- **Rapporto di verosimiglianza generalizzato, ottenuto massimizzando λ sulla ddp della fase di *a*:**

$$\ln\lambda = \frac{2}{\sigma_n^2}\,|a|\,\left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right|\cos\left[-\angle a+\angle \mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right]-\frac{|a|^2}{\sigma_n^2} > T\ ?$$

$$\max_{\angle a}\{\ln\lambda\} = \frac{2}{\sigma_n^2}\,|a|\,\left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right|-\frac{|a|^2}{\sigma_n^2} > T$$

$$\frac{\partial}{\partial|a|}\left[\max_{\angle a}\{\ln\lambda\}\right] = \frac{2}{\sigma_n^2}\left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right|-\frac{2|a|}{\sigma_n^2}=0$$

$$|a| = \left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right|$$

$$\max_{\angle a,|a|}\{\ln\lambda\} = \frac{2}{\sigma_n^2}\left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right|^2-\frac{\left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right|^2}{\sigma_n^2} = \frac{1}{\sigma_n^2}\left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right|^2 > T$$

Rivelazione GLRT (II)

– **Rapporto di verosimiglianza :**

$$\lambda = \exp\left\{\frac{2}{\sigma_n^2}\,|a|\,\left|\mathbf{s}_0^H[t_0, f_0]\mathbf{r}\right|\cos\left[-\angle a + \angle \mathbf{s}_0^H[t_0, f_0]\mathbf{r}\right]\right\}\exp\left\{-\frac{|a|^2}{\sigma_n^2}\right\}$$

$$\max_{\angle a, |a|}\{\ln\lambda\} = \frac{1}{\sigma_n^2}\left|\mathbf{s}_0^H[t_0, f_0]\mathbf{r}\right|^2$$

$$\Lambda > T_d \qquad \Leftrightarrow \qquad \left|\mathbf{s}_0^H[t_0, f_0]\mathbf{r}\right| > T$$

Rivelazione GLRT per sequenza

– **Rapporto di verosimiglianza per sequenza di N impulsi :**

- Se la fase delle riflessioni dal bersaglio è aleatoria e uniformemente distribuita, gli echi di ritorno a ciascun impulso inviato sono statisticamente indipendenti.
- Dunque la DDP della sequenza è ottenuta come prodotto delle DDP relative agli echi ai singoli impulsi.
- Di conseguenza, anche la funzione di verosimiglianza per la sequenza risulta pari al prodotto delle verosimiglianze per i singoli impulsi
- La massimizzazione del logaritmo della DDP congiunta rispetto alle ampiezze complesse degli echi bersagli, porta alla somma delle verosimiglianze generalizzate logaritmiche

r_0, r_1, r_2, $\vdots$, r_{N-1}

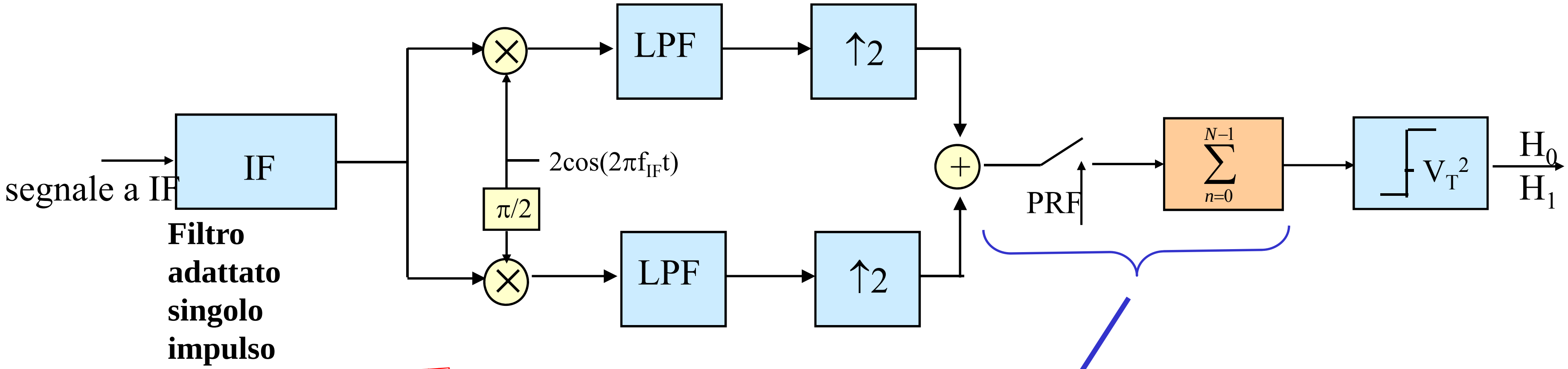
$$\lambda_{seq} = \prod_{n=0}^{N-1} \lambda_n = \prod_{n=0}^{N-1} \exp\left\{ \frac{2}{\sigma_n^2} |a_n| \left| \mathbf{s}_0^H[t_0, f_0]\mathbf{r}_n \right| \cos\left[-\angle a_n + \angle \mathbf{s}_0^H[t_0, f_0]\mathbf{r}_n \right] \right\} \exp\left\{ -\frac{|a_n|^2}{\sigma_n^2} \right\}$$

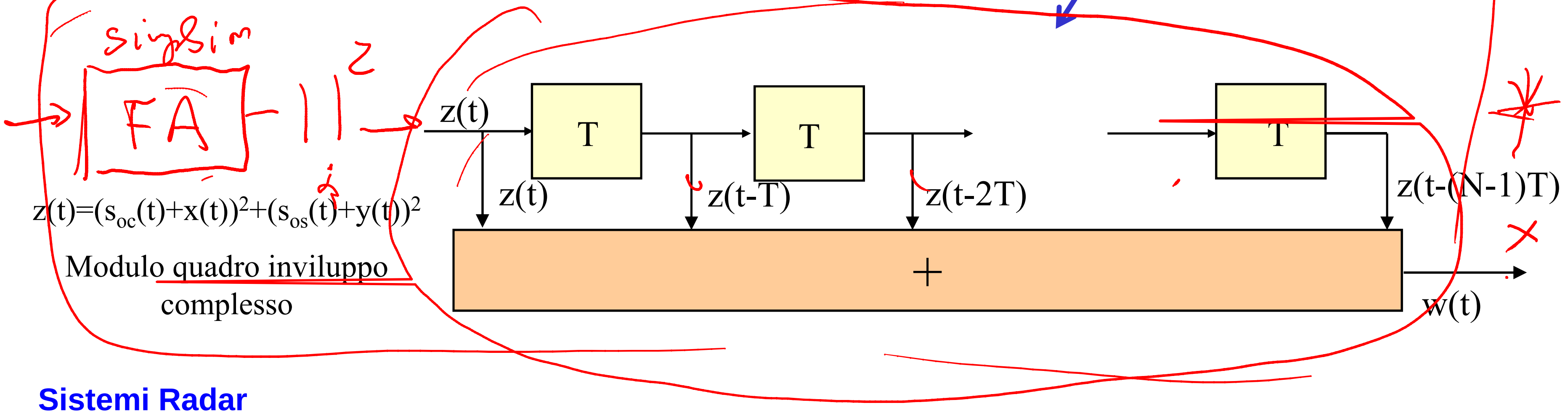
$$\max_{\angle a_0, |a_0|, \cdots, \angle a_{N-1}, |a_{N-1}|,} \{\ln \lambda_{seq}\} = \sum_{n=0}^{N-1} \max_{\angle a_n, |a_n|} \{\ln \lambda_n\} = \frac{1}{\sigma_n^2} \sum_{n=0}^{N-1} \left| \mathbf{s}_0^H[t_0, f_0]\mathbf{r}_n \right|^2$$

$$\Lambda > T_d \quad \Leftrightarrow \quad \sum_{n=0}^{N-1} \left| \mathbf{s}_0^H[t_0, f_0]\mathbf{r}_n \right|^2 > T$$

Integrazione incoerente quadratica (I)

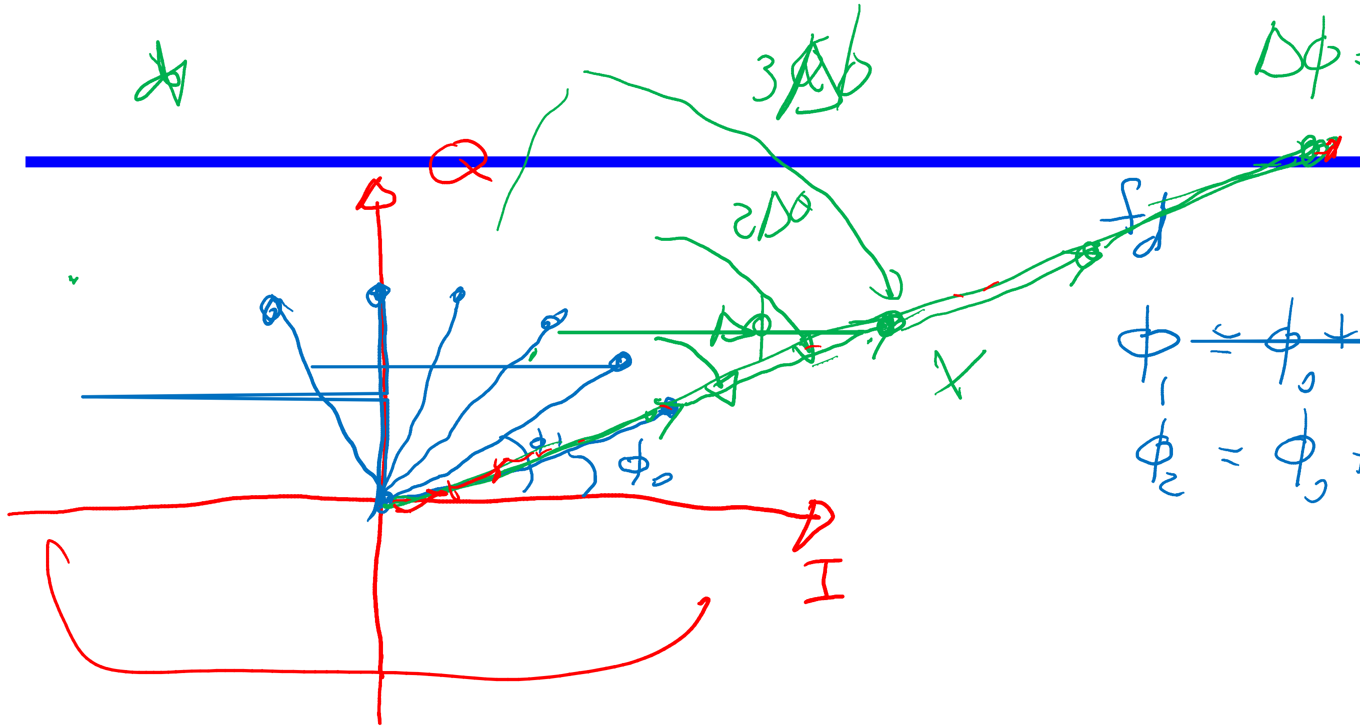
- Modulo quadro dell'inviluppo complesso;
- Somma dei moduli quadri degli inviluppi complessi associati agli N impulsi





$z(t)=(s_{oc}(t)+x(t))^2+(s_{os}(t)+y(t))^2$

Modulo quadro inviluppo complesso

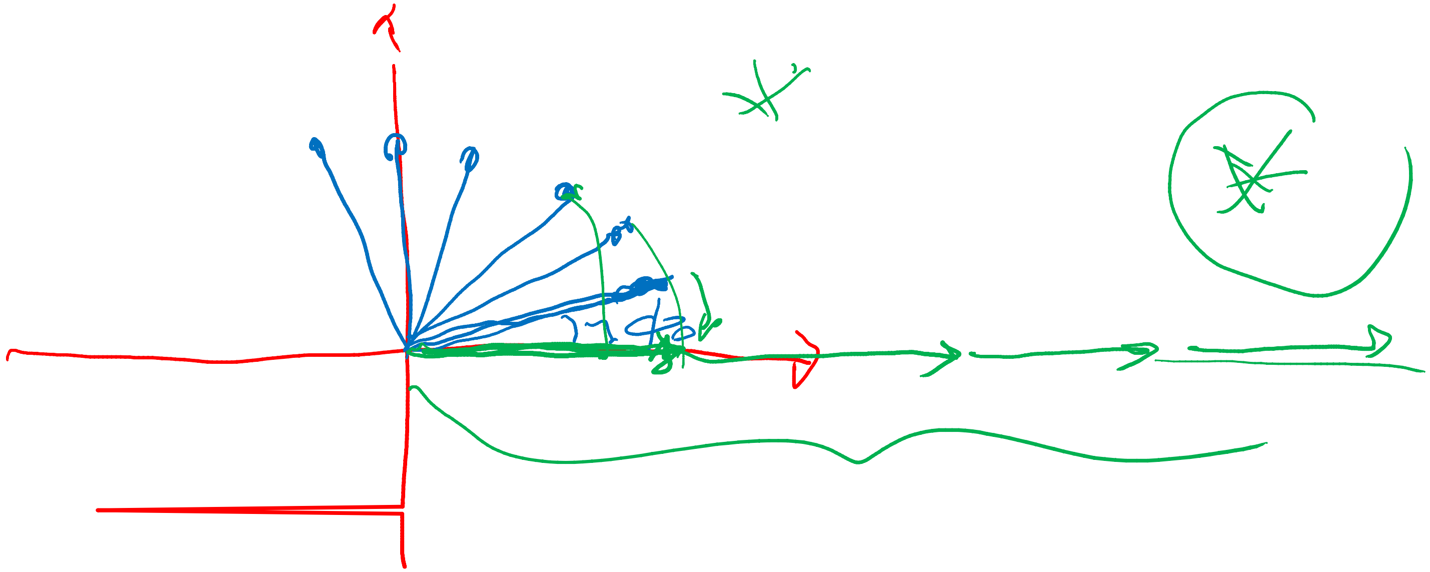


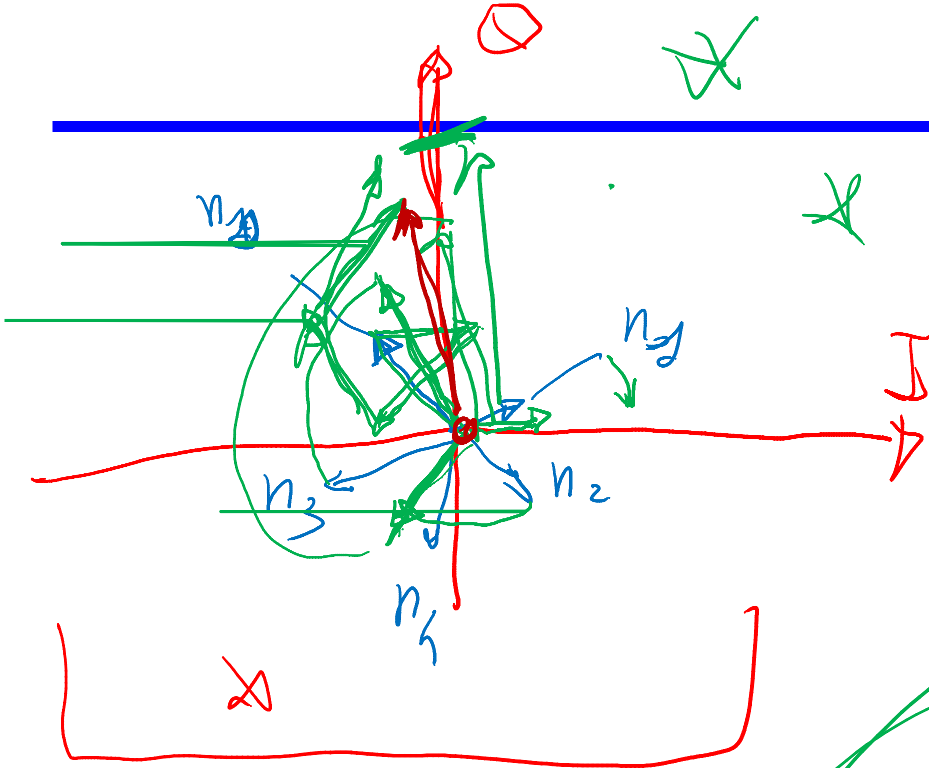
$$\Delta\phi = 2\pi f_d T$$

$$A_{sq} \simeq N A_0$$

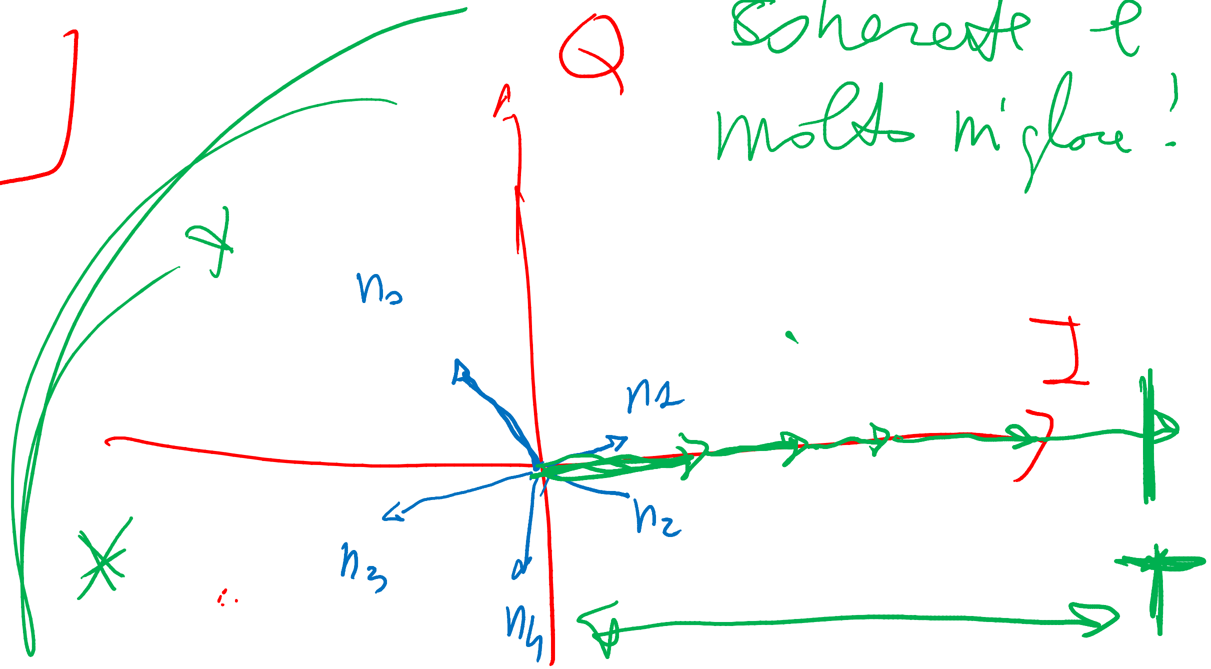
$$\phi_1 = \phi_0 + 2\pi f_d T$$

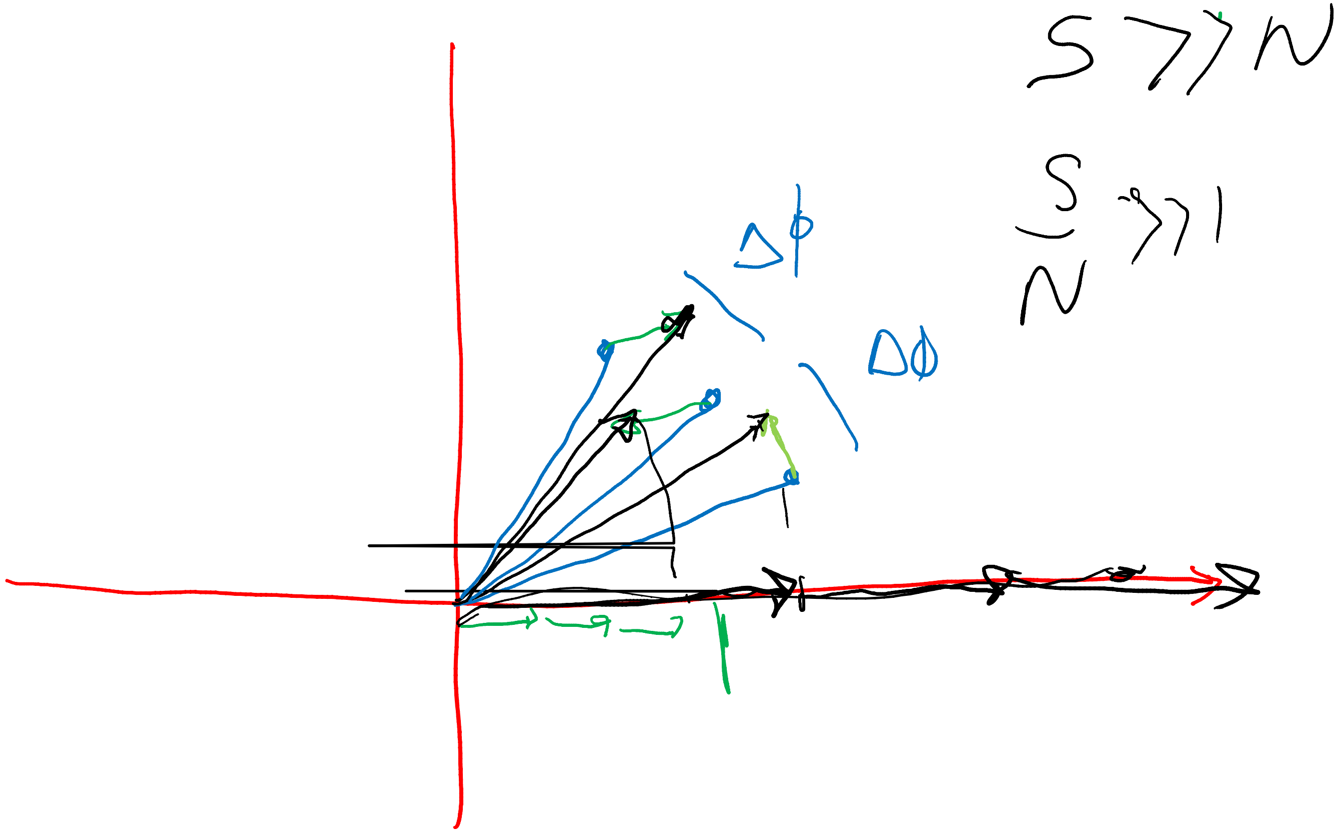
$$\phi_2 = \phi_0 + 2\pi f_d 2T$$



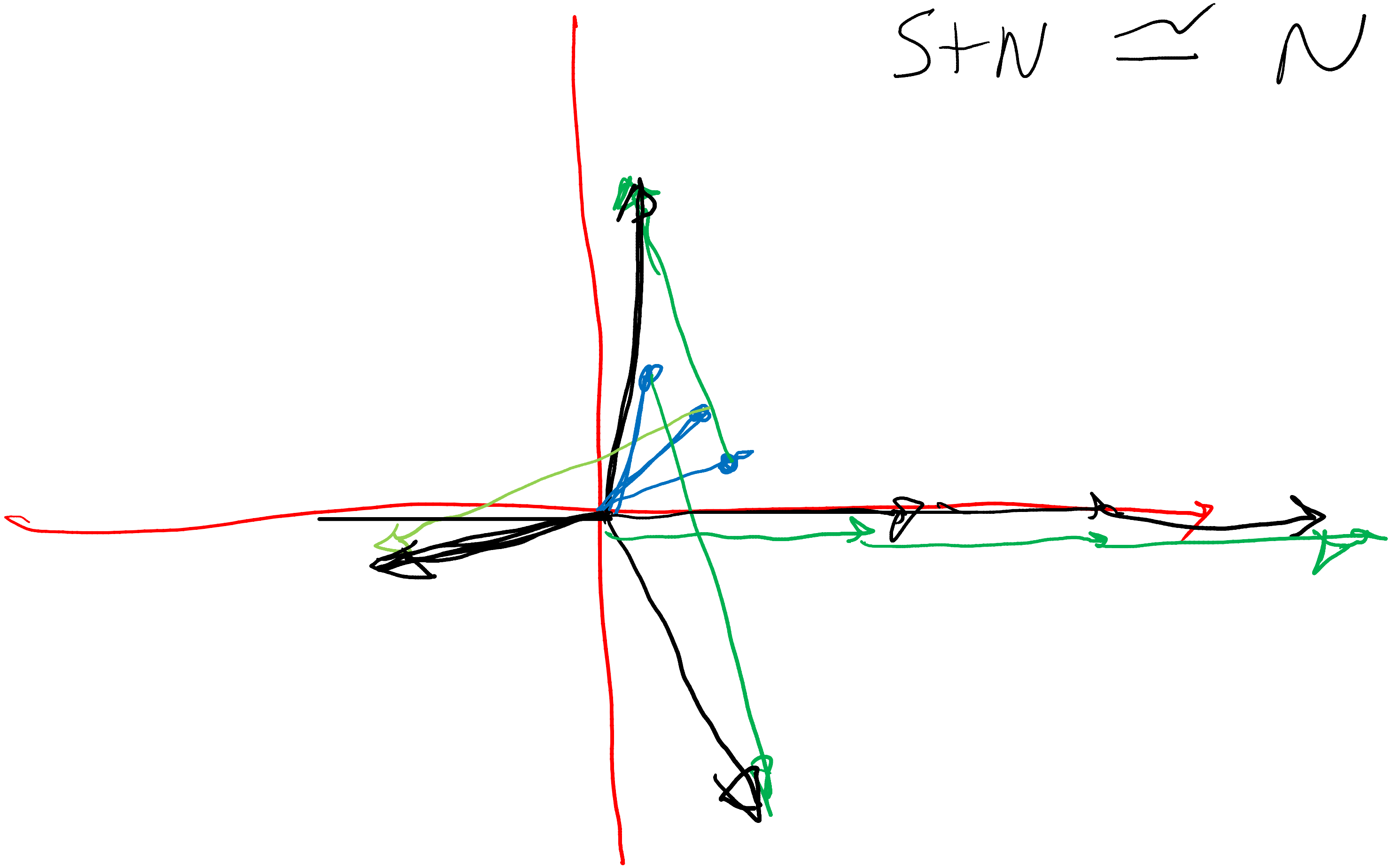


- in alto SNR
- quasi uguali
- in basso SNR coerente è molto migliore!




S >> N
S/N >> 1
Δφ
Δφ

$S+N \simeq N$



Rivelatore coerente

$$P_{fa} = e^{-\frac{T^2}{\sigma^2}}$$

$$P_d = \text{funzione di Marcum}\left(P_{fa},\ SNR^*\right)$$

$$N \cdot SNR_{o} \quad (si)$$

$$P_{fa} = 10^{-6} \quad P_d = 0{,}9 \Rightarrow \underline{SNR = 13\,dB} \Rightarrow 20$$

$$SNR_o = SNR_{si} = \frac{20}{N}$$

$$\boxed{SNR_o\big|_{dB} = SNR_{si}\big|_{dB} = 13 - 10\log_{10} N^*}$$

Rivelazione non coerente (NCI) (I)

– **Test:**

$$\Lambda > T_d \quad \Leftrightarrow \quad \left|\mathbf{s}_0^H[t_0, f_0]\mathbf{r}_0\right|^2 + \left|\mathbf{s}_0^H[t_0, f_0]\mathbf{r}_1\right|^2 + \cdots + \left|\mathbf{s}_0^H[t_0, f_0]\mathbf{r}_{N-1}\right|^2 > T^2$$

$\xi_n = z_n$

$$\xi = z^2 = \sum_{n=0}^{N-1} z_n^2 = \sum_{n=0}^{N-1} \xi_n = \sum_{n=0}^{N-1} \left|\mathbf{s}_0^H[t_0, f_0]\mathbf{r}_n\right|^2 > T^2$$

Sotto l'ipotesi $\mathbf{H_0}$ (a=0) ho un Falso Allarme

$$p_{\mathbf{r}_n}(\mathbf{r}_n | H_0) = \frac{1}{\pi^N \sigma_n^{2N}} \exp\left\{-\frac{1}{\sigma_n^2}|\mathbf{r}_n|^2\right\} \quad \Rightarrow \quad \boxed{p_{\xi_n}(\xi_n | H_0) = \frac{1}{\sigma_d^2} \exp\left\{-\frac{1}{\sigma_d^2}\xi_n\right\}}$$

$$p(\xi | H_0) = p_{\xi_0}(\xi | H_0) * p_{\xi_1}(\xi | H_0) \cdots * p_{\xi_{N-1}}(\xi | H_0)$$

$$\boxed{p_\xi(\xi | H_0) = \frac{1}{(N-1)!} \frac{\xi^{N-1}}{\sigma_d^{2N}} e^{-\frac{\xi}{\sigma_d^2}}}$$

Gamma

$$\boxed{P_{fa} = \mathrm{Prob}\left\{\xi > T^2 \mid H_0\right\} = \int_{T^2}^{\infty} p_\xi(\xi | H_0)\, d\xi = \sum_{n=0}^{N-1} \frac{1}{n!}\left(\frac{T^2}{\sigma_d^2}\right)^n e^{-\frac{T^2}{\sigma_d^2}}}$$

Richiamo DDP NCI (I)

$$p_{\xi_n}(\xi_n|H_0)=\frac{1}{\sigma_d^2}\exp\left\{-\frac{1}{\sigma_d^2}\xi_n\right\}$$

$$C_{\xi_n}(\omega|H_0)=\int_0^\infty \frac{1}{\sigma_d^2}\exp\left\{-\frac{1}{\sigma_d^2}\xi_n\right\}e^{-j\omega\xi_n}d\xi_n=\frac{1}{\sigma_d^2}\frac{1}{\frac{1}{\sigma_d^2}+\omega}=\frac{1}{1+\omega\,\sigma_d^2}$$

$$p(\xi|H_0)=p_{\xi_0}(\xi|H_0)*p_{\xi_1}(\xi|H_0)\cdots*p_{\xi_{N-1}}(\xi|H_0)$$

$$C_{\xi}(\omega|H_0)=\prod_{n=0}^{N-1}C_{\xi_n}(\omega|H_0)=C_{\xi_n}^N(\omega|H_0)=\frac{1}{(1+\omega\,\sigma_d^2)^N}$$

$$\boxed{p_{\xi}(\xi|H_0)=\frac{1}{(N-1)!}\frac{\xi^{N-1}}{\sigma_d^{2N}}e^{-\frac{\xi}{\sigma_d^2}}}$$

$$P_{fa}=\Pr ob\left\{\xi>T^2\,|\,H_0\right\}=\int_{T^2}^{\infty}p_{\xi}(\xi|H_0)\,dz$$

$$\int_x^{+\infty}\frac{1}{(N-1)!}t^{N-1}e^{-t}dt=\sum_{n=0}^{N-1}\frac{x^n}{n!}e^{-x}$$

$per\ \ N\ \ \text{int}\,ero>0$

$$\boxed{P_{fa}=\int_{T^2}^{\infty}\frac{1}{(N-1)!}\frac{\xi^{N-1}}{\sigma_d^{2N}}e^{-\frac{\xi}{\sigma_d^2}}d\xi=\int_{T^2/\sigma_d^2}^{\infty}\frac{1}{(N-1)!}t^{N-1}e^{-t}\,dt=\sum_{n=0}^{N-1}\frac{1}{n!}\left(\frac{T^2}{\sigma_d^2}\right)^n e^{-\frac{T^2}{\sigma_d^2}}}$$

Rivelazione non coerente (NCI) (II)

– **Test:**

$$\Lambda > T_d \quad \Leftrightarrow \quad \left|\mathbf{s}_0^H[t_0, f_0]\mathbf{r}_0\right|^2 + \left|\mathbf{s}_0^H[t_0, f_0]\mathbf{r}_1\right|^2 + \cdots + \left|\mathbf{s}_0^H[t_0, f_0]\mathbf{r}_{N-1}\right|^2 > T^2$$

$$\xi = z^2 = \sum_{n=0}^{N-1} z_n^2 = \sum_{n=0}^{N-1} \left|\tilde{z}_n\right|^2 = \sum_{n=0}^{N-1} \xi_n = \sum_{n=0}^{N-1} \left|\mathbf{s}_0^H[t_0, f_0]\mathbf{r}_n\right|^2 > T^2$$

Sotto l'ipotesi H_1 (a≠0) ho una rivelazione

$$p(\tilde{z}|a;H_1) = \frac{1}{\pi\sigma_d^2}\exp\left\{-\frac{1}{\sigma_d^2}\left|(\tilde{z} - a\cdot\chi(\tau,\nu)\right|^2\right\}$$

$$p_{\xi_n}(\xi_n|\sigma;H_1) = \frac{1}{\sigma_d^2}\exp\left\{-\frac{\xi_n + \sigma\cdot\left|\chi(\tau,\nu)\right|^2}{\sigma_d^2}\right\}\cdot I_0\left[\frac{2\sqrt{\xi_n\ \sigma}\cdot\left|\chi(\tau,\nu)\right|}{\sigma_d^2}\right]$$

$$p(\xi|H_1) = p_{\xi_0}(\xi|H_1) * p_{\xi_1}(\xi|H_1)\cdots * p_{\xi_{N-1}}(\xi|H_1)$$

$$p_\xi(\xi|\sigma;H_1) = \frac{1}{\sigma_d^2}\left(\frac{\xi}{N\sigma\cdot\left|\chi(\tau,\nu)\right|^2}\right)^{\frac{N-1}{2}}\exp\left\{-\frac{\xi + N\sigma\cdot\left|\chi(\tau,\nu)\right|^2}{\sigma_d^2}\right\}\cdot I_{N-1}\left[\frac{2\sqrt{\xi\,N\sigma}\cdot\left|\chi(\tau,\nu)\right|}{\sigma_d^2}\right]$$

$$\boxed{SNR = \frac{\sigma\cdot\left|\chi(\tau,\nu)\right|^2}{\sigma_d^2}}$$

$$\boxed{P_d(SNR) = \int_{T^2/\sigma_d^2}^{\infty}\left(\frac{t}{N\cdot SNR}\right)^{\frac{N-1}{2}}\exp\{-t - N\cdot SNR\}\cdot I_{N-1}\left[2\sqrt{t\ N\cdot SNR}\right]dt}$$

Richiamo DDP NCI (II)

$$p_{\xi_n}(\xi_n|\sigma;H_1)=\frac{1}{\sigma_d^2}\exp\left\{-\frac{\xi_n+\sigma\cdot|\chi(\tau,\nu)|^2}{\sigma_d^2}\right\}\cdot I_0\left[\frac{2\sqrt{\xi_n\,\sigma}\cdot|\chi(\tau,\nu)|}{\sigma_d^2}\right]$$

$$\int_0^\infty e^{-(x+a^2)}I_0\left[2a\sqrt{x}\right]dx=1$$

$$C_{\xi_n}(\omega|H_1)=\int_0^\infty \frac{1}{\sigma_d^2}\exp\left\{-\frac{\xi_n+\sigma\cdot|\chi(\tau,\nu)|^2}{\sigma_d^2}\right\}\cdot I_0\left[\frac{2\sqrt{\xi_n\,\sigma}\cdot|\chi(\tau,\nu)|}{\sigma_d^2}\right]e^{-j\omega\xi_n}d\xi_n=$$

$$=\frac{1}{\sigma_d^2}\exp\left\{-\frac{\sigma\cdot|\chi(\tau,\nu)|^2}{\sigma_d^2}\right\}\int_0^\infty \exp\left\{-\left(\frac{1}{\sigma_d^2}+j\omega\right)\xi_n\right\}\cdot I_0\left[\frac{2\sqrt{\xi_n\,\sigma}\cdot|\chi(\tau,\nu)|}{\sigma_d^2}\right]d\xi_n=$$

$$=\frac{1}{\sigma_d^2}\exp\left\{-\frac{\sigma\cdot|\chi(\tau,\nu)|^2}{\sigma_d^2}\right\}\frac{1}{\frac{1}{\sigma_d^2}+j\omega}\int_0^\infty \exp\{-x\}\cdot I_0\left[\frac{2\sqrt{\sigma}\cdot|\chi(\tau,\nu)|}{\sigma_d^2\sqrt{\frac{1}{\sigma_d^2}+j\omega}}\sqrt{x}\right]dx=$$

$$=\frac{1}{\sigma_d^2}\exp\left\{-\frac{\sigma\cdot|\chi(\tau,\nu)|^2}{\sigma_d^2}\right\}\frac{1}{\frac{1}{\sigma_d^2}+j\omega}\exp\left\{\frac{\sigma\cdot|\chi(\tau,\nu)|^2}{\sigma_d^4\left(\frac{1}{\sigma_d^2}+j\omega\right)}\right\}=$$

$$=\frac{1}{1+j\omega\sigma_d^2}\exp\left\{\frac{\sigma\cdot|\chi(\tau,\nu)|^2}{\sigma_d^2\left(1+j\omega\sigma_d^2\right)}\right\}$$

Richiamo DDP NCI (III)

$$p(\xi|H_1) = p_{\xi_0}(\xi|H_1) * p_{\xi_1}(\xi|H_1) \cdots * p_{\xi_{N-1}}(\xi|H_1)$$

$$C_\xi(\omega|H_1) = \prod_{n=0}^{N-1} C_{\xi_n}(\omega|H_1) = C_{\xi_n}^N(\omega|H_1) = \frac{1}{(1+\omega\,\sigma_d^2)^N}\exp\left\{\frac{N\sigma\cdot|\chi(\tau,\nu)|^2}{\sigma_d^2\left(1+j\omega\sigma_d^2\right)}\right\}$$

$$\boxed{p_\xi(\xi|\sigma;H_1) = \frac{1}{\sigma_d^2}\left(\frac{\xi}{N\sigma\cdot|\chi(\tau,\nu)|^2}\right)^{\frac{N-1}{2}}\exp\left\{-\frac{\xi+N\sigma\cdot|\chi(\tau,\nu)|^2}{\sigma_d^2}\right\}\cdot I_{N-1}\left[\frac{2\sqrt{\xi N\sigma}\cdot|\chi(\tau,\nu)|}{\sigma_d^2}\right]}$$

Richiamo DDP NCI (IV)

$$P_d(\sigma) = \Pr ob\left\{\xi > T^2 \mid \sigma;\, H_1\right\} = \int_{T^2}^{\infty} p_{\xi}(\xi \mid \sigma; H_1)\, d\xi$$

$$P_d(\sigma) = \int_{T^2}^{\infty} \frac{1}{\sigma_d^2} \left(\frac{\xi}{N\sigma \cdot |\chi(\tau,\nu)|^2} \right)^{\frac{N-1}{2}} \exp\left\{ -\frac{\xi + N\sigma \cdot |\chi(\tau,\nu)|^2}{\sigma_d^2} \right\} \cdot I_{N-1}\left[\frac{2\sqrt{\xi\, N\sigma \cdot |\chi(\tau,\nu)|}}{\sigma_d^2} \right] d\xi =$$

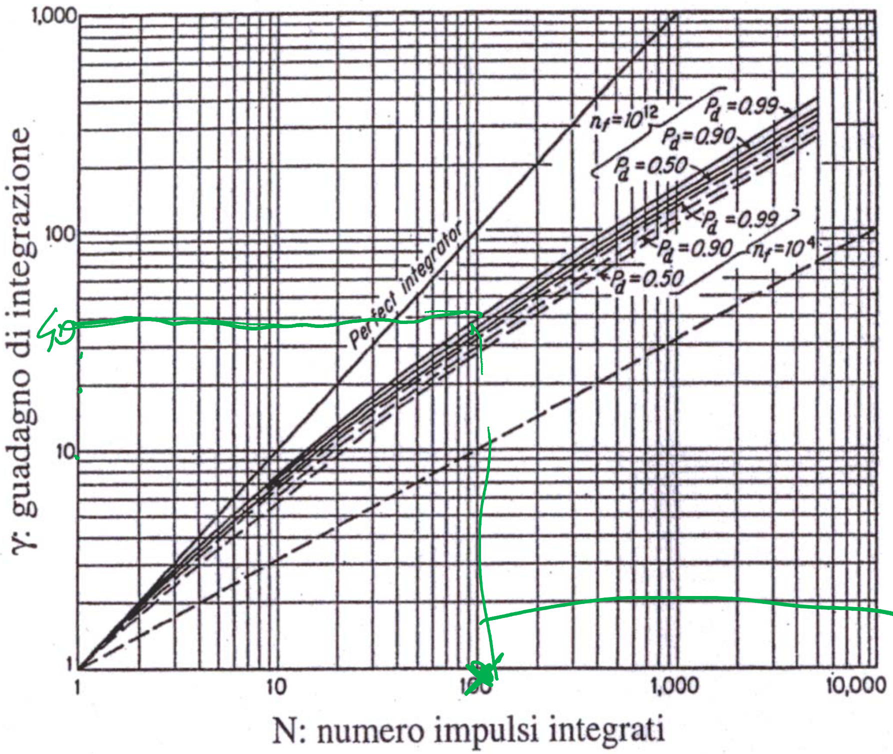
$$= \int_{T^2/\sigma_d^2}^{\infty} \left(\frac{t\, \sigma_d^2}{N\sigma \cdot |\chi(\tau,\nu)|^2} \right)^{\frac{N-1}{2}} \exp\left\{ -t - \frac{N\sigma \cdot |\chi(\tau,\nu)|^2}{\sigma_d^2} \right\} \cdot I_{N-1}\left[\frac{2\sqrt{t\, N\sigma} \cdot |\chi(\tau,\nu)|}{\sigma_d} \right] dt$$

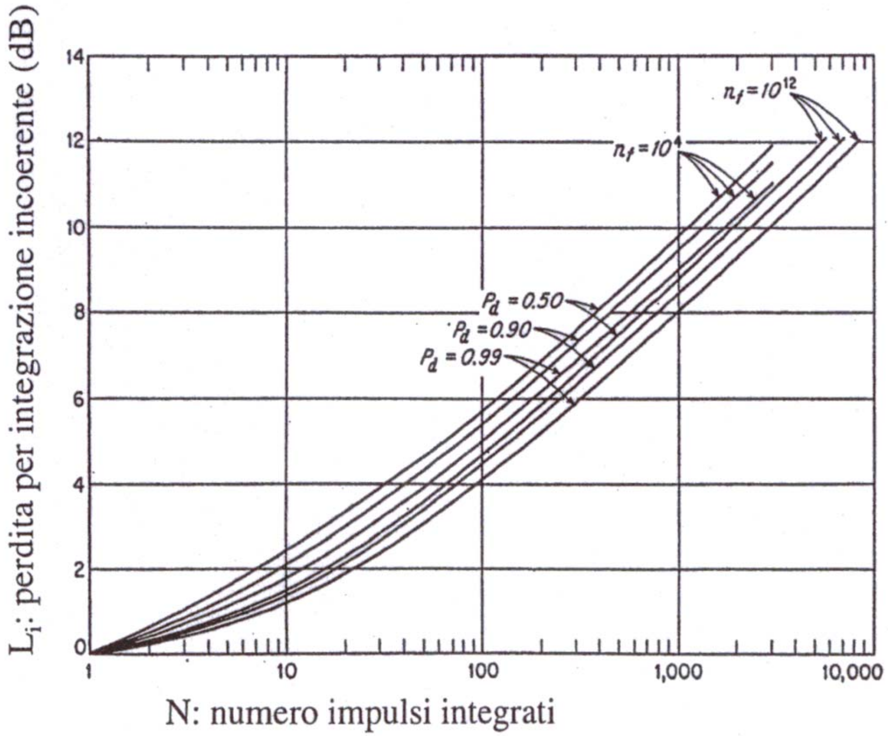
$$\Rightarrow \quad \boxed{P_d(SNR) = \int_{T^2/\sigma_d^2}^{\infty} \left(\frac{t}{N \cdot SNR} \right)^{\frac{N-1}{2}} \exp\left\{ -t - N \cdot SNR \right\} \cdot I_{N-1}\left[2\sqrt{t\, N \cdot SNR} \right] dt}$$

$$\boxed{SNR = \frac{\sigma \cdot |\chi(\tau,\nu)|^2}{\sigma_d^2}}$$

Integrazione incoerente quadratica (II)

integrazione coerente perfetta





$SNR_{si} = 13 - 10 \log_{10} 40$

per $N = 100$

Integrazione incoerente quadratica (III)

Portata radar in presenza di integrazione

$$SNR^N = \frac{1}{\gamma} SNR^1$$

Guadagno di integrazione γ

Nel caso di integrazione incoerente quadratica:

- γ≅N per elevati valori di rapporto segnale a rumore;
- γ≅√N per bassi valori di rapporto segnale a disturbo

$$L_i = 10\log_{10}\left(\frac{N}{\gamma}\right)$$

Perdita integrazione incoerente rispetto alla coerente L_i

- L_i≅0 dB per elevati valori di rapporto segnale a rumore;
- Li≅$10\log_{10}(\sqrt{N})$ per bassi valori di rapporto segnale a disturbo

$$R_{\max} = \left[\frac{E_t G A_e \sigma}{(4\pi)^2 L k T_0 F \, SNR^* / \gamma}\right]^{1/4}$$

Il guadagno di integrazione γ consente di

- Aumentare la portata di $\gamma^{1/4}$ se si lascia inalterata l'energia trasmessa fissata per il caso di decisione su singolo impulso
- Trasmettere un'energia γ volte inferiore rispetto a quella fissata per il caso di decisione su singolo impulso se si lascia inalterata la portata

Albersheim's Equation

Albersheim's Equation

Albersheim's equation is a closed-form approximation to the SNR required to achieve the specified detection and false alarm probabilities for a nonfluctuating target in independent and identically distributed Gaussian noise. The approximation is valid for a linear detector and is extensible to the noncoherent integration of N samples.

Let

$$A = \ln \frac{0.62}{P_{FA}}$$

and

$$B = \ln \frac{P_D}{1 - P_D}$$

where P_{FA} and P_D are the false alarm and detection probabilities.

Albersheim's equation for the required SNR in dB is:

$$\mathrm{SNR} = -5\log_{10} N + [6.2 + 4.54 / \sqrt{N + 0.44}]\log_{10}(A + 0.12AB + 1.7B)$$

where N is the number of noncoherently integrated samples.

$N=1 \rightarrow$ lavoro con 1 impulso

$N \rightarrow N \rightarrow$ lavoro con N impulsi in modo incoerente

– per integraz. coerente con N impulsi

metto ~~N=1 e so~~ttraggo $10\log_{10}N$

where N is the number of noncoherently integrated samples