Integrazione non coerente per bersagli fissi

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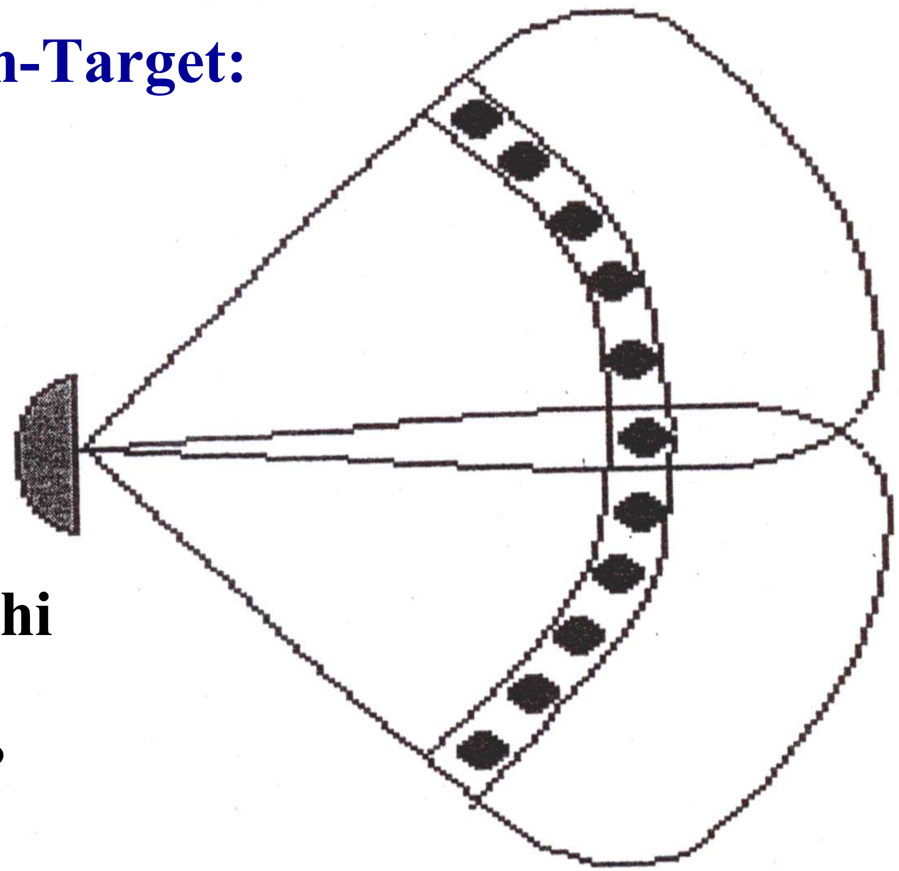
Integrazione di impulsi

– **N impulsi nel Time-on-Target:**



Come sfruttare gli N echi per massimizzare la capacità di rivelazione?

Rivelazione GLRT (I)

– **Rapporto di verosimiglianza :**

$$\lambda = \exp\left\{\frac{2}{\sigma_n^2}\,|a|\,\left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right|\cos\left[-\angle a + \angle \mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right]\right\}\exp\left\{-\frac{|a|^2}{\sigma_n^2}\right\}$$

- **Rapporto di verosimiglianza generalizzato, ottenuto massimizzando λ sulla ddp della fase di *a*:**

$$\ln\lambda = \frac{2}{\sigma_n^2}\,|a|\,\left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right|\cos\left[-\angle a + \angle \mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right] - \frac{|a|^2}{\sigma_n^2}$$

$$\max_{\angle a}\{\ln\lambda\} = \frac{2}{\sigma_n^2}\,|a|\,\left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right| - \frac{|a|^2}{\sigma_n^2}$$

$$\frac{\partial}{\partial|a|}\left[\max_{\angle a}\{\ln\lambda\}\right] = \frac{2}{\sigma_n^2}\left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right| - \frac{2|a|}{\sigma_n^2} = 0$$

$$|a| = \left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right|$$

$$\max_{\angle a,|a|}\{\ln\lambda\} = \frac{2}{\sigma_n^2}\left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right|^2 - \frac{\left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right|^2}{\sigma_n^2} = \frac{1}{\sigma_n^2}\left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right|^2$$

Rivelazione GLRT (II)

– **Rapporto di verosimiglianza :**

$$\lambda = \exp\left\{\frac{2}{\sigma_n^2}\,|a|\,\left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right|\cos\left[-\angle a + \angle\mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right]\right\}\exp\left\{-\frac{|a|^2}{\sigma_n^2}\right\}$$

$$\max_{\angle a,|a|}\{\ln\lambda\} = \frac{1}{\sigma_n^2}\left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right|^2$$

$$\Lambda > T_d \qquad \Leftrightarrow \qquad \left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}\right| > T$$

Rivelazione GLRT per sequenza

– **Rapporto di verosimiglianza per sequenza di N impulsi :**

- Se la fase delle riflessioni dal bersaglio è aleatoria e uniformemente distribuita, gli echi di ritorno a ciascun impulso inviato sono statisticamente indipendenti.
- Dunque la DDP della sequenza è ottenuta come prodotto delle DDP relative agli echi ai singoli impulsi.
- Di conseguenza, anche la funzione di verosimiglianza per la sequenza risulta pari al prodotto delle verosimiglianze per i singoli impulsi
- La massimizzazione del logaritmo della DDP congiunta rispetto alle ampiezze complesse degli echi bersagli, porta alla somma delle verosimiglianze generalizzate logaritmiche

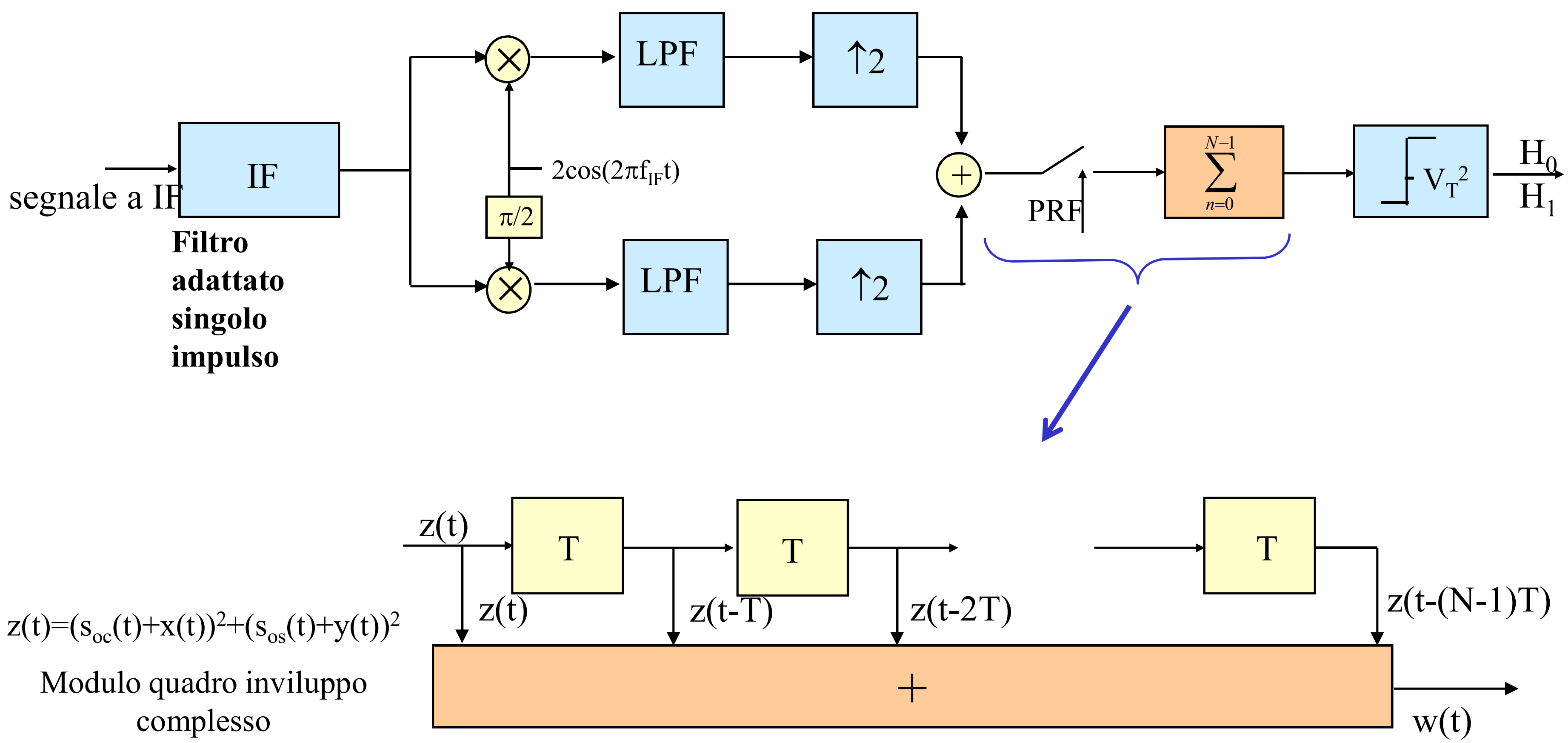
$$\lambda_{seq} = \prod_{n=0}^{N-1} \lambda_n = \prod_{n=0}^{N-1} \exp\left\{ \frac{2}{\sigma_n^2} |a| \left| \mathbf{s}_0^H[t_0, f_0]\mathbf{r}_n \right| \cos\left[-\angle a + \angle \mathbf{s}_0^H[t_0, f_0]\mathbf{r}_n \right] \right\} \exp\left\{ -\frac{|a|^2}{\sigma_n^2} \right\}$$

$$\max_{\angle a_0, |a_0|, \cdots, \angle a_{N-1}, |a_{N-1}|,} \{\ln \lambda_{seq}\} = \sum_{n=0}^{N-1} \max_{\angle a_n, |a_n|} \{\ln \lambda_n\} = \frac{1}{\sigma_n^2} \sum_{n=0}^{N-1} \left| \mathbf{s}_0^H[t_0, f_0]\mathbf{r}_n \right|^2$$

$$\Lambda > T_d \quad \Leftrightarrow \quad \sum_{n=0}^{N-1} \left| \mathbf{s}_0^H[t_0, f_0]\mathbf{r}_n \right|^2 > T$$

Integrazione incoerente quadratica (I)

- Modulo quadro dell'inviluppo complesso;
- Somma dei moduli quadri degli inviluppi complessi associati agli N impulsi



Rivelazione non coerente (NCI) (I)

– **Test:**

$$\Lambda > T_d \quad \Leftrightarrow \quad \left| \mathbf{s}_0^H[t_0, f_0]\mathbf{r}_0 \right|^2 + \left| \mathbf{s}_0^H[t_0, f_0]\mathbf{r}_1 \right|^2 + \cdots + \left| \mathbf{s}_0^H[t_0, f_0]\mathbf{r}_{N-1} \right|^2 > T^2$$

$$\xi = z^2 = \sum_{n=0}^{N-1} z_n^2 = \sum_{n=0}^{N-1} \xi_n = \sum_{n=0}^{N-1} \left| \mathbf{s}_0^H[t_0, f_0]\mathbf{r}_n \right|^2 > T^2$$

Sotto l'ipotesi H_0 (a=0) ho un Falso Allarme

$$p_{\mathbf{r}_n}(\mathbf{r}_n | H_0) = \frac{1}{\pi^N \sigma_n^{2N}} \exp\left\{ -\frac{1}{\sigma_n^2} |\mathbf{r}_n|^2 \right\} \quad \Rightarrow \quad p_{\xi_n}(\xi_n | H_0) = \frac{1}{\sigma_d^2} \exp\left\{ -\frac{1}{\sigma_d^2} \xi_n \right\}$$

$$p(\xi | H_0) = p_{\xi_0}(\xi | H_0) * p_{\xi_1}(\xi | H_0) \cdots * p_{\xi_{N-1}}(\xi | H_0)$$

$$p_\xi(\xi | H_0) = \frac{1}{(N-1)!} \frac{\xi^{N-1}}{\sigma_d^{2N}} e^{-\frac{\xi}{\sigma_d^2}}$$

$$P_{fa} = \mathrm{Pr}ob\left\{ \xi > T^2 \,|\, H_0 \right\} = \int_{T^2}^{\infty} p_\xi(\xi | H_0)\, d\xi = \sum_{n=0}^{N-1} \frac{1}{n!} \left(\frac{T^2}{\sigma_d^2} \right)^n e^{-\frac{T^2}{\sigma_d^2}}$$

Richiamo DDP NCI (I)

$$p_{\xi_n}(\xi_n|H_0)=\frac{1}{\sigma_d^2}\exp\left\{-\frac{1}{\sigma_d^2}\xi_n\right\} \qquad C_{\xi_n}(\omega|H_0)=\int_0^{\infty}\frac{1}{\sigma_d^2}\exp\left\{-\frac{1}{\sigma_d^2}\xi_n\right\}e^{-j\omega\xi_n}d\xi_n=\frac{1}{\sigma_d^2}\frac{1}{\frac{1}{\sigma_d^2}+\omega}=\frac{1}{1+\omega\,\sigma_d^2}$$

$$p(\xi|H_0)=p_{\xi_0}(\xi|H_0)*p_{\xi_1}(\xi|H_0)\cdots*p_{\xi_{N-1}}(\xi|H_0)$$

$$C_{\xi}(\omega|H_0)=\prod_{n=0}^{N-1}C_{\xi_n}(\omega|H_0)=C_{\xi_n}^N(\omega|H_0)=\frac{1}{(1+\omega\,\sigma_d^2)^N}$$

$$p_{\xi}(\xi|H_0)=\frac{1}{(N-1)!}\frac{\xi^{N-1}}{\sigma_d^{2N}}e^{-\frac{\xi}{\sigma_d^2}}$$

$$P_{fa}=\Pr ob\left\{\xi>T^2\mid H_0\right\}=\int_{T^2}^{\infty}p_{\xi}(\xi|H_0)\,dz$$

$$\int_x^{+\infty}\frac{1}{(N-1)!}t^{N-1}e^{-t}dt=\sum_{n=0}^{N-1}\frac{x^n}{n!}e^{-x}$$

$per\ N$ intero >0

$$P_{fa}=\int_{T^2}^{\infty}\frac{1}{(N-1)!}\frac{\xi^{N-1}}{\sigma_d^{2N}}e^{-\frac{\xi}{\sigma_d^2}}d\xi=\int_{T^2/\sigma_d^2}^{\infty}\frac{1}{(N-1)!}t^{N-1}e^{-t}\,dt=\sum_{n=0}^{N-1}\frac{1}{n!}\left(\frac{T^2}{\sigma_d^2}\right)^n e^{-\frac{T^2}{\sigma_d^2}}$$

Rivelazione non coerente (NCI) (II)

– **Test:**

$$\Lambda > T_d \qquad \Leftrightarrow \qquad \left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}_0\right|^2 + \left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}_1\right|^2 + \cdots + \left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}_{N-1}\right|^2 > T^2$$

$$\xi = z^2 = \sum_{n=0}^{N-1} z_n^2 = \sum_{n=0}^{N-1} \left|\tilde{z}_n\right|^2 = \sum_{n=0}^{N-1} \xi_n = \sum_{n=0}^{N-1} \left|\mathbf{s}_0^H[t_0,f_0]\mathbf{r}_n\right|^2 > T^2$$

Sotto l'ipotesi $\mathbf{H_1}$ (a≠0) ho una rivelazione

$$p(\tilde{z}\,|a;H_1) = \frac{1}{\pi\sigma_d^2}\exp\left\{-\frac{1}{\sigma_d^2}\left|(\tilde{z} - a\cdot\chi(\tau,\nu)\right|^2\right\} \qquad p_{\xi_n}(\xi_n\,|\sigma;H_1) = \frac{1}{\sigma_d^2}\exp\left\{-\frac{\xi_n + \sigma\cdot\left|\chi(\tau,\nu)\right|^2}{\sigma_d^2}\right\}\cdot I_0\left[\frac{2\sqrt{\xi_n\,\sigma}\cdot\left|\chi(\tau,\nu)\right|}{\sigma_d^2}\right]$$

$$\Rightarrow \qquad p(\xi\,|H_1) = p_{\xi_0}(\xi\,|H_1) * p_{\xi_1}(\xi\,|H_1)\cdots * p_{\xi_{N-1}}(\xi\,|H_1)$$

$$p_\xi(\xi\,|\sigma;H_1) = \frac{1}{\sigma_d^2}\left(\frac{\xi}{N\sigma\cdot\left|\chi(\tau,\nu)\right|^2}\right)^{\frac{N-1}{2}}\exp\left\{-\frac{\xi + N\sigma\cdot\left|\chi(\tau,\nu)\right|^2}{\sigma_d^2}\right\}\cdot I_{N-1}\left[\frac{2\sqrt{\xi\,N\sigma}\cdot\left|\chi(\tau,\nu)\right|}{\sigma_d^2}\right]$$

$$\boxed{SNR = \frac{\sigma\cdot\left|\chi(\tau,\nu)\right|^2}{\sigma_d^2}}$$

$$\boxed{P_d(SNR) = \int_{T^2/\sigma_d^2}^{\infty}\left(\frac{t}{N\cdot SNR}\right)^{\frac{N-1}{2}}\exp\left\{-t - N\cdot SNR\right\}\cdot I_{N-1}\left[2\sqrt{t\;N\cdot SNR}\right]dt}$$

Richiamo DDP NCI (II)

$$p_{\xi_n}(\xi_n|\sigma;H_1)=\frac{1}{\sigma_d^2}\exp\left\{-\frac{\xi_n+\sigma\cdot|\chi(\tau,\nu)|^2}{\sigma_d^2}\right\}\cdot I_0\left[\frac{2\sqrt{\xi_n\ \sigma}\cdot|\chi(\tau,\nu)|}{\sigma_d^2}\right]$$

$$\int_0^\infty e^{-(x+a^2)}I_0\left[2a\sqrt{x}\right]dx=1$$

$$C_{\xi_n}(\omega|H_1)=\int_0^\infty \frac{1}{\sigma_d^2}\exp\left\{-\frac{\xi_n+\sigma\cdot|\chi(\tau,\nu)|^2}{\sigma_d^2}\right\}\cdot I_0\left[\frac{2\sqrt{\xi_n\ \sigma}\cdot|\chi(\tau,\nu)|}{\sigma_d^2}\right]e^{-j\omega\xi_n}d\xi_n=$$

$$=\frac{1}{\sigma_d^2}\exp\left\{-\frac{\sigma\cdot|\chi(\tau,\nu)|^2}{\sigma_d^2}\right\}\int_0^\infty \exp\left\{-\left(\frac{1}{\sigma_d^2}+j\omega\right)\xi_n\right\}\cdot I_0\left[\frac{2\sqrt{\xi_n\ \sigma}\cdot|\chi(\tau,\nu)|}{\sigma_d^2}\right]d\xi_n=$$

$$=\frac{1}{\sigma_d^2}\exp\left\{-\frac{\sigma\cdot|\chi(\tau,\nu)|^2}{\sigma_d^2}\right\}\frac{1}{\frac{1}{\sigma_d^2}+j\omega}\int_0^\infty \exp\{-x\}\cdot I_0\left[\frac{2\sqrt{\sigma}\cdot|\chi(\tau,\nu)|}{\sigma_d^2\sqrt{\frac{1}{\sigma_d^2}+j\omega}}\sqrt{x}\right]dx=$$

$$=\frac{1}{\sigma_d^2}\exp\left\{-\frac{\sigma\cdot|\chi(\tau,\nu)|^2}{\sigma_d^2}\right\}\frac{1}{\frac{1}{\sigma_d^2}+j\omega}\exp\left\{\frac{\sigma\cdot|\chi(\tau,\nu)|^2}{\sigma_d^4\left(\frac{1}{\sigma_d^2}+j\omega\right)}\right\}=$$

$$=\frac{1}{1+j\omega\sigma_d^2}\exp\left\{\frac{\sigma\cdot|\chi(\tau,\nu)|^2}{\sigma_d^2\left(1+j\omega\sigma_d^2\right)}\right\}$$

Richiamo DDP NCI (III)

$$p(\xi|H_1) = p_{\xi_0}(\xi|H_1) * p_{\xi_1}(\xi|H_1) \cdots * p_{\xi_{N-1}}(\xi|H_1)$$

$$C_\xi(\omega|H_1) = \prod_{n=0}^{N-1} C_{\xi_n}(\omega|H_1) = C_{\xi_n}^N(\omega|H_1) = \frac{1}{(1+\omega\,\sigma_d^2)^N} \exp\left\{ \frac{N\sigma \cdot |\chi(\tau,\nu)|^2}{\sigma_d^2\left(1+j\omega\sigma_d^2\right)} \right\}$$

$$p_\xi(\xi|\sigma;H_1) = \frac{1}{\sigma_d^2}\left(\frac{\xi}{N\sigma\cdot|\chi(\tau,\nu)|^2}\right)^{\frac{N-1}{2}} \exp\left\{-\frac{\xi + N\sigma\cdot|\chi(\tau,\nu)|^2}{\sigma_d^2}\right\} \cdot I_{N-1}\left[\frac{2\sqrt{\xi N\sigma}\cdot|\chi(\tau,\nu)|}{\sigma_d^2}\right]$$

Richiamo DDP NCI (IV)

$$P_d(\sigma) = \Pr ob\left\{\xi > T^2 \mid \sigma;\, H_1\right\} = \int_{T^2}^{\infty} p_\xi(\xi \mid \sigma; H_1)\, d\xi$$

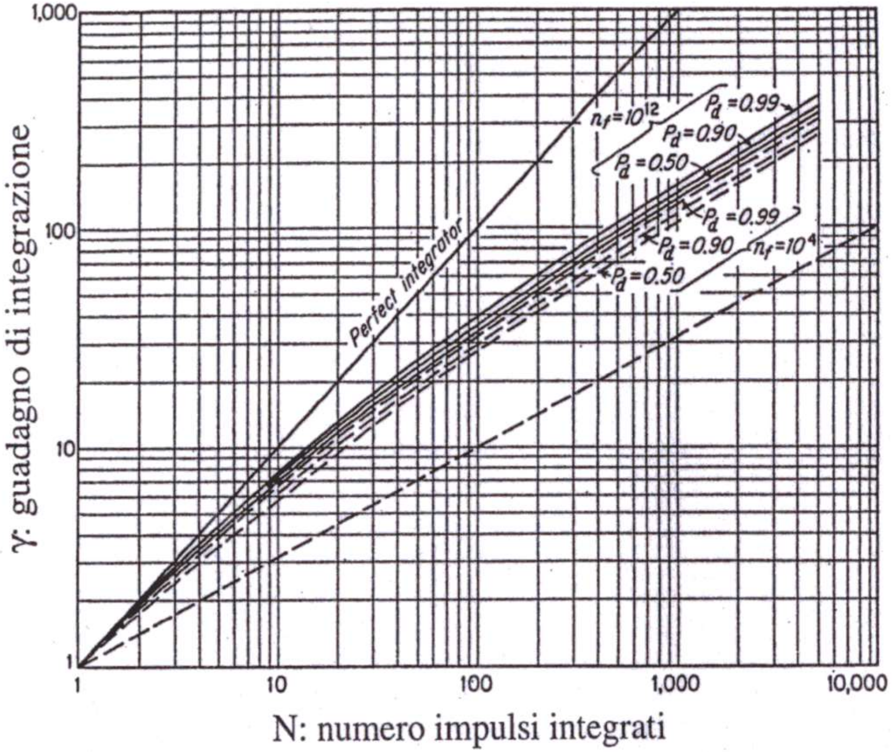
$$P_d(\sigma) = \int_{T^2}^{\infty} \frac{1}{\sigma_d^2}\left(\frac{\xi}{N\sigma\cdot|\chi(\tau,\nu)|^2}\right)^{\frac{N-1}{2}} \exp\left\{-\frac{\xi + N\sigma\cdot|\chi(\tau,\nu)|^2}{\sigma_d^2}\right\}\cdot I_{N-1}\left[\frac{2\sqrt{\xi N\sigma}\cdot|\chi(\tau,\nu)|}{\sigma_d^2}\right] d\xi =$$

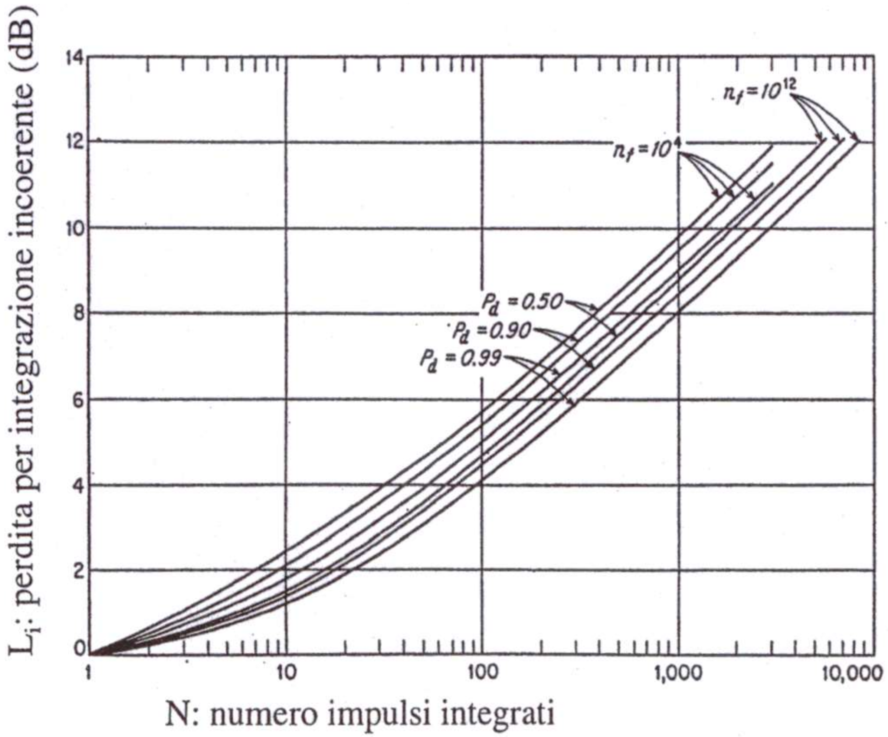
$$= \int_{T^2/\sigma_d^2}^{\infty} \left(\frac{t\,\sigma_d^2}{N\sigma\cdot|\chi(\tau,\nu)|^2}\right)^{\frac{N-1}{2}} \exp\left\{-t - \frac{N\sigma\cdot|\chi(\tau,\nu)|^2}{\sigma_d^2}\right\}\cdot I_{N-1}\left[\frac{2\sqrt{t\,N\sigma}\cdot|\chi(\tau,\nu)|}{\sigma_d}\right] dt$$

$$\Rightarrow \quad P_d(SNR) = \int_{T^2/\sigma_d^2}^{\infty} \left(\frac{t}{N\cdot SNR}\right)^{\frac{N-1}{2}} \exp\{-t - N\cdot SNR\}\cdot I_{N-1}\left[2\sqrt{t\,N\cdot SNR}\right] dt$$

$$SNR = \frac{\sigma\cdot|\chi(\tau,\nu)|^2}{\sigma_d^2}$$

Integrazione incoerente quadratica (II)





Integrazione incoerente quadratica (III)

Portata radar in presenza di integrazione

$$SNR^N = \frac{1}{\gamma} SNR^1$$

Guadagno di integrazione γ

Nel caso di integrazione incoerente quadratica:

- $\gamma \cong N$ per elevati valori di rapporto segnale a rumore;
- $\gamma \cong \sqrt{N}$ per bassi valori di rapporto segnale a disturbo

$$L_i = 10\log_{10}\left(\frac{N}{\gamma}\right)$$

Perdita integrazione incoerente rispetto alla coerente L_i

- $L_i \cong 0$ dB per elevati valori di rapporto segnale a rumore;
- $Li \cong 10\log_{10}(\sqrt{N})$ per bassi valori di rapporto segnale a disturbo

$$R_{max} = \left[\frac{E_t G A_e \sigma}{(4\pi)^2 L k T_0 F \, SNR^* / \gamma}\right]^{1/4}$$

Il guadagno di integrazione γ consente di

- Aumentare la portata di $\gamma^{1/4}$ se si lascia inalterata l'energia trasmessa fissata per il caso di decisione su singolo impulso
- Trasmettere un'energia γ volte inferiore rispetto a quella fissata per il caso di decisione su singolo impulso se si lascia inalterata la portata

Albersheim's Equation

Albersheim's Equation

Albersheim's equation is a closed-form approximation to the SNR required to achieve the specified detection and false alarm probabilities for a nonfluctuating target in independent and identically distributed Gaussian noise. The approximation is valid for a linear detector and is extensible to the noncoherent integration of N samples.

Let

$$A = \ln \frac{0.62}{P_{FA}}$$

and

$$B = \ln \frac{P_D}{1-P_D}$$

where P_{FA} and P_D are the false alarm and detection probabilities.

Albersheim's equation for the required SNR in dB is:

$$\mathrm{SNR} = -5\log_{10} N + [6.2 + 4.54 / \sqrt{N + 0.44}]\log_{10}(A + 0.12AB + 1.7B)$$

where N is the number of noncoherently integrated samples.

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