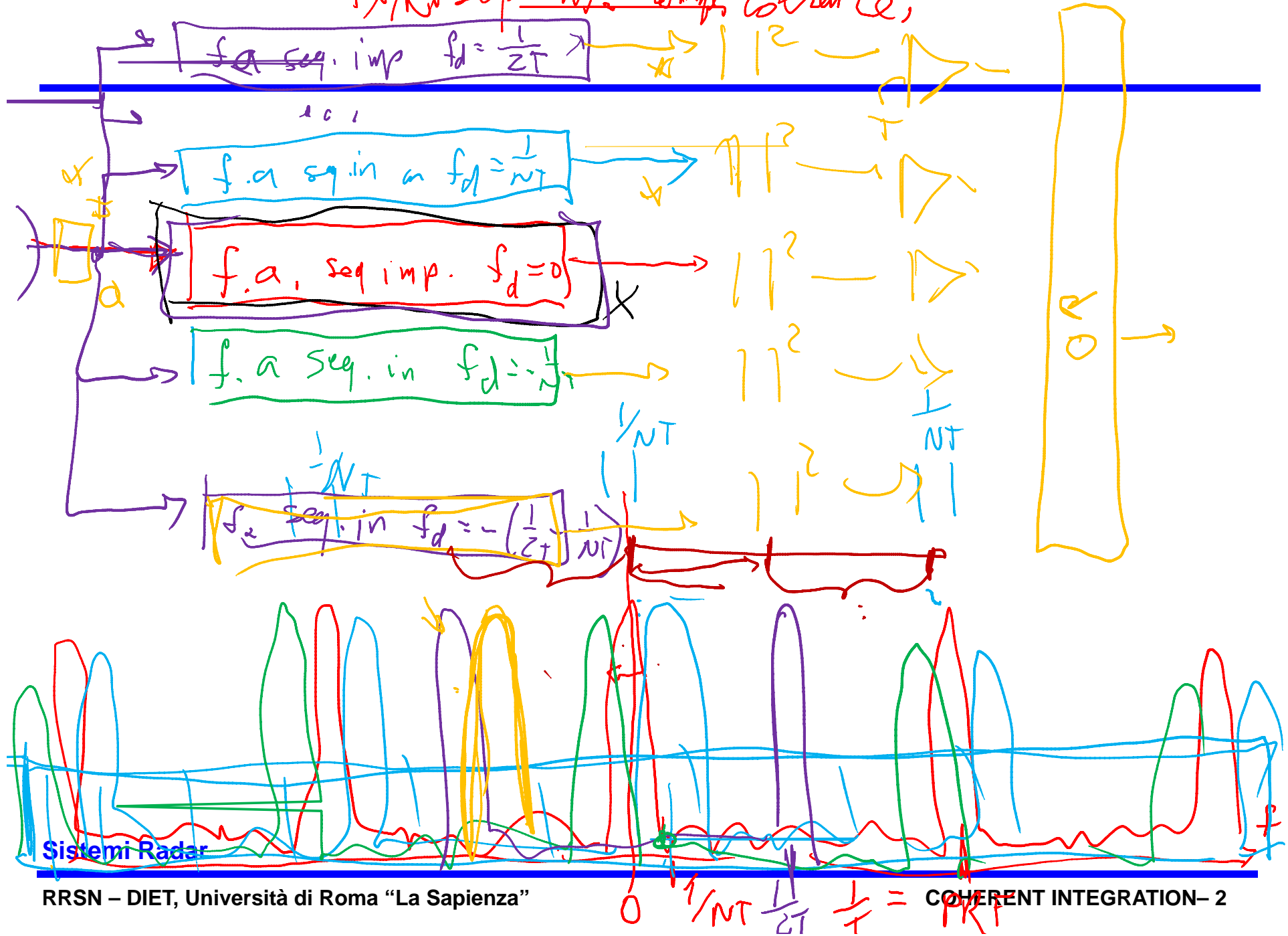
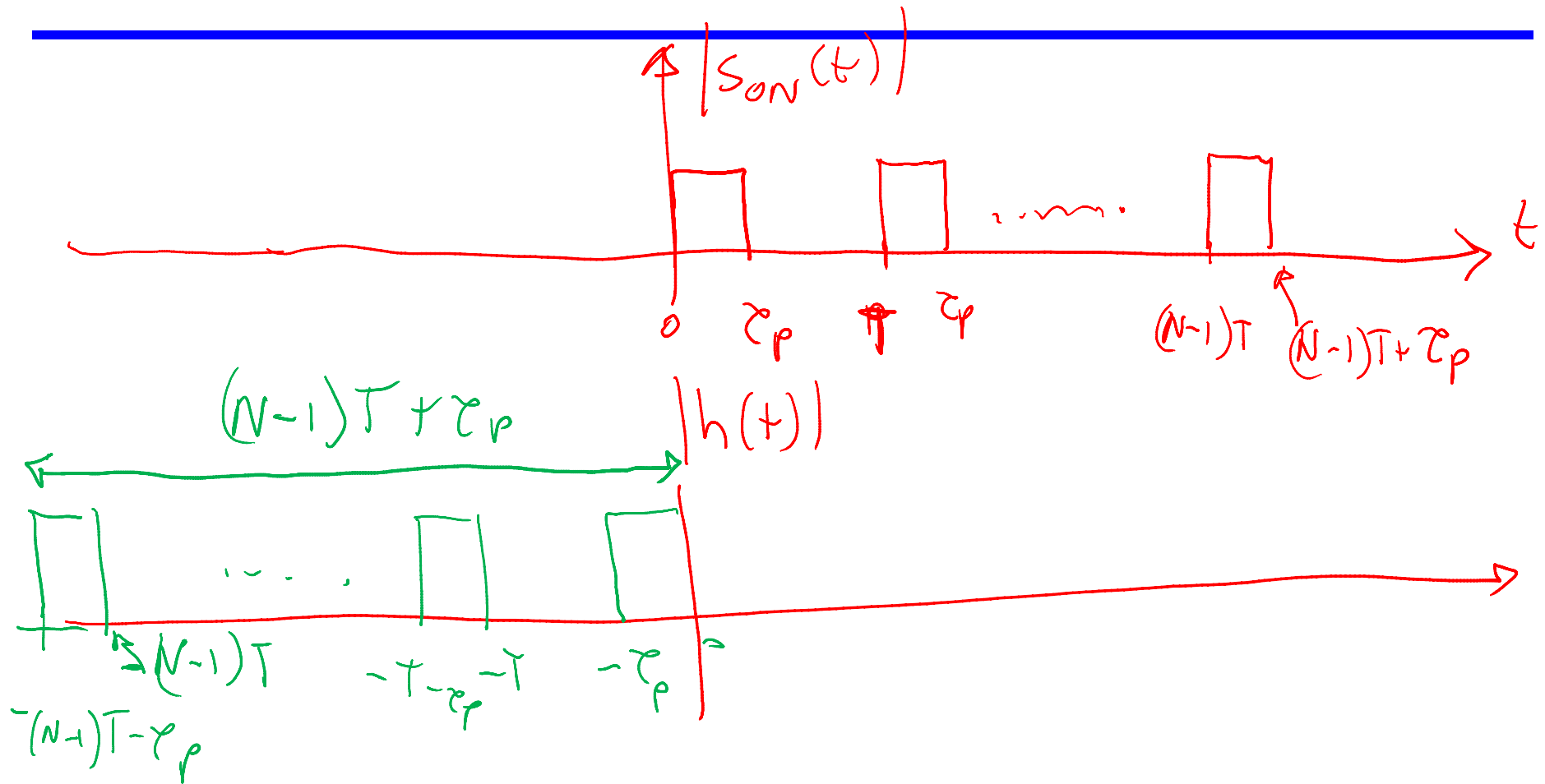

Integrazione coerente

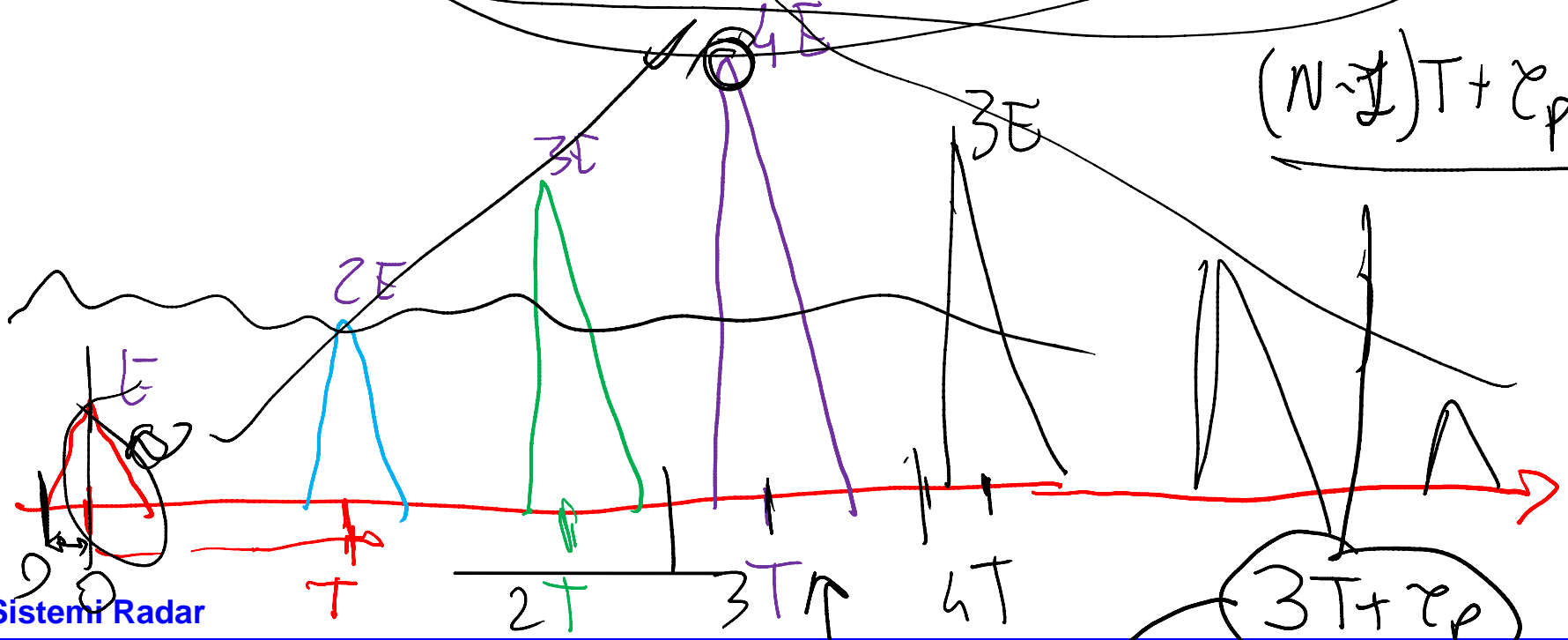
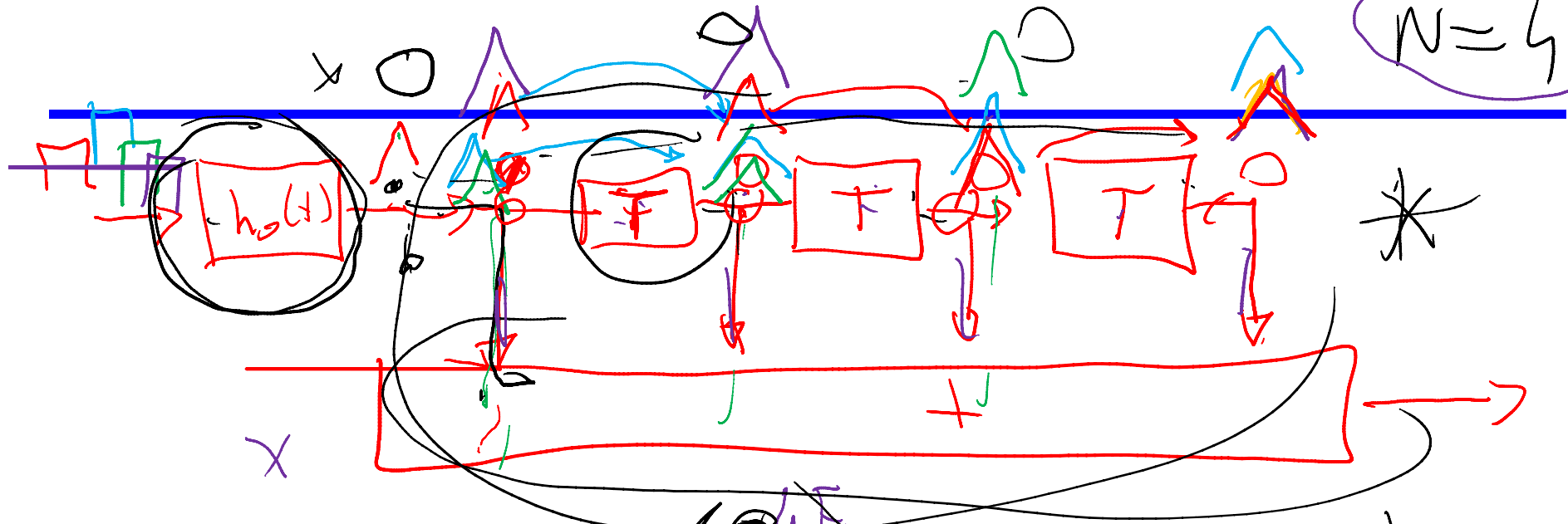
Pierfrancesco Lombardo

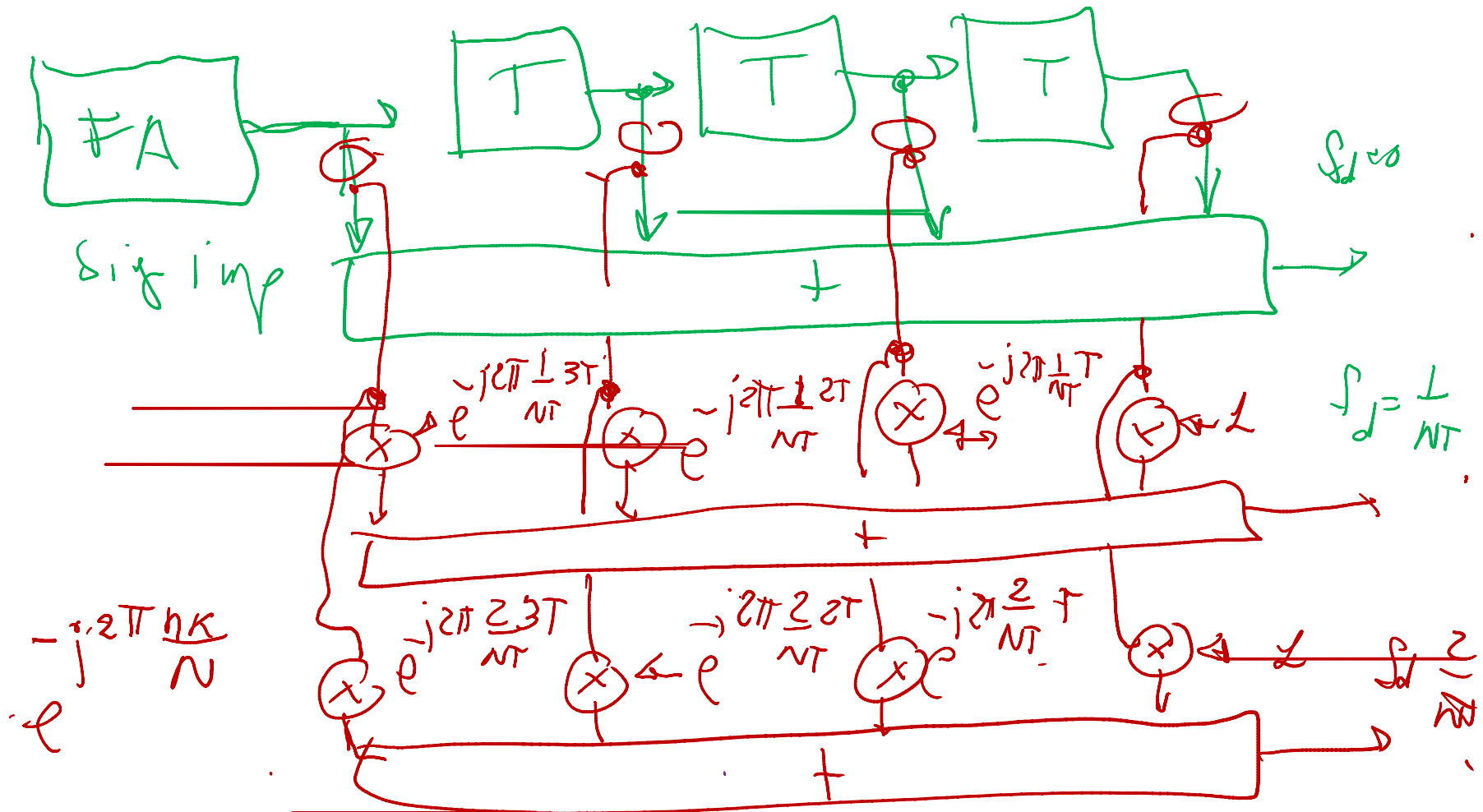
TX/RX Sequenz. δ imp. coerente,





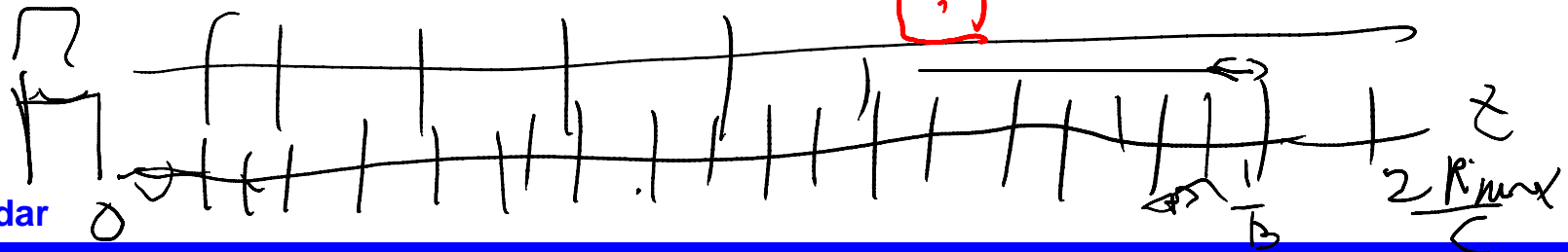
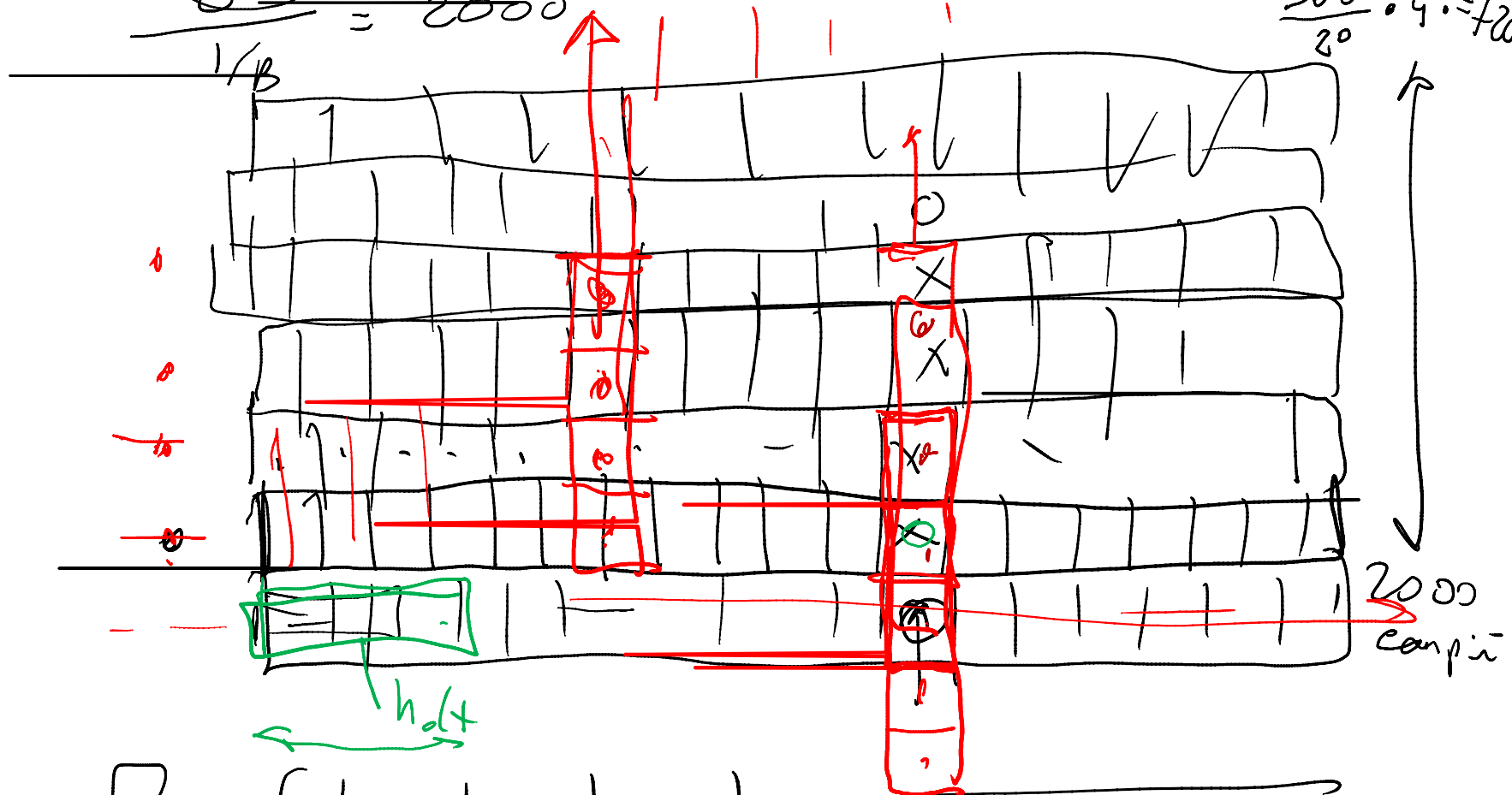
$N=4$

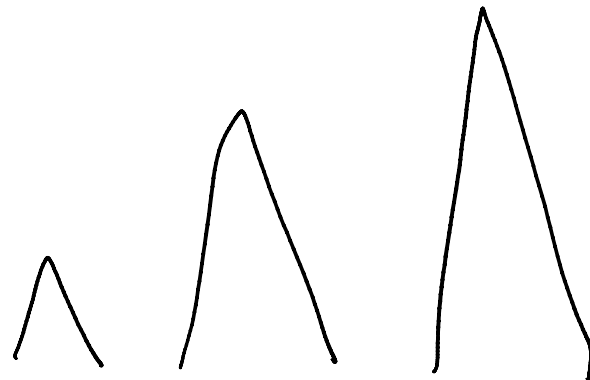
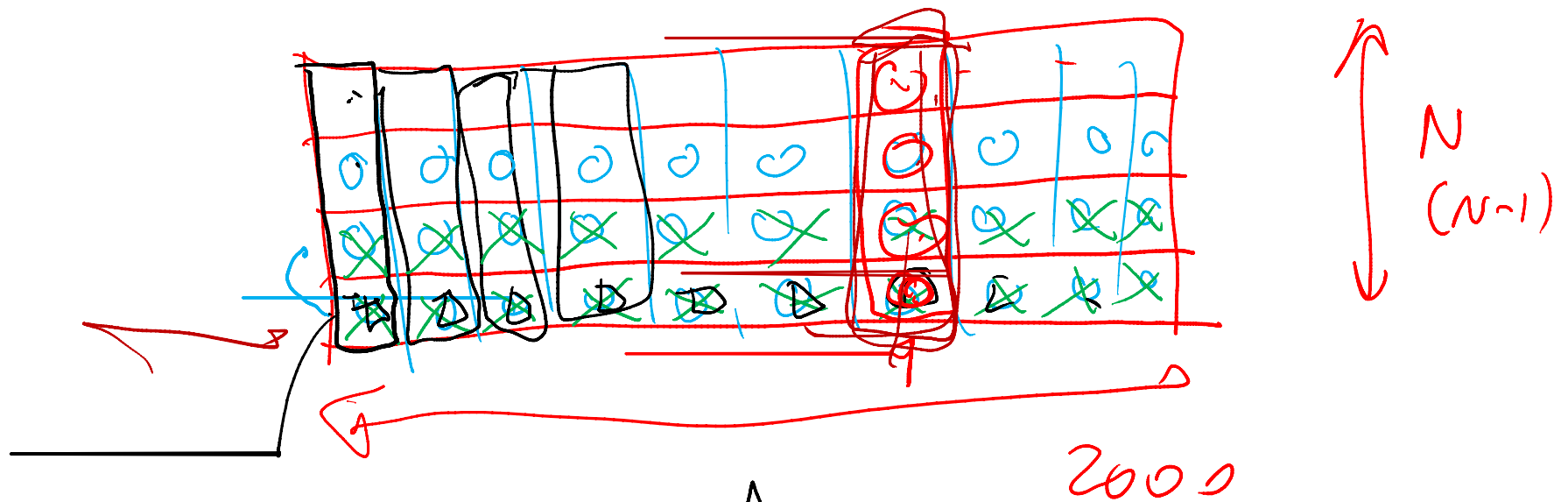
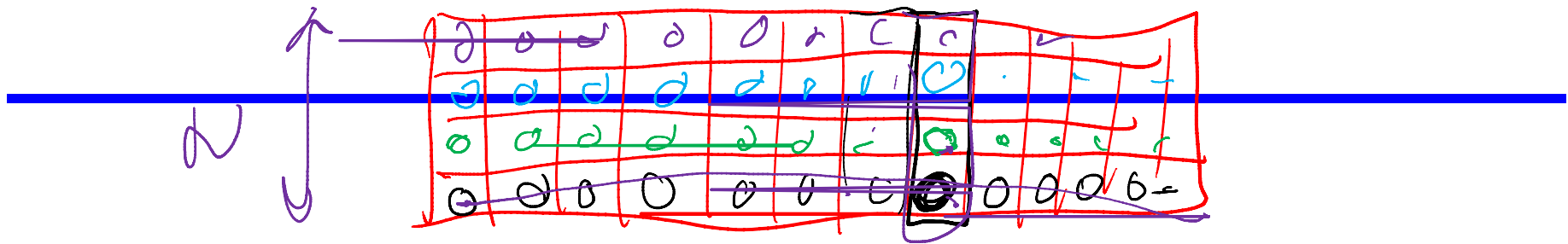




Sistemi Radar

$R_{\text{max}} = 300 \text{ km}$, $r_{\text{is dist}} = 150 \text{ m}$ — $\frac{300 \cdot 10^3}{150} = 2000$ $\frac{360^\circ}{20^\circ} \cdot N$
 $\frac{2R_{\text{max}}}{C} = 2000$ $\frac{360^\circ}{20^\circ} \cdot 4 = 720$

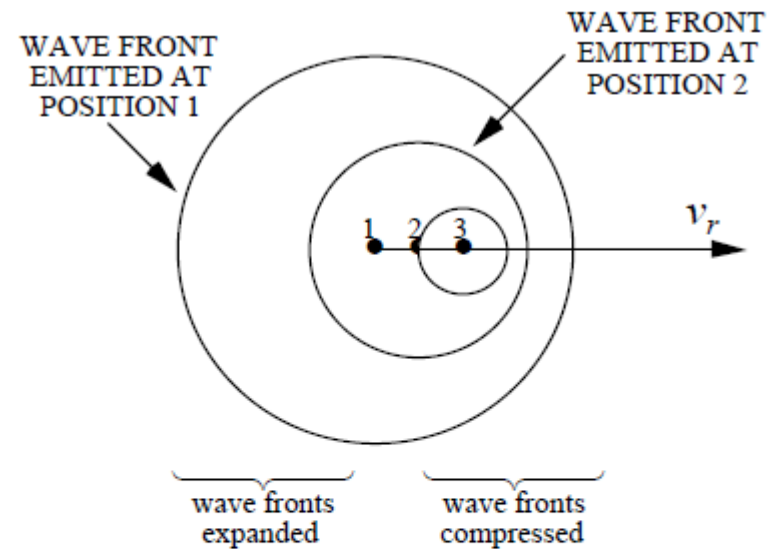
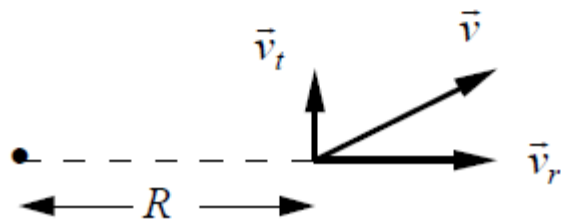




Doppler shift

- Targets in motion relative to the radar cause the return signal frequency to be shifted.
- A Doppler shift only occurs when the relative velocity vector has a radial component. In general there will be both radial and tangential components to the velocity

$$f_d = -2v_r / \lambda$$



$$R \text{ decreasing} \Rightarrow \frac{dR}{dt} < 0 \Rightarrow f_d > 0 \text{ (closing target)}$$

$$R \text{ increasing} \Rightarrow \frac{dR}{dt} > 0 \Rightarrow f_d < 0 \text{ (receding target)}$$

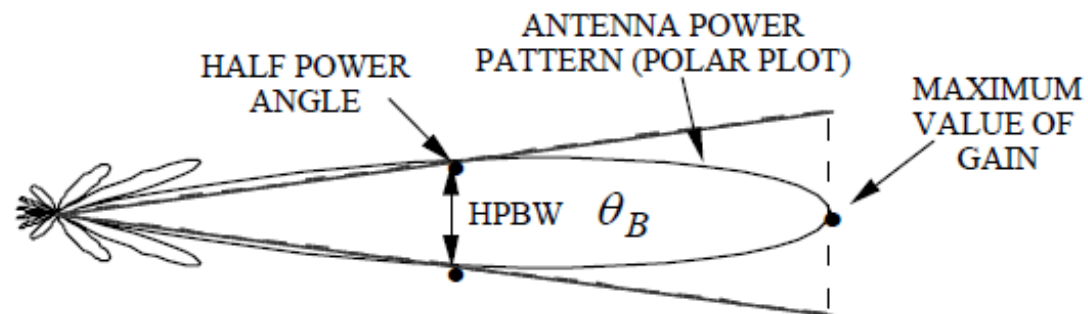
PRF & number of pulses

- Simple antenna model: constant gain inside the half power beamwidth (HPBW), zero outside. If the aperture has a diameter D with uniform illumination $\theta_B \approx \lambda / D$.
- The time that the target is in the beam (dwelt time, look time, or time on target) is t_{ot}

$$t_{ot} = \theta_B / \dot{\theta}_s$$

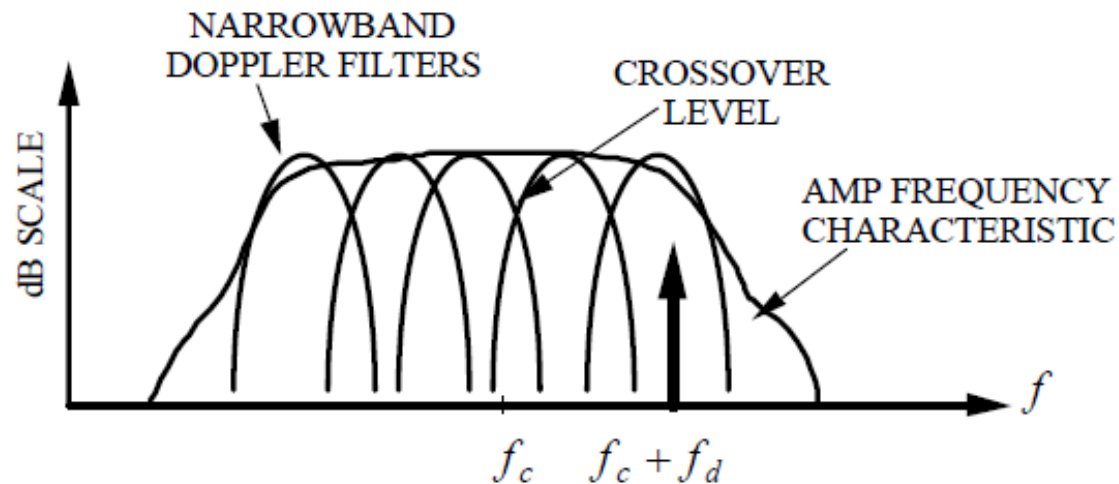
- The beam scan rate is ω_s in revolutions per minute or $\frac{d\theta_s}{dt} = \dot{\theta}_s$ in degrees per second.
- The number of pulses that will hit the target in this time is

$$n_B = t_{ot} f_p$$



PRF & number of pulses

- The radar's operating band is divided into narrow sub-bands. Ideally there should be no overlap in sub-band frequency characteristics.
- The noise bandwidth of the Doppler filters is small compared to that of the radar's total bandwidth, which improves the SNR.
- Velocity estimates can be made by monitoring the power out of each filter.
- If a signal is present in a filter, the target's velocity range is known.



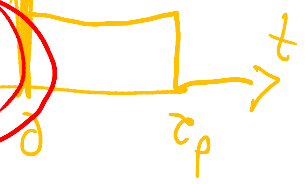
$$h(t) = s_{0N}^* (-(t - \tau_p)) = s_{0N}^* (\tau_p - t)$$

Filtro adattato alla sequenza (I)

$$s_{0N}(t) = \sum_{n=0}^{N-1} s_0(t - n \cdot T)$$

$$s_0(t) = \frac{1}{\sqrt{\tau_p}} e^{j\phi(t)} \text{rect}_{\tau_p} \left(t - \frac{\tau_p}{2} \right)$$

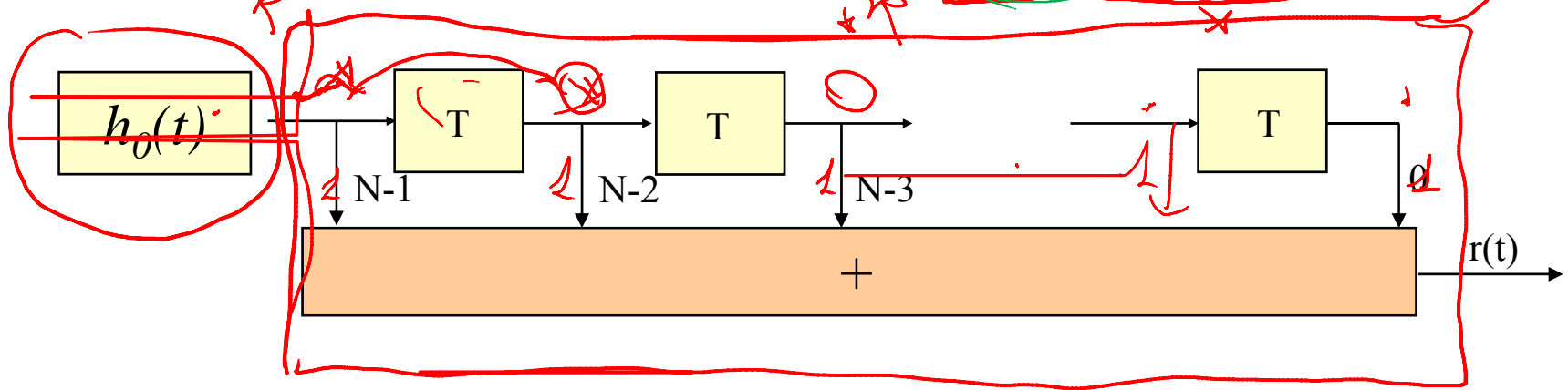
$$h_0(t) = s_0^*(\tau_p - t)$$



$$h(t) = s_{0N}^* [(N-1)T + \tau_p - t] = \sum_{n=0}^{N-1} s_0^* [(N-1)T + \tau_p - t - n \cdot T]$$

rit

$$h(t) = s_0^*[\tau_p - t] * \sum_{n=0}^{N-1} \delta[t - (N-1-n)T] = h_0(t) * \sum_{n=0}^{N-1} \delta[t - (N-1-n)T]$$

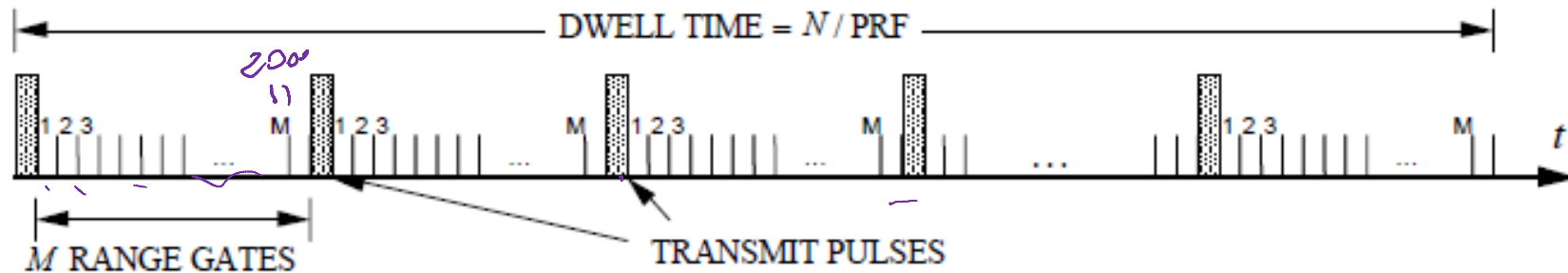


$$h_0(t) = s_0^*(\tau_p - t) = \frac{1}{\sqrt{\tau_p}} e^{-j\phi(\tau_p - t)} \text{rect}_{\tau_p} \left(\frac{\tau_p}{2} - t \right)$$

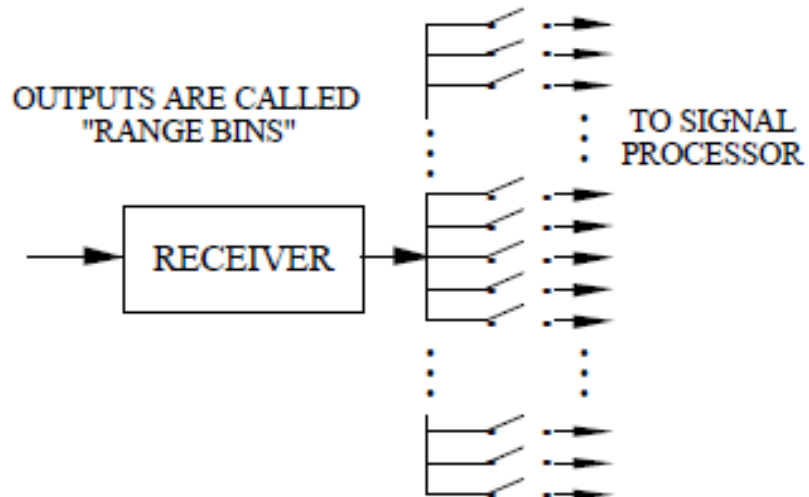
Sistemi Radar

Range gates

- Typical pulse train and range gates



- Analog implementation of range gates

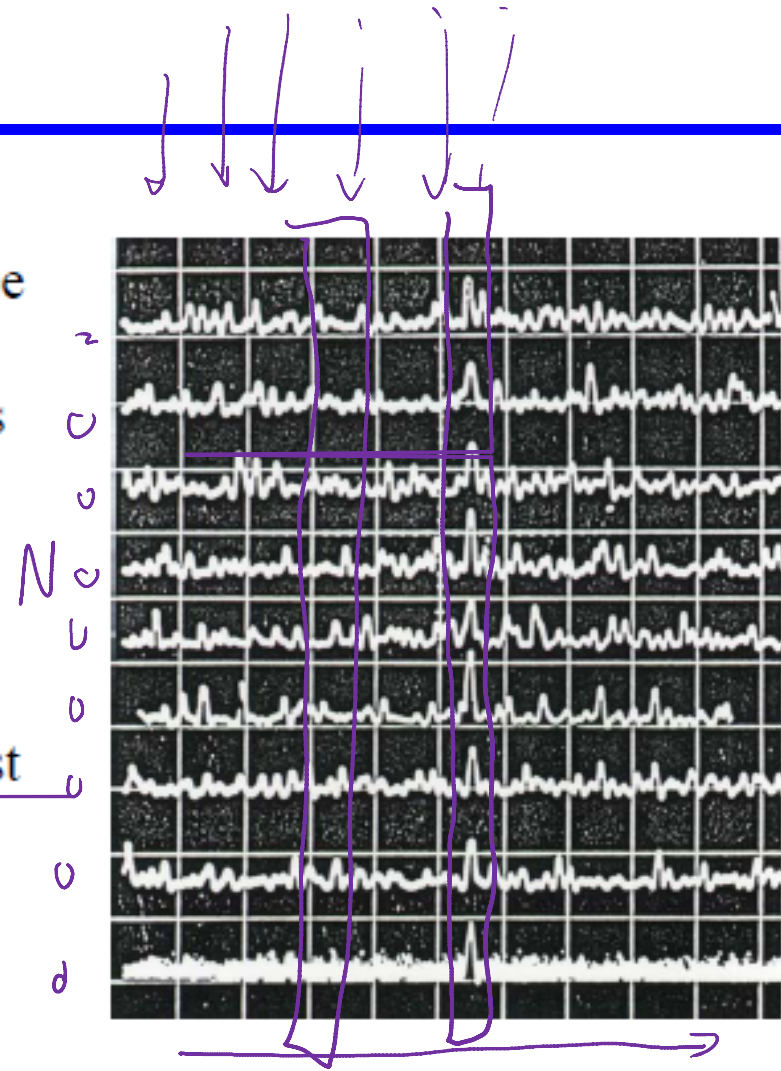


- Gates are opened and closed sequentially
- The time each gate is closed corresponds to a range increment
- Gates must cover the entire interpulse period or the ranges of interest
- For tracking a target a single gate can remain closed until the target leaves the bin

25

Integration of pulses

- Noncoherent integration (postdetection integration): performed after the envelope detector. The magnitudes of the returns from all pulses are added. SNR increases approximately as \sqrt{N} .
- Coherent integration (predetection integration): performed before the envelope detector (phase information must be available). Coherent pulses must be transmitted. The SNR increases as N .
- The last trace shows a noncoherent integrated signal.
- Integration improvement an example of processing gain.



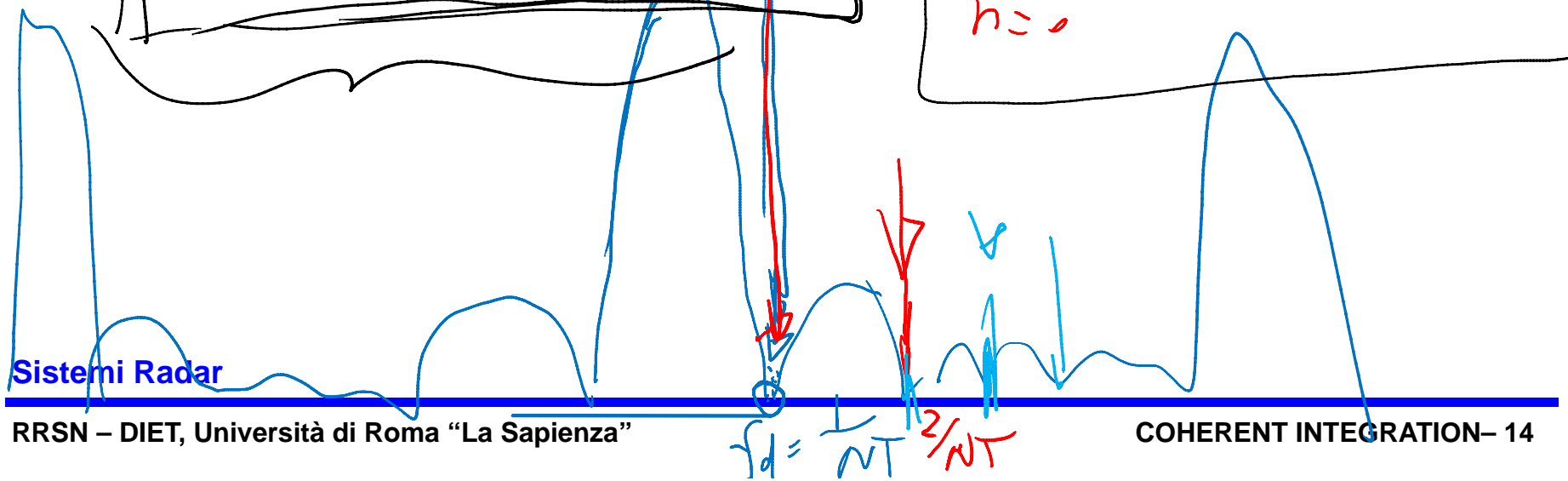
From Byron Edde, *Radar: Principles, Technology, Applications*, Prentice-Hall

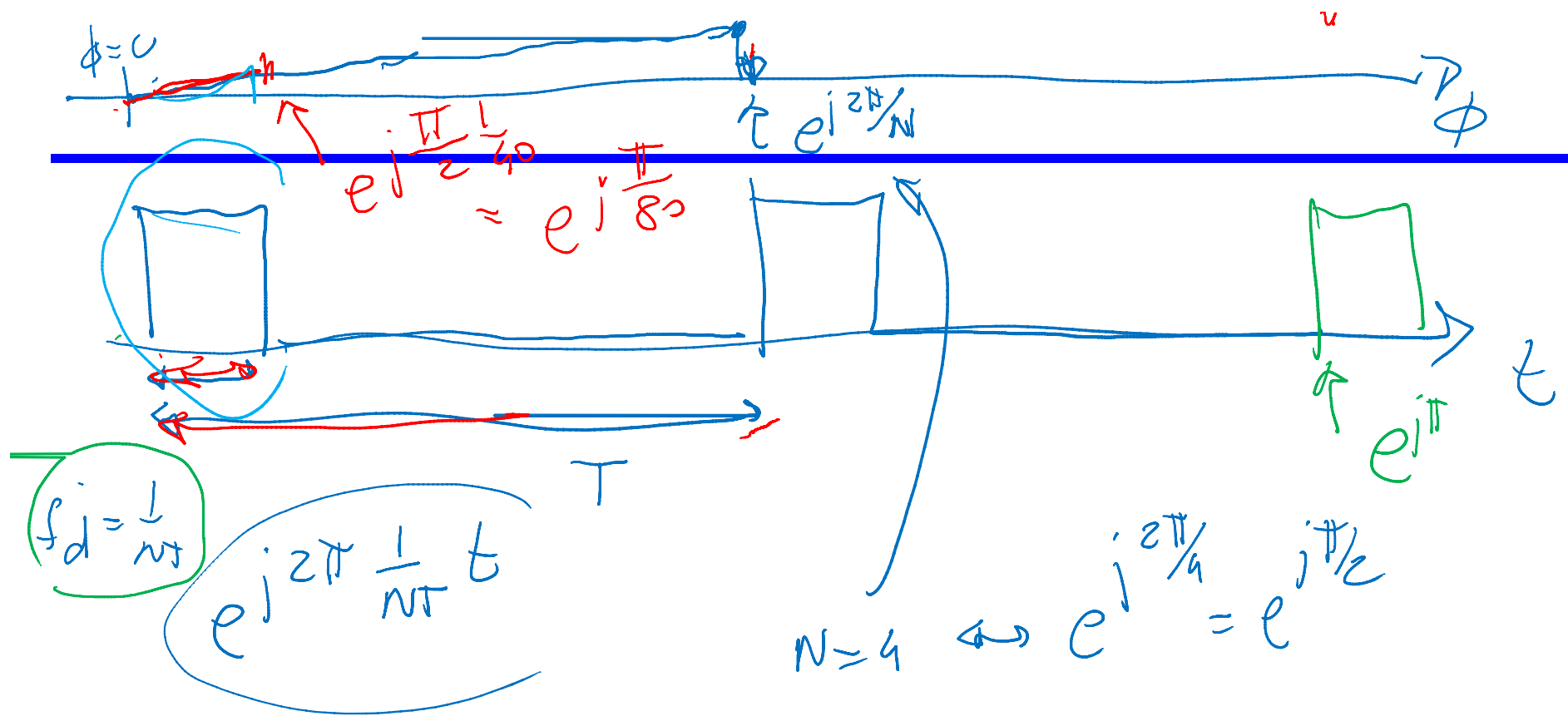
$$S_{ON}(t) = \sum_{n=0}^{N-1} s_o(t-nT) \cdot e^{+j 2\pi f_d t}$$

$$h(t) = \sum_{n=0}^{N-1} s_o^* \left((N-1)T + \tau_p - t - nT \right) e^{j 2\pi f_d \left((N-1)T + \tau_p - t - nT \right)}$$

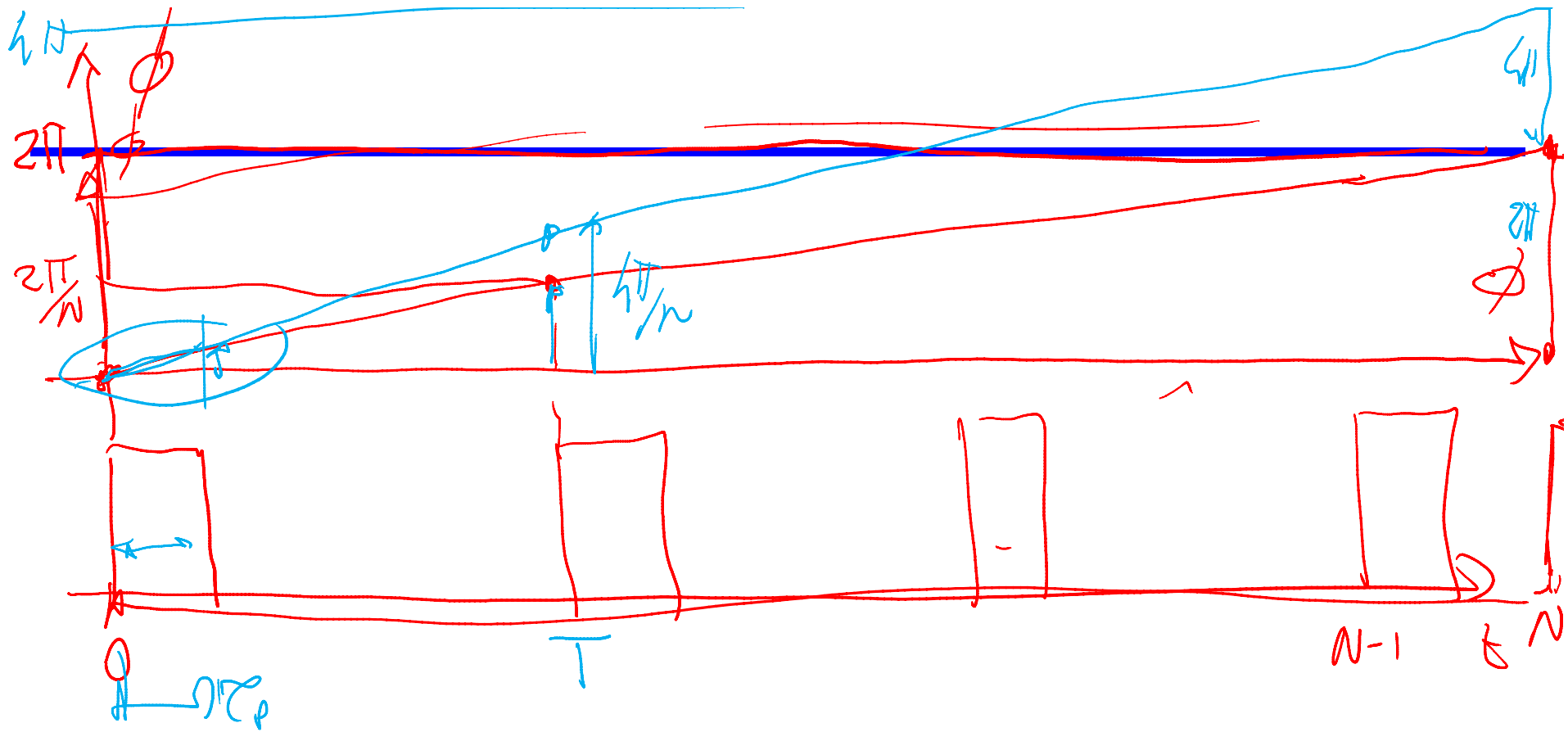
$$s_o^* \left(\tau_p - t \right) e^{j 2\pi f_d \left(\tau_p - t \right)}$$

$$\sum_{n=0}^{N-1} e^{j 2\pi f_d \left((N-1-n)T \right)} s_o \left(t - (N-1-n)T \right)$$

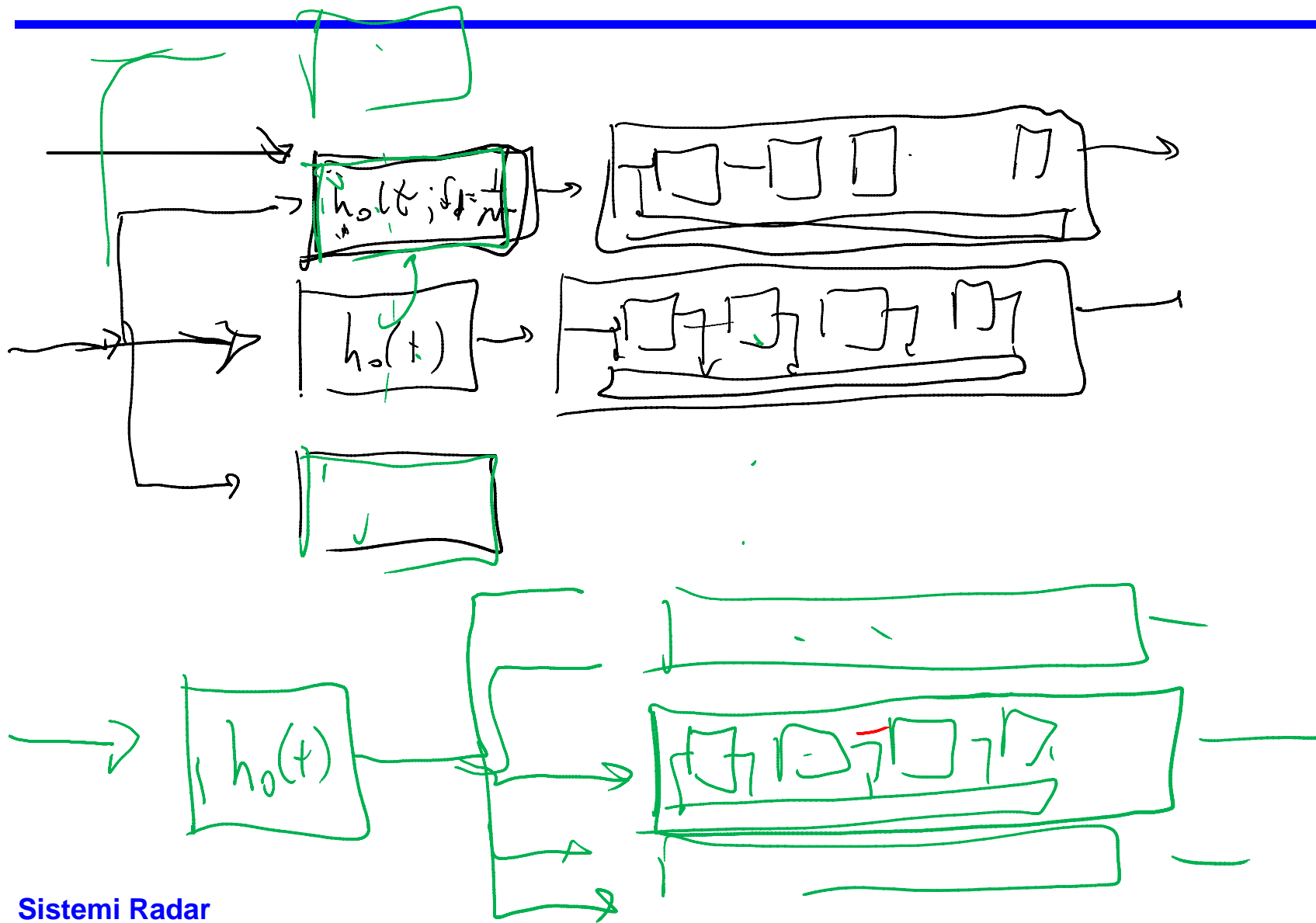


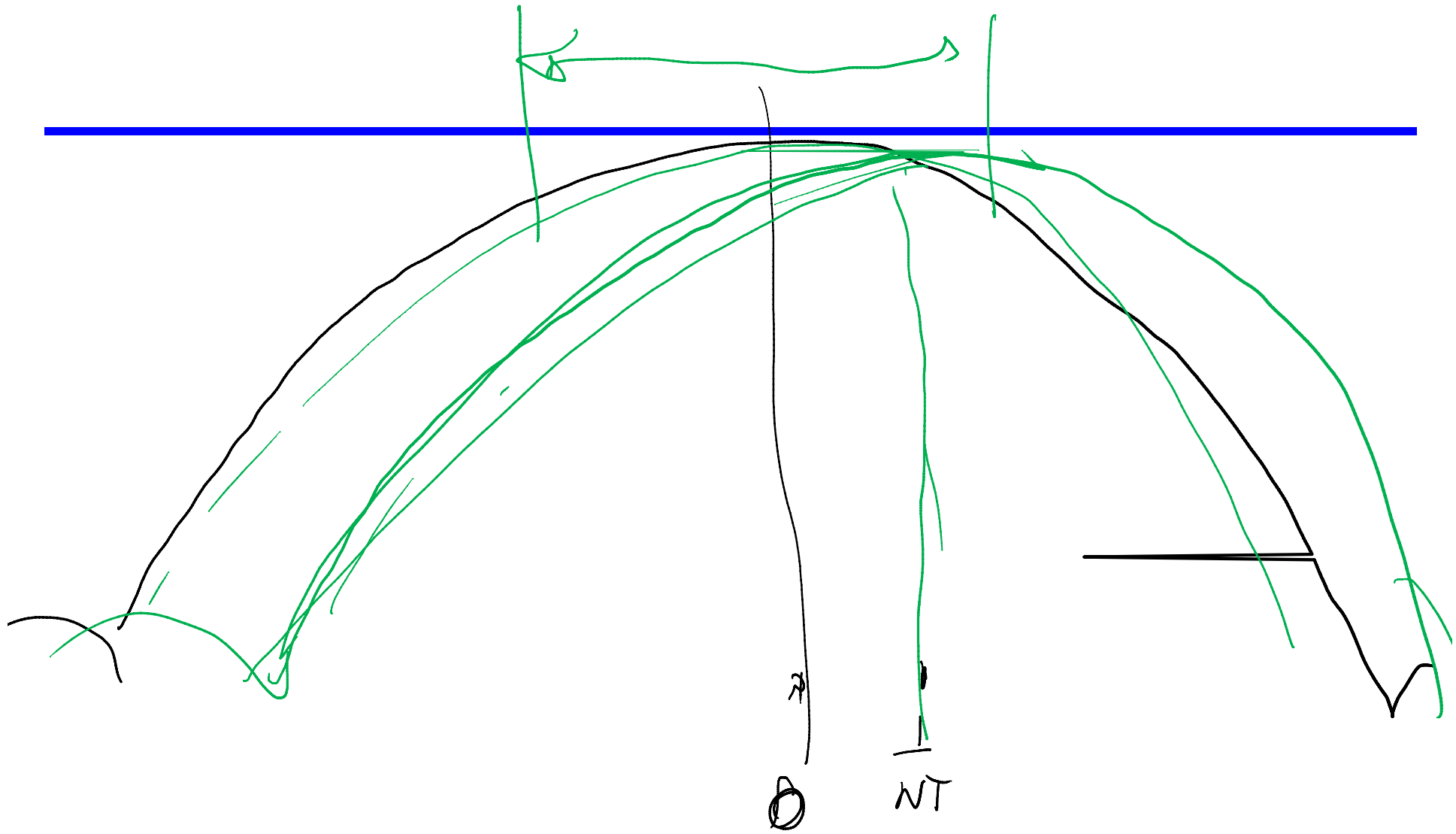


$$\frac{1}{T_p} = \frac{50 \text{ ms}}{2 \text{ ms}} = \frac{50}{200} = \frac{1}{40}$$



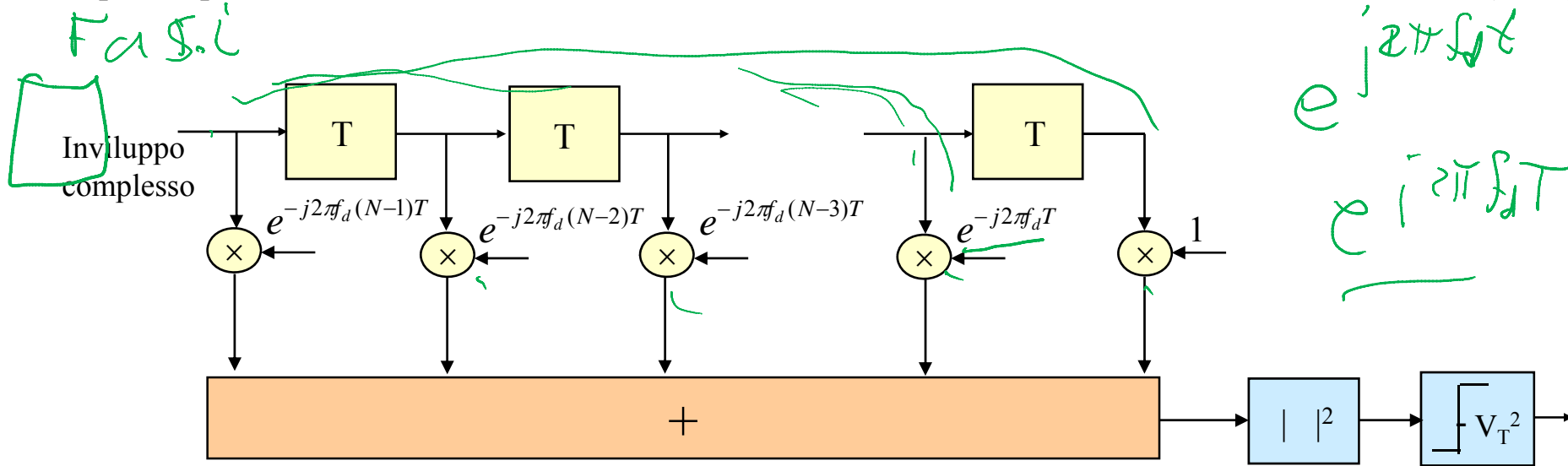
$$f_d = \frac{1}{NT} \quad e^{i 2\pi f_d t} = e^{i \frac{2\pi}{NT} t} = \begin{cases} \frac{2\pi}{N}, & t = T \\ 2\pi, & t = N \end{cases}$$



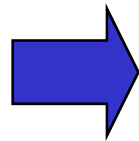


Filtro adattato alla sequenza (V)

Per poter operare coerentemente è necessario riallineare in fase i ritorni cioè considerare lo schema (filtro):



frequenza Doppler f_d
del bersaglio incognita



- Si considera un insieme discreto di valori della frequenza Doppler e si compensano i valori selezionati

Insieme dei valori considerati i cui corrispondenti sfasamenti sono compensati (N schemi come sopra ciascuno relativo a uno degli N valori di f_d presi in considerazione: **banco di N filtri**).

$$f_d = \frac{k}{NT} \quad k = 0 \dots N-1$$

$$\Rightarrow e^{-j2\pi f_d nT} = e^{-j2\pi \frac{kn}{N}}$$

Generico termine di riallineamento della fase dell'n-esimo impulso

Sistemi Radar

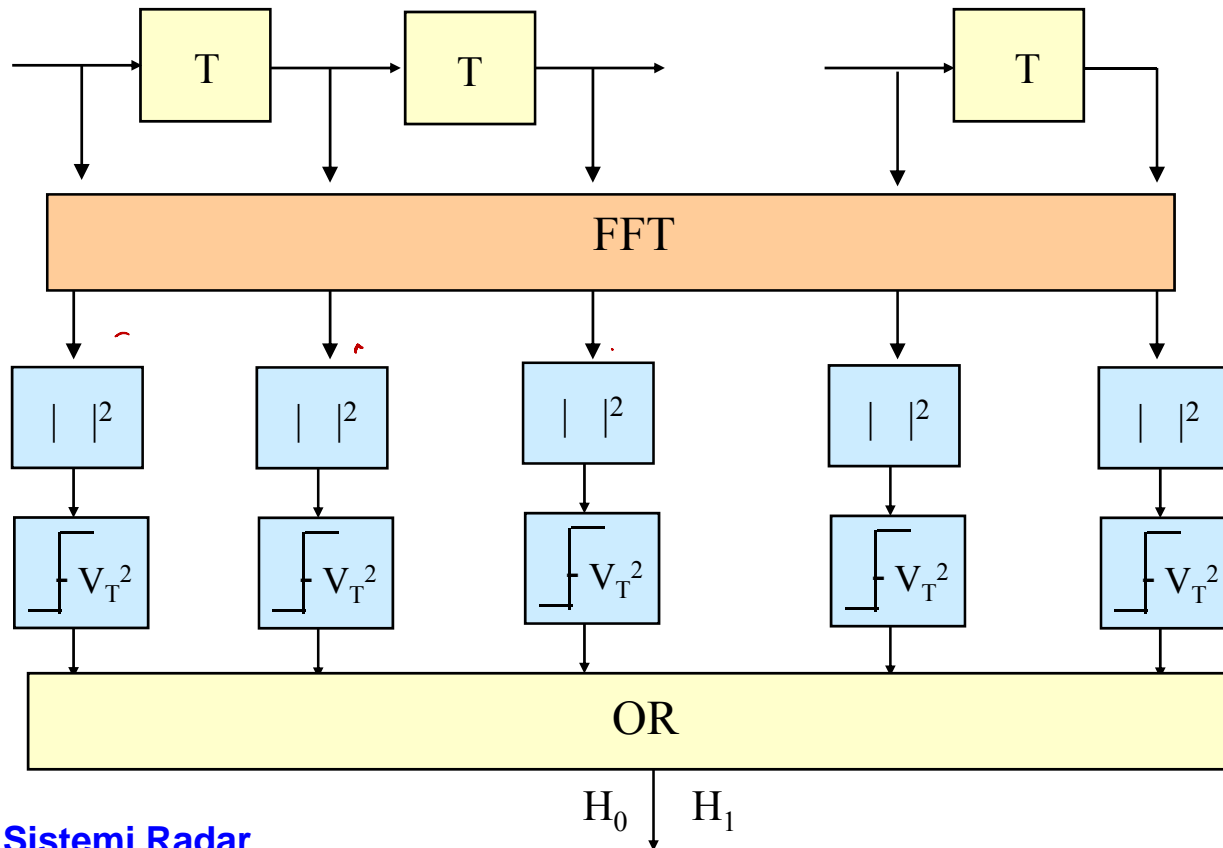
Filtro adattato alla sequenza (VI)

Sono considerati N diversi rami ciascuno dei quali calcola:

$$\sum_{n=0}^{N-1} \bar{w}(t-nT) e^{-j2\pi \frac{kn}{N}} \quad k = 0 \dots N-1$$

Trasformata discreta di Fourier valutata alla frequenza k/N

Lo schema di elaborazione per l'integrazione coerente diviene:



- Si decide per H_1 se almeno una delle uscite da 1 (OR)
- Dalla conoscenza dell'uscita che da 1 è possibile stimare la Doppler del bersaglio e quindi la sua velocità radiale

Filtro adattato alla sequenza (VII)

A meno della portante la forma d'onda trasmessa è del tipo:

$$\sum_{n=0}^{N-1} f(t-nT)$$

Per il nostro caso particolare $f(t)=\text{rect}_\tau(t)$ ma il discorso che segue è valido qualunque sia la forma d'onda trasmessa sulla singola interrogazione.

La funzione di trasferimento del filtro adattato è quindi data da:

$$H_{MF}(f) = F^*(f) \left[\sum_{n=0}^{N-1} e^{-j2\pi f n T} \right]^* = F^*(f) \left[\frac{1 - e^{-j2\pi f N T}}{1 - e^{-j2\pi f T}} \right]^*$$

$F^*(f) e^{j\pi f (N-1) T}$

$\frac{\sin(\pi f N T)}{\sin(\pi f T)}$

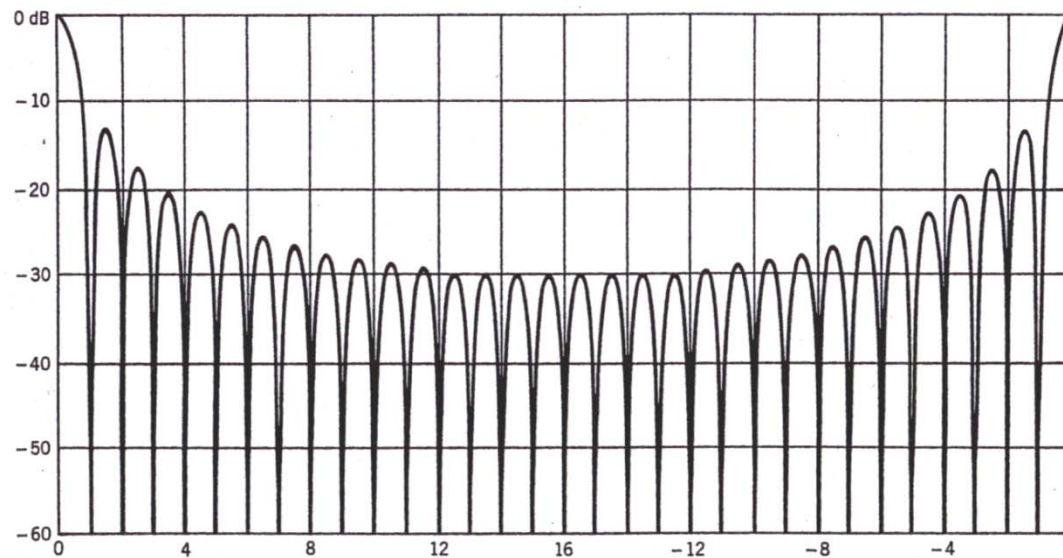
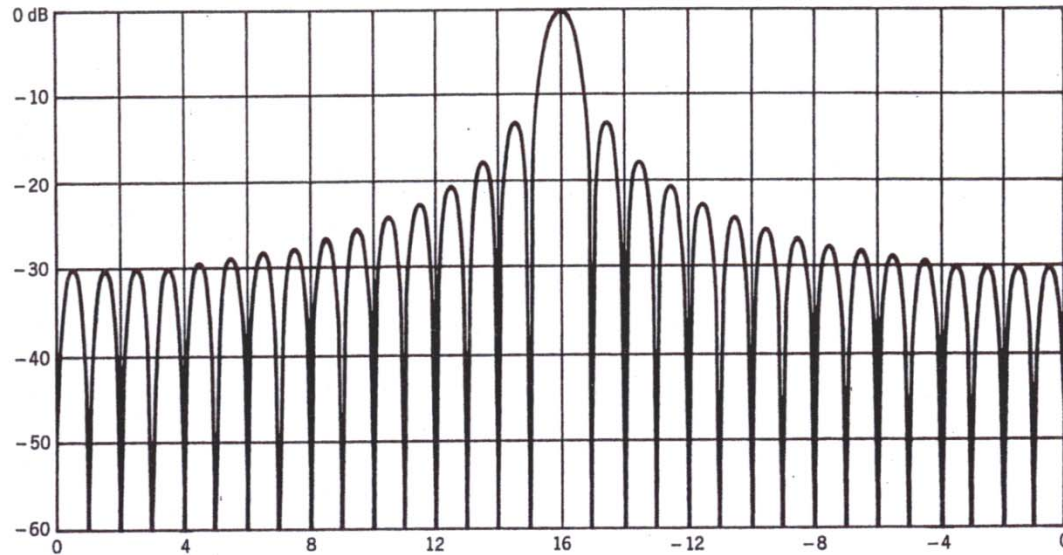
Filtraggio adattato alla
forma d'onda
trasmessa

Filtraggio
azimutale

Il filtro adattato ad un treno di interrogazioni si decompone nel filtro adattato alla forma d'onda trasmessa sulla singola interrogazione ($F^*(f)$) e nel filtro che opera l'integrazione azimutale ($\sin(\pi f N T)/\sin(\pi f T)$): nel caso che la forma d'onda trasmessa sia un impulso rettangolare si ha

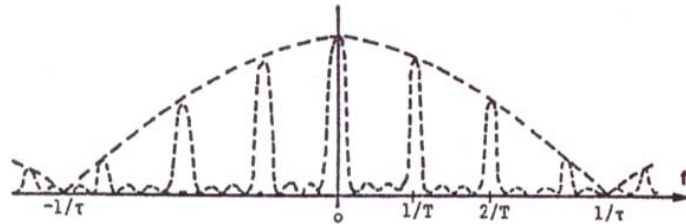
$$H_{MF}(f) = \text{sinc}(\pi f \tau) \frac{\sin(\pi f N T)}{\sin(\pi f T)}$$

Filtro adattato alla sequenza (VIII)



Filtro adattato alla sequenza (IX)

FILTRO A PETTINE



filtro adattato un treno di impulsi rettangolari con Doppler nulla \Rightarrow per tenere conto delle diverse Doppler si utilizza un banco di N filtri centrati sulle frequenze k/NT con $k=0\dots N-1$ ciascuno largo $1/NT$

Lettura in termini del teorema del campionamento:

- ▶ frequenza di campionamento PRF: lo spettro è periodico con periodo PRF (quindi si considera una sola campata tra 0 e PRF ad es.);
- ▶ campionamento tramite impulsi rettangolari di durata τ anziché impulsi di Dirac: lo spettro è involuppato dal $\text{sinc}(\pi f\tau)$;
- ▶ N campioni anziché infiniti campioni: lo spettro ha le righe costituite da forme di tipo $\text{sinc}(\pi fNT)$ (è il generico filtro del banco: quando $N \rightarrow \infty$ il $\text{sinc}(\pi fNT)$ diventa un Dirac e il potere risolutore in Doppler $\rightarrow 0$);

(sezione della FdA per ritardo nullo)