1. Base $|\ell_A m_A; \ell_B m_B\rangle \chi_S^{S_z}$

(a) $|00;00\rangle \chi_0^0$ deg $= 1$ $E = -\frac{3\epsilon}{4}$

(b) $\frac{1}{\sqrt{2}}\left(|1m;00\rangle + |00;1m\rangle\right)\chi_0^0$ deg $= 3$ $E = \frac{5\epsilon}{4}$

(c) $\frac{1}{\sqrt{2}}\left(|1m;00\rangle - |00;1m\rangle\right)\chi_1^{S_z}$ deg $= 9$ $E = \frac{9\epsilon}{4}$

2. a) ψ_1 è autofunzione, ψ_2 non è autofunzione

b) base $|\ell_A, \ell_B; \ell_{TOT}, m_{TOT}\rangle_T$

$$\psi_1 = \frac{1}{\sqrt{2}}\left(|1,1;1,0\rangle_T + |1,1;1,1\rangle_T\right)\chi_1^0$$

$$\psi_2 = N\left(|1,1;2-2\rangle_T + 4\pi|0,0;0,0\rangle_T\right)\chi_0^0 \qquad N = [1+16\pi^2]^{-1}$$

c) possibili valori $J^2 = 0, 2\hbar^2, 6\hbar^2$ (ψ_1)

$= 0, 6\hbar^2$ (ψ_2)

$$\psi_1 \begin{cases} P(6\hbar^2) = 7/12 \\ P(2\hbar^2) = 1/4 \\ P(0) = 1/6 \end{cases} \qquad \psi_2 = \begin{cases} P(6\hbar^2) = N^2 \\ P(0) = (4\pi)^2 N^2 \end{cases}$$

3. a) non è unitario

3. b) $|\phi\rangle = \frac{1}{\sqrt{2}}\left(e^{\pi/2}|1,1;1,1\rangle_T + e^{\pi}|1,1;1,0\rangle_T\right)$

$|\phi\rangle$ non è normalizzato:

stato normalizzato:

$$|\phi'\rangle = K\left(|1,1;1,1\rangle_T + e^{\pi/2}|1,1;1,0\rangle_T\right)$$

$$K = (1+e^{\pi})^{-1/2}$$

$$P(L_{tz}=0) = e^{\pi}K^2 \qquad P(L_{tz}=\hbar) = K^2$$

$$\psi_2(t=0) = N\left(|1,1;2,-2\rangle_T + 4\pi\,|0,0;0,0\rangle_T\right)\chi_0^0$$

$$\psi_2(t) = N\left(|1,1;2,-2\rangle_T + 4\pi\,e^{-4i\epsilon t/\hbar}\,|0,0;0,0\rangle_T\right)\chi_0^0$$

$$\langle\psi_2(t)|\frac{x_A x_B}{r^2}|\psi_2(t)\rangle =$$

$$= N^2\left({}_T\langle -2,2;1,1| + 4\pi\,e^{4i\epsilon t/\hbar}\,{}_T\langle 0,0;00|\right) x_A x_B$$

$$\left(|1,1;2,-2\rangle_T + 4\pi\,e^{-4i\epsilon t/\hbar}\,|0,0,00\rangle_T\right)$$

$$= N^2\,{}_T\langle -2,2;1,1|x_A x_B|1,1;2,-2\rangle_T \longrightarrow \phi \text{ [dispari sotto } x_A\to -x_A]$$

$$+ N^2(4\pi)^2\,{}_T\langle 0,0;0,0|x_A x_B|0,0;00\rangle_T \longrightarrow \phi \text{ [dispari sotto } x_A\to -x_A)$$

$$+ 4\pi N^2 e^{4i\epsilon t/\hbar}\,{}_T\langle 0,0;0,0|x_A x_B|1,1;2,-2\rangle_T$$

$$+ 4\pi N^2 e^{-4i\epsilon t/\hbar}\,{}_T\langle -2,2;1,1|x_A x_B|0,0;00\rangle_T$$

$$= 4\pi N^2 e^{4i\epsilon t/\hbar}\,\langle 0,0;0,0|x_A x_B|1,1;2,-2\rangle + \text{compl. coniugato}$$

$$\langle 0,0;0,0|x_A x_B|1,1;2,-2\rangle =$$

$$\int d\Omega_A\, d\Omega_B\, Y_{A0}^{0*}\, Y_{B0}^{0*}\, x_A x_B\, Y_{A1}^{-1}\, Y_{B1}^{-1} =$$

$$\left[\int d\Omega\, Y_0^{0*}\, x\, Y_1^{-1}\right]^2 = \left[\int d\cos\theta\, d\phi\, \frac{1}{\sqrt{4\pi}}\sin\theta\cos\phi\,\sqrt{\frac{3}{8\pi}}\,e^{-i\phi}\sin\theta\right]^2$$

$$= \frac{3}{32\pi^2}\left[\int_{-1}^{1} d\cos\theta\,(1-\cos^2\theta)\int_0^{2\pi} d\phi\,\frac{1}{2}\left(e^{i\phi}+e^{-i\phi}\right)e^{-i\phi}\right]^2 = \frac{3}{32\pi^2}\cdot\left(\frac{4}{3}\cdot\pi\right)^2$$

Risultato per l'elemento di matrice:

$$\frac{4\pi}{3}N^2\cos\frac{4\epsilon t}{\hbar}$$