PhD Handout: Optimal fiscal and monetary policy - The Ramsey problem

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1 Competitive equilibrium

The representative household (i) maximizes the following utility function

$$U = E_{t=0} \sum_{t=0}^{\infty} \beta^t u\left(c_{t,i}, l_{t,i}\right); \quad u\left(c_{t,i}, l_{t,i}\right) = \ln c_{t,i} + \eta \ln\left(1 - l_{t,i}\right) \tag{1}$$

where $\beta \in (0,1)$ is the intertemporal discount rate, $c_{t,i} = \left(\int_0^1 c_{t,i}(j)^{\rho} dj\right)^{\frac{1}{\rho}}$ is a consumption bundle, $l_{t,i}$ denotes the individual labor supply. Note $u_c\left(c_t, l_t\right) > 0$ and $u_{cc}\left(c_t, l_t\right) < 0$; $u_l\left(c_t, l_t\right) < 0$ and $u_{ll}\left(c_t, l_t\right) > 0$; $u_{cl}\left(c_t, l_t\right) = 0$.

The flow budget constraint in period t is given by

$$c_{t,i}\left(1 + s(v_{t,i})\right) + \frac{M_{t,i}}{P_t} + \frac{B_{t,i}}{P_t} = \left(1 - \tau_t\right) w_{t,i} l_{t,i} + \frac{M_{t-1,i}}{P_t} + \theta_t + \frac{R_{t-1} B_{t-1,i}}{P_t} + t_t \tag{2}$$

where $P_t = \left(\int_0^1 p_t(j)^{\frac{\rho}{\rho-1}} di\right)^{\frac{\rho-1}{\rho}}$ is the consumption price index and $\rho = \frac{1+d'}{d'} \in (0,1)$ ($d' < -1$: elasticity of demand); $w_{t,i}$ is the real wage; τ_t is the labor income tax rate; t_t denotes real fiscal transfers; θ_t are firms profits; R_t is the gross nominal interest rate, $B_{t,i}$ is a nominally riskless bond that pays one unit of currency in period $t+1$. $M_{t,i}$ defines nominal money holdings to be used in period $t+1$ in order to facilitate consumption purchases.

Consumption purchases are subject to a transaction cost:

$$s(v_{t,i}), \quad s'(v_{t,i}) > 0 \text{ for } v_{t,i} > v^* \tag{3}$$

where $v_{t,i} = \frac{P_{t,i} c_{t,i}}{M_{t,i}}$ is the household's consumption-based money velocity.

The features of $s(v_{t,i})$ are such that a satiation level of money velocity ($v^* > 0$) exists where the transaction cost vanishes and, simultaneously, a finite demand for money is associated to a zero nominal interest rate.

The transaction cost is parameterized as:

$$s(v_{t,i}) = Av_{t,i} + \frac{B}{v_{t,i}} - 2\sqrt{AB} \tag{4}$$

The first-order conditions of the household's maximization problem are:

$$c_t(j) = c_t \left(\frac{p_t(j)}{P_t}\right)^{\frac{1}{\rho-1}} \tag{5}$$

$$\lambda_t = \frac{u_c\left(c_t, l_t\right)}{1 + s(v_t) + v_t s'(v_t)} \tag{6}$$

$$\lambda_t = \beta E_t \left(\frac{\lambda_{t+1} R_t}{\pi_{t+1}}\right) \tag{7}$$

$$\lambda_t \left(1 - \tau_t\right) w_t = -u_l\left(c_t, l_t\right) \tag{8}$$

$$\frac{R_t - 1}{R_t} = s'(v_t) v_t^2 \tag{9}$$

When solving its optimization problem, the household takes as given goods and bond prices. As usual, we also assume that the household is subject to a solvency constraint that prevents him from engaging in Ponzi schemes.

- Equation (5) is the demand for the good j.
- Equation (6) states that the transaction cost introduces a wedge between the marginal utility of consumption and the marginal utility of wealth that vanishes only if $v = v^*$.
- Equation (7) is a standard Euler condition where $\pi_{t+1} = P_{t+1}/P_t$ denotes the gross inflation rate.
- Equation (8) defines the individual labor supply condition.
- Equation (9) implicitly defines the money demand function, such that

$$\frac{M_t}{P_t} = \left(\frac{R_t - 1}{R_t A} + \frac{B}{A}\right)^{-\frac{1}{2}} c_t \tag{10}$$

In the supply side, each firm (j) produces a differentiated good:

$$y_t(j) = z_t l_{t,j}, \tag{11}$$

where z_t denotes a productivity shock. We assume that $\ln z_t$ follows an $AR(1)$ process.

We assume a sticky price specification based on a quadratic cost of nominal price adjustment:

$$\frac{\xi_p}{2} y_t \left(\pi_t - 1\right)^2 \tag{12}$$

where $\xi_p > 0$ is a measure of price stickiness. We assume that the re-optimization cost is proportional to output.

In a symmetrical equilibrium the price adjustment rule satisfies:

$$\pi_t \left(\pi_t - 1\right) = E_t \beta \frac{y_{t+1}\lambda_{t+1}}{y_t \lambda_t} \left[\pi_{t+1}\left(\pi_{t+1} - 1\right)\right] + \frac{z_t \left(mc_t - \rho\right)}{\xi_p \left(1 - \rho\right)} \tag{13}$$

where

$$mc_t = \frac{1}{z_t} w_t \tag{14}$$

From (5) it would be straightforward to show that $\frac{1}{\rho} = \mu^p$ defines the price markup that obtains under flexible prices. Note that $\xi_p \to 0$ (flexible prices), $mc_t = \rho = \frac{1}{\mu^p}$, i.e., $\mu^p mc_t = 1$.

The aggregate resource constraint closes the model:

$$y_t = c_t \left(1 + s(v_t)\right) + g_t + \frac{\xi_p}{2} y_t \left(\pi_t - 1\right)^2 \tag{15}$$

2 Government problem

The government supplies an exogenous, stochastic and unproductive amount of public good g_t and implements exogenous transfers t_t. Government financing is obtained through a labor-income tax, money creation and issuance of one-period, nominally risk free bonds. The government's flow budget constraint is then given by

$$R_{t-1} b_{t-1} + g_t + t_t = \tau_t w_t l_t + \frac{M_t - M_{t-1}}{P_t} + b_t \tag{16}$$

where $b_t = \frac{B_t}{P_t}$ defines real debt; $\ln(g_t/y_t)$, is assumed to evolve exogenously following an independent $AR(1)$ process. We assume instead that the level of the real transfer (t_t/y_t) is non-stochastic.

The Ramsey policy is a set of plans $\{c_t, l_t, \lambda_t, mc_t, \pi_t, v_t, R_t, \tau_t, b_t\}_{t=0}^{+\infty}$ that maximizes the expected value of (1) subject to the government's flow budget constraint, (16), competitive equilibrium [(6), (7), (8), (9), (11), (13), (14), (15))], and to the exogenous fiscal and technology shocks.

Given (6), (8) and (14), labor tax revenues may be written as

$$\tau_t w_t l_t = \left(z_t mc_t + \frac{u_l\left(c_t, l_t\right)\left(1 + s(v_t) + v_t s'(v_t)\right)}{u_c\left(c_t, l_t\right)} \right) l_t. \tag{17}$$

Condition (17), simply states that government fiscal revenues are equivalent to the wedge between the firm's wage cost and the household's desired wage rate.

The Lagrangean of the Ramsey Planner problem can be written as follows

$$\begin{aligned}
\mathcal{L} \;=\; & E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u\left(c_t, l_t\right) + \right. \\
& + \boldsymbol{\lambda}_t^{AR} \left[z_t l_t - c_t \left(1 + s_t(v_t)\right) - g_t - \frac{\xi_p z_t l_t \left(\pi_t - 1\right)^2}{2} \right] + \\
& + \boldsymbol{\lambda}_t^{B} \left[\lambda_t - \beta \frac{\lambda_{t+1} R_t}{\pi_{t+1}} \right] + \lambda_t^{GBC} \left[\frac{c_t}{v_t} + \frac{B_t}{P_t} + \right. \\
& + \left(z_t mc_t + \frac{u_l\left(c_t, l_t\right)\left[1 + s(v_t) + v_t s'(v_t)\right]}{u_c\left(c_t, l_t\right)} \right) l_t + \\
& \left. - R_{t-1} \frac{B_{t-1}}{P_{t-1}} - \frac{c_{t-1}}{\pi_t v_{t-1}} - g_t - t_t \right] + \\
& + \boldsymbol{\lambda}_t^{Ph} \left[\frac{\beta y_{t+1} \lambda_{t+1} \xi_p \pi_{t+1} \left(\pi_{t+1} - 1\right)}{y_t \lambda_t} - \frac{z_t \left(\rho - mc_t\right)}{1 - \rho} - \xi_p \pi_t \left(\pi_t - 1\right) \right] + \\
& \left. + \boldsymbol{\lambda}_t^{MUC} \left[\frac{u_c\left(c_t, l_t\right)}{1 + s(v_t) + v_t s'(v_t)} - \lambda_t \right] \right\}
\end{aligned}$$

where R and $s(v)$ are defined in defined in (4) and (9) respectively.

3 Finding the Ramsey solution: An overview

A step-by-step solution involves the following process:

1. **Derive the Optimality Conditions:** Start by formulating the (non-linear) optimality conditions using the Lagrangian, which integrates the government's objectives with the constraints of the economy.

2. **Determine the Steady State:** Solve for the (non-stochastic) steady state, representing a balanced economic situation where all variables remain constant over time.

3. **Linearize Around the Steady State:** Approximate the (nonlinear) optimality conditions by linearizing them around the steady state. This simplifies the analysis of small deviations from equilibrium.

4. **Simulate the Economy:** Perform numerical simulations to explore how the economy responds to external changes, such as unexpected government spending or technology shocks.

5. **Analyze Economic Dynamics:** Use the simulation results to study key properties of the economy, including: The average behavior of variables (e.g., consumption, labor); How different variables interact or move together (correlations); the influence of past values on future outcomes (autocorrelations); responses to sudden changes, such as shocks, captured through impulse response functions (IRFs).