Optimal Policy: The Ramsey Approach

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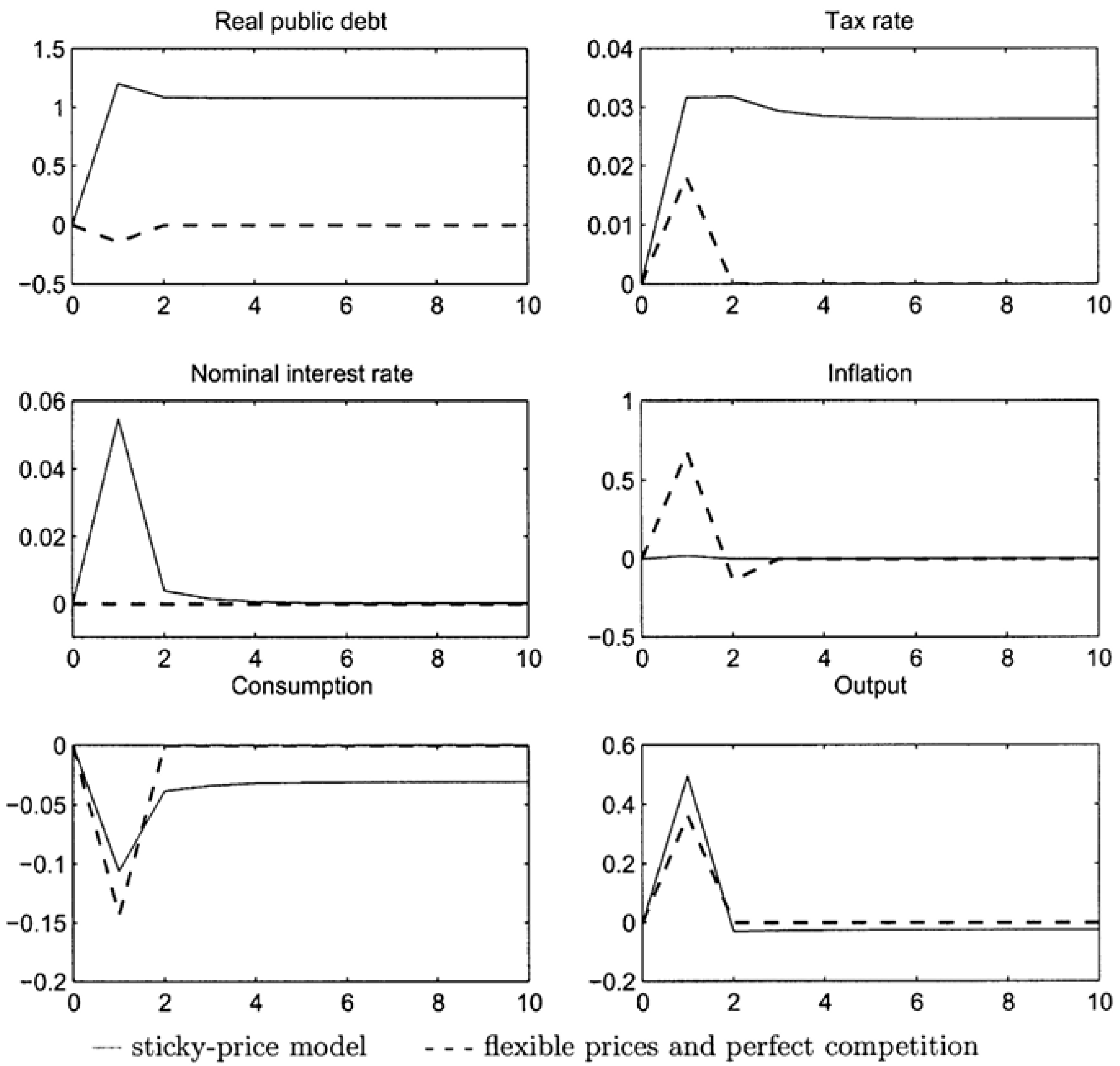
Schmitt-Grohe and Uribe (2002, JET)



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Schmitt-Grohe and Uribe (2002, JET)

Variable	Mean	Std. dev.	Auto. corr.	Mean	Std. dev.	Auto. corr.
	Exact solution			Log-linear approximation		
Flexible prices and perfect competition						
τ	18.8	0.0491	0.88	18.7	0.044	0.834
π	−3.39	7.47	−0.0279	−3.66	6.04	−0.0393
R	0	0	—	0	0	—
Flexible prices and imperfect competition						
τ	26.6	0.042	0.88	25.8	0.0447	0.616
π	−1.46	7.92	−0.0239	−1.82	6.8	−0.0411
R	1.95	0.0369	0.88	1.83	0.0313	0.797
	Log-quadratic approximation			Log-linear approximation		
Baseline sticky-price economy						
τ	25.2	1.04	0.75	25.1	0.998	0.743
π	−0.16	0.18	0.03	−0.16	0.171	0.0372
R	3.83	0.56	0.86	3.85	0.562	0.865

Note: τ, π and R are expressed in percentage points.

Optimal monetary policy: Friedman rule

- Social Planner's problem (MIU model):

$$\max U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

- Subject to

$$C_t = A_t {N_t}^{1-\alpha}$$

- Optimality conditions:

$$-\frac{U_{n,t}}{U_{c,t}} = (1-\alpha) A_t {N_t}^{\alpha}$$

$$U_{m,t} = 0$$

- It follows optimal monetary policy (**Friedman rule**):

$$i_t = 0$$

Friedman rule and inflation

- Optimal monetary policy: $i_t = 0$ (**Friedman rule**).
- At the equilibrium:

$$i - \pi = \rho$$

- where ρ is the real natural interest rate determined independently of nominal values.
- Then substituting the optimal $i = 0$, we have:

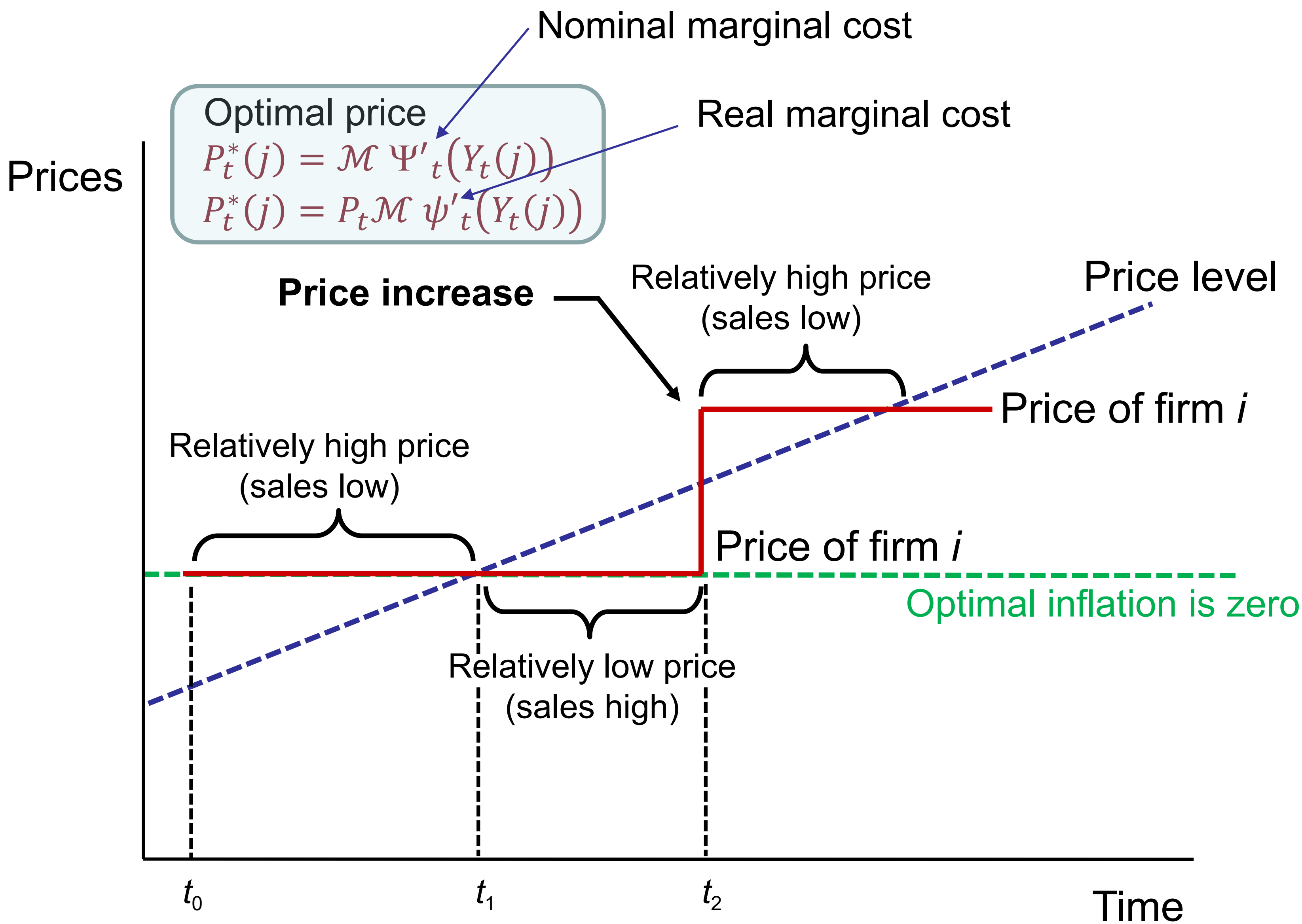
$$-\pi = \rho$$

- Average optimal inflation is hence negative:

$$\pi = -\rho < 0$$

- The Friedman rule implies a **negative optimal inflation rate**.

Distortions associated with nominal rigidities



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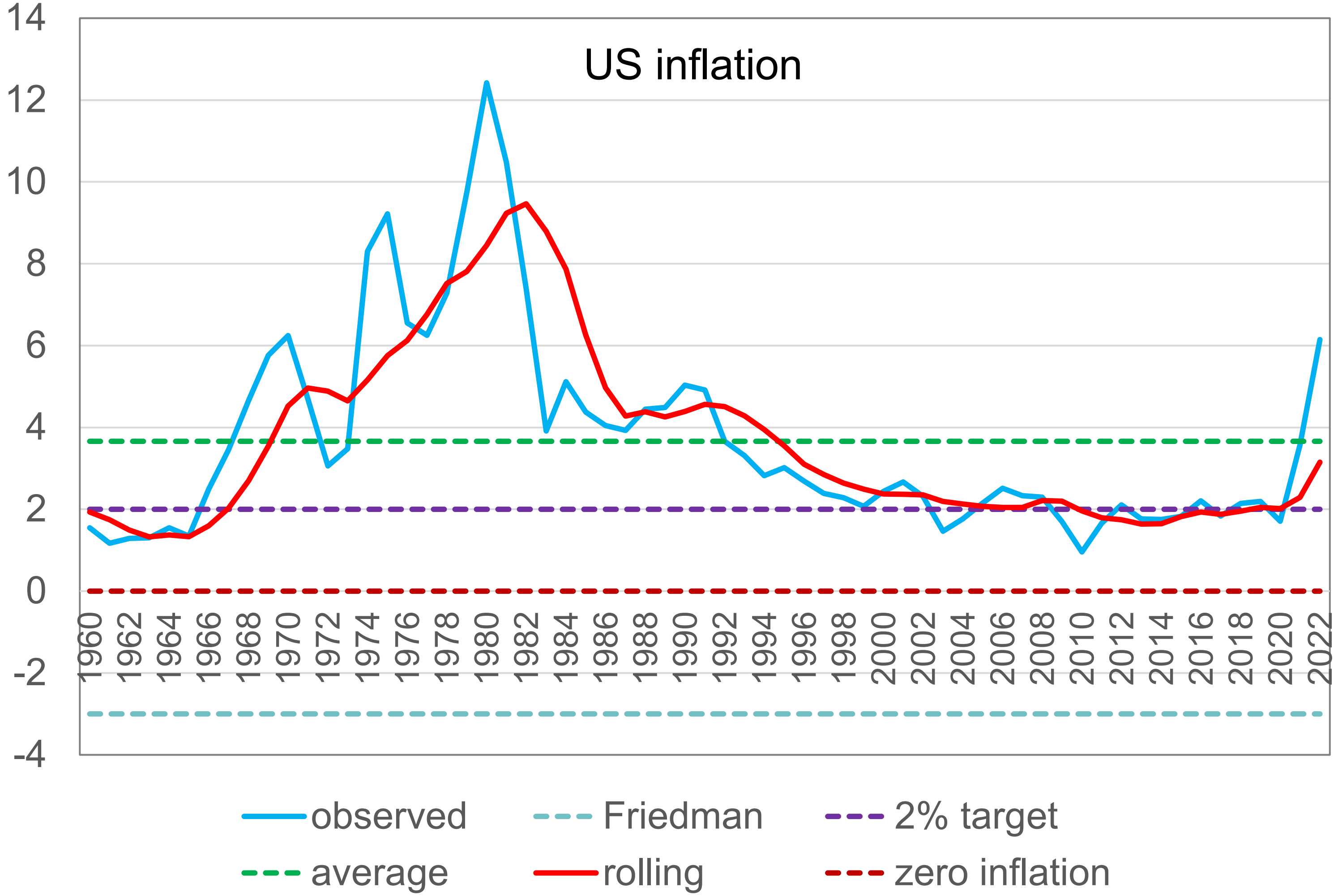
Phelps-Tobin arguments

- The ability of the government to finance expenditures by issuing money is the `seigniorage' associated with its sovereign monetary monopoly. Both explicit and implicit taxes are distortionary. The distortion of the inflation tax is the diversion of resources or loss of utility associated with the scarcity of money, already mentioned. But there are also distortions in explicit taxes; lump-sum taxes are not available. The problem is to optimize the choice of taxes, given the necessity of government expenditure. This formulation correctly connects the money-supply process to the government budget. Tobin (1986: 11).
- Government budget constraint:

$$\frac{M_t - M_{t-1}}{P_t} + \frac{B_t}{P_t} + \tau_t(w_t l_t) = g_t + t_t + R_{t-1}\frac{B_{t-1}}{P_{t-1}}$$

Inflation puzzle



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The debate: Optimal trend inflation

- Bailey (1957) and Friedman (1969) first raised the issue of the optimal monetary policy: $r = \cancel{i} - \pi \Rightarrow \pi < 0$.
- Khan et al. (2003) and Schmitt-Grohé and Uribe (2004), two key frictions driving the optimal inflation rate.
  - Adjustment costs of goods prices invariably drive the optimal inflation rate to zero: $\xi_p \;\Rightarrow \pi = 0$.
  - Monetary transaction costs that arise unless the central bank implements the Friedman rule

  Stickiness implies about zero inflation: $\pi \cong 0$.
- Phelps (1973), to alleviate the burden of distortionary taxation, governments should resort to monetary financing, driving a wedge between the private and the social cost of money: $g > 0 \Rightarrow \pi > 0$.

The model in a nutshell

- A standard framework (non-linear, second-order approximation)
  - Households: Consumption Euler equation (labor and consumption separable), money transaction cost (Friedman rule) proportional to the velocity
  - Good producers: Price stickiness *a la* Rotemberg (zero inflation), monopolistic competition
  - Labor market: a competitive market with distortionary labor tax (seigniorage)
  - Augmented with public transfers
- Policies: Optimal Ramsey fiscal and monetary policy to finance an exogenous stream of public expenditure (timeless perspective, given steady state debt level)

The household's problem

- Assuming: $u(c_{t,i}, l_{t,i}) = \ln\ c_{t,i} + \eta \ln\left(1 - l_{t,i}\right)$
- The household maximizes:

$$U = E_{t=0} \sum_{t=0}^{\infty} \beta^t u(c_{t,i}, l_{t,i})$$

- Subject to:

$$c_{t,i}\left(1 + s(v_{t,i})\right) + \frac{M_{t,i}}{P_t} + \frac{B_{t,i}}{P_t} = (1 - \tau_t) w_{t,i} l_{t,i} + \frac{M_{t-1,i}}{P_t} + \theta_t + \frac{R_{t-1} B_{t-1,i}}{P_t} + t_t$$

- Money velocity $v_{t,i} = \frac{P_t c_{t,i}}{M_{t,i}}$.
- Transaction cost $s(v_{t,i})$ [$s(v^*) = 0$ and $s'\left(v_{t,i}\right) > 0$ for $v_{t,i} > v^* > 0$].

First-order conditions

- Good j's demand:

$$c_t(j) = c_t \left(\frac{p_t(j)}{P_t}\right)^{\frac{1}{\rho-1}}$$

- Lagrange multiplier:

$$\lambda_t = \frac{u_c(c_t, l_t)}{1 + s(v_t) + v_t s'(v_t)}$$

- Euler equation:

$$\lambda_t = \beta E_t \left(\frac{\lambda_{t+1} R_t}{\pi_{t+1}}\right)$$

- Labor supply:

$$\lambda_t(1-\tau_t)w_t = -u_l(c_t, l_t)$$

- Money demand:

$$\frac{R_t - 1}{R_t} = s'(v_t)v_t^2 \quad \Rightarrow \quad \frac{M_t}{P_t} = \left(\frac{1}{A}\frac{R_t - 1}{R_t} + \frac{B}{A}\right)^{-\frac{1}{2}} c_t$$

The firm's problem

- Given the technology: $y_t(j) = z_t l_{t,j}$
- Real wages: $w_t = z_t mc_t$
- Flexible prices are set to equate the marginal cost to the inverse markup: $mc_t = \rho$
- Sticky price are due to Rotenberg's adjustment costs:

$$\frac{\xi_p}{2} y_t (\pi_t - 1)^2$$

- Price dynamics:

$$\frac{z_t(\rho - mc_t)}{1-\rho} + \xi_p \pi_t (\pi_t - 1) = E_t \beta \Lambda_t \xi_p [\pi_{t+1}(\pi_{t+1} - 1)]$$

$$\Lambda_t = \frac{y_{t+1}\lambda_{t+1}}{y_t \lambda_t}$$

Phillips curve

- Price dynamics:

$$\frac{z_t(\rho - mc_t)}{1-\rho} + \xi_p \pi_t(\pi_t - 1) = E_t \beta \Lambda_t \xi_p [\pi_{t+1}(\pi_{t+1} - 1)]$$

- Flexible prices ($\xi_p = 0$) are set to equate the marginal cost to the inverse markup:

$$mc_t = \rho$$

- By setting $\Pi_t = \pi_t(\pi_t - 1)$, the price dynamics can be written as

$$\Pi_t = E_t \beta \Lambda_t \Pi_{t+1} + \frac{z_t}{\xi_p(1-\rho)}(mc_t - \rho)$$

Other equations

- Aggregate resource constraint:

$$z_t l_t - c_t(1+s_t) - g_t - \frac{\xi_p z_t l_t (\pi_t - 1)^2}{2} = 0$$

- Shocks (AR(1)) technology (z_t) and public consumption (g_t) and a given public transfer (t_t)
- Government budget constraint:

$$\tau_t w_t l_t + \frac{M_t - M_{t-1}}{P_t} + \frac{B_t}{P_t} = R_{t-1}\frac{B_{t-1}}{P_{t-1}} + g_t + t_t$$

- That is

$$\tau_t w_t l_t + \frac{c_t}{v_t} - \frac{c_{t-1}}{\pi_t v_{t-1}} + \frac{B_t}{P_t} - R_{t-1}\frac{B_{t-1}}{P_{t-1}} - g_t - t_t = 0$$

Friedman rule

- Money demand and velocity are

$$v_{t,i} = \frac{P_t c_{t,i}}{M_{t,i}} \quad \text{and} \quad \frac{R_t - 1}{R_t} = s'(v_t) v_t^2$$

- If inflation were costless ($\xi_p = 0$) and money were not needed for public finance ($g = 0$), then, since $s(v^*) = 0$, it would be optimal

$$v_{t,i} = v^*$$

$$\frac{R_t - 1}{R_t} = 0 \Rightarrow R_t = 1$$

- The Euler equation in the steady:

$$1 = \beta \left(\frac{R}{\pi}\right) \Rightarrow \pi = \beta < 1$$

- Note that π is the gross inflation rate, if $\pi < 1$, prices are falling then the net rate is negative

Price adjustment cost

2

- Note that in the steady state $y = l$, the aggregate resource constraint in the steady state is then:

$$y\left(1 - \frac{\xi_p}{2}(\pi - 1)^2\right) = c(1 + s) + g$$

- This implies a wedge between the production and the "gross" private and public consumption
- If money were not needed for public finance (e.g., $g = 0$) and transaction cost are zero $s = 0$, then

$$y\left(1 - \frac{\xi_p}{2}(\pi - 1)^2\right) = c$$

- and it would be optimal to set the gross inflation rate:

$$\pi = 1$$

Tobin's argument

3

- Government budget constraint:

$$\frac{M_t - M_{t-1}}{P_t} + \frac{B_t}{P_t} - R_{t-1}\frac{B_{t-1}}{P_{t-1}} = g_t + t_t - \tau_t w_t l_t$$

- That is

$$\frac{1}{v}\frac{c}{y}\left(1 - \frac{1}{\pi}\right) + \frac{b}{y}(1 - r - \pi) = \frac{g}{y} + \frac{t}{y} - \tau w$$

- As $\tau w = \left(mc + \frac{u_l[1+s+vs']}{u_c}\right)$, we obtain for $s = 0$, $t = 0$ and $b = 0$:

$$\frac{g}{y} = \frac{\pi - 1}{\pi}\frac{m}{y} + \left(mc + \frac{u_l}{u_c}\right)$$

Social planner's (Ramsey) problem

- Maximize the utility concerning
  - Aggregate resource constraint
  - Euler equation
  - Government budget constraint
  - Phillips curve
  - Consumer multiplier (marginal utility of consumption)
- Implicitly
  - Money demand: $\frac{R_t - 1}{R_t} = s'(v_t)v_t^2 \Rightarrow R_t = \frac{1}{1 - s'(v_t)v_t^2}$
  - Fiscal revenues are the wedge between the firm's wage cost and the household's desired wage rate:

$$\tau_t w_t l_t = \left(z_t m c_t + \frac{u_l(c_t, l_t)\left[1 + s(v_t) + v_t s'(v_t)\right]}{u_c(c_t, l_t)} \right) l_t$$

Social planner: Lagrangian ($\mathcal{L}$)

$$
\begin{aligned}
E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ & u(c_t, l_t) + \lambda_t^B \left[\lambda_t - \beta \frac{\lambda_{t+1} R_t(v_t)}{\pi_{t+1}} \right] \\
& + \lambda_t^{AR} \left[z_t l_t - c_t(1 + s_t) - g_t - \frac{\xi_p z_t l_t (\pi_t - 1)^2}{2} \right] + \\
& + \lambda_t^{GBC} \left[\frac{c_t}{v_t} + \frac{B_t}{P_t} - R_{t-1}(v_t) \frac{B_{t-1}}{P_{t-1}} - \frac{c_{t-1}}{\pi_t v_{t-1}} - g_t - t_t + \right. \\
& \left. + \left(z_t mc_t + \frac{u_l(c_t, l_t)[1 + s(v_t) + v_t s'(v_t)]}{u_c(c_t, l_t)} \right) l_t \right] + \\
& + \lambda_t^{Ph} \left[\frac{\beta y_{t+1} \lambda_{t+1} \xi_p \pi_{t+1} (\pi_{t+1} - 1)}{y_t \lambda_t} - \frac{z_t(\rho - mc_t)}{1 - \rho} + \right. \\
& \left. - \xi_p \pi_t (\pi_t - 1) \right] + \lambda_t^{MUC} \left[\frac{u_c(c_t, l_t)}{1 + s(v_t) + v_t s'(v_t)} - \lambda_t \right] \Bigg\}
\end{aligned}
$$

Euler

Aggregate resources

Budget constraint

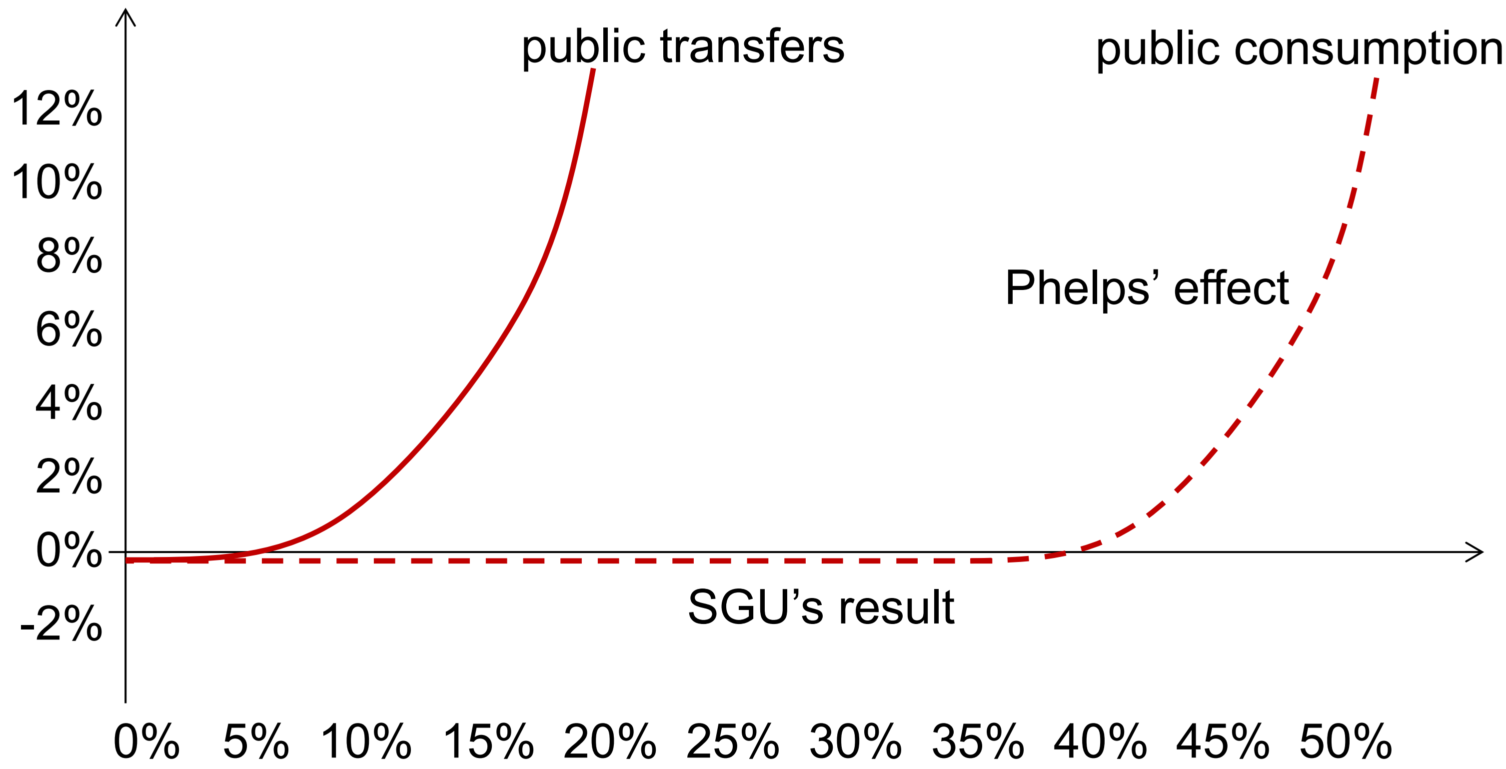
Prices

Multiplier

Baseline calibration (SGU, 2004)

- The time unit is meant to be a year
  - Subjective discount rate is 0.96 (consistent with a steady-state real rate of return of 4% per year)
  - Transaction cost parameters estimated (SGU, 2004)
  - The debt-to-GDP ratio is assumed to be 0.44 percent.
  - In the goods market, monopolistic competition implies a gross markup of 1.2, and the annualized Rotemberg price adjustment cost is set to have 8 month-contract duration (on average).
  - No indexation, competitive labor markets.

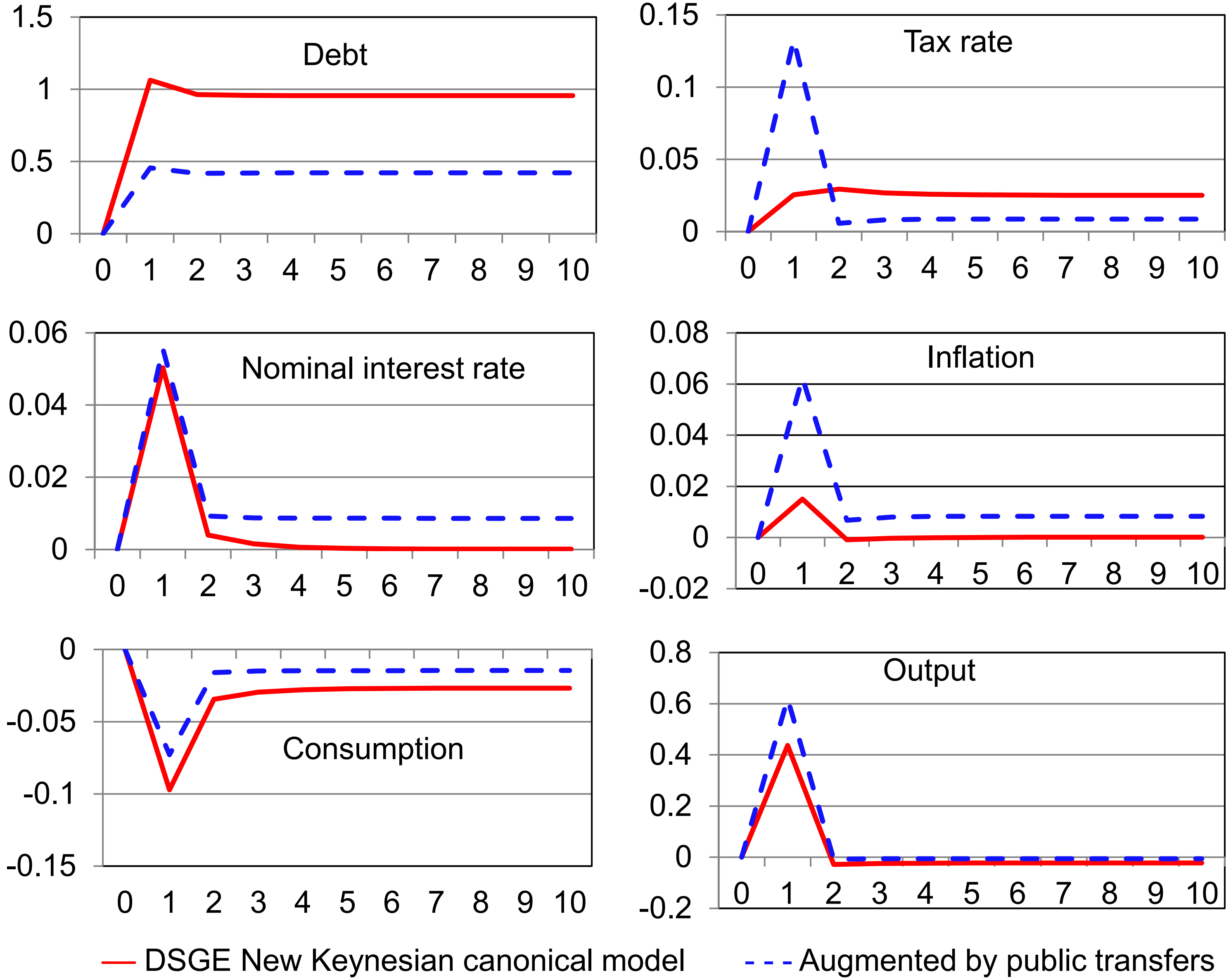
Optimal inflation



Result summary: Optimal trend inflation

- **Fact A1 (SGU-trend inflation):** Assuming that public transfers are neglected ($t_t = 0$), the optimal steady state inflation rate is about zero if the price-adjustment costs are not zero ($\xi_p > 0$).
- **Fact A2 (SGU-trend inflation):** Assuming that public transfers are neglected ($t_t = 0$), in a flexible price economy ($\xi_p = 0$), the Friedman rule always applies.
- **Fact B (DTA-trend inflation):** Assuming that public transfers are not neglected ($t_t > 0$), the optimal steady state inflation rate is positive as the Pheps' effect dominates the other rationales for determining the optimal inflation rate.

Stabilization policies



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Result summary: Optimal policy mix

- **Fact C1 (SGU-optimal-policy mix):** In a sticky price economy ($\xi_p > 0$), if the effects of public transfers are neglected ($t_t = 0$), the optimal response to a government consumption shock is a mix of tax and debt rather than the inflation tax.
- **Fact C2 (DTA-optimal-policy mix):** In a sticky price economy ($\xi_p > 0$), if the effects of public transfers are not neglected ($t_t = 0$), the optimal response to a government consumption shock is a mix of tax and inflation rather than the debt.

Dynamic properties (2nd or. approx.)

	mean	st. dev.	auto. corr.	corr(x,y)	corr(x,g)	corr(x,z)
Without fiscal transfers						
τ	25.19	1.062	0.759	-0.305	0.436	-0.236
π	-0.16	0.177	0.034	-0.108	0.374	-0.275
R	3.82	0.566	0.863	-0.942	-0.044	-0.962
y	0.21	0.007	0.820	1.000	0.204	0.938
l	0.21	0.003	0.823	-0.085	0.590	-0.402
c	0.17	0.007	0.824	0.940	-0.123	0.954
With fiscal transfers						
τ	42.69	2.860	-0.053	-0.110	0.284	-0.356
π	1.46	0.962	-0.054	-0.062	0.304	-0.309
R	5.50	0.489	0.775	-0.790	0.142	-0.926
y	0.17	0.005	0.823	1.000	0.408	0.884
l	0.17	0.003	0.714	-0.237	0.699	-0.651
c	0.13	0.005	0.783	0.851	-0.091	0.985

First-order conditions in the steady state

$$l = [1+s(v)]c + g_c l + \frac{\xi_p}{2} l(\pi-1)^2$$

$$1 = \beta r(v)\frac{1}{\pi}$$

$$\frac{c}{v} + b + [mc + Z\gamma(v)]l = \frac{r(v)b}{\pi} + \frac{c}{v\pi} + (g_{PC} + g_{PT})l$$

$$\xi_p(1-\beta)\pi(\pi-1) = \frac{mc-\rho}{1-\rho}$$

$$u_c(c,l) = \lambda\gamma(v)$$

$$u_c(c,l) - \lambda^{AR}[1+s(v)] + \left[\frac{1}{v}\left(1-\frac{\beta}{\pi}\right) - \frac{\delta}{1-l}l\gamma(v)\right]\lambda^{GBC} + \lambda^{MUC}u_{cc} = 0$$

$$u_l(c,l) + \lambda^{AR}\left(1-\frac{\xi_p}{2}(\pi-1)^2\right) + \lambda^{GBC}\left[mc - \left(\frac{\delta c}{1-l} + \frac{\delta c}{(1-l)^2}l\right)\gamma(v)\right] = 0$$

$$\left(1-\frac{r(v)}{\pi}\right)\lambda^B + (1-\beta)\frac{\lambda^{Ph}}{\lambda}\pi(\pi-1) - \lambda^{MUC}\gamma(v) = 0$$

$$-\lambda^{AR}s'(v)c - \lambda^B\frac{\beta r'(v)\lambda}{\pi} - \lambda^{GBC}\left[\left(1-\frac{\beta}{\pi}\right)\frac{c}{v^2} + \frac{\delta c}{1-l}l\gamma'(v) + \frac{\beta b r'(v)}{\pi}\right] - \lambda^{MUC}\lambda\gamma'(v) = 0$$

$$-\lambda^{AR}\xi_p(\pi-1)l + \frac{1}{\pi^2}\left[\lambda^B r(v)\lambda + \lambda^{GBC}\left(r(v)b + \frac{c}{v}\right)\right] = 0$$

$$\pi = \beta r(v)$$

$$\xi_p\lambda^{GBC}l = -\frac{\lambda^{Ph}}{1-\rho}$$

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Two-equation-simplified model

- By some simplification, the Ramsey problem reduces to

Government budget locus (GBC)

$$\frac{c}{lv}\frac{\pi-1}{\pi}+\left\{\rho-\frac{\delta c}{1-l}\left[1+2\left(A\frac{\pi-1}{\pi}\right)^{\frac{1}{2}}\right]\right\}=g_{PC}+g_{PT}$$

$$\frac{\delta c}{1-l}=\frac{\frac{1-\pi^2}{\pi^4}-\frac{\delta l}{1-l}+\frac{\rho}{2}-\delta c\left(\frac{1}{2}+\left(A\frac{\pi-1}{\pi}\right)^{\frac{1}{2}}\right)/(1-l)^2}{\left(\frac{1-\pi^2}{\pi^4}-\frac{\delta l}{1-l}\right)\left[1+\left(A\frac{\pi-1}{\pi}\right)^{\frac{1}{2}}\right]+\frac{1}{2}\left(A\frac{\pi-1}{\pi}\right)^{\frac{1}{2}}-\frac{\delta l\left(\frac{1}{2}+\left(A\frac{\pi-1}{\pi}\right)^{\frac{1}{2}}\right)}{1-l}}.$$

Marginal rate of substitution (MRS)

- Moreover

$$c=\frac{1-g_{PC}}{1+\left(A\frac{\pi-1}{\pi}\right)^{\frac{1}{2}}}l$$

From the aggregate resource constraint

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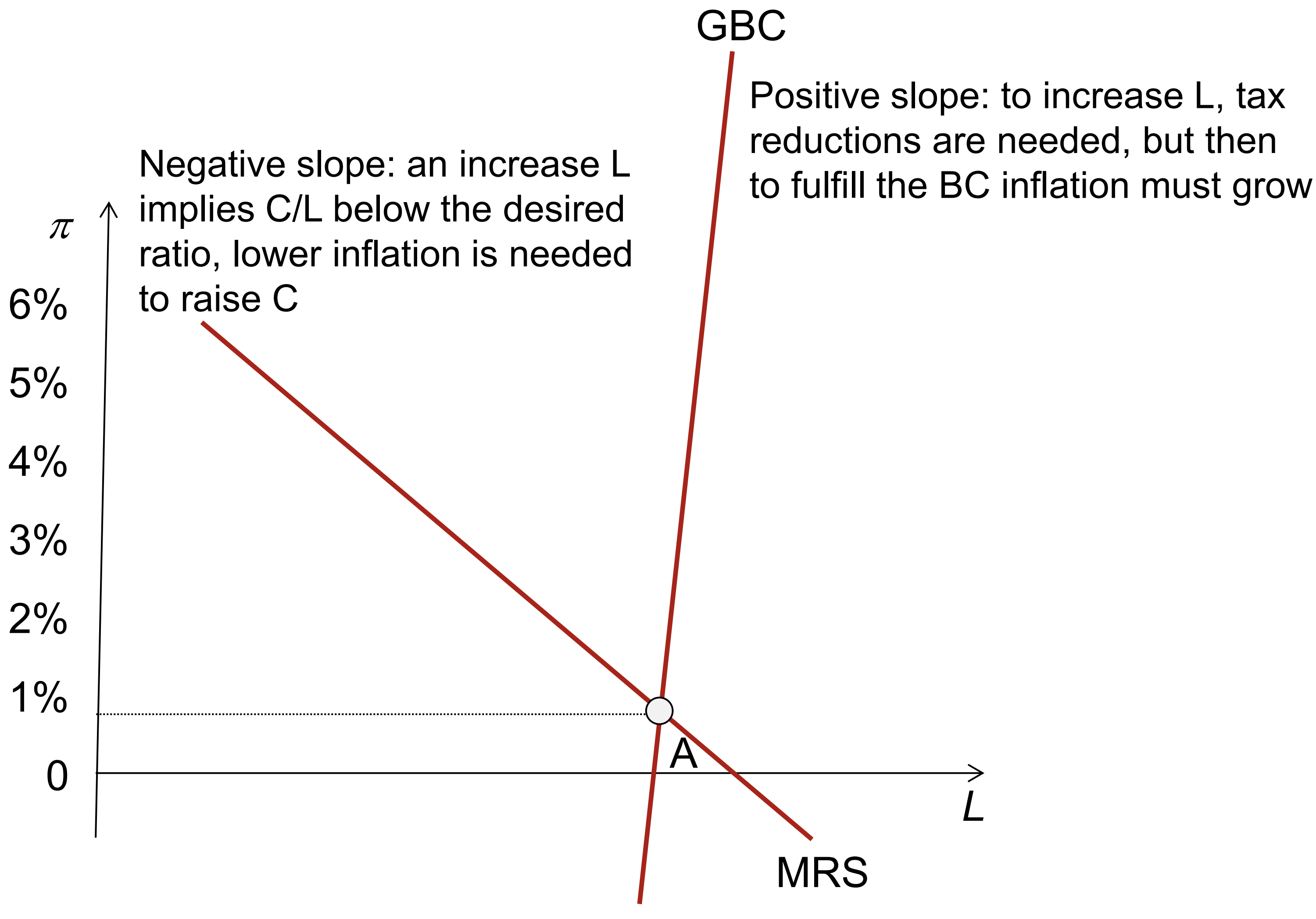
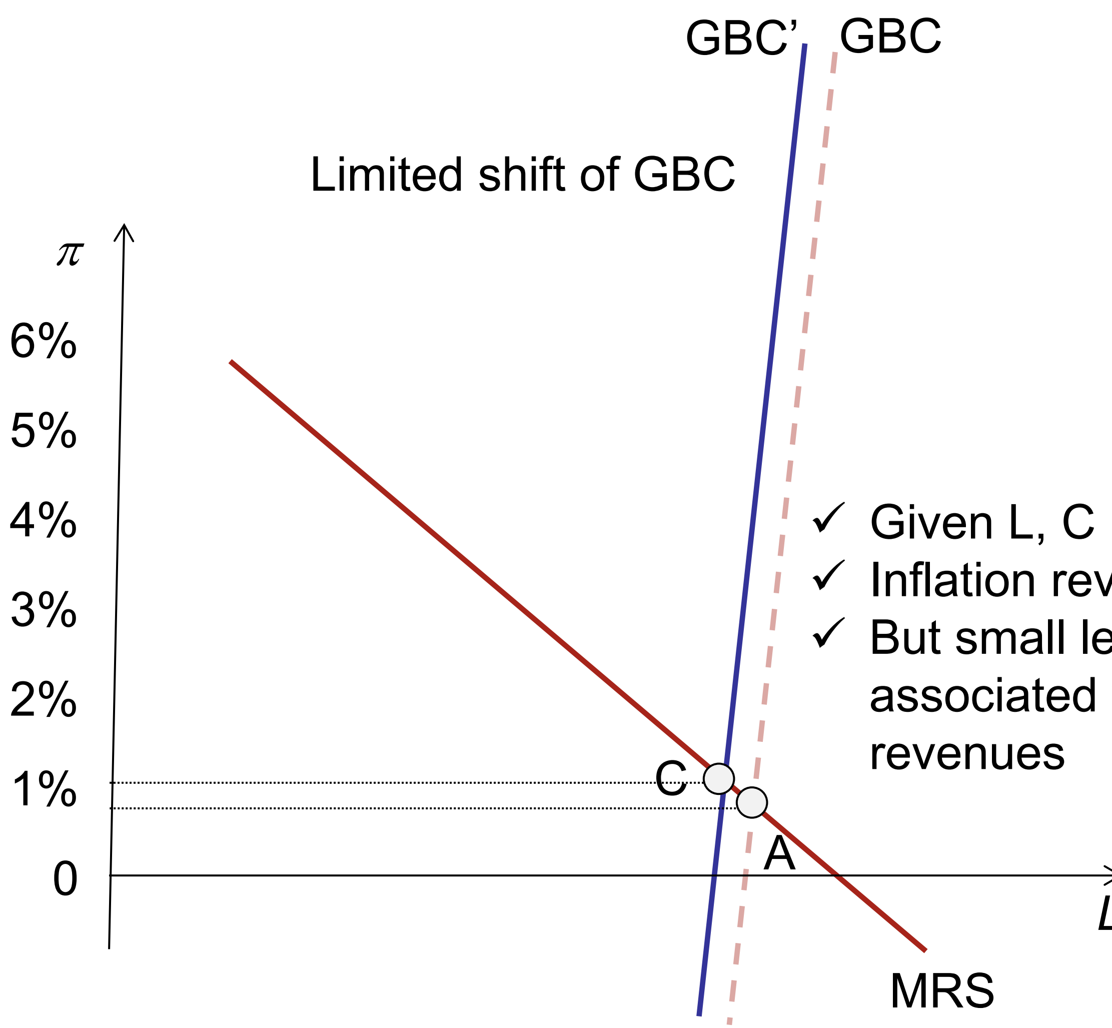
Ramsey equilibrium

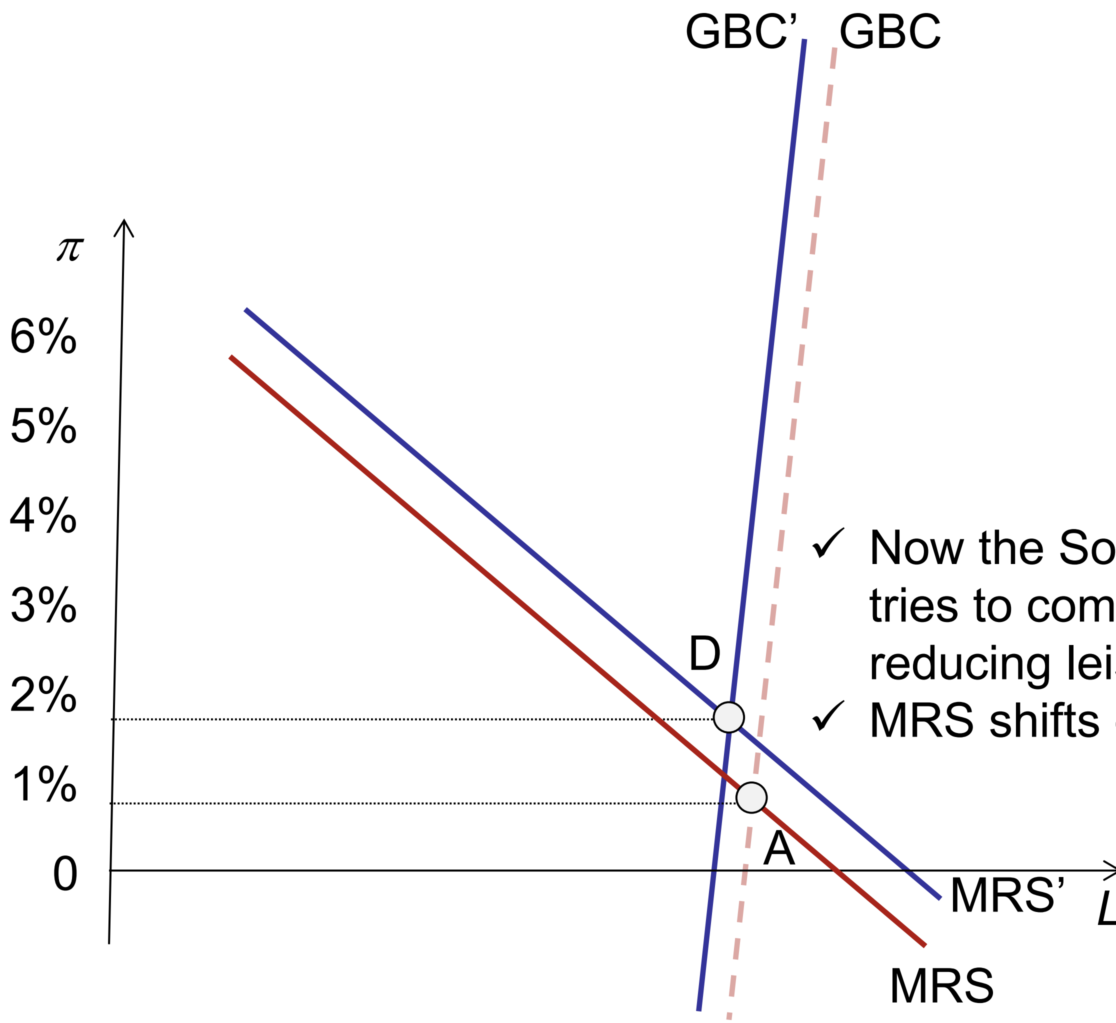


Intuition: Public consumption



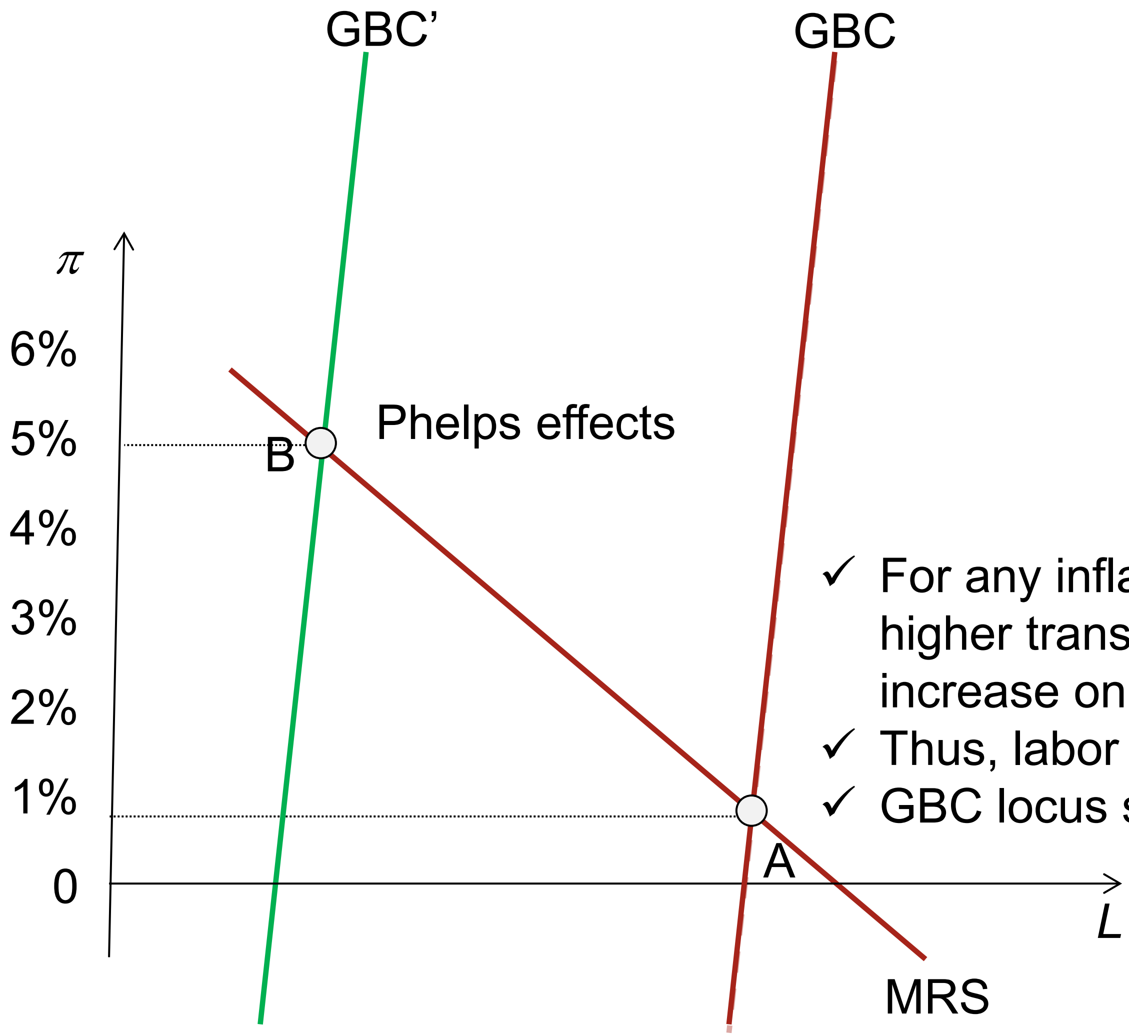
- ✓ Given L, C must fall → M/P falls
- ✓ Inflation revenues fall with M/P
- ✓ But small levels of C given L are associated with larger fiscal revenues

Intuition: Public consumption



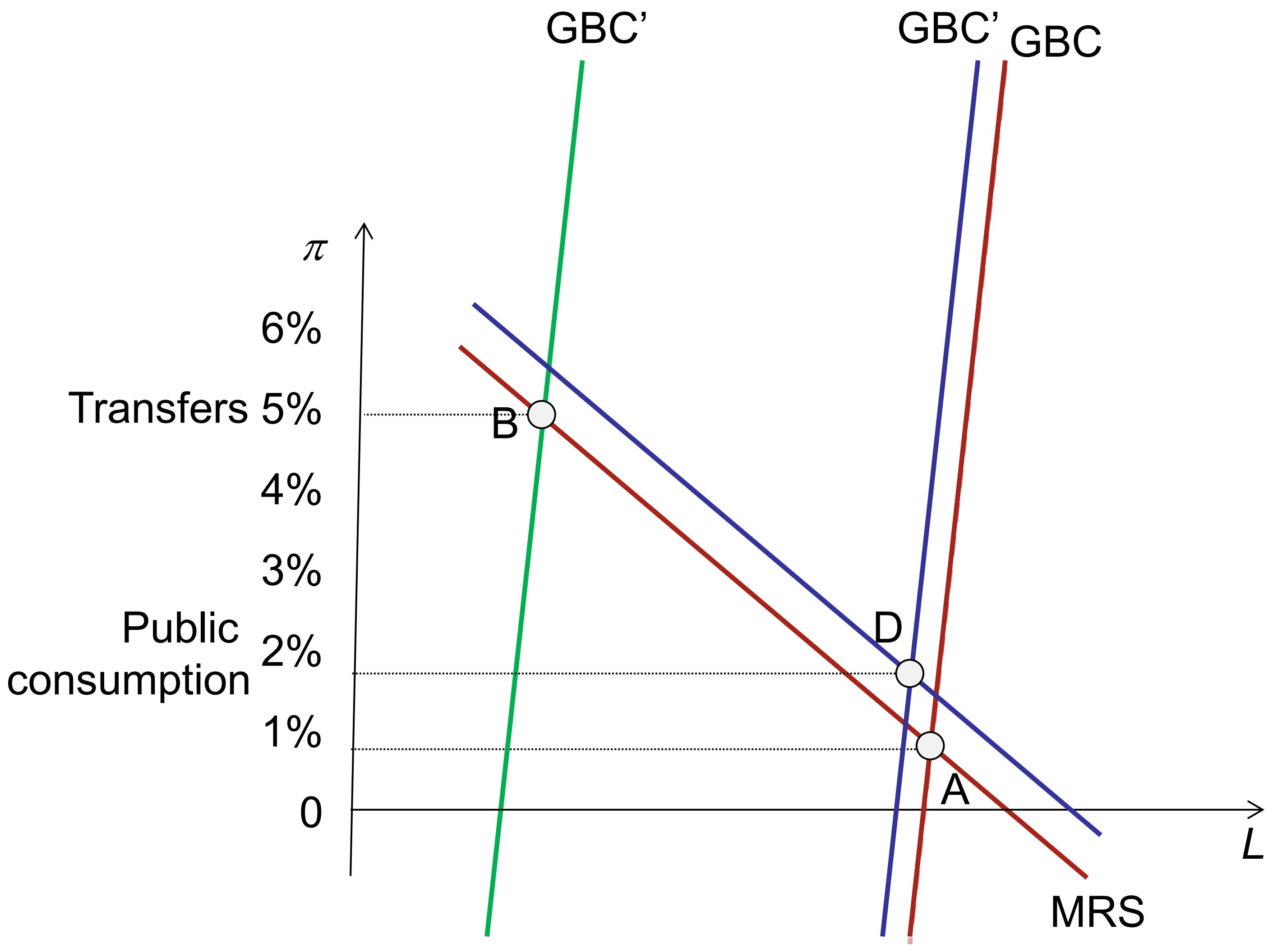
- ✓ Now the Social Planner also tries to compensate C fall by reducing leisure
- ✓ MRS shifts on the right

Intuition: Public transfers



- ✓ For any inflation rate, the higher transfer requires an increase on tax to be financed
- ✓ Thus, labor must fall
- ✓ GBC locus shifts leftwards

Phelps' effect: Comparison



Result summary: Optimal policy mix

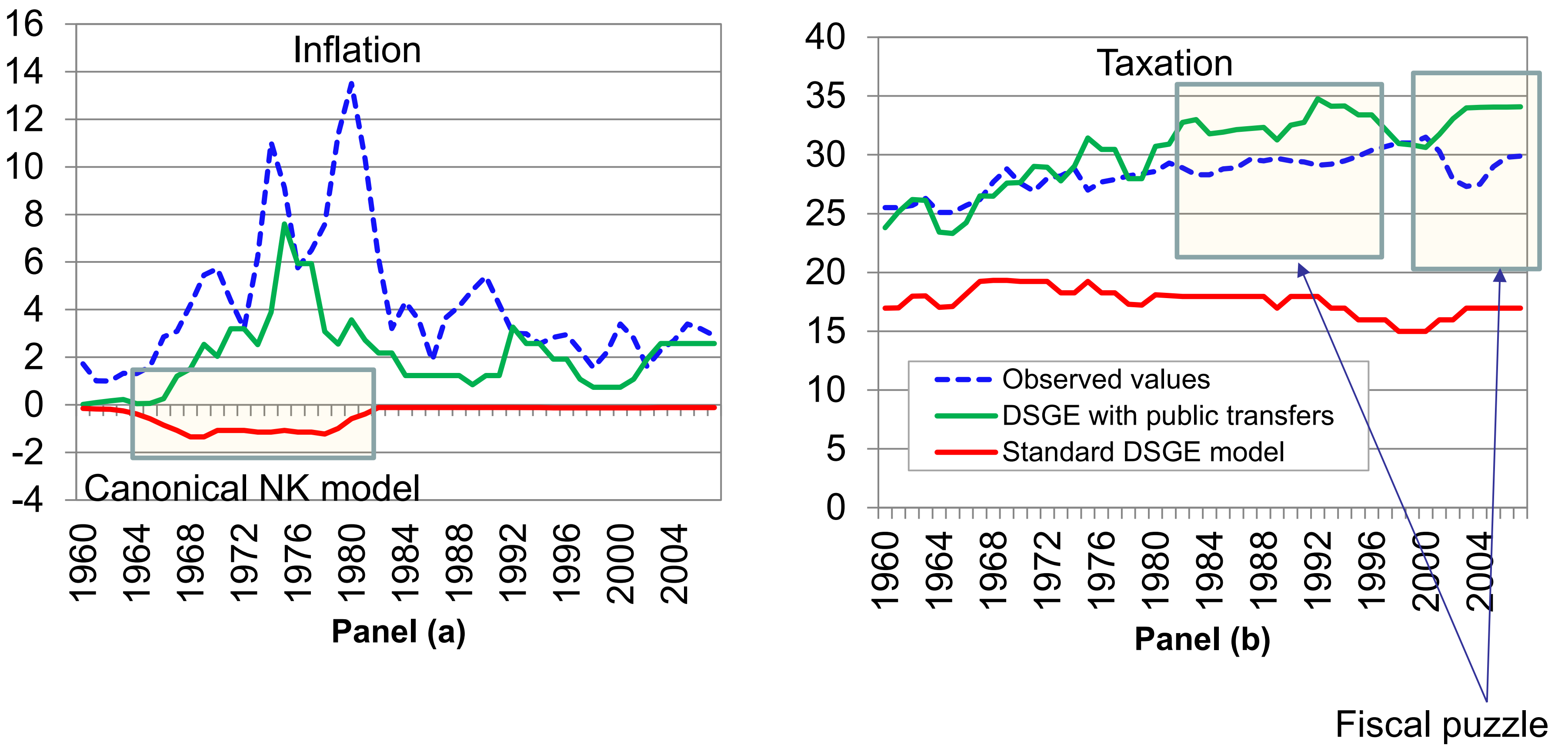
- **Fact D1 (public consumption):** If the effects of public transfers are neglected, in a sticky-price economy ($\xi_p > 0$), the optimal response to a government consumption shock is to use a mix of tax and debt rather than the inflation tax.
- **Fact D2 (public transfers):** If the effects of public transfers are not neglected , in a sticky-price economy ($\xi_p > 0$), the optimal response to a government consumption shock is to use a mix of tax and inflation rather than the debt.

The optimal policy in the US

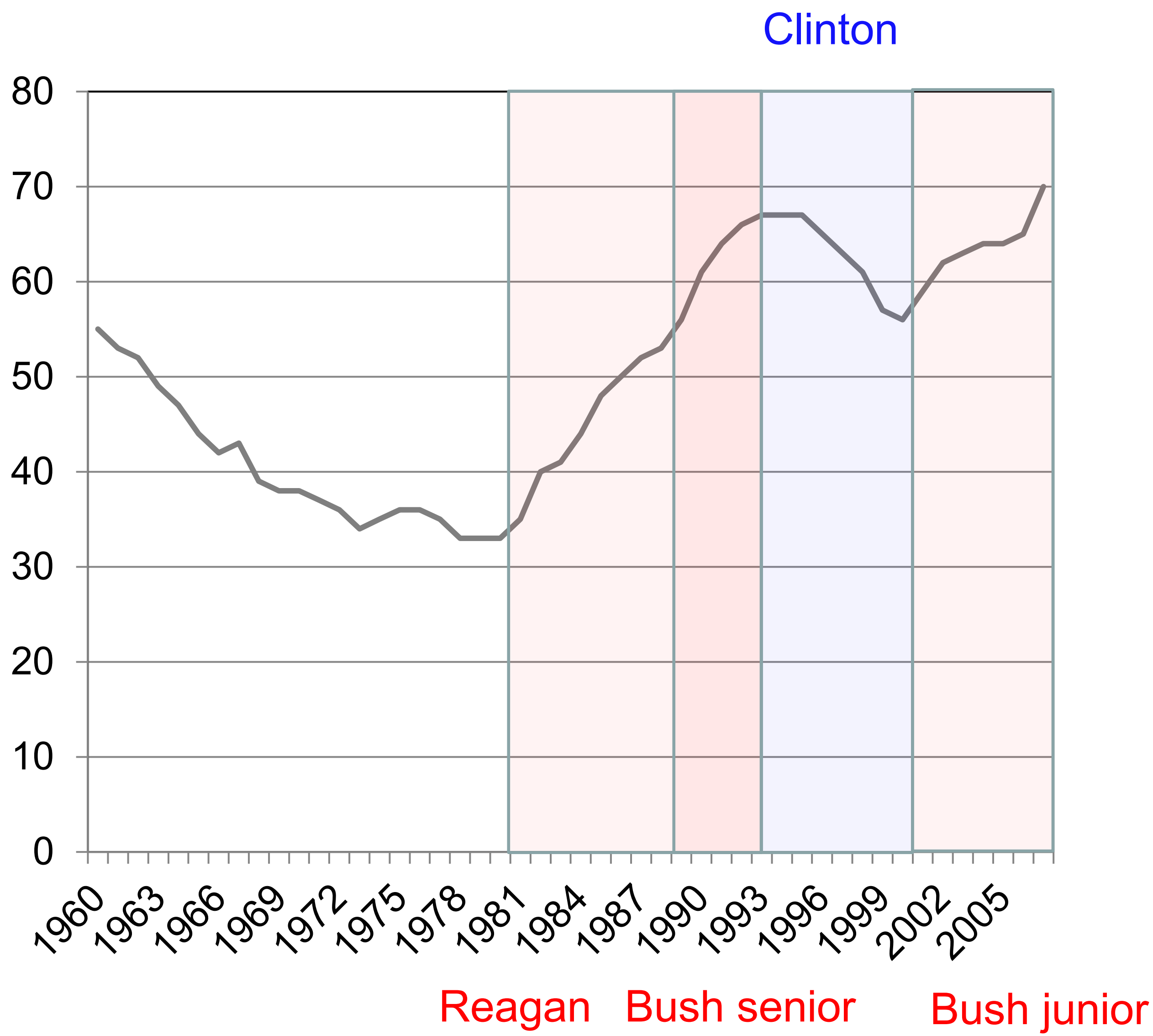
- The US economy (1960-2008)
- Reconsider the model by adding
  - Sticky price and wage
  - Time varying Price and wage indexation (high in 70s)
  - Time varying public consumption
  - Time varying public transfers

Fiscal and monetary policy dynamics

- By introducing nominal wage rigidities, time varying indexation (high in the 70s) and public expenditure.



U.S. debt on GDP



Result summary: The US macro-dynamics

- **Fact E1 (Great Inflation):** The outcomes of the Great Inflation are inconsistent with the optimal model's outcomes without public transfers. By contrast. the model augmented with public transfers fairly matches the macro-data.
- **Fact E2 (fiscal puzzle):** Although the model augmented with public transfers reasonably matches the data (as inflation), a fiscal puzzle is observed. It's hard to explain the tax revenue path.
- **Fact E3 (political economy):** Republicans' governments experienced suboptimal low taxes and increasing sovereign debt. Democrats' governments experienced sub-optimal high taxes and decreasing sovereign debt.