Laboratorio di decisioni statistiche - Lab. 09 – 2024-2025 - 27 novembre 2024

Nome, cognome e matricola: ____

1. **AfE - stima puntuale 1** Assume that $\Theta|\mathbf{z}_n \sim \text{Beta}(5,8)$ and that $L(a,\theta) = |a-\theta|$.
   (a) Define the function `rho.abs.fun(a)` in order to obtain the values of $\rho(a,\mathbf{z}_n)$ as a varies in $[0,1]$.
   (b) Define a grid of values `a.val` for a and plot $\rho(a,\mathbf{z}_n)$ in $[0,1]$.
   (c) Determine numerically the minimum a^* of $\rho(a,\mathbf{z}_n)$ and check that it is equal to the posterior median, i.e. with `qbeta(0.5,5,8)`.
   (d) Determine $\rho(a^*,\mathbf{z}_n)$.

2. **AfE - stima puntuale 2** Assume again that $\Theta|\mathbf{z}_n \sim \text{Beta}(5,8)$ but consider now the asymmetric loss function $(b,c>0)$:

$$L(a,\theta) = \begin{cases} b(\theta-a) & a \le \theta \\ c(a-\theta) & a \ge \theta \end{cases}.$$

   (a) Define the function `rho.asy.fun(a)` in order to obtain the values of $\rho(a,\mathbf{z}_n)$ as a varies in $[0,1]$.
   (b) Set $b=1, c=2$ and define a grid of values `a.val` for a and plot $\rho(a,\mathbf{z}_n)$ in $[0,1]$.
   (c) Determine numerically the minimum a^* of $\rho(a,\mathbf{z}_n)$ and check that it coincides with the posterior quantile of level $b/(b+c)$, i.e. with `qbeta(b/(b+c),5,8)`.
   (d) Determine $\rho(a^*,\mathbf{z}_n)$.
   (e) Check that, for $b=c=1$ we obtain the same results obtained with the absolute loss function.

3. **AfN - stima puntuale con rischio di Bayes** Consider $X_i|\theta \sim \text{Ber}(\theta)$ i.i.d. and $\Theta \sim \text{Beta}(\alpha,\beta)$. Consider $d(\mathbf{Z}_n) = \overline{X}_n$. Using the quadratic loss function we have that $R(d,\theta) = \frac{\theta(1-\theta)}{n}$. Assume $\alpha=3, \beta=6, n=10$. Compute the Monte Carlo approximation of $r_\pi(d) = \mathbb{E}[R(d,\Theta)]$