RADIATION PHYSICS : PHOTONS AND CHARGED HADRONS

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1. INTRODUCTION

As an Introduction to this Workshop on '*Nuclear technologies and clinical innovation in radiation oncology*' I have been asked to review the basic physics phenomena that form the background of the two most modern radiotherapy techniques: *Intensity Modulated Radiation Therapy* (IMRT) and *Intensity Modulated Hadron Therapy* (IMHT).

DIA 1 Title of the presentation

To perform the task I have chosen to present a simple, but quantitative, view of the most relevant phenomena, so that I shall use some equations to make clear the physical origin of well-known behaviours of hadrons (that are all particles composed of quarks) and photons propagating in matter. In doing so I have chosen the risky path of being too pedagogic and to become somewhat boring.

However it is my hope that at least the written version of my intervention will be later useful to those who do not like to use the output of large computer programs and Montecarlo codes without qualitatively understanding what is behind them.

2. RUTHERFORD FORMULA

Both photons and charged hadrons transfer their energy to matter through the interaction of charged particles with atomic electrons. Thus the phenomenon to be considered first is the collision of a charged particle with an atomic electron.

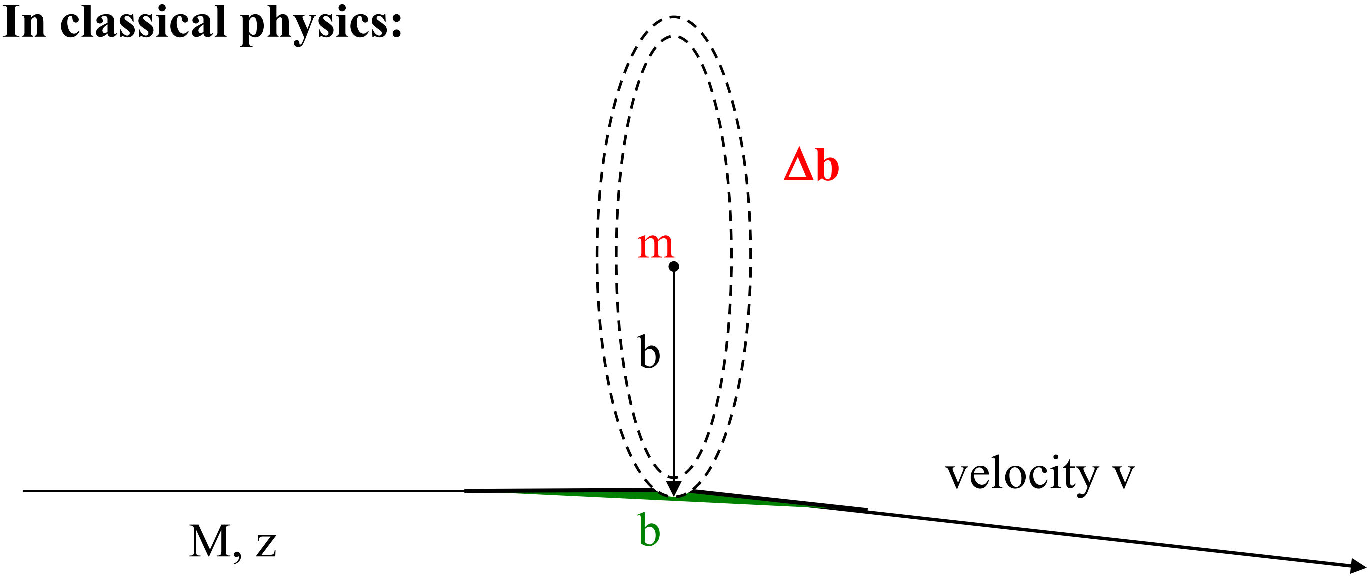
Rutherford formula describes classically such a scattering phenomenon. The formula is obtained by considering the collision of a particle M - of velocity v – that passes with a closest distance of approach b from an initially standing electron. Given this 'impact' parameter b, the impulse passed to the electron is the product of the duration of the collision b/2v with the intensity of the Coulomb force, itself proportional to $z\ e^2/\ b^2$. Here ze is the charge of the passing-by particle, that has mass M, and e is the charge of the electron of mass m.

DIA 2 Figure giving the derivation of the Rutherford formula and deducing the probability of a energy transferred in the range that goes from E to E+ ΔE.

The momentum p_t transferred to the recoiling electron equals the impulse and is thus proportional to z/bv. The energy $E_t = m\ v^2/2 = p_t^2/2m$ transferred to the electron is thus proportional to $z^2/(2m\ b^2\ v^2)$. All collisions in which the energy transferred E_t to the electron

RUTHERFORD SCATTERING

In classical physics:



$$\text{time} \cong \frac{b}{v} \qquad \text{force} = \frac{z\,e^2}{\varepsilon_0\, b^2}$$

- p_t = **momentum trasferred to electron m** $\propto \dfrac{z}{bv}$
- $E_t = \dfrac{p_t^2}{2\,m}$ = **energy transferred** $\propto \dfrac{z^2}{2mb^2 v^2}$
- $\left(\text{PROBABILITY OF } E_t > E\right) = \pi b^2 \propto \dfrac{z^2}{2mv^2 E}$
- $\left(\text{PROBABILITY OF } E < E_t < E+\Delta E\right) \propto \dfrac{z^2}{2mv^2} \dfrac{\Delta E}{E^2}$

is *larger* than E have an impact parameter *smaller* than b, so that the probability that such a collision takes place is proportional to the surface of a disk of area π $b^2 = z^2/(2m\ v^2\ E^2)$. This probability is inversely proportional to E^2.

Finally the *probability* that the energy imparted to an ensemble of Z independent atomic electrons practically at rest by the particle M falls in the interval that goes from E to (E+ ΔE) is the differential $\Delta E\ /E^2$ of 1/E:

$$\frac{Z z^2}{2 m v^2} \frac{\Delta E}{E^2}$$

This formula was derived using classical mechanics, while one should have used *quantum mechanics,* since the atomic electrons are waves and the incoming particle is also described by a wave.

DIA 2 Impact parameter of a collision with an atom described by electronic waves. Quantum mechanically the impact parameter can be defined only with respect to the nucleus. But the atom can be seen as the ensemble of Z classical oscillators.

But the most interesting fact happens: the *same* Rutherford formula is obtained in quantum mechanics. This happens only for a force field that decreases as $1/r^2$. I call it the 'Gauss miracle' since it allowed Rutherford to use a classical formula in 1911 to prove the existence of atomic nuclei. However, as we shall see in a moment, quantum mechanics cannot be forgotten in the description of the collective behaviour of the electrons of the target atoms.

The Rutherford formula is the basis of all what we shall say on the passage of charged hadrons in matter and it has to be examined very carefully.

To start, the probability is quite naturally proportional to the atomic number Z of the atom, but *the mass M of the incoming particle does not enter*.

The properties of the incoming particle enter only through the proportionality to *the square* of its charge. Thus a Carbon nucleus (z = 6) transfers to atomic electrons 36 times more energy than an electron (z = -1) that has the *same* speed v. Once the factor z^2 is taken into account, electrons and heavy nuclei of the same velocity behave (almost) identically. Indeed the probability is *inversely proportional to* v^2. This is due to the fact that the *slower* is the particle the *longer* is the time of interaction and thus the *greater* is the momentum and energy transferred. These two facts are fundamental for hadrontherapy.

Finally, the probability decreases as the square of the energy transferred to the electron, so that the spectrum of the electrons extracted from the atom (the so-called 'delta rays') is proportional to $1/\ E^2$: small-energy delta rays are much more probable than high-energy ones. This point is further discussed in the next Section.

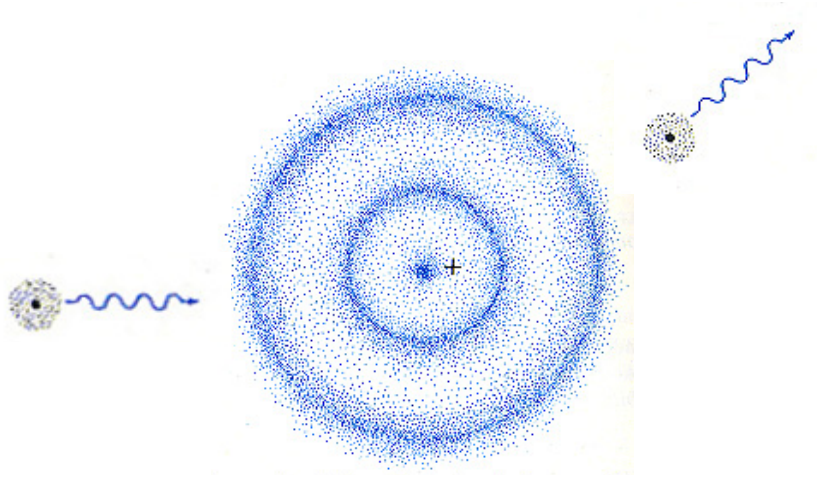
3. EXCITATIONS, IONISATIONS AND LOCAL ENERGY DEPOSITION

We now want to understand the main physical arguments leading to the mathematical expression of the *Linear Energy Transfer* (LET), that is defined as the global loss per unit path of the particle (M, z) in a piece of matter having atomic number Z.

In the following, to be concrete, all examples will concern water.

in classical physics:

$$\left(\begin{array}{c}\text{PROBABILITY FOR ATOM Z} \\ \text{OF } E \le E_t \le E+\Delta E\end{array}\right) \propto \frac{z^2}{2mv^2} \, \frac{Z}{E^2}$$



in quantum physics:

b=impact parametre can be defined <u>only</u> with respect to the nucleus surrounded by Z electrons but <u>still</u> ("Gauss miracle"):

$$\left(\begin{array}{c}\text{PROBABILITY FOR ATOM Z} \\ \text{OF } E \le E_t \le E+\Delta E\end{array}\right) \propto \frac{z^2}{2mv^2} \, \frac{Z}{E^2}$$

semi-classical model:

ATOM = ensemble of Z oscillators of $\begin{cases} \text{frequency } f = \frac{I}{h} \\ \text{period } T = \frac{h}{I} \end{cases}$

(water: I =75 ev)

Precise LET calculations can be only performed by applying quantum mechanics that intervenes, as shown in the figure, in two places.. Firstly, the incoming particle is a wave, a “wavicle” as Edington would say. Secondly, the atomic electrons are waves surrounding the nucleus; in the figure I have represented the waves of the K and L shells of an Oxygen atom of water. The shell K is occupied by two electrons and the larger shell L is occupied by six electrons. Both facts have to be taken into account.

The wave nature of the incoming particle puts limitations on the definition of the impact parameter of a collision. In particular, onecannot neglect the fact that, in quantum mechanics, *by principle* the impact parameter of an electron-electron collision cannot be defined. Instead, the impact parameter of the incoming particle with respect to the nucleus can *always* be defined (Bohr 1948 – Kgl.Danske Videnskab. Selskab, Mat.Fys.Medd.,18, n.8).

Thus one is allowed to speak of the impact parameter b of the moving particle *with respect to the nucleus* of an atom. Then ‘*close collisions*’ can be distinguished from ‘*distant collisions*’ by comparing the impact parameter b with the radius R of the atom.

We have already seen that *impact parameter* and *transferred energy* are complementary, in a loose sense of the word. This implies that close and distant collisions can be separated by looking to the transferred energy in an encounter between the particle and an atom. When the energy transferred is much larger than all the binding energies of the atomic electrons, the collision is surely ‘close’ and the atomic structure is irrelevant. Instead, when the particles passes far away from the atom, its structure is important in so much that the atom absorbs energy in a phenomenon similar to the absorption of a low energy photon belonging to a packet of electromagnetic radiation. At such large distances the atom reacts to the passage of the fast particle as a whole and it cannot be described as an ensemble of Z independent electrons. This is the second effect of quantum physics on the phenomenon that interests us.

So the main question is: how to describe simply an atom with all its complicated occupied orbitals? To do this, since the middle of the last century, physicists use a semiclassical model that identifies the quantum mechanical atom with an *ensemble of Z charged classical oscillators.* It can be shown that this system of oscillators absorbs energy from an electromagnetic wave as the real atom if the frequency of the Z oscillators is I/h, where I is a properly chosen quantity usually named ‘*mean excitation energy*’.

The name is somewhat confusing because, with reference to the part of the graph of DIA 4 that is below 150 eV, it is clear that the quantity I has to be a ‘*mean energy spent in excitations and ionisations*’ produced in distant collisions. Only taking into account the ionisations one can lump in a single number the complicated behaviour of the graph for transferred energies smaller than 150 eV.

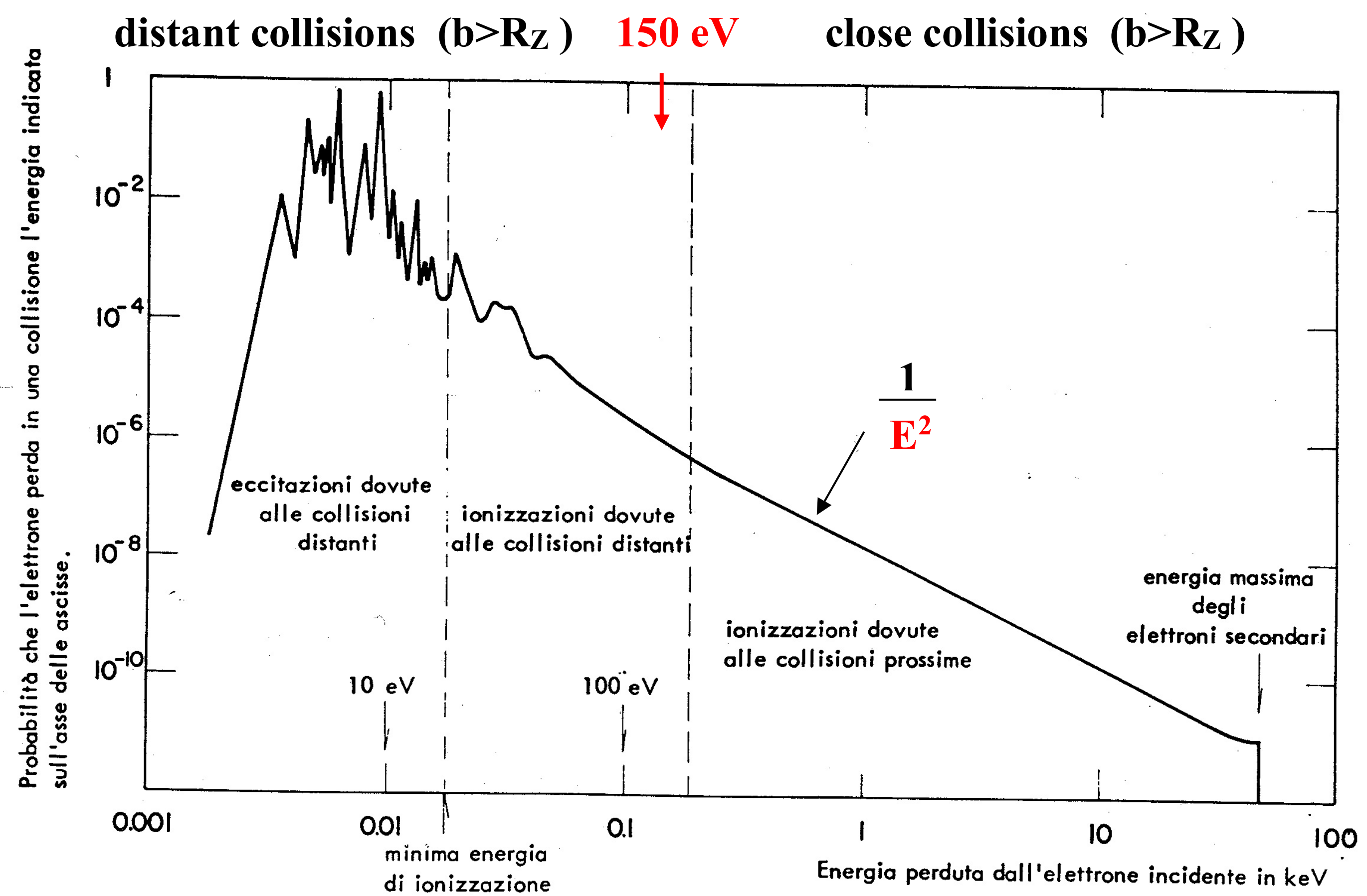
DIA 4 Distribution of the energy transferred to the atoms of water plotted versus the energy lost by a 100 KeV electron

The main point is that the quantity I can be computed with approximate quantum descriptions of the atom.. The conclusion is that I is proportional to the atomic number Z of the atom and is given with good approximation by the simple formula

$$I = (10\ Z)\ \text{eV}.$$

How can we justify it? Since I has to summarise the average energy spent in ionisations and in excitations, it has to be close to the average binding energy of the atomic electrons. For the inner K shell the binding energy is of the order of 10 Z^2. For the most external electrons the binding energy is equal to 10 eV and does not depend on the Z value. One cannot be

WATER
- close collisions $E_c > 150$ ev
- distant collisions $E_c < 150$ ev



For $E_t < 150$ eV : local deposition with
- excitations
- ion clusters

surprised if the atomic calculation give for I the logarithmic average between this two numbers.

In Oxygen I=80 eV and in Hydrogen I=10 eV. But only 2 of the 10 electrons of a water molecule H_2O belong to Hydrogen atoms, so that in water the value I =75 eV is usually adopted.

In summary, the quantity I is an average number that describes the properties of an atom when one cannot neglect the binding energy of the atomic electrons. It is the relevant parameter when the atom reacts as a whole to a charged particle that passes at a distances that is greater than few atomic diameters. To transfer a energy much larger than I, the particle has to interact mainly with a single electron in what we have called a 'close collision'. It is thus natural to take *twice the value of I* as the energy that separates the 'close' from the 'distant' collisions. Thus for water the separation between close and distant collisions is set at 150 eV, but none of the arguments we shall present is sensitive to such a choice.

This limit is indicated in the figure, where the distribution of the energy transfers in water is plotted versus the energy lost by an incoming electron of 50 KeV. For energy transferred smaller than 150 MeV, the molecules of water are either ionised or excited. The excitations of particular molecular states depend on the precise distribution of the atomic orbitals. They are represented by the peaks drawn qualitatively in the figure.

In parallel with the excitations, that rapidly decay, many close collisions produce ionised molecules, which correspond to larger transferred energies. Since it is known that to create in matter an ion pair on average 30-40 eV have to be spent, the closest of the distant collisions (in which an energy equal to 150 eV is transferred) at maximum will produce 3 or 4 ion pairs. The corresponding electrons have practically no kinetic energy, so that they remain close to the original molecule and often recombine. In a few cases they will be recognised as a *cluster of ions*.

When the energy transferred in water is larger than about 150 eV, we speak of close collisions: the atomic electrons react practically as isolated and practically at rest targets. When set in motion as the consequence of an encounter they take the name of '*delta rays'*. The Rutherford formula gives their energy spectrum, that is simply proportional to $1/E^2$. This implies that 2 keV delta rays are 4 times less numerous than 1 keV delta rays.

The distribution of the figure is most important because it shows that small energy transfers are much more probable than large ones and, at the same time, makes clear that the energy span of the rare close collisions is much larger than the one of distant collisions. *These two facts balance each other* so that the average losses in close and distant collision are practically equal, as we now pass to show. To show it we must first compute the energy dependence of the LET.

4. LET IN WATER

The LET is obtained by summing the formula obtained above on all possible energies transferred to the atoms of the traversed piece of matter. In this sum the maximum E_{max} is obtained, of course, in the *closest* of all the close collisions. In this case the electrons react independently and the *maximum* transfer can be computed applying classical mechanics to the collision with an atomic electron.

When non-relativistic electrons move in matter, the maximum energy of the delta rays is equal to the energy of the incoming electron v divided by square root of two. This happens when the two indistinguishable final electrons are moving after the collision at 45°. In this case the energy is ($m\ v^2/2$). For non-relativistic hadrons the electron velocity is equal to twice the velocity of the hadron itself ($v_e = 2v$), as it happens when a ball is launched against the motor of an advancing truck, so that

$$E_{max} = m\ v_e^2/2 = 2\ mv^2.$$

Corrections have to be applied when the incoming particles are relativistic, but for qualitative arguments it is enough to recall that the maximum electron energy is always of the order of mv^2. In the formula we shall use the ($2\ mv^2$) valid for non-relativistic hadrons, but eventually in the electron case a small correction will be applied. This implies that a hadron of about 200 MeV per nucleon ($v/c = 0.5$) puts in motion electrons of energy smaller than about 100 keV. This limit does not depend upon the value of the hadron mass M:

The lower limit of the energy E_{min}, is transferred in the *most distant* of all distant collisions.

DIA 5 Computation of the maximum energy transferred to an atom described as ensemble of classical oscillators.

The *minimum* can thus be computed within the *oscillator model* by computing the maximum impact parameter in distant collisions. This is determined by the *adiabatic limit*: the electromagnetic field of the passing particle varies so slowly (with respect to the time $T = h/I$) that the passage distorts only adiabatically the electronic orbitals that eventually are left back in the initial condition without any transfer of energy. This is similar to what happens when a spring is compressed and released very slowly and the energy spent is completely given back to the hand acting on the spring. Thus the atom cannot absorb energy if the duration of the electromagnetic pulse is equal to h/I.

As shown above, classically the duration of the collision is b_{max}/v. But one has to take into account the fact that its electromagnetic field is squeezed longitudinally by the relativistic effects. This multiplies b_{max} by the usual relativistic shortening factor $(1- v^2/c^2)^{½}$. Equating this time to h/I, the maximum impact parameter is obtained

$$b_{max} = h\ v\ (1- v^2/c^2)^{-½} / I\ .$$

By computing the numerical value one finds *that the charged particles passes at hundreds of atomic diameters from the nucleus*: distant collisions are very loose encounters indeed.

Given the maximum impact parameter, the minimum momentum is computed by applying the *uncertainty principle* that fixes the momentum transfer

$$p_{min} = h/\ b_{max} = I\ (1 - v^2/c^2)^{½} / v.$$

Note that in taking this ratio the constant h has disappeared. One can conclude that quantum mechanics is needed to compute the losses in matter, but eventually leaves no sign on the final formula. This is the second miracle that allows some great simplifier to deduce an approximate formula for the losses of charged particles in matter without even mentioning quantum physics.

Knowing the momentum transferred to an electron, since this electron is non-relativistic the energy transferred can be computed with the formula of classical physics:

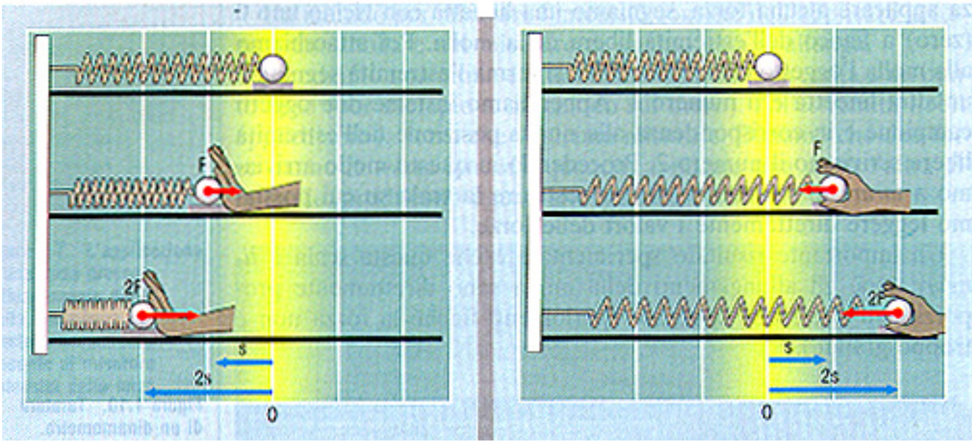
$$E_{min} = p_{min}^2 / 2m = I^2\ (1- v^2/c^2) / (2m\ v^2).$$

The Linear Energy Transfer is obtained by first multiplying the Rutherford probability for having an energy loss E by the energy lost E and then by summing this product between E_{min}

$E_{max} \Rightarrow 2mv^2$ computed with classical mechanics

E_{min} must be computed with quantum mechanics

forced oscillator



TIME OF INTERACTION IN THE ADIABATIC LIMIT $= \dfrac{b}{v} > T = \dfrac{h}{I}$

$\Downarrow$

$(1-v^2/c^2)^{1/2} \Rightarrow$ (relativity) $\quad b_{max} = \dfrac{vh}{I}$

$\Downarrow$ quantum mechanics

$$p_{min} = \frac{h}{b_{max}} = \frac{I\,(1-v^2/c^2)^{1/2}}{v}$$

$\Downarrow$

$$E_{min} = \frac{p^2}{2m} = \frac{I^2\,(1-v^2/c^2)}{2mv^2}$$

and E_{max}. Since the probability is proportional to $1/E^2$, the multiplication by E give a function that is proportional to 1/E.

DIA 6 Integral to obtain the LET and formula of LET in water

Then the integral of dE/E gives the natural logarithm of the ratio (E_{max} / E_{min}) of the two limits. By introducing the numerical factors, left out for simplicity in the above derivation, the *Linear Energy Transfer* comes out to be

$$\text{in water:} \quad \left(0.0076 \frac{\text{keV}}{\mu\text{m}}\right) \frac{z^2}{v^2/c^2} \ln\left(\frac{(2\,mv^2)^2}{I^2\,(1-v^2/c^2)}\right)$$

The numerical factor has the following expression that was not derived only for brevity:

$$(e^4 N_A) / (16\,\pi\,\varepsilon\, m\, c^2).$$

This is the formula we have been looking for. The most important fact is that *the LET is a function of v only*, so that it is the same (a part the simple factor z^2) for all charged particles having the same speed. This is true also for electrons, but for the small correction discussed above that reduces the value of the logarithm, typically in the range 10-20, by about 10%.

DIA 7 Plot of LET of the model compared with the exact calculation.

In the figure the results of the exact calculations, plotted as a function of (K/Mc^2), are compared with the above formul. K is the kinetic energy of the particle of rest energy equal to Mc^2. (The fraction $k = K/Mc^2$ is related to the quantity γ often used by physicists: $\gamma = k + 1$.)

One can express the LET as a function of K/Mc^2, the *fractional kinetic energy* of the incoming particle, because the only relevant parameter is the square of the velocity, and v^2 can be always expressed as a function of K/Mc^2.

The rest energies of the charged particles used in radiotherapy are collected in the table.

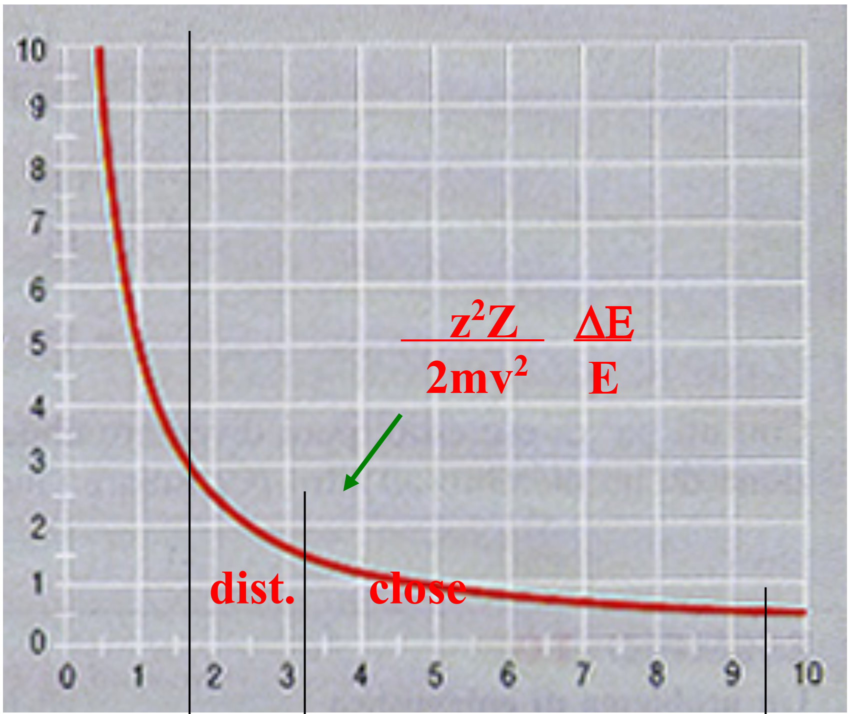
Table 1. *Properties of the charged particles used in hadrontherapy*

Charged particle	**z**	**Rest energy Mc^2 [MeV]**	**Kinetic energy for which $K/Mc^2 = 0.40$ [MeV]**	**Kinetic energy per nucleon for which $K/Mc^2 = 0.40$ [MeV/u]**
electron	-1	0.51	0.20	-
proton	1	938	375	375
Helium ion	2	3'700	1'480	370
Carbon ion	6	11'170	4'470	370
Neon ion	10	18'600	7'4500	370

In the double-logarithmic graph of the figure, for fractional kinetic energies k smaller than about 0.4 (i.e. for $v/c<0.7$) the LET is practically represented by a straight line with an accuracy that is definitely better than 5% over almost the full range of energies. This implies that a simple power law can be used to describe the LET for the kinetic energies used in hadrontherapy – as shown in the last two column of the table. The simple expression that can be used is

$$\text{LET in water:} \quad 0.12\, z^2\ \text{keV} / \mu\text{m}$$

$$\Delta K = \sum_{E\ min}^{E\ max} \left(\begin{matrix} \text{PROBABILITY OF LOSS E} \\ \propto \dfrac{z^2 Z}{2mv^2} \dfrac{\Delta E}{E^2} \end{matrix} \right) \times E$$

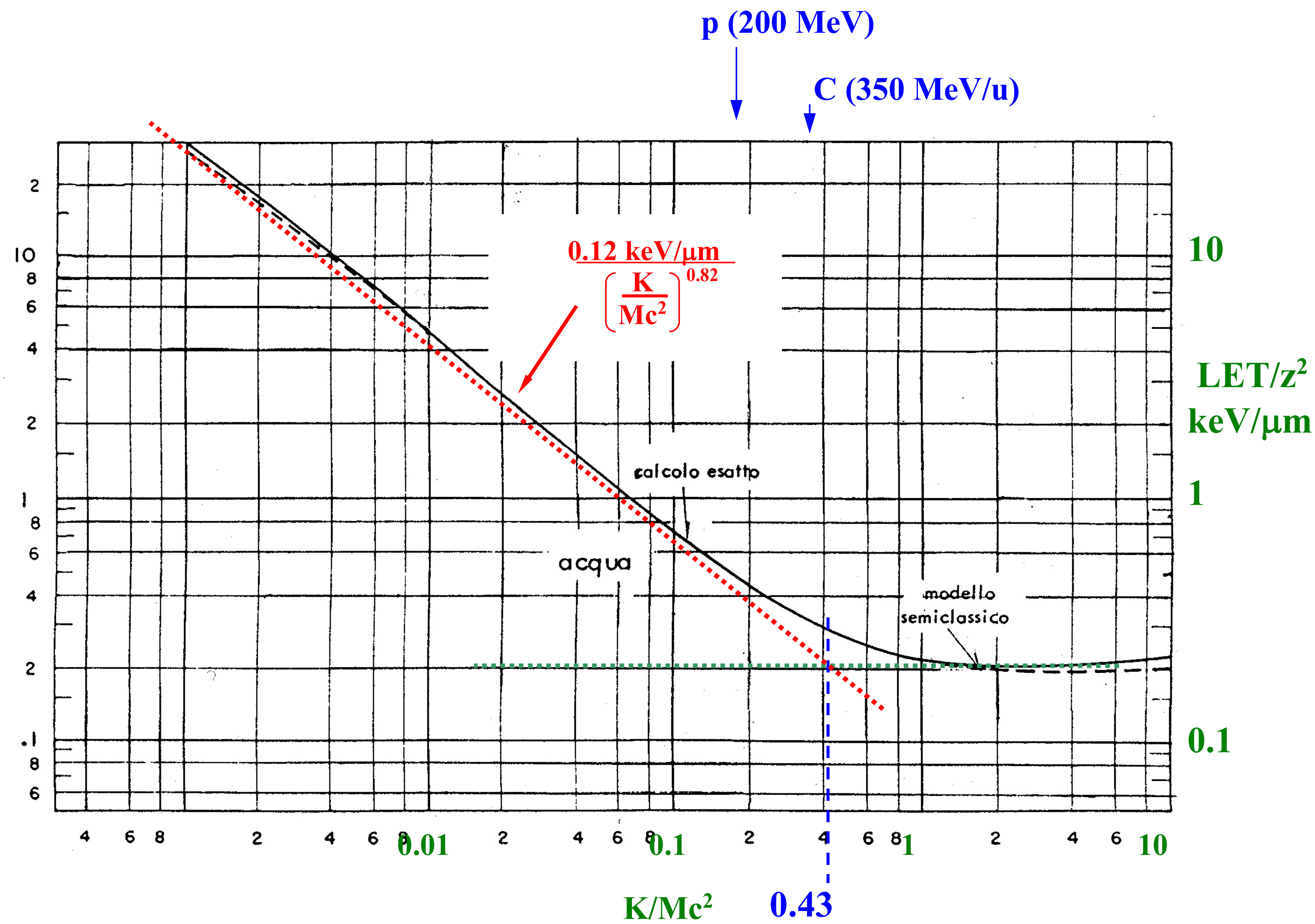


$2\,I = 150$ eV $\qquad 2\,mv^2 = 250$ keV

$$\frac{I^2\,(1-v^2/c^2)^{1/2}}{2mv^2} = 0.02 \text{ eV} \qquad v/c = \tfrac{1}{2},\ K_{proton} = 150 \text{ MeV}$$

$$LET_{water} = \left(0.0076\ \frac{\text{keV}}{\mu\text{m}}\right) \frac{z^2}{v^2/c^2} \ln\left(\frac{(2mv^2)^2}{I^2\,(1-v^2/c^2)}\right)$$

the only parameter is v!



p (200 MeV)
C (350 MeV/u)
0.12 keV/µm / (K/Mc²)^0.82
calcolo esatto
acqua
modello semiclassico
LET/z²
keV/µm
10
1
0.1
0.01
0.1
1
10
K/Mc²
0.43

$$(K / Mc^2)^{0.82}$$

For k = (K /.Mc²) larger than 1 the particles are relativistic and the velocity does not increase any longer with energy. The LET is practically constant and equal to 0.21 keV/μm.. In the range between 0.4 and 1 no simple rule applies.

Note that a classical calculation (no relativity and no quantum physics) would have given in the denominator k^1 instead than $k^{0.82}$. The power 1 corresponds to a power 2 in the speed v of the moving charge, due to the fact that the faster the particle goes the smaller is the impulse p_t given to an atomic electron. The energy transferred then goes as v^2. This is the physical origin of the k^{-1} law of the LET in classical physics. All what the relativistic and quantum complications have done is to modify a 1 in a 0.82!

The above formula (reduced by 10%) is also valid for electrons up to k = 0.4, i.e. kinetic energies less than about 200 KeV. Above 0.5 MeV the loss is practically constant and equal to 0.19 keV/μm in water, 10% less than for hadrons.

Now we can go back to the problem of the fraction of the lost energy that is spent in distant collisions. In the semiclassical model the fraction $F_{distant}$ is simply as the ratio of two logarithms which are proportional to the integrals of the losses from E_{min} to the energy 2I separating close and far collisions and from E_{min} to E_{max}

$$\ln\left(E_{min} / 2I\right) \;/\; \ln\left(E_{max} / E_{min}\right)$$

This is essentially the fraction $F_{distant}$ plotted in the figure for electrons. The graph shows that the fraction of the energy lost by a charged particle in distant collisions is *equal to 60%* of all the losses independently of the kinetic energy of the fast particle.

DIA 8 Ratio of the energy lost in close collision to the total energy lost.

We are interested in $F_{distant}$ because it quantifies the fraction of all the losses that are *deposited locally,* i.e. very close to the molecule that reacted to the far away passage of the charged particle.

The large fraction of ionisation and excitations without production of delta rays should be always taken into account when using the usual statement that fast electrons, for instance, are '*sparsely ionising*' radiations.

5. MULTIPLE SCATTERING

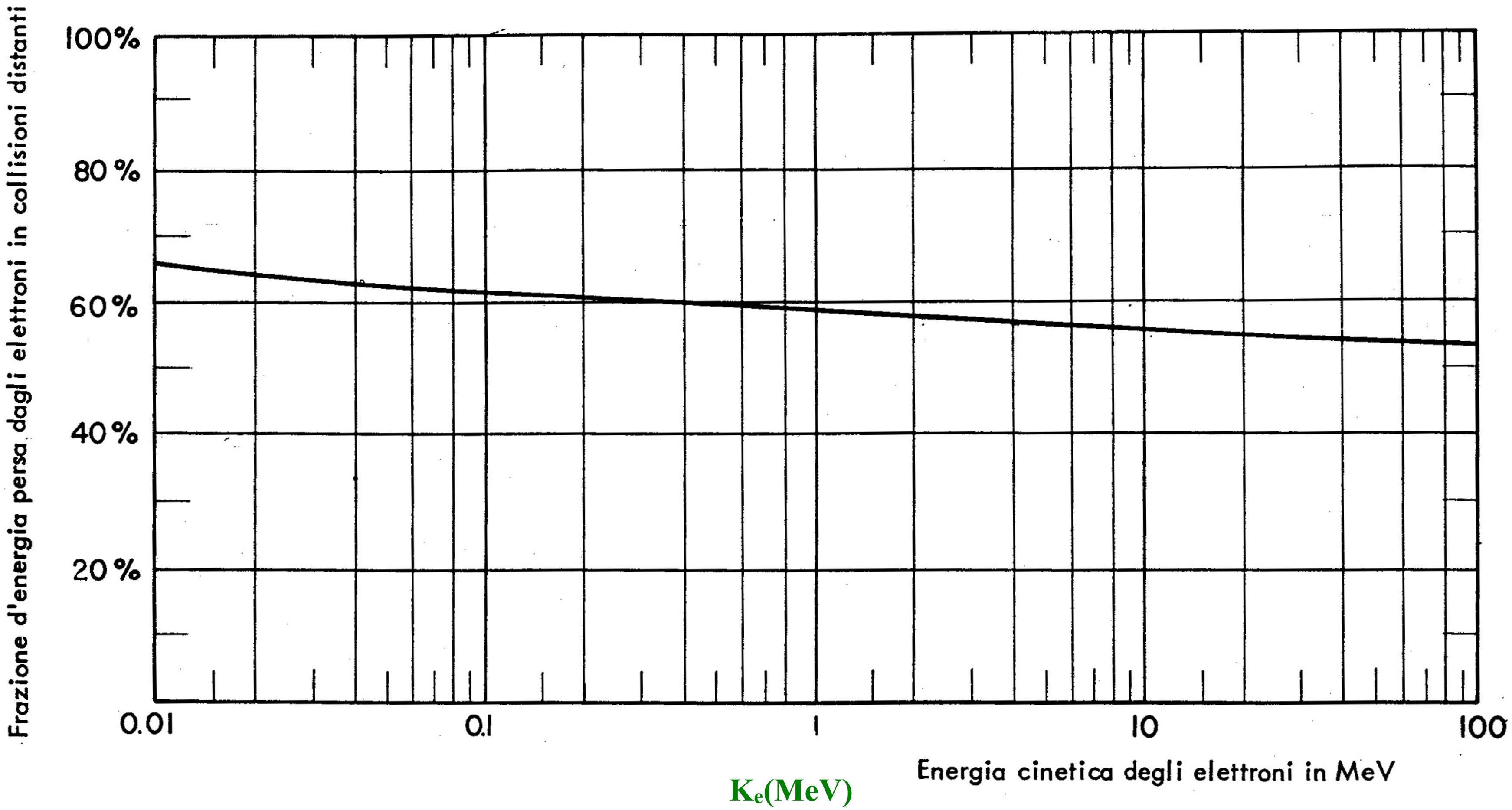
In the interactions with the nuclei of the traversed matter electrons are much more disturbed than hadrons because of the much smaller mass with respect to the masses of the target nuclei.

A pencil beam of hadrons acquires in matter a radius that increases with the depth, as shown if the figure. Note that the heavier nuclei disperse much less than the protons. Anyway, the widening of a pencil beam is small with respect to the depth reached, so that the penetration in matter practically equals the length of the actual path followed by the average particle. This fact is used in the computation of the range of hadrons in matter.

DIA 9 Widening of a pencil beam of hadrons and high energy electrons in water. Electrons in matter. plural and multiple scattering. Practical range

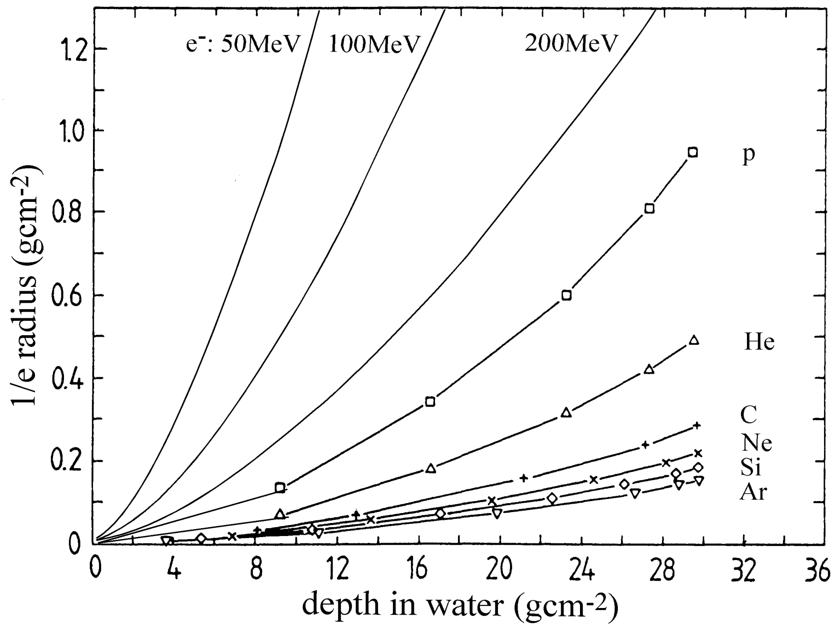
As far as the electrons are concerned, it is interesting to remark that electrons of many hundred MeV, i.e electrons having energies similar to the ones of the protons used in therapy,

$$\frac{\Delta E \text{ dist. coll}}{\Delta E \text{ collision}} = \frac{\ln (E_{min} / 2I)}{\ln (E_{max} / E_{min})} \cong 60\% \Longrightarrow LET_{150} \cong 0.6 \times LET$$



60% of the energy is deposited “locally”

SCATTERING OF CHARGED PARTICLES
(mainly on nuclei)



HADRONS

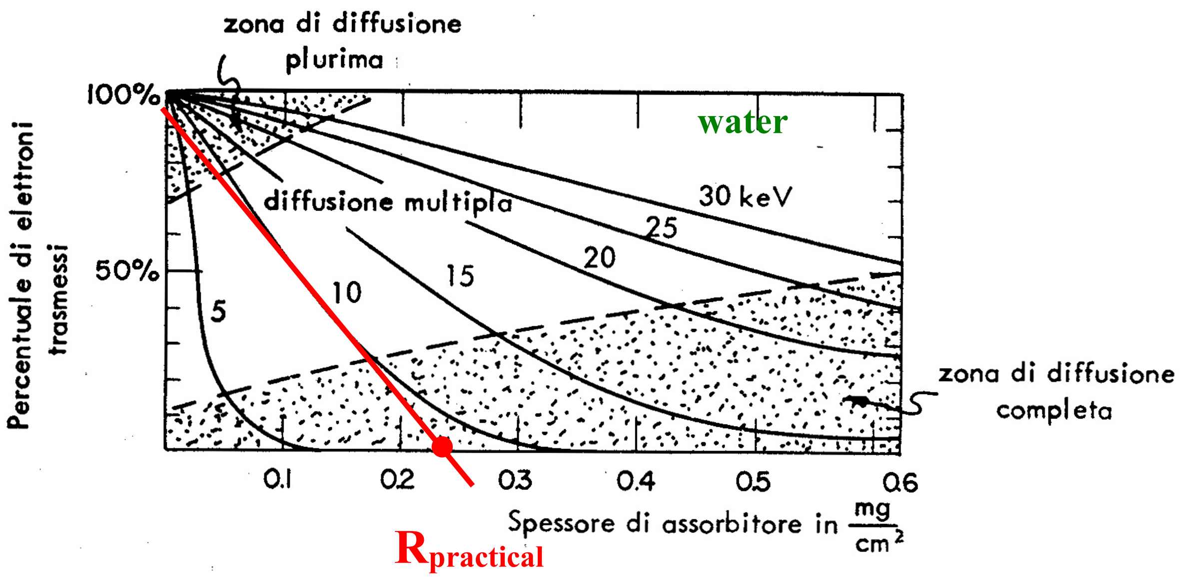
ELECTRONS

Three regimes

plural scattering

multiple scattering

complete diffusion



K_e = 20 keV **R_p = 0.7 10^{-3} cm = 7 μm**

do not behave very differently from protons as far as the dispersion in light matter is concerned.

The behaviour of low energy electrons is shown in the next figure.

Three different regimes can be recognised in the case of low energy electrons. In the first layers few large angle scattering are important. This is called '*plural*' scattering. At greater depth many small angle scattering combine statistically so that the average angle of the initially collimated beam increases as the square root of the thickness. This is called 'multiple' scattering and statistical formula exists to compute the spreading of a beam. Finally, when the electrons have spent almost all of their kinetic energy in collisions, the memory of the initial direction is lost and the phenomenon is called of 'complete diffusion'.

It is not possible to define an average range of electrons in matter, as it is done for hadrons. The quantity used is then the '*practical range'* R_p, obtained by extrapolating (in the almost linear region) the graph of the number of electrons as a function of depth. For instance from the figure one can deduce that the practical range of 20 keV electrons in water is 0.7 10^{-3} cm, i.e 7 microns.

6. RANGE OF HADRONS IN MATTER

By neglecting multiple scattering, the range of hadrons in matter obtained by remarking that a particle looses the kinetic energy ΔK in a distance $\Delta x = \Delta K$ / LET. The range is then computed as the sum of a large number of small steps.

DIA 10 Integral of LET to obtain the range of hadrons. Good to 5%

For the energies of interest in therapy ($K/Mc^2 \leq 0.4$) it is enough to integrate the approximate expression that gives the LET as proportional to The integral of $K^{3/4}$ is 7 $K^{7/4}$/ 4 so that, taking into account the numerical factors,

$$R^{water} = (425 \text{ cm}) (A/z^2) (K /.Mc^2)^{1.82} \qquad \text{(hadrons).}$$

The mass number A enters because Mc^2 has been expressed as A×931 MeV.

***Table 2**. Properties and ranges of the charged particles used in hadrontherapy*

Charged particle	z	Rest energy Mc^2 [MeV]	Kinetic energy for R_{water} = 20.0 cm (exact calculation) [MeV/u]	k= $K /.Mc^2$	Range computed with simple formula [cm]
proton	1	938	172	938	19.4
Helium ion	2	3'700	173	3'700	20,3
Carbon ion	6	11'170	330	11'170	21.3
Neon ion	10	18'600	410	18'600	19.2

This is a simple formula valid to better than 5%, as shown by comparing the number listed in the last column with the value 20.0 cm obtained in a complete calculation. The accuracy improves when the energy is reduced and the range is smaller than 20 cm in water.

7. THE BRAGG PEAK

As shown in the DIA 11 by taking the power 1/1.82 of the formula of the range in water one obtains the kinetic energy as a function of the range:

RANGE OF CHARGED HADRONS

The range in matter is equal to the path length because the trajectors are straight

$$R = \Sigma \frac{\Delta K}{\frac{\Delta K}{\Delta x}} = \Sigma \frac{\Delta K}{LET}$$

$$R = \frac{1}{z^2\, 0.12 \ \mathrm{MeV/mm}} \int \left(\frac{K}{Mc^2}\right)^{0.82} \frac{dK}{Mc^2} \cdot Mc^2$$

$$= (425 \ \mathrm{cm}) \ \frac{A}{z^2} \left(\frac{K}{Mc^2}\right)^{1.82}$$

	A	z	Mc^2 (MeV)	K/A (MeV)	K(MeV)	R(cm)
p	1	1	938	172	172	**19.4**
He	4	2	3'700	174	695	**20.3**
C	12	6	11'200	330	3'960	**21.3**
Ne	20	10	18'600	410	8'200	**19.2**
					average	20.0

R_{exact} = 20.0 cm

good to 5%

DIA 11 Plot of the Bragg peak in water for protons.

The result is

$$K / Mc^2 = (z^2/A)^{1/1.82} (R^{water}/R^*)^{1/1.82}.$$

Here a convenient unit of length, that comes out *naturally* in this phenomenon, has been introduced: R*= 425 cm.

By substituting this formula in the expression giving the LET, one can express the LET as a function of the *residual range*

$$LET = L^* (z^{1.1} A^{0.45}) (R^*/ R^{water})^{0.45}.$$

The power 0.45 is the ratio 0.82/1.82 and the exponent 1.1 of z is (2- 2 0.45). In this formula L* is the *natural* unit of energy loss: L*= 0.12 keV/µm.

We conclude that, if the LET can be expressed as a power law of K, all Bragg peaks of light ions in water *have the same shape*, but the numerical values are higher than the proton one by the factor ($z^{1.1}$ $A^{0.45}$) compared with the exact calculation in Table 3. This dependence is clearly superior to the one (z $A^{0.5}$).that could be derived with simple classical arguments.

Table 3. *Ion dependence of the height of the Bragg curve*

	proton	**He ion**	**Carbon ion**	**Neon ion**
Value at 20.0 cmm (Table 2)	1	4.02	23.0	47.6
$z^{1.1}$ $A^{0.45}$	1	4.00	22.0	48.5
z $A^{0.5}$	1	4.00	20.8	44.7

Hadrons interact through the strong force with the nuclei of matter. These nuclear interactions cannot be described analytically, but are very important for the dose deposited by the nuclear fragments and the break-up of the nuclei of matter. One has to use Montecarlo simulations that are usually normalised to a total cross section determined by Sihver et al.

DIA 12 Nuclear interactions of Carbon in water.

I have no time to discuss this important subject.

8. RANGE OF ELECTRONS IN MATTER

The practical range of electrons is plotted in the next figure, which shows that - at variance with what happens in Aluminium - in such a light material the practical range R_p is approximately equal to the range measured along the path. This range can be computed using the LET by reducing the hadron losses by 10%, but here the situation is more complicated

DIA 13 Practical range of electrons.

THE BRAGG PEAK

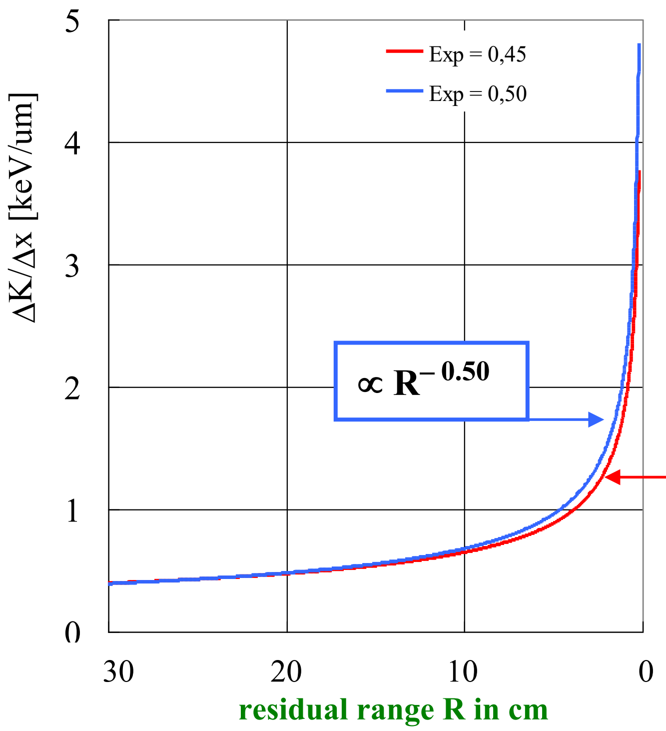
In water (I=75 eV) with $k = K/Mc^2$

$$R = R_\circ \frac{A}{z^2} k^{1.82} \quad \Rightarrow \quad k = \left(\frac{z^2}{A}\frac{R}{R^\circ}\right)^{\frac{1}{1.82}}$$

$$\frac{0.82}{1.82} = 0.45$$

$$LET = L_\circ \frac{z^2}{k^{0.82}} \quad \Rightarrow \quad LET = L_\circ z^{1.1} A^{0.45} \left(\frac{R_\circ}{R}\right)^{0.45}$$

[natural units : $R_\circ = 425$ cm, $L_\circ = 0.12$ keV/µm]



$$0.12 \frac{keV}{\mu m} \left(\frac{425\ cm}{R}\right)^{0.45}$$

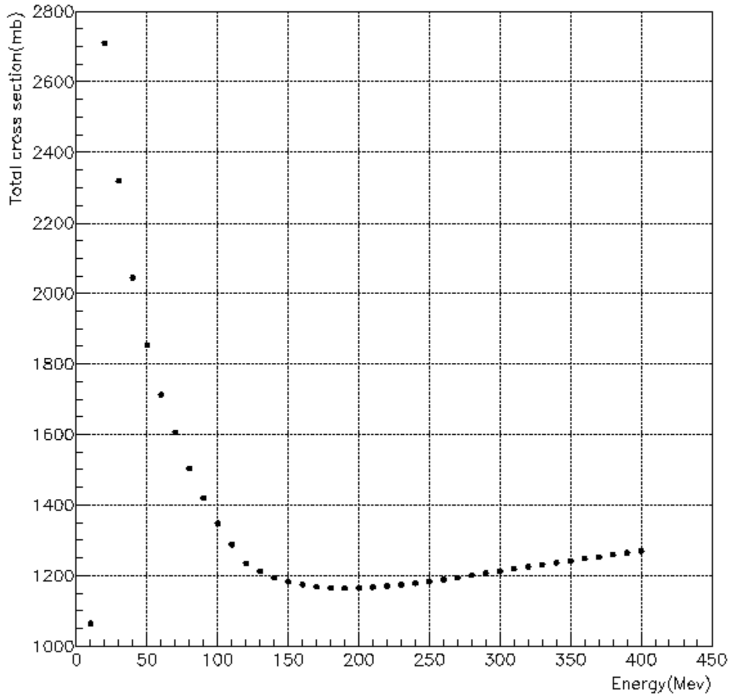
All Bragg peaks are equal in shape, but the losses are proportional

to: $z^{1.1} A^{0.45}$

	p	He	C	Ne
$z^{1.1} A^{0.45}$	1	4	22	48

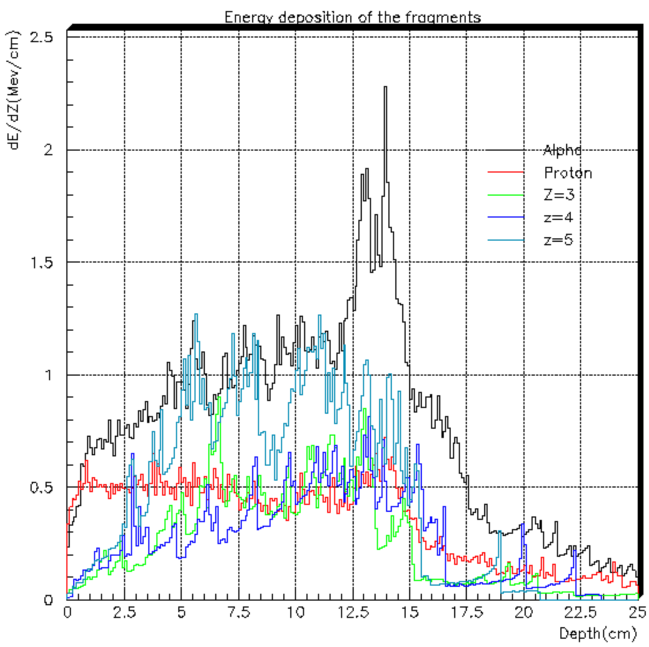
NUCLEAR INTERACTIONS

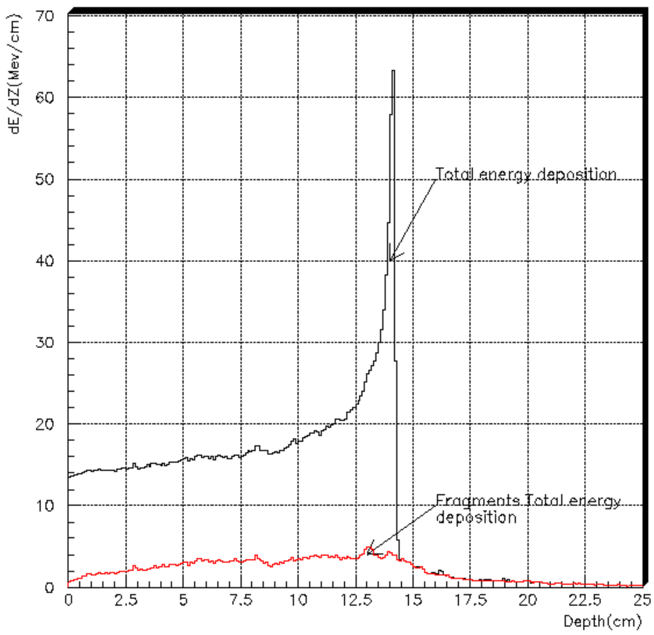
The nuclear cross section varies with energies



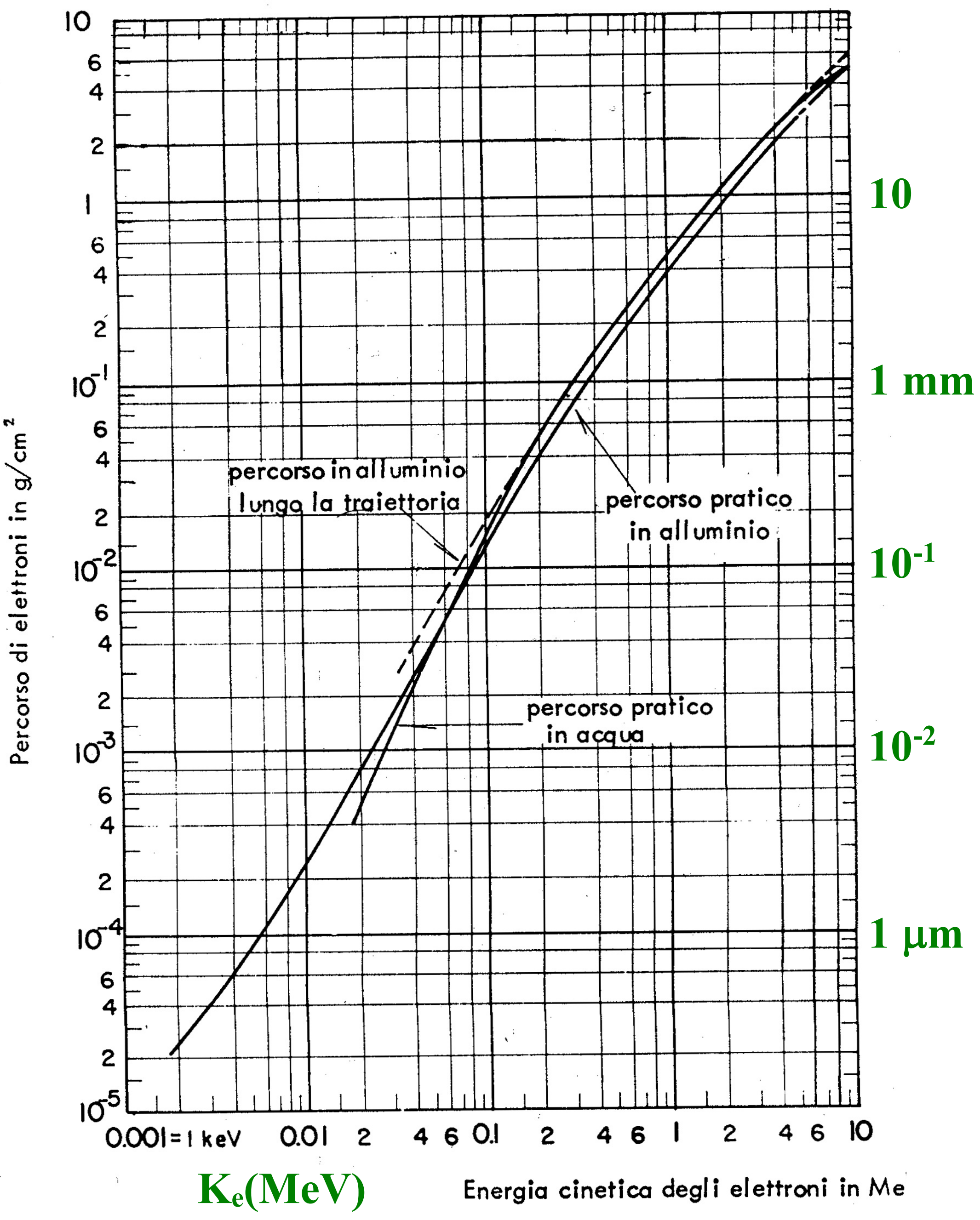
[Sihver et al]

Monte Carlo results (Geant 3)





PRACTICAL RANGE OF ELECTRONS



E (keV)	R_p
1	0.1 μm ≅ 100 nm
10	2.5 μm
25	10 μm

Indeed we have to recall that

(i) the only quantity that counts is the fractional kinetic energy K/Mc^2 and

(ii) for $K/Mc^2>1$ electrons have an almost constant value of LET: 0.18 keV/μm.

On the other side, for $K/Mc^2<0.4$ the same formula derived for the hadrons applies with a reduction of 10% and the substitution A= m/M= (0.51 MeV) / (931 MeV) = 5.5 10^{-4}:

$R_p^{water} = (0.21\ cm)\ (K\ /.0.5\ MeV)^{1.82} + (K - 0.25\ MeV)\ /(1.9\ MeV/cm)$ (electrons).

DIA 14 Formula to obtain the practical range of electrons, with table.

To compute the range at all energies we use the hadron formula up to $K/Mc^2 = 0.5$ and assume a constant LET at higher energies. The practical range in water can thus be written as the sum of two terms, which are given in Table 3 for four different energies.

One can conclude that the semi-classical model gives electron ranges that are good to about 15%. This is an excellent result, taking into account the fact that the very definition of practical range and its identification with R are subject to experimental and theoretical uncertainties.

Table 4. *Practical ranges of electrons in water*

Electron energy	Two terms [cm]	Range computed with simple formula	Range read from the figures
0.020 MeV= 20 keV	0.0006 + 0	0.0006 cm = 6 μm	7 μm
0.20 MeV= 200 keV	0.04 + 0	0.04 cm = 400 μm	450 μm
2 MeV	0.06 + 0.92	0.98 cm	1.1 cm
10 MeV	0.06 + 5.1	5.2 cm	5.0 cm

9. CONVENTIONAL RADIATIONS

The above arguments, centred on the LET and the range of charged particles in matter, have *two* important consequences for conventional radiotherapy.

The first point I want to discuss is the possibility to compute the practical range of the electron beams used in radiotherapy. The figure shows the dose distributions measured with 6 MeV and 18 MeV electrons. The measured ranges agree well with the one computed by the simple formula derived above, that give 3.1 cm and 9.4 cm respectively.

DIA 15 Ranges and radiation due to electrons used in radiotherapy.

To discuss the second consequence one should recall that electrons irradiate energy in the form of high-energy photons. The filtered energy spectrum of these photons goes from about 0.5 MeV to the energy of the electrons.

These photons transfer energy to the traversed tissues because they set in motion atomic electrons by the so-called *Compton effect*. This is just the scattering of two particles, a high-energy-photon and an electron practically at rest in matter. For reasons that I shall not discuss here, the energy transferred by a beam of X-ray - irradiated by electron in a far away heavy target -decreases exponentially with the depth. This quantity is called KERMA for *Kinetic Energy Released in Matter*.

RANGE OF ELECTRONS

For E < 0.25 MeV $$R = 0.21\ \text{cm}\left(\frac{K}{0.5\ \text{MeV}}\right)^{1.82}$$

For E > 0.25 MeV $$R = 0.06\ \text{cm} + \frac{K - 0.25\ \text{MeV}}{1.9\ \text{MeV/cm}}$$

Scattering and diffusion make

$$R_p < R$$

BUT

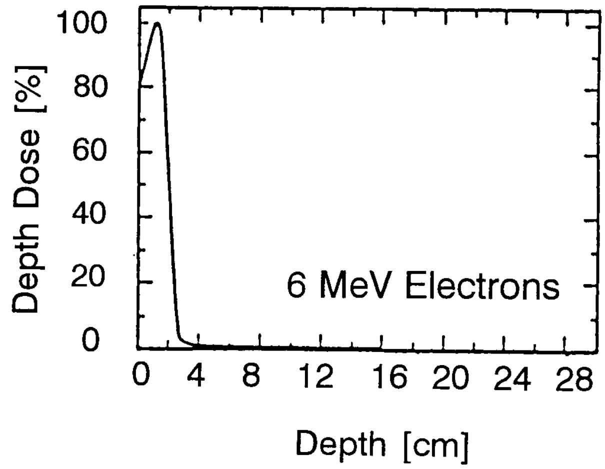
in light material (water)

$$R_p \cong R$$

Electron energy	Range computed with simple formula	Practical range
0.020 MeV= 20 keV	6 μm	7 μm
0.20 MeV= 200 keV	400 μm	450 μm
2 MeV	0.98 cm	1.1 cm
10 MeV	5.2 cm	5.0 cm

↑ **good to 15%**

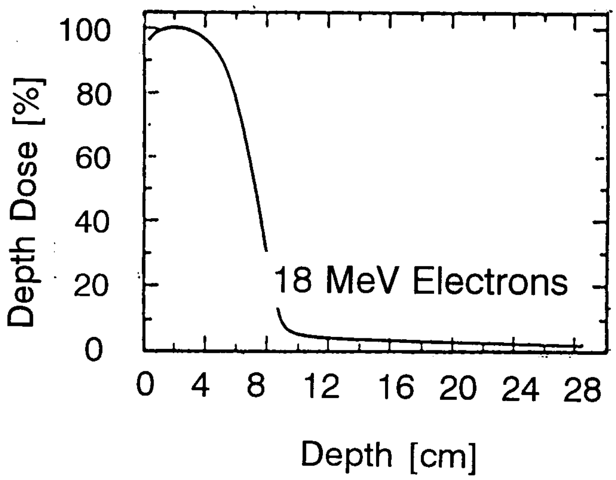
CONVENTIONAL RADIOTHERAPY



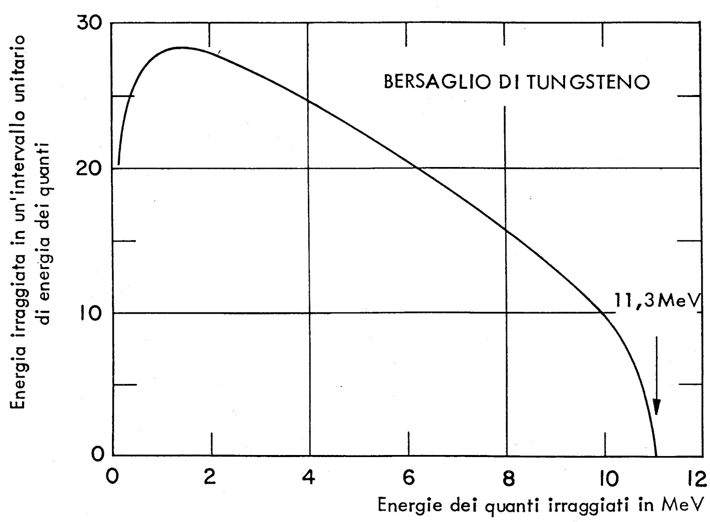
$$R = 0.06\ \text{cm} + \frac{K - 0.25\ \text{MeV}}{1.9\ \text{MeV/cm}}$$

R(6 MeV) = 3.1 cm

R(18 MeV) = 9.4 cm



Electrons radiate a continuum spectrum of "X-rays"



for the radiated spectrum

effective energy $E_X \cong \dfrac{2\ K_e}{5}$

DIA 16 Figure on the build up effect

The second consequence is an understanding of the reason for which high-energy X-rays spare the surface tissues. As well known, the dose increases in the first few centimetres due to the ‘build-up’ effect.

If the X-ray beam - produced by electrons of energy K_e - was not attenuated in matter, and all the electrons moved forward with the *same* range R, the position D of the peak of the dose would be equal to R.

But there are many phenomena that play important roles when one is faced with the understanding of experimental data.

DIA 17 Figure on the build up effect

The photons have energy E_X in the range that goes from 0.5 MeV and K_e and the Compton electrons have energy between zero and E_X. All ranges are thus present and one can only say that D is expected to be smaller than the range computed using the energy K_e of the electrons radiating the photons in the heavy target. This is shown in the Table

Table 5. *Peak of the dose in depth for different energies of the radiating electrons*

K_e (MeV)	**D (cm)**	**K(D)** (MeV)	K(D) / K_e
2	0.5	1.1	0.55
8	2.2	4.2	0.52
20	4.5	8.6	0.43
70	11	21	0.30

The fact that the ratio decreases with the energy shows that the phenomenon is complicated.

10. DISTRIBUTIONS OF THE LINEAR ENERGY TRANSFER

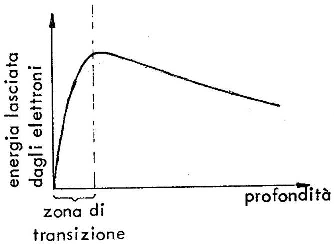
A somewhat unfashionable but still (in my opinion) useful way of representing the way in which energy is transferred by radiation to matter makes use of the spectra of LEY. In such a graph the area under each part of the curve represents how much energy is transferred by particles that have the value of the LET represented on the horizontal axis.

DIA 18 LET spectra for three beams of X-rays of increasing energy

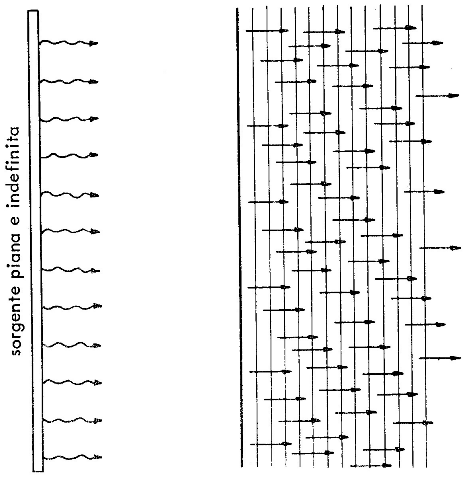
When comparing the effects of hadrons with photons an important reference point is the LET vale of 10 keV/μm.

Indeed radiobiological studies have shown that for LET values larger than this the cell killing follows from radiobiological processes different from the ones acting for LET in the range 0.2 – 1 keV/μm. This delicate, and somewhat controversial, point will be discussed in the next presentation. With this last figure I want only to underline that, due to the many low energy electrons and delta rays, high-LET depositions are present also in a field of conventional radiations. Hadrontherapy is different from conventional radiotherapy, but in reality the underlying microscopic phenomena are the same. What changes is the relative weight.

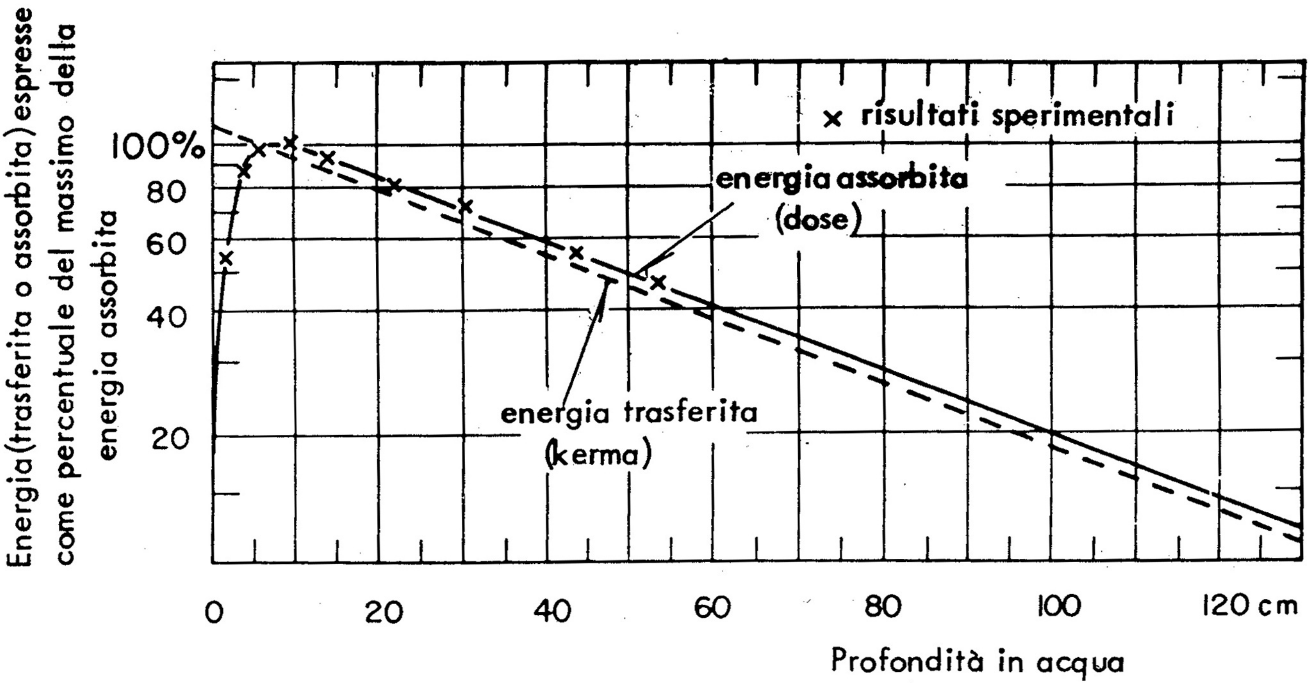
RADIOTHERAPY WITH X-RAYS



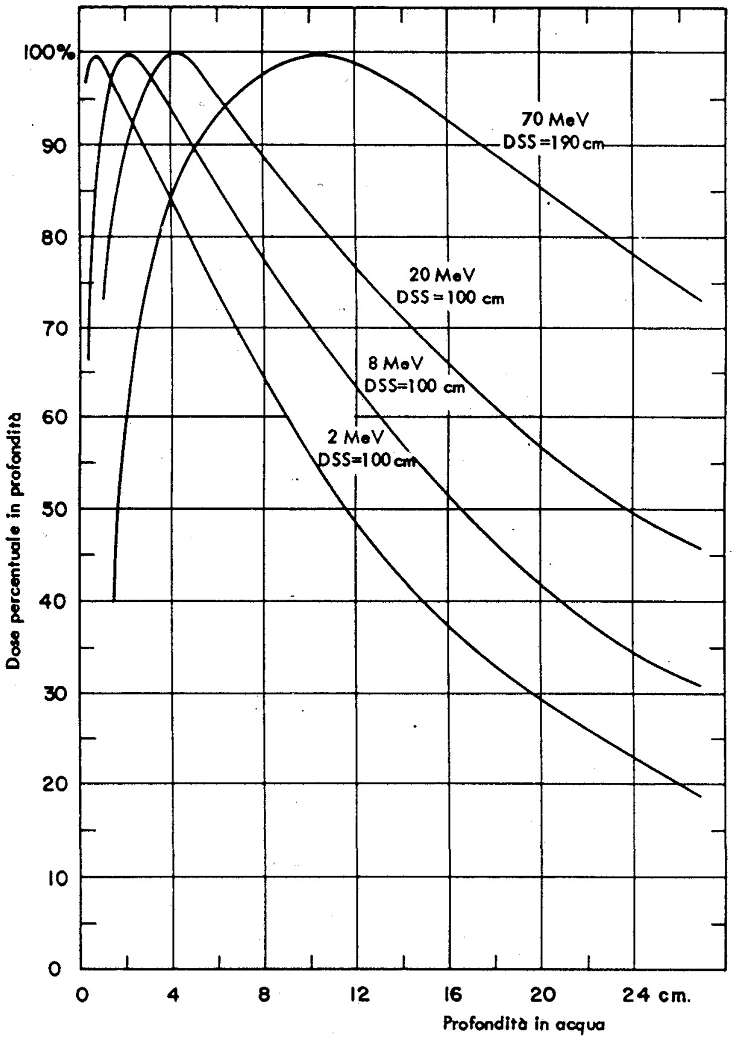
TRANSITION REGION



KERMA = **K**inetic **E**nergy **R**eleased in **MA**tter



The finite range of the electrons produced in
COMPTON SCATTERING
spares the skin



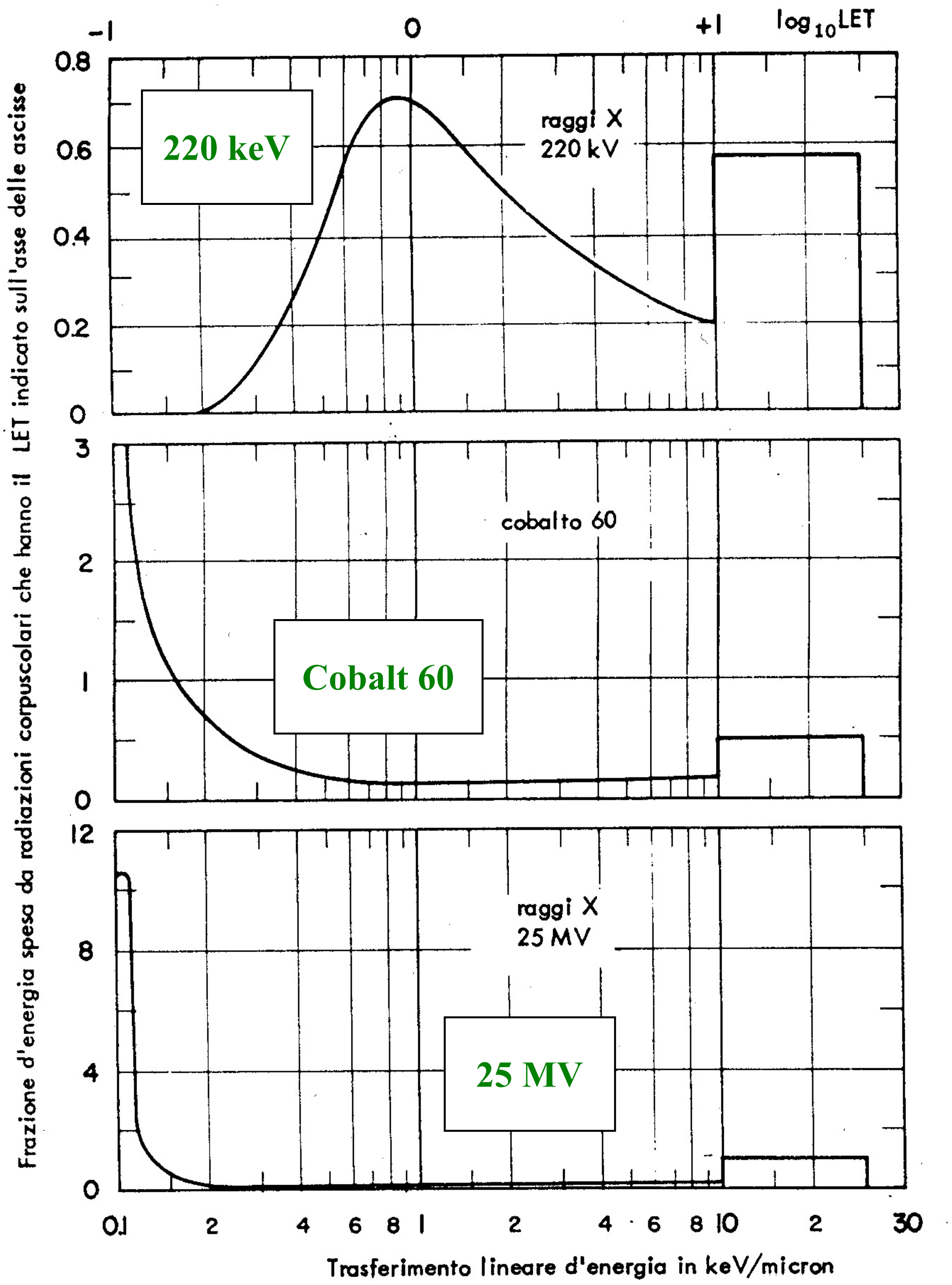
The depth D of the maximum is proportional to the range of the Compton electrons set in motion in the first layers

The phenomenon is complicated

K_e (MeV)	D (cm)	K(D)	K(D)/ K_e
2	**0.5**	**1.1**	**0.55**
8	**2.2**	**4.2**	**0.52**
20	**4.5**	**8.6**	**0.43**
70	**11**	**21**	**0.30**

SPECTRA OF THE LET DUE TO X-RAYS

By increasing the energy of the X-rays the energy deposited with LET > 10 keV/µm decreases



also

"sparsely ionizing" radiations deposit energy with "high" LET