Elenco di Formule utili - Eugenio Montefusco

Lo spazio vettoriale $\mathbb{R}^n$

$$\mathbb{R}^n = \{x = (x_1, x_2, ..., x_n) : x_i \in \mathbb{R} \text{ per ogni } i\}$$
$$x + y = (x_1, x_2, ..., x_n) + (y_1, y_2, ..., y_n)$$
$$= (x_1 + y_1, x_2 + y_2, ..., x_n + y_n)$$
$$\lambda x = \lambda(x_1, x_2, ..., x_n) = (\lambda x_1, \lambda x_2, ..., \lambda x_n)$$
$$x \cdot y = x_1 y_1 + x_2 y_2 + ... + x_n y_n$$
$$\|x\|_2 := x_1^2 + x_2^2 + ... + x_n^2 \qquad x \in \mathbb{R}^n$$
$$\|x\|_1 := |x_1| + |x_2| + ... + |x_n| \qquad x \in \mathbb{R}^n$$
$$\|x\|_\infty := \max_{i=1,...,n} |x_i| \qquad x \in \mathbb{R}^n$$
$$\text{se } n = 3 \qquad x \wedge y = (x_2 y_3 - x_3 y_2, x_3 y1 - x_1 y_3, x_1 y_2 - x_2 y_1)$$

Lo spazio vettoriale $C^0[a, b]$

$$C^0[a, b] = \{f : [a, b] \longrightarrow \mathbb{R} : f \text{ continua}\}$$
$$(f + g)(x) = f(x) + g(x)$$
$$(\lambda f)(x) = \lambda f(x)$$
$$\|f\|_{\infty,[a,b]} := \max_{x \in [a,b]} |f(x)|$$

Lo spazio vettoriale $L^p(a, b)$, $p \in ([1, +\infty)$

$$L^p(a, b) = \left\{ f : (a, b) \longrightarrow \mathbb{R} : \int_{(a,b)} |f(x)|^p dx < +\infty \right\}$$
$$\|f\|_{L^p} := \left[\int_{(a,b)} |f(x)|^p dx \right]^{1/p}$$

Derivazione e differenziazione

$$u : A \subseteq \mathbb{R}^n \longrightarrow \mathbb{R}$$
$$\partial_i u(x) = \partial_{x_i} u(x) \qquad \text{per } i = 1, ..., n$$
$$\partial_{jk} u(x) = \partial_{x_k x_j} u(x) \qquad \text{per } j, k = 1, ..., n$$
$$\nabla u(x) = (\partial_1 u(x), \partial_2 u(x), ..., \partial_n u(x))$$
$$Hu(x) = \left(\partial_{jk} u(x)\right)_{j,k=1,...,n}$$
$$\Delta u(x) = \nabla \cdot \nabla u(x) = (\partial_{11} u(x), \partial_{22} u(x), ..., \partial_{nn} u(x))$$

$$u : A \subseteq \mathbb{R}^n \longrightarrow \mathbb{R}^p$$
$$\partial_i u_j(x) = \partial_{x_i} u_j(x) \qquad \text{per } i = 1, ..., n \text{ e } j = 1, ..., p$$
$$Ju(x) = \left(\partial_j u_k(x)\right)_{k,j=1,...,n}$$
$$J(g(f))(x) = \left[Jg(y)\right]_{y=f(x)} Jf(x)$$
$$\text{se } p = n \qquad \mathrm{div}[u](x) = \nabla \cdot u(x) = (\partial_1 u_1(x), \partial_2 u(x), ..., \partial_n u_n(x))$$
$$\text{se } p = n = 3 \qquad \mathrm{rot}[u](x) = \nabla \wedge u(x) = (\partial_2 u_3 - \partial_3 u_2, \partial_3 u_1 - \partial_1 u_3, \partial_1 u_2 - \partial_2 u_1)$$

Integrali multipli

se $s(x)=\sum c_k \chi_{E_k}$ allora $\int s(x)dx=\sum c_k m_n(E_k)$

$m_2(E)=\int m_1(E_{x_j})dx_j$ dove $E_{x_j}=\{x_j : x=(x_1,x_2)\in E\}$ con $j=1,2$

$m_3(E)=\int m_2(E_{x_j})dx_j$ dove $E_{x_j}=\{x_j : x=(x_1,x_2,x_3)\in E\}$ con $j=1,2,3$

$m_3(E)=\int m_1(E_{x_j,x_k})dx_j dx_k$ dove $E_{x_j,x_k}=\{(x_j,x_k) : x=(x_1,x_2,x_3)\in E\}$ con $j,k=1,2,3$ e $j<k$

$\int_{\mathbb{R}^{n+p}} f(x,y)dxdy=\int_{\mathbb{R}^n}\left[\int_{\mathbb{R}^p} f(x,y)dy\right]dx=\int_{\mathbb{R}^p}\left[\int_{\mathbb{R}^n} f(x,y)dx\right]dy$ per $(x,y)\in\mathbb{R}^n\times\mathbb{R}^p$

$\int_E f(y)dy=\int_{g^{-1}(E)} f(g(x))|\det[Jg(x)]|dx$

dove $A,B\subseteq\mathbb{R}^n$ aperti, $g:A\longrightarrow B$ è un differomorfismo e $E\subseteq B$ è misurabile

Coordinate ellittiche (polari $a=b=1$)

$$\begin{cases} x_1 = ar\cos(\theta) \\ x_2 = br\sin(\theta) \end{cases} \qquad (r,\theta)\in[0,+\infty)\times[0,2\pi) \qquad |\det(J)|=abr$$

Coordinate sferiche

$$\begin{cases} x_1 = r\sin(\phi)\cos(\theta) \\ x_2 = r\sin(\phi)\sin(\theta) \\ x_3 = r\cos(\phi) \end{cases} \qquad (r,\theta,\phi)\in[0,+\infty)\times[0,2\pi)\times[0,\pi] \qquad |\det(J)|=r^2\sin(\phi)$$

Curve e integrali

$x(t)=(x_1(t),x_2(t),x_3(t)):[a,b]\longrightarrow\mathbb{R}^3$ $\qquad \gamma=x([a,b])$

$$T(t)=\frac{x'(t)}{\|x'(t)\|_2}$$

$$L(\gamma)=\int_{[a,b]}\|x'(t)\|_2 dt$$

$$\int_\gamma f(x)ds=\int_{[a,b]} f(x(t))\|x'(t)\|_2 dt$$

$$\int_\gamma F(x)\cdot ds=\int_\gamma F(x)\cdot T(x)ds=\int_a^b F(x(t))\cdot x'(t)dt$$

$$\int_\gamma \omega=\int_\gamma [a(x)dx_1+b(x)dx_2+c(x)dx_3]=\int_a^b\Big[a(x(t))x_1'(t)+b(x(t))x_2'(t)+c(x(t))x_3'(t)\Big]dt$$

Superfici e integrali

$$x(u) = (x_1(u_1,u_2), x_2(u_1,u_2), x_3(u_1,u_2)) : K \longrightarrow \mathbb{R}^3 \qquad \Sigma = x(K)$$

$$n(u) = \frac{\partial_1 x(u) \wedge \partial_2 x(u)}{\|\partial_1 x \wedge \partial_2 x\|_2}$$

$$E(u) = \partial_1 x(u) \cdot \partial_1 x(u) \qquad F(u) = \partial_1 x(u) \cdot \partial_2 x(u) \qquad G(u) = \partial_2 x(u) \cdot \partial_2 x(u)$$

$$A(\Sigma) = \int_\Sigma d\sigma = \int_K \|\partial_1 x(u) \wedge \partial_2 x(u)\|_2 du = \int_K \left[E(u)G(u) - F^2(u)\right]^{1/2} du$$

$$\int_\Sigma f(x) d\sigma = \int_K f(x(u)) \|\partial_1 x(u) \wedge \partial_2 x(u)\|_2 du$$

$$\int_\Sigma f(x) \cdot n(x) d\sigma = \int_K f(x(u)) \cdot (\partial_1 x(u) \wedge \partial_2 x(u)) du$$

Formule di Gauss-Green

$$\int_D \partial_x B(x,y) dx dy = \int_{\partial^+ D} B(x,y) dy$$

$$\int_D \partial_y A(x,y) dx dy = -\int_{\partial^+ D} A(x,y) dx$$

Teoremi della divergenza e del rotore

$$\int_D \operatorname{div}[F](x) dx = \int_{\partial^+ D} F(x) \cdot n d\sigma$$

$$\int_\Sigma [\operatorname{rot}[F](x) \cdot n(x)] \, d\sigma = \int_{\partial^+ \Sigma} F(x) \cdot ds$$