Determinare $c \in \mathbb{R}$ t.c. $\lim_{x\to+\infty} \left(\frac{x+c}{x-c}\right)^{x} = 4$

$x \to +\infty$

$\frac{x+c}{x-c} \to 1$

$$\left(\frac{x+c}{x-c}\right)^x = \left(\frac{x-c+2c}{x-c}\right)^x = \left(1+\boxed{\frac{2c}{x-c}}\right)^x =$$

$\frac{2c}{x-c} \to 0$

$(1+t)^{1/t} \to e$ per $t \to 0$

$$= \underbrace{\left[\left(1+\frac{2c}{x-c}\right)^{\frac{x-c}{2c}}\right]}_{\to e}^{\boxed{\frac{2c}{x-c}\cdot x}} \xrightarrow[x\to+\infty]{} e^{2c} = 4$$

$\frac{2c}{x-c}\cdot x \to 2c$

$\Updownarrow$

$$2c = \log 4 = 2\log 2$$

$$\boxed{c = \log 2}$$

In alternativa:

$$\left(\frac{x+c}{x-c}\right)^x = e^{x\log\left(\frac{x+c}{x-c}\right)} \to e^{2c}$$

$$x\log\left(\frac{x+c}{x-c}\right) = x\,\overbrace{\log\left(1+\boxed{\frac{2c}{x-c}}\right)}^{\sim\frac{2c}{x-c}} \sim x\,\frac{2c}{x-c} \to 2c$$

$-c+c$

$\frac{2c}{x-c} \to 0$ $x \to +\infty$

$\log(1+t) \sim t$ per $t \to 0$

$$\lim_{x\to 0}\left(\log\left(3^{1/x}\right) - \frac{1}{x}\log\left(\left(x+\frac{1}{3}\right)(1-2x)\right)\right) =$$

$$= \lim_{x\to 0}\left(-\frac{1}{x}\left[\log 3 + \log\left(x+\frac{1}{3}\right)(1-2x)\right]\right) =$$

$$= \lim_{x\to 0}\left(-\frac{1}{x}\log\left[(3x+1)(1-2x)\right]\right) =$$

$$= \lim_{x\to 0}\left(-\frac{1}{x}\left[\log(1+3x) + \log(1-2x)\right]\right) =$$

$$= \lim_{x\to 0}\left(\boxed{-\frac{\log(1+3x)\,3}{3x}}\ \boxed{-\frac{\log(1-2x)}{x}}\right) = -1$$

$$\downarrow -3 \qquad \downarrow +2$$

$$-\frac{1}{x}\log(1+\underbrace{x-6x^2}_{\to 0}) \sim -\frac{1}{x}(x-6x^2) = -1+6x \to -1$$

$$\lim_{x\to 4^+}\frac{\log\left(5e^{x-4}-x\right)-\sin(3x-12)}{x-4} = \left[\begin{array}{l} x-4=y\to 0^+ \\ x=4+y \end{array}\right]$$

$$= \lim_{y\to 0^+}\frac{\log(5e^y-4-y)-\sin(3y)}{y} =$$

$$= \lim_{y\to 0^+}\left[\boxed{\frac{\log(5e^y-4-y)}{y}} - \frac{\sin(3y)}{y}\right] = 4-3 = 1.$$

$$\searrow 3$$

$$\| $$

$$\frac{\log(1+\overbrace{(5e^y-5-y)}^{\to 0})}{y} \sim \frac{5e^y-5-y}{y}$$

$$\log(1+t)\sim t \quad \text{per } t\to 0$$

$$\|$$

$$5\left(\boxed{\frac{e^y-1}{y}}\right)-1 \to 4$$

$$\searrow 1$$

$$\lim_{x\to0}\frac{\log\left((1+\operatorname{arctg}x)^x\right)}{e-e^{\cos^4x}}=\lim_{x\to0}\frac{x\log(1+\operatorname{arctg}x)}{DEN}=$$

$$=\lim_{x\to0}\frac{x^2}{2e\,x^2}=\frac{1}{2e}$$

$$x\log(1+\overbrace{\operatorname{arctg}x}^{\to0})\sim x\,\underbrace{\operatorname{arctg}x}_{\sim x}\sim x^2$$

$$e-e^{\cos^4x}=e\left(1-e^{\boxed{\cos^4x-1}\,\to0}\right)=-e\left(e^{\overbrace{\cos^4x-1}^{\to0}}-1\right)\sim$$

$$\sim-e\left(\cos^4x-1\right)=e\left(1-\cos^4x\right)=e\left(1-\cos^2x\right)\left(1+\cos^2x\right)=$$

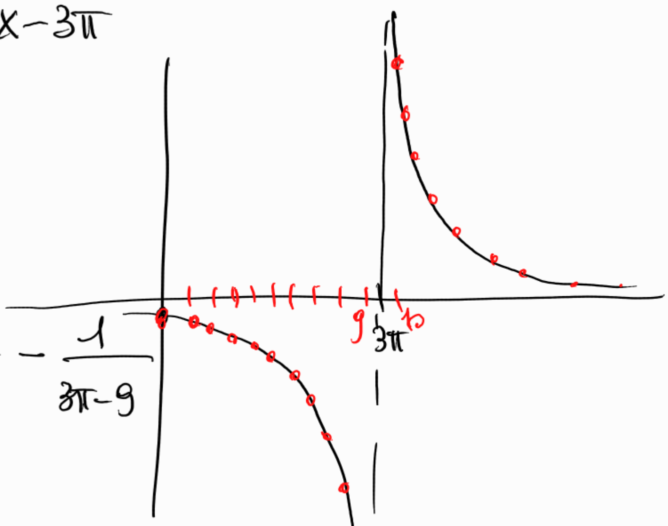
$$=e\underbrace{(1-\cos x)}_{\sim\frac{x^2}{2}}\underbrace{(1+\cos x)}_{\to2}\underbrace{(1+\cos^2x)}_{\to2}\sim2e\,x^2$$

$e^t-1\sim t$ per $t\to0$ $\quad t=\cos^4x-1\to0$

Calcolare sup e inf di $c_n=\operatorname{arctg}\left(\frac{1}{n-3\pi}\right)$.

osservando che $\operatorname{arctg}x$ è strettamente crescente, la crescenza e la decrescenza di $\{c_n\}$ equivale alla cresc/decr. di $\boxed{a_n=\frac{1}{n-3\pi}}$

$$f(x)=\frac{1}{x-3\pi}$$



Si deduce che

$$\sup a_n=\max a_n=a_{10}=\frac{1}{10-3\pi}$$

$$\inf a_n=\min a_n=a_9=\frac{1}{9-3\pi}=-\frac{1}{3\pi-9}$$

$$\sup c_n = \max c_n = c_{10} = \operatorname{arctg} \frac{1}{10-3\pi}$$

$$\inf c_n = \min c_n = c_9 = \operatorname{arctg} \frac{1}{9-3\pi} = -\operatorname{arctg} \frac{1}{3\pi-9}$$

$$\operatorname{arctg} x + \operatorname{arctg} \frac{1}{x} = \frac{\pi}{2} \qquad \forall x>0.$$

$$= -\frac{\pi}{2} \qquad \forall x<0.$$

$$\operatorname{arctg} \frac{1}{10-3\pi} = \frac{\pi}{2} - \operatorname{arctg}(10-3\pi)$$

$$\lim_{x\to+\infty} \Big(\underbrace{\sqrt[3]{2+x^3}}_{\sim x} - \underbrace{\sqrt[3]{1+2x^2+x^3}}_{\sim x} \Big) = (+\infty - \infty)$$

$$= \lim_{x\to+\infty} x \left(\sqrt[3]{1+\frac{2}{x^3}} - \sqrt[3]{1+\frac{2}{x}+\frac{1}{x^3}} \right) =$$

$$= \lim_{x\to+\infty} x \left(\underbrace{\left(\sqrt[3]{1+\frac{2}{x^3}} - 1\right)}_{=\frac{2}{3x^3}(1+o(1))} + \underbrace{\left(1 - \sqrt[3]{1+\frac{2}{x}+\frac{1}{x^3}}\right)}_{=-\frac{1}{3}\left(\frac{2}{x}+\frac{1}{x^3}\right)(1+o(1))} \right) =$$

$$-\frac{1}{3}\left(\frac{2}{x}+\frac{1}{x^3}\right)(1+o(1)) = -\frac{2}{3x}(1+o(1))$$

$$(1+t)^{1/3} - 1 \sim \frac{t}{3} \quad \text{per } t\to 0.$$

$$= \lim_{x\to+\infty} \cancel{x} \cdot \frac{1}{\cancel{x}} \underbrace{\left(\underbrace{x \underbrace{\left(\sqrt[3]{1+\frac{2}{x^3}} - 1\right)}_{\sim \frac{2}{3x^3}}}_{\to 0} + \underbrace{\left(1 - \sqrt[3]{1+\frac{2}{x}+\frac{1}{x^3}}\right) x}_{\to -2/3} \right)}_{\to -2/3} = -\frac{2}{3}$$

$$\lim_{x\to 0} \frac{\overbrace{e^{tg^3 x}-1}^{\sim x^3}}{\underbrace{x\left(\cos x - e^{x^2}\right)}_{\sim -\frac{3}{2}x^3}} = \left(\frac{0}{0}\right) = -\frac{2}{3}$$

$$e^{\overbrace{tg^3 x}^{\to 0}} - 1 \sim tg^3 x \sim x^3$$

$e^t - 1 \sim t$ per $t \to 0$

$$\cos x - e^{x^2} = \underbrace{(\cos x - 1)}_{\sim -\frac{x^2}{2}} + \underbrace{\left(1 - e^{x^2}\right)}_{\sim -x^2} = x^2\left(\underbrace{\boxed{\frac{\cos x - 1}{x^2}}}_{\to -1/2} + \underbrace{\boxed{\frac{1-e^{x^2}}{x^2}}}_{\to -1}\right) \sim$$

$\to -3/2$

$$\sim -\frac{3}{2}x^2$$

$$\text{den} = x\left(\cos x - e^{x^2}\right) \sim -\frac{3}{2}x^3$$

Trovare l'ordine di infinito / infinitesimo delle seguenti funzioni

$\log\left(e^x + \sqrt{x}\right)$ per $x \to 0^+$

è un infinitesimo.

$$\log\left(e^x + \sqrt{x}\right) = \log\left(1 + \underbrace{\left(e^x - 1 + \sqrt{x}\right)}_{\to 0}\right) \sim \underbrace{\left(e^x - 1\right)}_{\sim x} + \sqrt{x} =$$

$$= \sqrt{x}\left(1 + \underbrace{\boxed{\frac{e^x - 1}{\sqrt{x}}}}_{\to 0}\right) \sim \sqrt{x}$$ infinitesimo di ordine $\frac{1}{2}$.

$\left(x^2 - x\right)^{10} - x^{20}$ per $x \to +\infty$.

$$= x^{20}\left[\left(1 - \frac{1}{x}\right)^{10} - 1\right] \sim x^{20}\left(-\frac{10}{x}\right) \sim -10\,x^{19}$$

$$(1+t)^{10}-1 \sim 10t \quad (t\to 0)$$

$$\left(1-\frac{1}{x}\right)^{10}-1 \sim -\frac{10}{x} \qquad x\to +\infty$$

E' un infinito di ordine 19.

$$\sqrt{4+\operatorname{tg}x}-\sqrt{4+\sin x} = \qquad x\to 0$$

$$= \left(\sqrt{4+\operatorname{tg}x}-\sqrt{4+\sin x}\right)\frac{\left(\sqrt{4+\operatorname{tg}x}+\sqrt{4+\sin x}\right)}{\sqrt{4+\operatorname{tg}x}+\sqrt{4+\sin x}} =$$

$$= \frac{\cancel{4}+\operatorname{tg}x-(\cancel{4}+\sin x)}{\underbrace{\sqrt{4+\operatorname{tg}x}+\sqrt{4+\sin x}}_{\to 4}} \sim \frac{\operatorname{tg}x-\sin x}{4} =$$

$$= \frac{\overset{\sim x}{\sin x}}{4}\left(\frac{1}{\cos x}-1\right) \sim \frac{x}{4}\,\frac{\overset{\sim \frac{x^2}{2}}{1-\cos x}}{\underset{\to 1}{\cos x}} \sim \frac{x^3}{8}$$

Infinitesimo di ordine 3

Ordine di infinitesimo di

$\log(-\sin x)$ per $x \to -\frac{\pi}{2}$

Per semplicità, prendiamo $x \to -\frac{\pi}{2}^+$

Infinitesimo campione per $x \to -\frac{\pi}{2}^+$ è $x + \frac{\pi}{2}$

Devo cercare $\alpha > 0$ t.c.

$$\lim_{x \to \left(-\frac{\pi}{2}\right)^+} \frac{\log(-\sin x)}{\left(x+\frac{\pi}{2}\right)^\alpha} \in \mathbb{R} \setminus \{0\}.$$

$$y = x + \frac{\pi}{2} \to 0^+$$

$$x = \left(y - \frac{\pi}{2}\right)$$

$$= \lim_{y \to 0^+} \frac{\log\left(-\sin\left(y - \frac{\pi}{2}\right)\right)}{y^\alpha} = \lim_{y \to 0^+} \frac{\log\left(\sin\left(\frac{\pi}{2} - y\right)\right)}{y^\alpha} =$$

$$= \lim_{y \to 0^+} \frac{\log(\cos y)}{y^\alpha} = \lim_{y \to 0^+} \frac{\overbrace{\log(1 + (\cos y - 1))}^{\sim \cos y - 1}}{y^\alpha}$$

$\cos y - 1 \to 0$

$$= \lim_{y \to 0^+} \frac{\cos y - 1}{y^\alpha} = -\frac{1}{2}. \qquad \boxed{\alpha = 2}$$

Ordine di infinitesimo di

$$\underbrace{\sqrt{x^2+3} - \sqrt{x^2+1}}_{} + \underbrace{2e^{-\sqrt{x}}}_{} \quad \text{per } x \to +\infty.$$

$2e^{-\sqrt{x}}$ = inf^{mo} di ordine superiore a $\frac{1}{x^\alpha}$ $\forall \alpha > 0$.

$\sqrt{x^2+3} - \sqrt{x^2+1}$ =

$$x\left(\sqrt{1+\frac{3}{x^2}} - \sqrt{1+\frac{1}{x^2}}\right)$$

$$= x\left(\left(\sqrt{1+\frac{3}{x^2}} - 1\right) + \left(1 - \sqrt{1+\frac{1}{x^2}}\right)\right) = \frac{x}{x^2}\left(\frac{\sqrt{1+\frac{3}{x^2}} - 1}{1/x^2} + \frac{1-\sqrt{1+1/x^2}}{1/x^2}\right.$$

$$\underbrace{\qquad\qquad}_{\sim \frac{3}{2x^2}} \qquad \underbrace{\qquad\qquad}_{\sim -\frac{1}{2x^2}}$$

$$= \frac{1}{x}\left(\frac{3}{2} - \frac{1}{2} + o(1)\right) \sim \frac{1}{x}$$

$$\sqrt{x^2+3} - \sqrt{x^2+1} + 2e^{-\sqrt{x}} =$$

$$= \frac{1}{x}\left(\underbrace{\boxed{\frac{\sqrt{x^2+3} - \sqrt{x^2+1}}{1/x}}}_{\to 1} + \underbrace{\boxed{2\frac{x}{e^{\sqrt{x}}}}}_{\to 0}\right) \sim \frac{1}{x}$$

inf^{mo} di ordine 1