$$\lim_{x\to 0} \frac{\sin(4x)}{x^2-x} = \left(\frac{0}{0}\right) = \lim_{x\to 0} \underbrace{\frac{\sin(4x)}{4x}}_{\downarrow\, 1} \cdot \frac{4}{\underbrace{(x-1)}_{\downarrow\, -1}} = -4$$

Oppure:

$$\frac{\overbrace{\sin(4x)}^{\sim 4x}}{\underbrace{x^2-x}_{\sim -x}} \sim \frac{4\cancel{x}}{-\cancel{x}} = -4$$

oss per $x\to 0$
$x^2 = o(x)$

$$x^2-x = x\underbrace{(x-1)}_{\sim -1} \sim -x$$

$$\lim_{x\to 0}\frac{\sin(4x)}{4x} = \lim_{y\to 0}\frac{\sin y}{y} = 1.$$

$y = 4x \to 0$

---

oss $\sin(\alpha(x)) \sim \alpha(x)$ per $x \to x_0$ se $\lim_{x\to x_0} \alpha(x) = 0$

$$\sin\frac{1}{n} \sim \frac{1}{n} \qquad n\to+\infty$$

$$\lim_{x\to 0} \frac{\overbrace{2\,\mathrm{tg}\,x}^{\sim 2x} - \overbrace{\sin x}^{\sim x}}{x^3} = \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{\cancel{x}\ 1}{x^{\cancel{3}\,2}} = +\infty.$$

$$2\,\mathrm{tg}\,x - \sin x = x\underbrace{\left(\underbrace{\frac{2\,\mathrm{tg}\,x}{x}}_{\downarrow\, 2} - \underbrace{\frac{\sin x}{x}}_{\downarrow\, 1}\right)}_{2-1=1} \sim x$$

In alternativa: $\operatorname{tg} x = \begin{cases} x(1+o(1)) \\ x+o(x) \end{cases}$ $\quad \sin x = \begin{cases} x(1+o(1)) \\ x+o(x) \end{cases}$

$$2\operatorname{tg} x - \sin x = 2x(1+o(1)) - x(1+o(1)) =$$

$$= x\left[2(1+o(1)) - 1 + o(1)\right] =$$

$$= x\,[2 + \underbrace{2o(1) - 1 + o(1)}_{=o(1)}] = x(1+o(1))$$

$$\lim_{x\to 0} \frac{x(1+o(1))}{x^3} = \lim_{x\to 0} \frac{1}{x^2}(1+o(1)) = +\infty$$

$$\frac{1}{x^2}\underbrace{\left(\frac{2\operatorname{tg} x}{x} - \frac{\sin x}{x}\right)}_{\to 1}$$

$$\underset{\substack{\parallel \\ \frac{\sin x}{\cos x}}}{2\operatorname{tg} x} - \sin x = \underbrace{\sin x}_{\sim x}\left(\underbrace{\frac{2}{\cos x}}_{\to 2} - 1\right) \sim x$$

$$\underbrace{\frac{2}{\cos x} - 1}_{\to 1}$$

$$\lim_{x\to 0} \frac{\operatorname{tg} x - \sin x}{x^3} = \frac{1}{2}$$

$\boxed{x \to 0}$

$$\cancel{\frac{\overset{\sim x}{\operatorname{tg} x} - \overset{\sim x}{\sin x}}{x^3} \sim \frac{x - x}{x^3} = 0}$$

Errato: ho sostituito asintoticamente equivalenti in una somma.

$$\frac{\overbrace{\operatorname{tg}x}^{=x+o(x)} - \overbrace{\sin x}^{=x+o(x)}}{x^3} = \frac{\cancel{x}+o(x)-\cancel{x}+o(x)}{x^3} = \frac{o(x)}{x^3}$$

corretto ma non permette di concludere

$$\frac{\operatorname{tg}x - \sin x}{x^3} = \frac{\sin x\left(\frac{1}{\cos x}-1\right)}{x^3} = \underbrace{\boxed{\frac{\sin x}{x}}}_{\to 1}\frac{\left(\frac{1}{\cos x}-1\right)}{x^2} =$$

$$= (1+o(1))\underbrace{\boxed{\frac{1-\cos x}{x^2}}}_{\to 1/2}\underbrace{\boxed{\cos x}}_{\to 1} \to \boxed{\frac{1}{2}}$$

Risultato corretto

$$\lim_{x\to-\infty}\left(\sqrt{x^2+1}-\sqrt{x^2+x}\right) = (+\infty-\infty)$$

$\boxed{x\to-\infty}$

$$\sqrt{x^2+1}-\sqrt{x^2+x} = (-x)\left(\underbrace{\sqrt{1+\frac{1}{x^2}}}_{\to 1}-\underbrace{\sqrt{1+\frac{1}{x}}}_{\to 1}\right)$$

OSS $\sqrt{x^2}=|x|=-x$ ($x<0$)

$$=(\infty\cdot 0)$$

Quindi così non funziona

$$\frac{\left(\sqrt{x^2+1}-\sqrt{x^2+x}\right)\left(\sqrt{x^2+1}+\sqrt{x^2+x}\right)}{\sqrt{x^2+1}+\sqrt{x^2+x}} = \frac{(\cancel{x^2}+1)-(\cancel{x^2}+x)}{\sqrt{x^2+1}+\sqrt{x^2+x}} =$$

$$= \frac{\overbrace{-x+1}^{\sim -x}}{\sqrt{x^2+1}+\sqrt{x^2+x}} = \frac{\cancel{(-x)}(1+o(1))}{\cancel{(-x)}\underbrace{\left(\sqrt{1+\frac{1}{x^2}}+\sqrt{1+\frac{1}{x}}\right)}_{\to 2}} \to \frac{1}{2}$$

$$\lim_{x\to 0^+} \frac{x^{3(1+\alpha)}-1}{x^\alpha+1} \qquad \alpha\in\mathbb{R}$$

<u>Den</u>

$$x^\alpha+1 \begin{cases} \sim 1 & \alpha>0 \\ = 2 & \alpha=0 \\ \sim x^\alpha & \alpha<0 \end{cases}$$

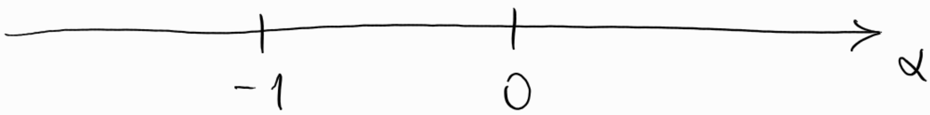
$\alpha<0$

$$x^\alpha+1 = x^\alpha\left(1+\frac{1}{x^\alpha}\right)\sim x^\alpha$$

$\frac{1}{x^\alpha} \to 0$

<u>Num</u>

$$x^{3(1+\alpha)}-1 \begin{cases} \sim -1 & \alpha>-1 \\ = 0 & \alpha=-1 \\ \sim x^{3(1+\alpha)} & \alpha<-1 \end{cases}$$



$\alpha<-1$ $\quad f(x)\sim \dfrac{x^{3(1+\alpha)}}{x^\alpha} = x^{3+2\alpha}$ $\qquad 3+2\alpha>0 \Leftrightarrow \alpha>-\frac{3}{2}$

Se $\alpha<-\frac{3}{2}$ $\quad f(x)\to+\infty$

$\alpha=-\frac{3}{2}$ $\quad f(x)\to 1$

$-\frac{3}{2}<\alpha<-1$ $\quad f(x)\to 0$

$\alpha=-1$ $\quad f(x)\equiv 0$

$-1<\alpha<0$ $\quad f(x)\sim -\dfrac{1}{x^\alpha} \to 0$

$\alpha=0$ $\quad f(x)\sim \dfrac{-1}{2} \to -\dfrac{1}{2}$

$\alpha > 0 \qquad f(x) \sim \frac{-1}{1} \to -1$

$$\lim_{x\to 0} f(x) = \begin{cases} +\infty & \text{se } \alpha < -\frac{3}{2} \\ 1 & \text{se } \alpha = -3/2 \\ 0 & -\frac{3}{2} < \alpha < 0 \\ -1/2 & \alpha = 0 \\ -1 & \alpha > 0 \end{cases}$$

$$\lim_{x\to 1} \frac{\boxed{\sin^2(x^2-3x+12)}}{\cos(x^3-1)-1} = -\infty$$

$\sin^2(10) > 0$

$\to 0^-$

$$\lim_{x\to 1} \boxed{\frac{\sin^2(x^2-3x+2)}{\cos(x^3-1)-1}} = \left(\frac{0}{0}\right)$$

$f(x)$

$\boxed{\text{NUM}}$

OSS $\quad x^2-3x+2 \to 0 \quad$ per $x \to 1$

$\Rightarrow \quad \sin(x^2-3x+2) \sim x^2-3x+2 \qquad x \to 1$

$\sin^2(\qquad) \sim (x^2-3x+2)^2 = (x-1)^2(x-2)^2$

$x^2-3x+2=0 \iff (x=1) \vee (x=2)$

$x^2-3x+2 = (x-1)(x-2)$

$\boxed{\text{DEN}} \quad \cos(\underbrace{x^3-1}_{\to 0})-1 \sim -\frac{(x^3-1)^2}{2} = -\frac{1}{2}(x-1)^2(x^2+x+1)^2$

$t \to 0 \qquad 1-\cos t \sim \frac{t^2}{2}$

$$x^3-1=(x-1)(x^2+x+1)$$

Per $x\to 1$

$$f(x)\sim\frac{\cancel{(x-1)^2}(x-2)^2}{-\frac{1}{2}\cancel{(x-1)^2}(x^2+x+1)^2}=-2\,\frac{\overbrace{(x-2)^2}^{\to 1}}{\underbrace{(x^2+x+1)^2}_{\to 9}}\to-\frac{2}{9}$$

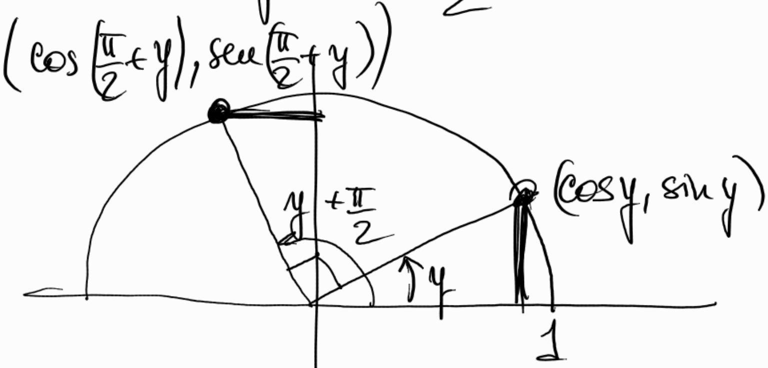
$$\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{2x-\pi}=\left(\frac{0}{0}\right)=$$

$$\boxed{x=\frac{\pi}{2}+y}$$

$$y=x-\frac{\pi}{2}\to 0.$$

$$=\lim_{y\to 0}\frac{\cos\left(\frac{\pi}{2}+y\right)}{2y}=$$

$$=\lim_{y\to 0}\frac{-\sin y}{2y}=-\frac{1}{2}$$



$$\cos\left(\frac{\pi}{2}+y\right)=-\text{sen}\,y$$

$$\lim_{x\to+\infty}\frac{\overbrace{\sqrt{x^6+4}}^{\sim x^3}\ \overbrace{\left(\cos\frac{2}{x}-1\right)}^{\sim -\frac{2}{x^2}}}{x}$$

$$\boxed{x\to+\infty}$$

$$=\lim_{x\to+\infty}\frac{x^3\left(-\frac{2}{x^2}\right)}{x}=-2$$

$$\frac{2}{x}\to 0\ \Rightarrow\ 1-\cos\frac{2}{x}\sim\frac{1}{2}\cdot\frac{4}{x^2}=\frac{2}{x^2}$$

$$\cos\frac{2}{x}-1\sim-\frac{2}{x^2}$$

$$\sqrt{x^6+4}=x^3\underbrace{\sqrt{1+\frac{4}{x^6}}}_{\to 1}\sim x^3$$

$$\cos\left(\frac{2}{x}\right)-1=-\left(1-\cos \overset{\to 0}{t}\right)\sim-\frac{1}{2}t^2=-\frac{2}{x^2} \qquad \frac{2}{x}=t\to 0$$

$$\lim_{x\to+\infty}\frac{\cos\left(\frac{2}{x}\right)-1}{\frac{4}{x^2}}=\lim_{t\to 0}\frac{\cos t-1}{t^2}=-\frac{1}{2}$$

$\frac{2}{x}=t\to 0$

$$\Rightarrow \cos\left(\frac{2}{x}\right)-1\sim-\frac{1}{2}\cdot\frac{4}{x^2}=-\frac{2}{x^2}$$

$$\lim_{n\to+\infty}\frac{n\overbrace{\left(1-\cos\frac{1}{n}\right)}^{\sim\frac{1}{2n^2}}}{\underbrace{\sin\frac{1}{n}}_{\sim\frac{1}{n}}}=$$

OSS per $n\to+\infty$, $\frac{1}{n}\to 0$

$$=\lim_{n\to+\infty}\frac{n\,\frac{1}{2n^2}}{1/n}=\frac{1}{2}$$

$$\lim_{n\to+\infty} e^n\underbrace{\boxed{\frac{\sin\left(\frac{1}{n^{100}}\right)}{\left(1/n^{100}\right)}}}_{\to 1}\cdot\frac{1}{n^{100}}=\lim_{n\to+\infty}\frac{e^n}{n^{100}}=+\infty$$

$$\lim_{n\to+\infty}\cos^n\frac{1}{n}=\lim_{n\to+\infty}\left(\cos\frac{1}{n}\right)^n=\left(1^{+\infty}\right)$$

$$=\lim_{n\to+\infty}\left(1+\underbrace{\left(\cos\frac{1}{n}-1\right)}_{\to 0}\right)^n=$$

$$= \lim_{n\to+\infty} \Bigg[\underbrace{\left(1 + \frac{1}{\left(\frac{1}{\cos\frac{1}{n}-1}\right)}\right)^{\frac{1}{\cos\frac{1}{n}-1}}}_{\substack{b_n \to -\infty \\ \downarrow \\ e}} \Bigg]^{\underbrace{\left(\cos\frac{1}{n}-1\right)n}_{\substack{\downarrow \\ 0}}} = 1$$

Vediamo l'esponente: $\underbrace{\left(\cos\frac{1}{n} - 1\right)}_{\sim -\frac{1}{2n^2}} n \sim -\frac{1}{2n} \to 0$

$$\lim_{n\to+\infty} \left(\cos\frac{1}{n}\right)^{n^2} = \lim_{n\to+\infty} \underbrace{\Big[\qquad \Big]}_{\substack{\downarrow \\ e}}^{\underbrace{\left(\cos\frac{1}{n}-1\right)n^2}_{\substack{\downarrow \\ -1/2}}} = e^{-1/2} = \frac{1}{\sqrt{e}}$$

Verifica di limite:

$$\lim_{x\to(-1)^+} \frac{1}{1-e^{\underbrace{1+x}_{\substack{\downarrow \\ 0^+}}}} = -\infty \qquad \text{Verifica}$$

Devo provare: $\forall M>0$ cerco $\delta > 0$ t.c.

$\forall x$ verificante $-1 < x < -1+\delta$ si ha

$$\boxed{\frac{1}{1-e^{1+x}} < -M} \iff -1 < x < -1+\delta \qquad \boxed{x > -1}$$

posso imporre solo condizione del tipo $x < -1+\delta$

con $\delta = \log\left(1+\frac{1}{M}\right)$

$$\frac{1}{\underbrace{e^{1+x}-1}_{\substack{\vee \\ 0}}} > M$$

$$\Updownarrow$$

$$e^{1+x}-1 < \frac{1}{M}$$

$$\Updownarrow$$

$$e^{1+x} < 1+\frac{1}{M}$$

$$\Updownarrow$$

$$1+x < \log\left(1+\frac{1}{M}\right)$$

$$\Updownarrow$$

$$x < -1+\underbrace{\log\left(1+\frac{1}{M}\right)}_{\delta>0}$$

$$\lim_{x\to 2}(x^2-x+5)=7$$

$\forall \varepsilon>0,\ \exists \delta>0$ t.c. $|x^2-x+5-7|<\varepsilon$ $\forall x$ verificante $0<|x-2|<\delta$

$$|x^2-x-2|<\varepsilon$$

$$-\varepsilon<x^2-x-2<\varepsilon$$

$$|x-2||x+1|<\varepsilon$$

$$|x-2|\,|x+1| \le 4|x-2| < \varepsilon$$

$\boxed{|x-2|<1}$ $\boxed{|x-2|<\frac{\varepsilon}{4}}$

Basta prendere $|x-2|<\min\left\{1,\frac{\varepsilon}{4}\right\}$

OSS se $|x-2|<1 \Rightarrow 1<x<3 \Rightarrow 2<x+1<4$

$\Rightarrow |x+1|<4$