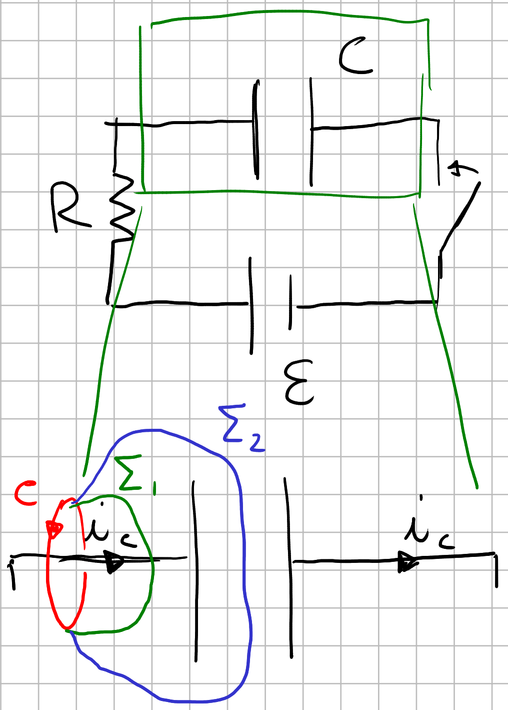
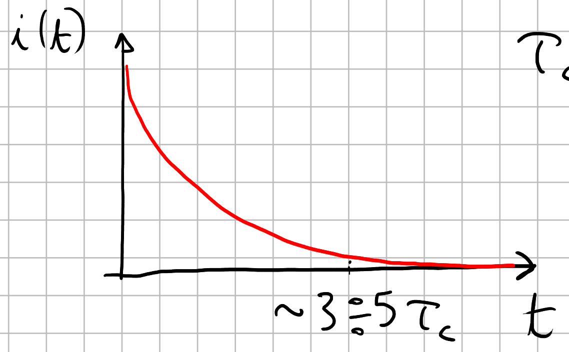
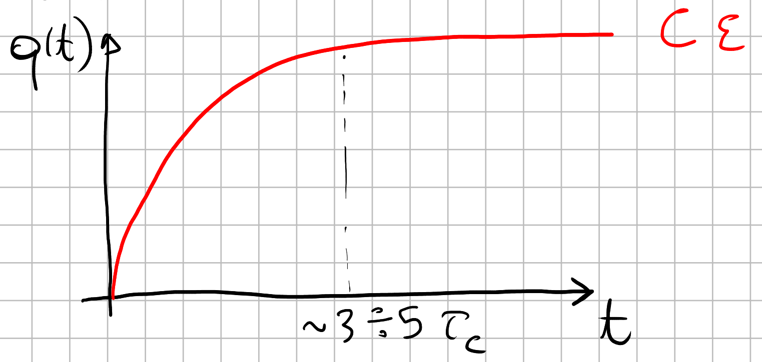
# LEGGE DI AMPERE - MAXWELL




$\tau_c = RC$




$$\oint_c \vec{B}\cdot d\vec{s} = \mu_0 i_c = \mu_0 \int_{\Sigma_1(c)} \vec{J}_c\cdot\hat{n}\, d\Sigma \overset{?}{=} \mu_0 \int_{\Sigma_2(c)} \vec{J}_c\cdot\hat{n}\, d\Sigma$$

$E(t) = \dfrac{\sigma(t)}{\varepsilon_0} = \dfrac{q(t)}{S\,\varepsilon_0}$ , $S$ superficie delle armature , $q(t) = C\mathcal{E}\left(1 - e^{-t/\tau_c}\right)$

$$\frac{dq}{dt} = \frac{d}{dt}\left(S\varepsilon_0 E(t)\right) = \varepsilon_0 \frac{d}{dt}\overbrace{\left(S E(t)\right)}^{\Phi_S(\vec{E})} \Rightarrow \frac{dq}{dt} = \varepsilon_0 \frac{d\Phi_S(\vec{E})}{dt} \equiv i_s \Rightarrow$$

CORRENTE DI SPOSTAMENTO

$$\oint_c \vec{B} \cdot d\vec{s} = \mu_0 i_{TOT} = \mu_0 i_c + \mu_0 i_s = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\Phi_s(\vec{E})}{dt} = \mu_0 i_c + \frac{1}{c^2} \frac{d\Phi_s(\vec{E})}{dt}$$

SPOSTAMENTO

CONDUZIONE

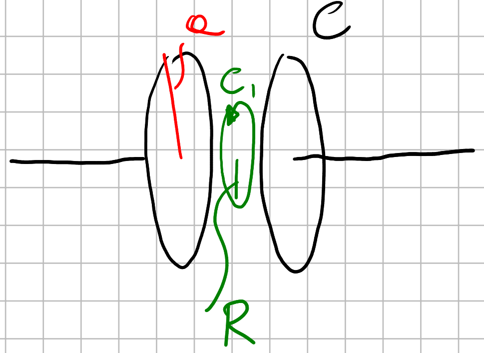
LEGGE DI AMPERE - MAXWELL

IN ASSENZA DI CORRENTE DI CONDUZIONE,

$$\oint_c \vec{B} \cdot d\vec{s} = \frac{1}{c^2} \frac{d\Phi_{\Sigma(c)}(\vec{E})}{dt} \longleftrightarrow \oint_c \vec{E} \cdot d\vec{s} = -\frac{d\Phi_{\Sigma(c)}(\vec{B})}{dt}$$

LEGGE DI FARADAY

ESEMPIO




$$q(t) = C\mathcal{E}\left(1 - e^{-t/\tau_C}\right), \quad \tau_C = \underbrace{R_E}C$$

→ RESISTENZA DEL CIRCUITO

$$E(t) = \frac{q(t)}{\pi a^2 \varepsilon_0}$$

$$\oint_{C_1} \vec{B}\cdot d\vec{s} = B\oint_{C_1} ds = B\,2\pi R, \quad \Phi_{\Sigma(C_1)}(\vec{E}) = \int_{\Sigma(C_1)} \vec{E}\cdot\hat{n}\,d\Sigma = E\int_{\Sigma(C_1)} d\Sigma = E(t)\,\pi R^2 \Rightarrow$$

$$\frac{d\Phi(\vec{E})}{dt} = \pi R^2 \frac{d}{dt}\frac{C\mathcal{E}}{\pi a^2 \varepsilon_0}\left(1 - e^{-t/R_E C}\right) = \cancel{\pi} R^2 \frac{\cancel{C}\mathcal{E}}{\cancel{\pi} a^2 \varepsilon_0}\frac{1}{R_E \cancel{C}} e^{-t/R_E C} = \frac{R^2 \mathcal{E}}{a^2 \varepsilon_0}\frac{1}{R_E} e^{-t/R_E C}$$

$$\Rightarrow B\,2\pi \cancel{R} = \mu_0 \cancel{\varepsilon_0}\frac{R^{\cancel{2}}\mathcal{E}}{a^2 \cancel{\varepsilon_0}}\frac{1}{R_E} e^{-t/R_E C} \Rightarrow B(t) = \frac{\mu_0 R\,\mathcal{E}}{2\pi a^2 R_E} e^{-t/R_E C}$$

# EQUAZIONI DI MAXWELL

$$\oint_\Sigma \vec{E}\cdot\hat{n}\,d\Sigma = \frac{Q_{INT}}{\varepsilon_0} \qquad \oint_c \vec{E}\cdot d\vec{s} = -\frac{d\,\Phi_\Sigma(\vec{B})}{dt}$$

$$\oint_\Sigma \vec{B}\cdot\hat{n}\,d\Sigma = 0 \qquad \oint_c \vec{B}\cdot d\vec{s} = \frac{1}{c^2}\frac{d\,\Phi_\Sigma(\vec{E})}{dt} + \mu_0 i$$

## FORMA LOCALE

STOKES

$$\oint_c \vec{E}\cdot d\vec{s} = \int_{\underbrace{\Sigma(c)}} \vec{\nabla}\times\vec{E}\cdot\underbrace{\hat{n}\,d\Sigma} = -\int_{\underbrace{\Sigma(c)}} \frac{\partial}{\partial t}\vec{B}\cdot\underbrace{\hat{n}\,d\Sigma} \Rightarrow \boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$$

STOKES

$$\oint_c \vec{B}\cdot d\vec{s} = \int_{\underbrace{\Sigma(c)}} \vec{\nabla}\times\vec{B}\cdot\underbrace{\hat{n}\,d\Sigma} = \frac{1}{c^2}\int_{\underbrace{\Sigma(c)}} \frac{\partial\vec{E}}{\partial t}\cdot\underbrace{\hat{n}\,d\Sigma} + \mu_0\int_{\underbrace{\Sigma(c)}} \vec{J}\cdot\underbrace{\hat{n}\,d\Sigma} \Rightarrow \boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}}$$

# EQUAZIONI DI MAXWELL

$$\begin{cases} \vec{\nabla} \cdot \vec{E} = \dfrac{\rho}{\varepsilon_0} & \vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \cdot \vec{B} = 0 & \vec{\nabla} \times \vec{B} = \mu_0 \left( \vec{J} + \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t} \right) \end{cases}$$

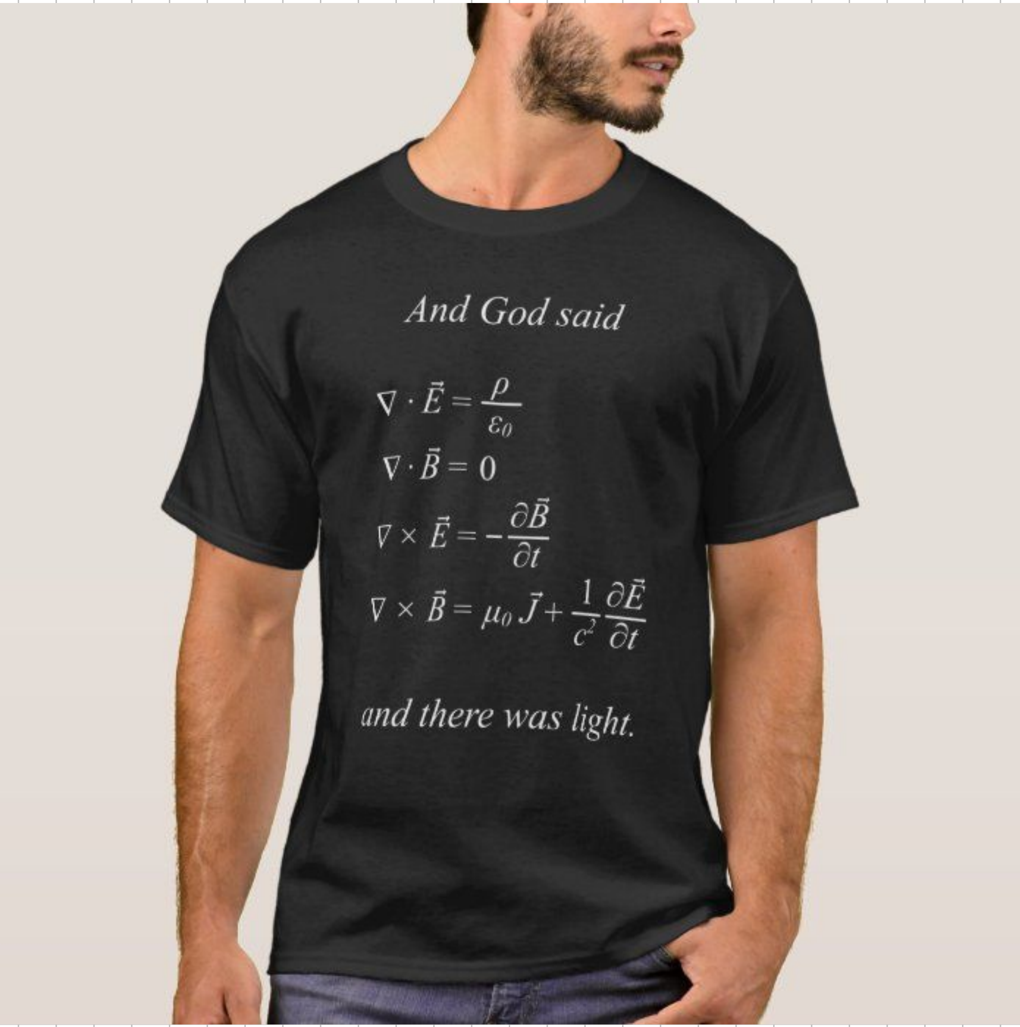
in assenza di sorgenti di campo

$$\begin{cases} \vec{\nabla} \cdot \vec{E} = 0 & \vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \cdot \vec{B} = 0 & \vec{\nabla} \times \vec{B} = \dfrac{1}{c^2} \dfrac{\partial \vec{E}}{\partial t} \end{cases}$$

$\nearrow$ $\vec{E} = 0, \quad \vec{B} = 0$

$\searrow$ $\vec{E} = \vec{E}(x, y, z, t)$

$\vec{B} = \vec{B}(x, y, z, t)$


And God said
$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$
$\nabla \cdot \vec{B} = 0$
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
$\nabla \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$
and there was light.

# CONSERVAZIONE DELLA CARICA

$$[\vec{a} \cdot (\vec{a} \times \vec{b}) = 0]$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} \implies \text{prendiamone la divergenza}$$

$$\vec{\nabla} \cdot \underbrace{(\vec{\nabla} \times \vec{B})}_{=0} = \mu_0 \vec{\nabla} \cdot \vec{J} + \frac{1}{c^2} \frac{\partial \vec{\nabla} \cdot \vec{E}}{\partial t} \implies \mu_0 \vec{\nabla} \cdot \vec{J} = -\frac{1}{c^2} \frac{\partial \vec{\nabla} \cdot \vec{E}}{\partial t} = -\mu_0 \cancel{\varepsilon_0} \frac{\partial}{\partial t} \left( \frac{\rho}{\cancel{\varepsilon_0}} \right) \implies$$

$$\boxed{\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}}$$

EQUAZIONE DI CONTINUITÀ

(1) $\displaystyle\int_\tau \nabla \cdot \vec{J} \, d\tau = \oint_{\Sigma(\tau)} \vec{J} \cdot \hat{n} \, d\Sigma = i$ , (2) $\displaystyle -\int_\tau \frac{\partial \rho}{\partial t} d\tau = -\frac{\partial}{\partial t} \int_\tau \rho \, d\tau = -\frac{\partial Q_{INT}}{\partial t} \implies$

$$\boxed{i = -\frac{\partial Q_{INT}}{\partial t}}$$