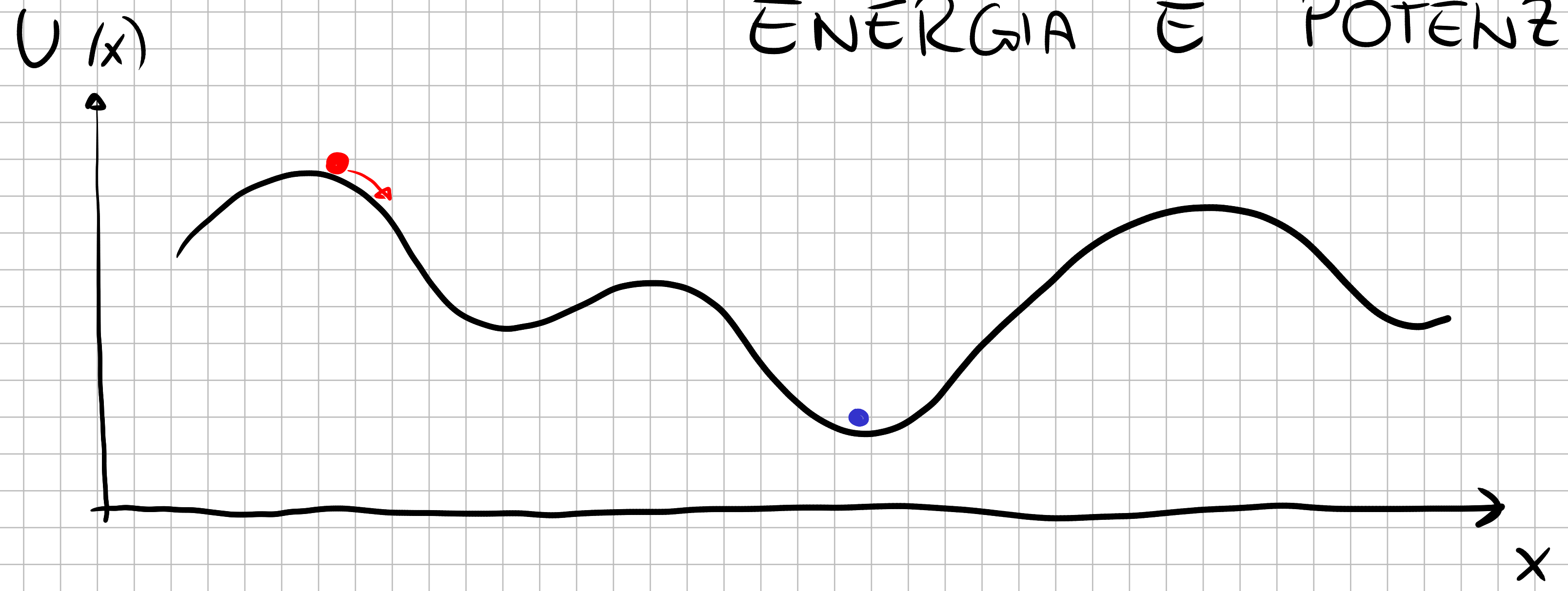
VENERDÍ 13

NON SI FARÀ

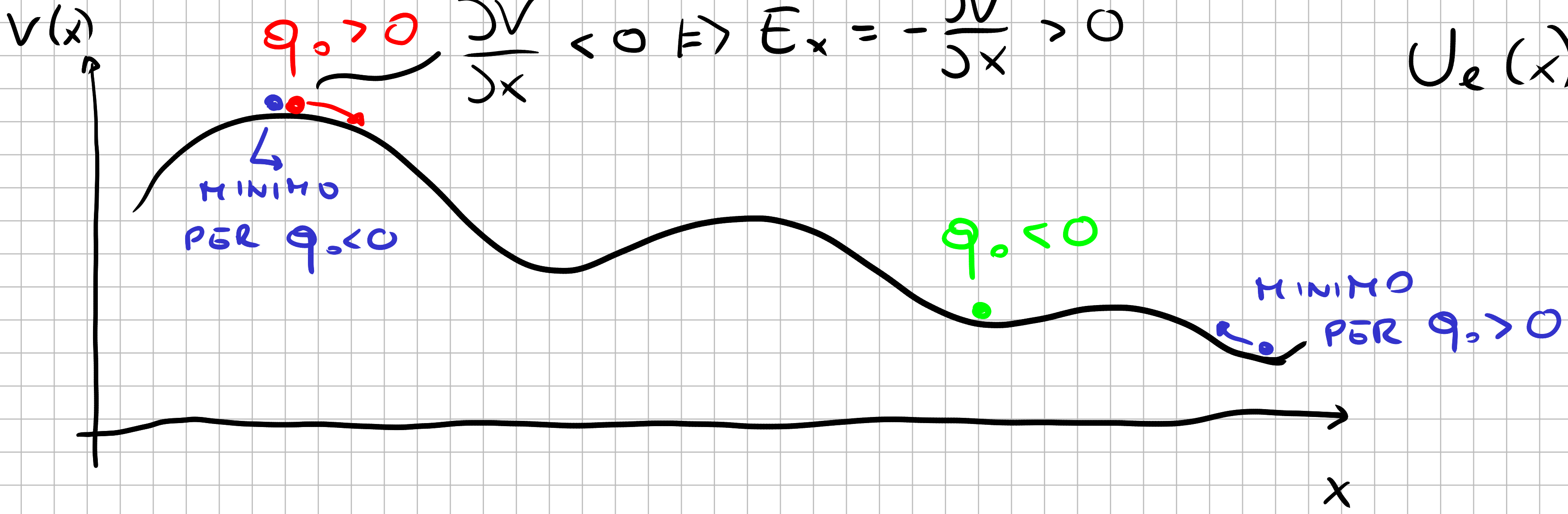
LEZIONE

ENERGIA E POTENZIALE

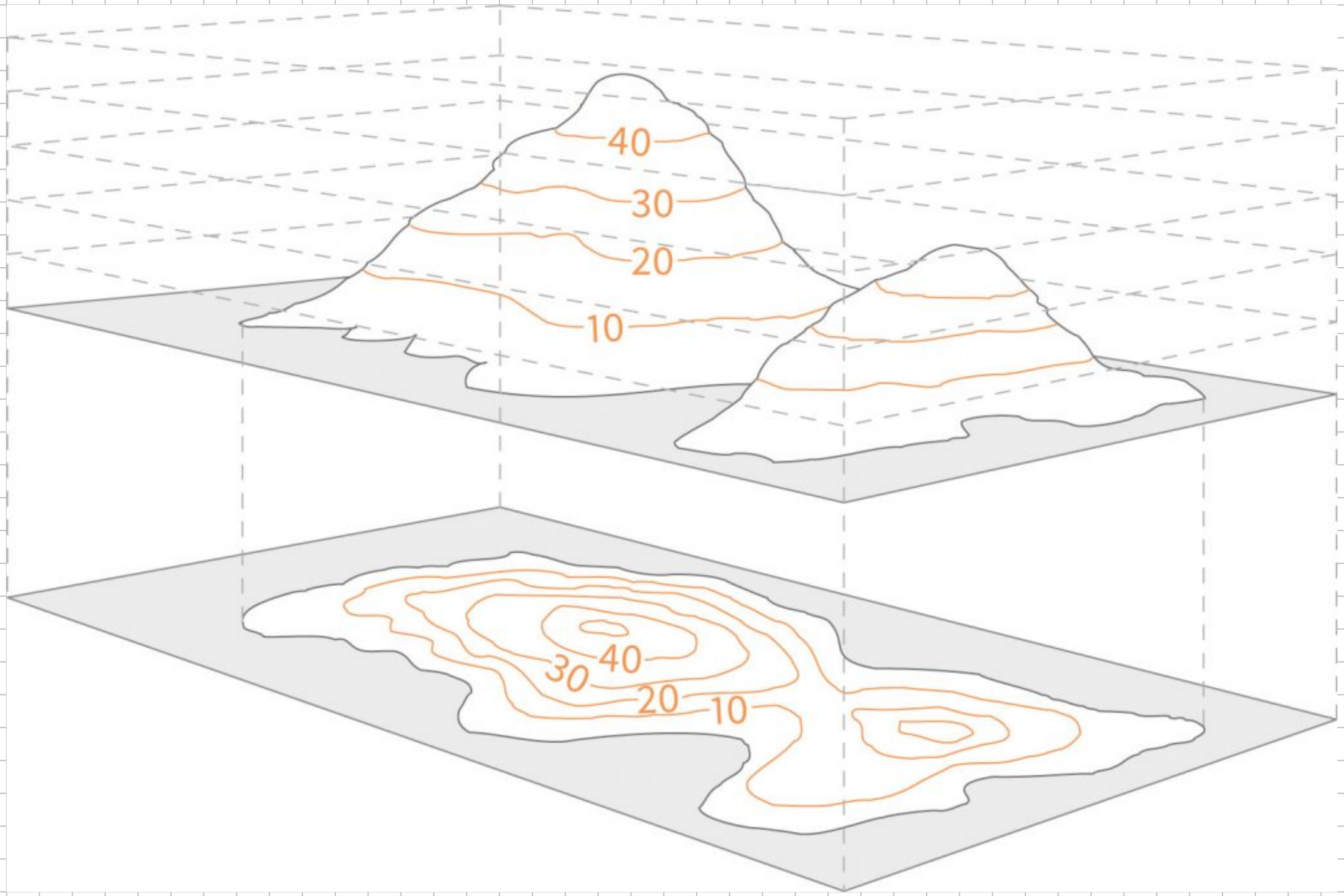


$q_0 > 0 \quad \frac{\partial V}{\partial x} < 0 \Rightarrow E_x = -\frac{\partial V}{\partial x} > 0$

$$U_e(x) = \boxed{q_0} V(x)$$

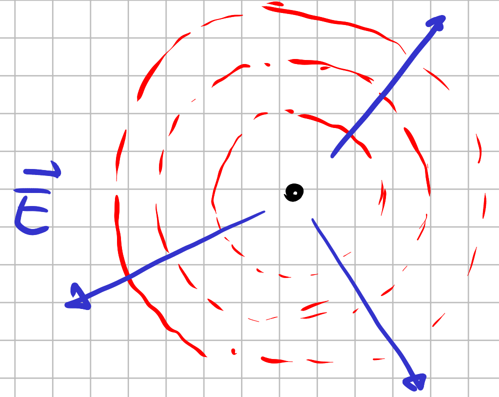


SUPERFICI EQUIPOTENZIALI



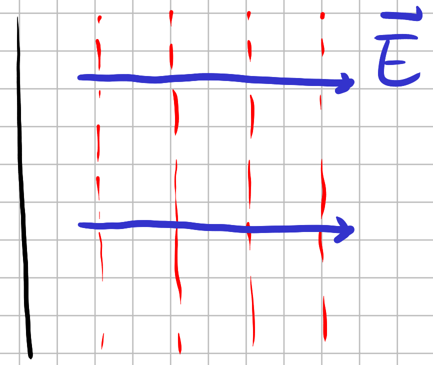
CARICA PUNTIFORME

$$V(r) = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}$$

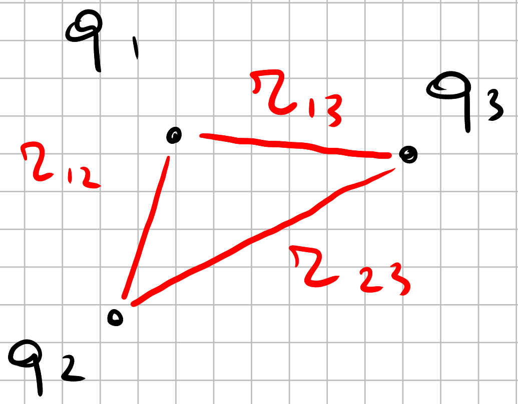


PIANO

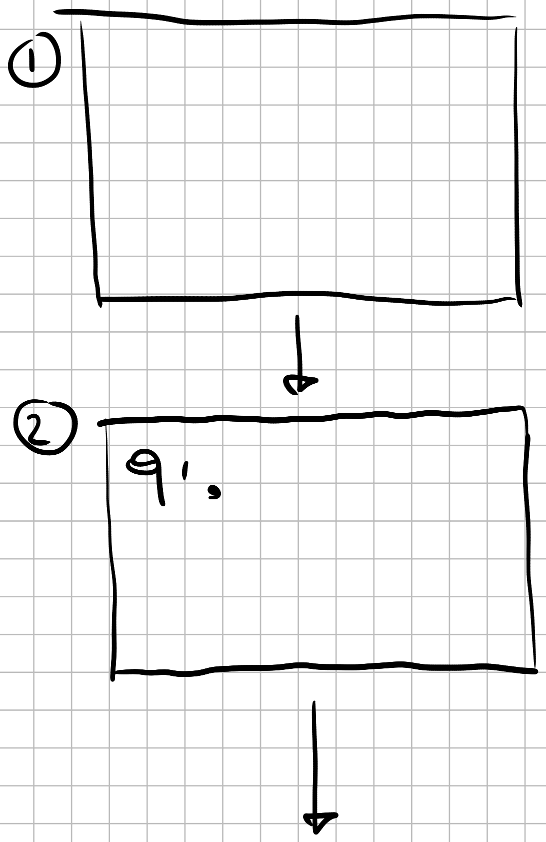
$$V(x) = -\frac{\sigma}{\varepsilon_0}x$$



INTERPRETAZIONE DELL'U_e



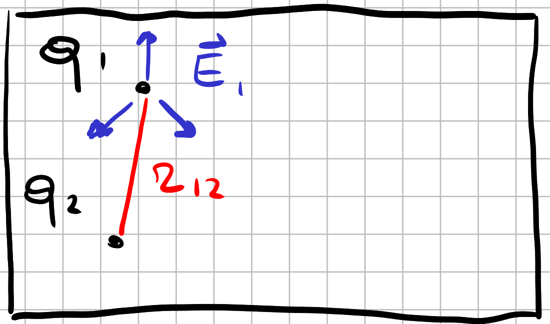
STADIO FINALE



STADIO INIZIALE

OPERAZIONE "GRATUITA"

③



$$W_{ext}^{(3)} = \int_{\infty}^{r_{12}} \vec{F}_{ext} \cdot d\vec{s} = \qquad \vec{F}_{ext} = -\vec{F}_{12} = -q_2 \vec{E}_1$$

$$= -\int_{\infty}^{r_{12}} \vec{F}_{12} \cdot d\vec{s} = -\int_{\infty}^{r_{12}} q_2 \vec{E}_1 \cdot d\vec{s} =$$

$$= -\frac{q_1 q_2}{4\pi\varepsilon_0} \int_{\infty}^{r_{12}} \frac{\hat{r}}{r_{12}^2} \cdot d\vec{s} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{1}{r} \Bigg|_{\infty}^{r_{12}} =$$

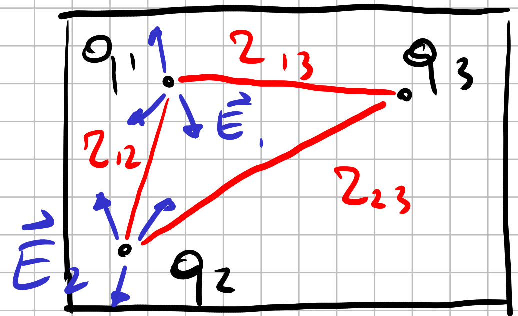
$$= \underbrace{\frac{q_1 q_2}{4\pi\varepsilon_0} \frac{1}{r_{12}}}_{\Delta V_1} = q_2 \Delta V_1 = \Delta U_e$$

DIFFERENZA DI ENERGIA POTENZIALE ELETTROSTATICA

$W_{ext}^{(3)} = \Delta U_e$ ← DALLE FORZE ESTERNE

$(W = -\Delta U_e)$ ← IL LAVORO COMPIUTO DALLE FORZE ELETTROSTATICHE

4



$$\vec{E}_{TOT} = \vec{E}_1 + \vec{E}_2 ,$$

$$W_{ext}^{(4)} = \int \vec{F}_{ext} \cdot d\vec{s} = -q_3 \int_{\infty}^{r_3} (\vec{E}_1 + \vec{E}_2) \cdot d\vec{s} =$$

$$= -q_3 \int_{\infty}^{r_{13}} \vec{E}_1 \cdot d\vec{s} - q_3 \int_{\infty}^{r_{12}} \vec{E}_2 \cdot d\vec{s} =$$

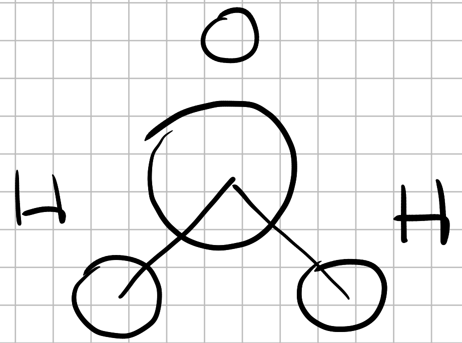
$$= \frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}} + \frac{q_2 q_3}{4\pi\varepsilon_0 r_{23}} = \Delta U_e^{(4)} \Rightarrow$$

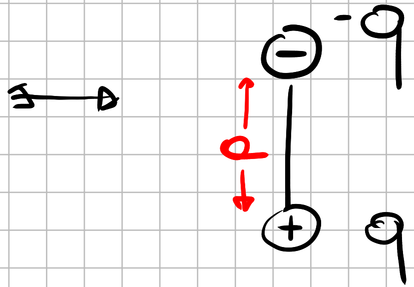
$$\Delta U_e = \underbrace{\cancel{\Delta U_e^{(1)}}}_{=0} + \underbrace{\cancel{\Delta U_e^{(2)}}}_{=0} + \Delta U_e^{(3)} + \Delta U_e^{(4)} =$$

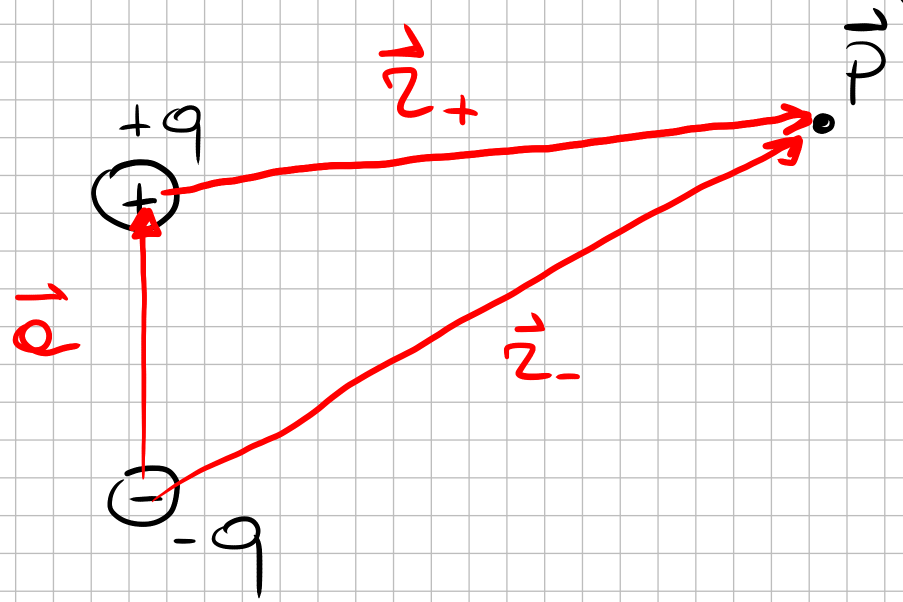
$$= \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{1}{r_{12}} + \frac{q_1 q_3}{4\pi\varepsilon_0} \frac{1}{r_{13}} + \frac{q_2 q_3}{4\pi\varepsilon_0} \frac{1}{r_{23}} \Rightarrow$$ PER UN SISTEMA GENERICO

$$\boxed{\Delta U_e = \frac{1}{2} \sum_{i=1}^{N} \sum_{i \neq j}^{N} \frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}} = \sum_{i>j} \frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}}}$$ (CFR. HAMILTONIANA)

DIPOLO ELETTRICO







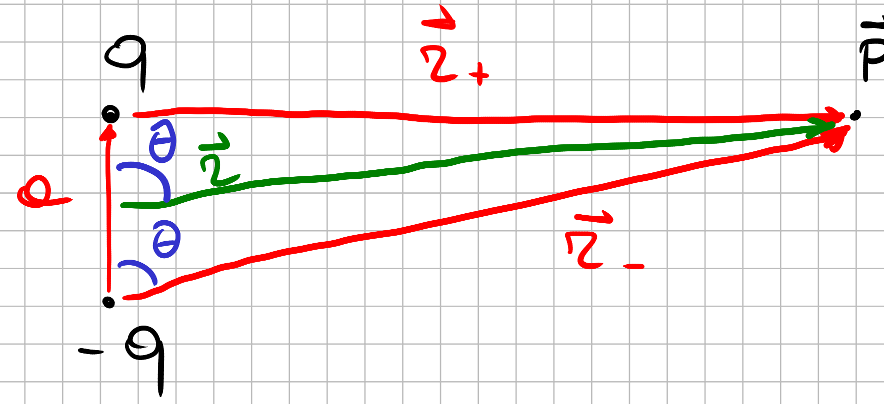
$$V(\vec{P}) = \frac{q}{4\pi\varepsilon_0}\frac{1}{r_+} - \frac{q}{4\pi\varepsilon_0}\frac{1}{r_-} =$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right) =$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{r_- - r_+}{r_+ r_-}$$

APP. DI DIPOLO

consideriamo distanze $\gg a$, cioè $\boxed{r_- \gg a, \quad r_+ \gg a}$



$$V(\vec P) = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_- - r_+}{r_- r_+}\right)$$

$$r_- r_+ \approx r^2$$

$$\vec r_- = \vec a + \vec r_+ \Rightarrow \vec r_+ = \vec r_- - \vec a \Rightarrow r_+ = \sqrt{\vec r_+ \cdot \vec r_+} =$$

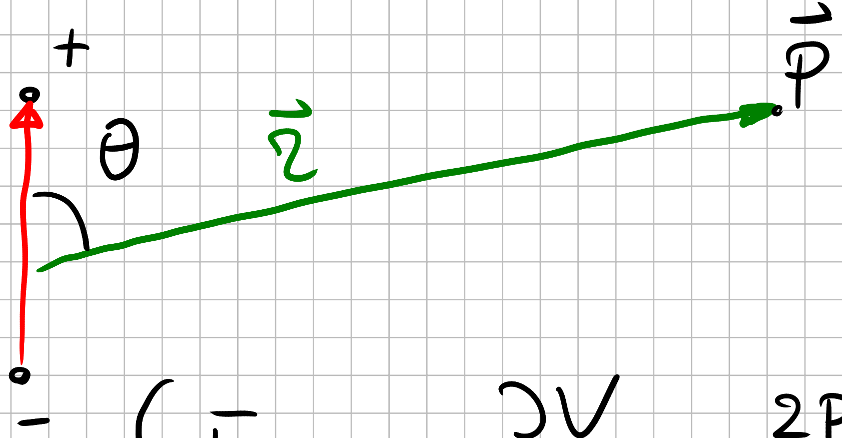
$$= \sqrt{(\vec r_- - \vec a)\cdot(\vec r_- - \vec a)} = \sqrt{r_-^2 - 2\vec a\cdot\vec r_- + a^2} = r_-\sqrt{1 - \frac{2\vec a\cdot\vec r_-}{r_-^2} + \overset{0}{\boxed{\frac{a^2}{r_-^2}}}} =$$

$$\approx r_-\sqrt{1 - \frac{2\vec a\cdot\vec r_-}{r_-^2}} = r_-\sqrt{1 - \frac{2a\cancel{r_-}\cos\theta}{r_-^{\cancel{2}}}} \underset{\sqrt{1-2\varepsilon}\,\approx\, 1-\varepsilon}{\approx} r_-\left(1 - \frac{a\cos\theta}{r_-}\right)$$

$$r_+ \approx r_-\left(1 - \frac{a\cos\theta}{r_-}\right) = r_- - a\cos\theta$$

$$r_- - r_+ = a\cos\theta \Rightarrow$$

$$V(\vec{P}) = \frac{q}{4\pi\varepsilon_0}\,\frac{a\cos\theta}{r^2} = \frac{\overbrace{qa}^{p\,=}\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r_2} = \boxed{\frac{\vec{p}\cdot\hat{r}}{4\pi\varepsilon_0 r^2}}$$



$$\vec{E} = -\vec{\nabla} V$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$\begin{cases} E_r = -\dfrac{\partial V}{\partial r} = \dfrac{2p\cos\theta}{4\pi\varepsilon_0 r_3} \\[2ex] E_\theta = -\dfrac{1}{r}\dfrac{\partial V}{\partial \theta} = \dfrac{p\sin\theta}{4\pi\varepsilon_0 r^3} \end{cases}$$