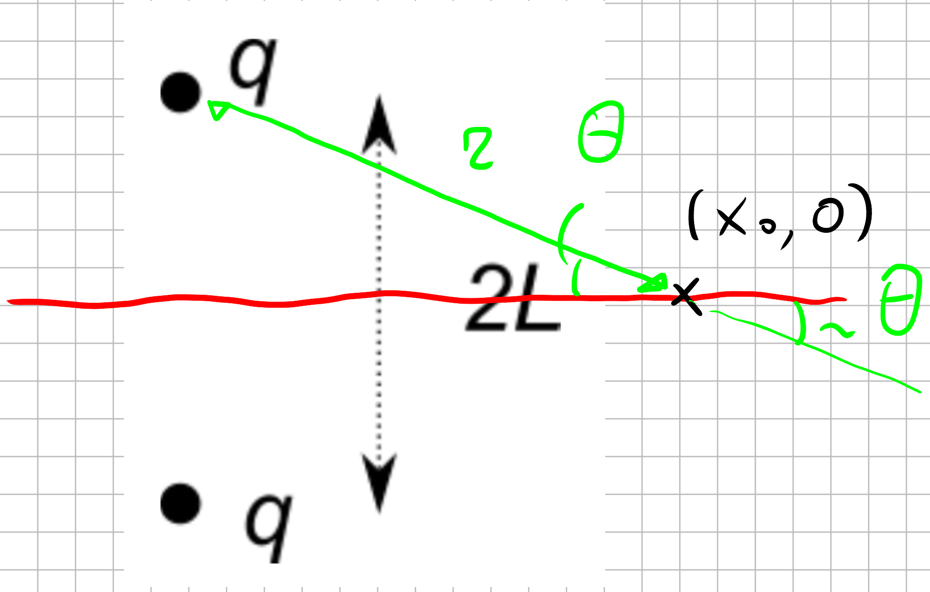
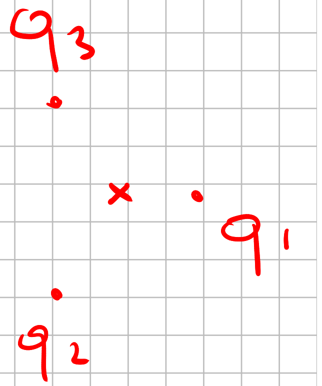
ESERCIZIO 3

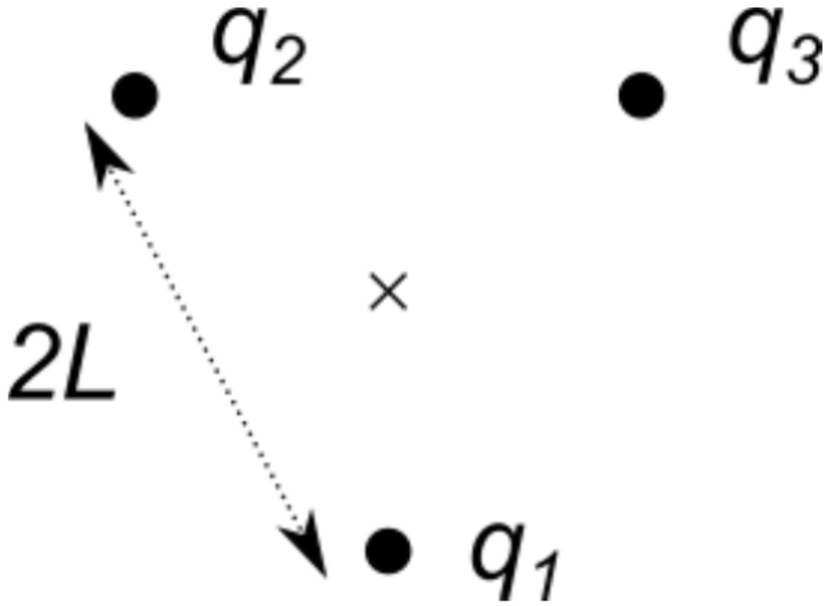


$$\vec{E}(x_0, 0) = (E_x, 0)$$

$$E_x = \frac{q}{2\pi\varepsilon_0}\frac{x_0}{r^3} = \frac{q}{2\pi\varepsilon_0}\frac{\cos\theta}{r^2}$$

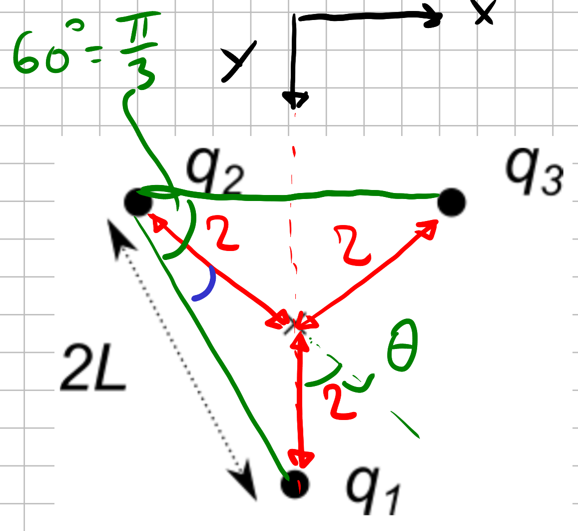
ESERCIZIO 4

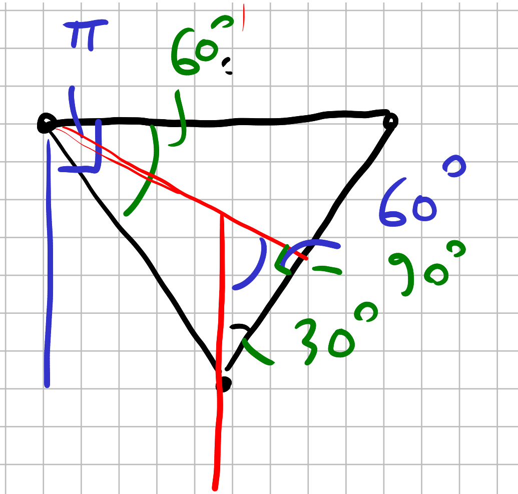




Tre cariche fisse q_1, q_2 e q_3 sono poste sui vertici di un triangolo equilatero di lato $2L$. Poniamo $q_2 = q_3 = q$.

1. Calcolare l'espressione del campo elettrostatico $\vec{E} = (E_x, E_y)$ nel centro $\vec{O}$ del triangolo.
2. Possiamo muovere la carica q_1 lungo l'asse che la congiunge al centro. Se $q_1 = 2q$, dove dobbiamo posizionare q_1 per far sì che il campo si annulli nel punto $\vec{O}$?





SVOLGIMENTO

$$\vec{E} = \vec{E}^{2+3} + \vec{E}^{1}$$

$$\vec{E} = (0, E_y)$$

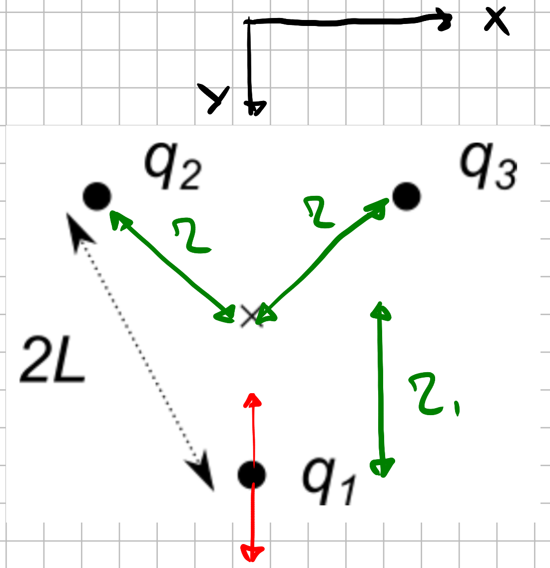
$$E_y^{2+3} = \frac{q}{2\pi\varepsilon_0}\frac{\cos\theta}{r^2}, \quad \cos\theta = \cos\frac{\pi}{3} = \frac{1}{2} \Rightarrow$$

$$E_y^{2+3} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}$$

$$\vec{E}^{1} = \frac{q_1}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r}_1 = \frac{q_1}{4\pi\varepsilon_0}\frac{1}{r^2}(-\hat{y}) = -\frac{q_1}{4\pi\varepsilon_0}\frac{\hat{y}}{r^2}$$

$$\Rightarrow E_y^{1} = -\frac{q_1}{4\pi\varepsilon_0}\frac{1}{r^2} \Rightarrow$$

$$E_y = E_y^{2+3} + E_y^{1} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2} - \frac{q_1}{4\pi\varepsilon_0}\frac{1}{r^2} = \frac{q-q_1}{4\pi\varepsilon_0}\frac{1}{r^2}$$



SECONDA PARTE

$q_2 = q_3 = q \quad , \quad q_1 = 2q \quad ,$

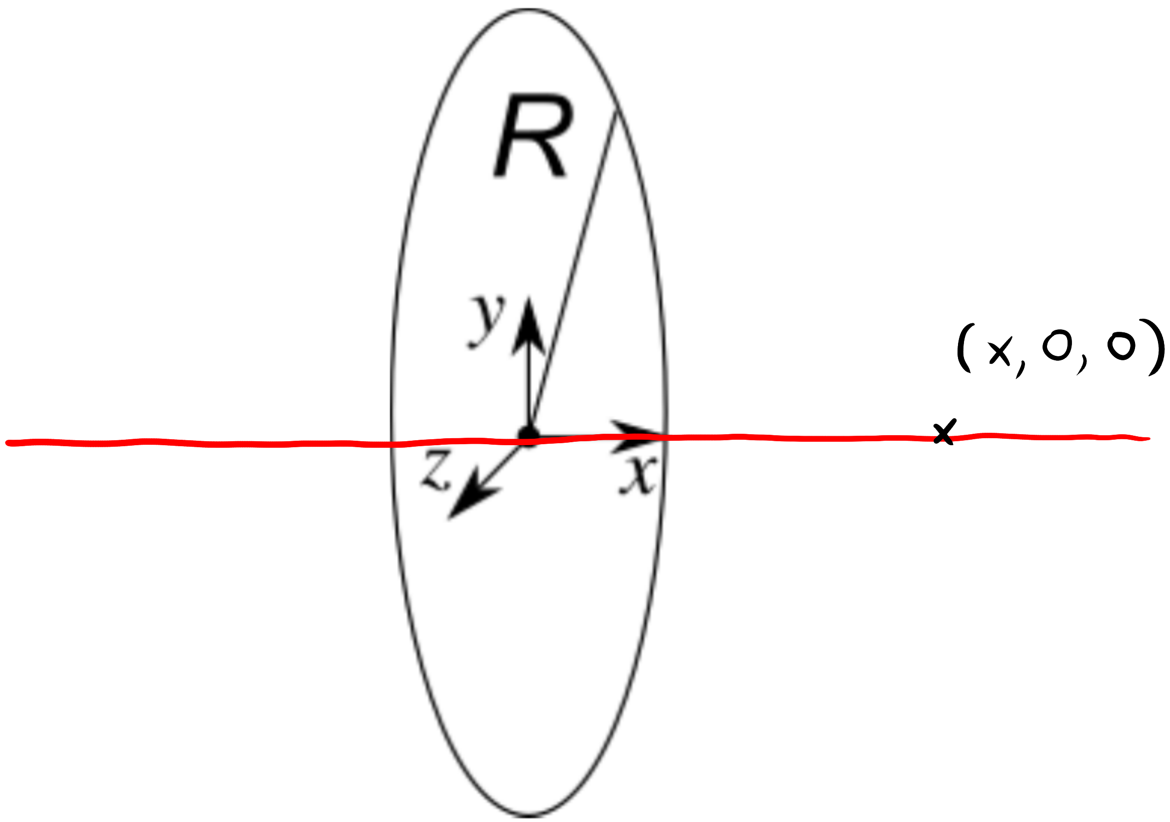
per quale posizione di q_1, $\vec{E} = 0$?

$$E_y = E_y^{2+3} + E_y^{1} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2} - \frac{q_1}{4\pi\varepsilon_0}\frac{1}{r_1^2} =$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2} - \frac{2q}{4\pi\varepsilon_0}\frac{1}{r_1^2} = 0 \implies$$

$$\frac{\cancel{q}}{\cancel{4\pi\varepsilon_0}}\frac{1}{r^2} = \frac{2\cancel{q}}{\cancel{4\pi\varepsilon_0}}\frac{1}{r_1^2} \implies r_1^2 = 2r^2 \implies \boxed{r_1 = \sqrt{2}\, r}$$

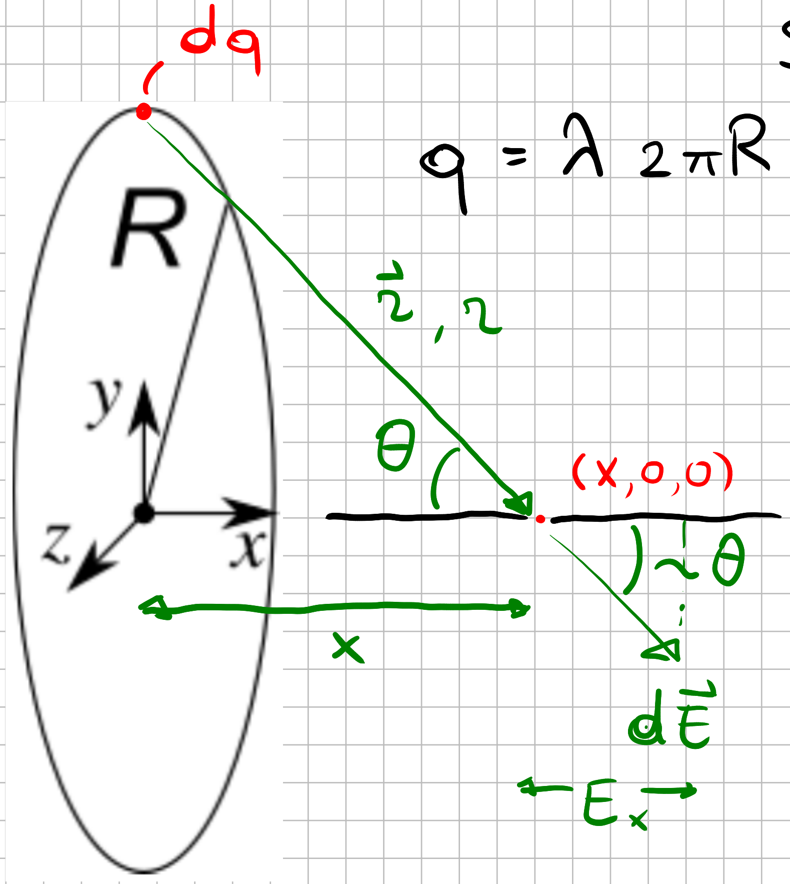
ESERCIZIO 5



Una carica q è distribuita uniformemente su un *sottile* anello di raggio R.

1. Senza calcolarlo, discutere qualitativamente il comportamento del campo elettrostatico lungo l'asse dell'anello (che prendiamo coincidente con l'asse x) al variare di x.
2. Calcolare il campo elettrico in un generico punto $(x, 0, 0)$.

SVOLGIMENTO



$$q = \lambda\, 2\pi R \Rightarrow \lambda = \frac{q}{2\pi R}$$

$$r\cos\theta = x \Rightarrow \cos\theta = \frac{x}{r}$$

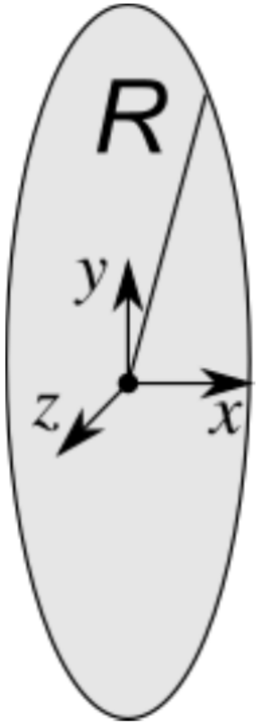
$$dE_x = \frac{dq}{4\pi\varepsilon_0}\frac{\cos\theta}{r^2} = \frac{dq}{4\pi\varepsilon_0}\frac{x}{r^3} =$$

$$= \frac{dq}{4\pi\varepsilon_0}\frac{x}{(x^2+R^2)^{3/2}} \Rightarrow$$

$$E_x = \int_{\text{ANELLO}} dE_x = \frac{1}{4\pi\varepsilon_0}\int_{\text{ANELLO}} \frac{x\,dq}{(x^2+R^2)^{3/2}} =$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{x}{(x^2+R^2)^{3/2}}\int_{\text{ANELLO}} dq = \boxed{\frac{q}{4\pi\varepsilon_0}\frac{x}{(x^2+R^2)^{3/2}}}$$

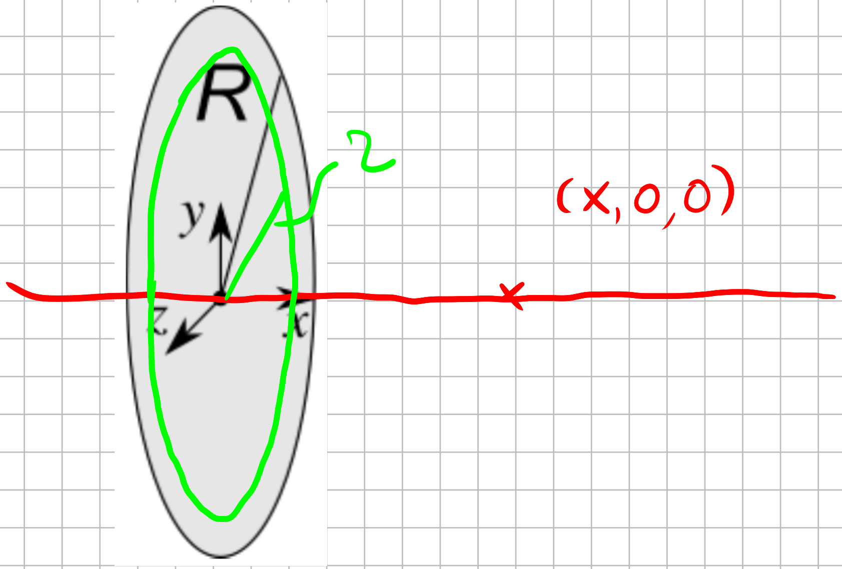
ESERCIZIO 6



Una carica q è distribuita uniformemente su un sottile disco di raggio R. Consideriamo il sistema di riferimento che ha l'origine nel centro del disco e $\hat{x}$ orientato in maniera concorde all'asse del disco.

1. Calcolare il modulo del campo elettrico in un generico punto $(x, 0, 0)$.
2. Cosa succede quando $R \to \infty$?
3. Discutere il verso del campo lungo l'asse x.

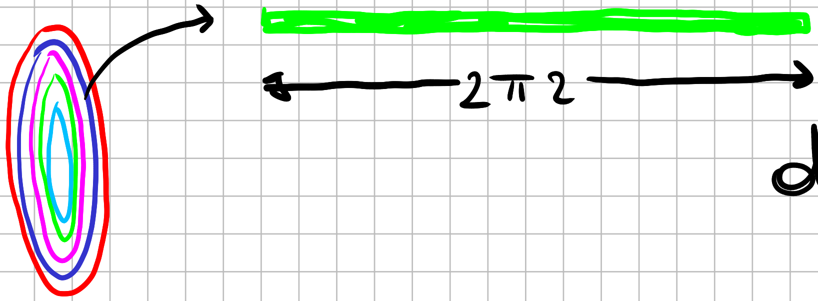
SVOLGIMENTO



$$q = \sigma \pi R^2 \Rightarrow \sigma = \frac{q}{\pi R^2}, \quad dq = \sigma \underbrace{d\Sigma}_{\text{AREA DELL'ELEMENTO DI SUPERFICIE}}$$

$$dE_x = \frac{dq}{4\pi\varepsilon_0} \frac{x}{(x^2+r^2)^{3/2}} =$$

$$= \frac{\sigma\, d\Sigma}{4\pi\varepsilon_0} \frac{x}{(x^2+r^2)^{3/2}}$$



$$: dr \qquad d\Sigma = 2\pi r\, dr \Rightarrow$$

$$dE_x = \frac{\sigma \cancel{2\pi} r\, dr}{2\cancel{4}\cancel{\pi}\varepsilon_0} \frac{x}{(x^2+r^2)^{3/2}} \Rightarrow$$

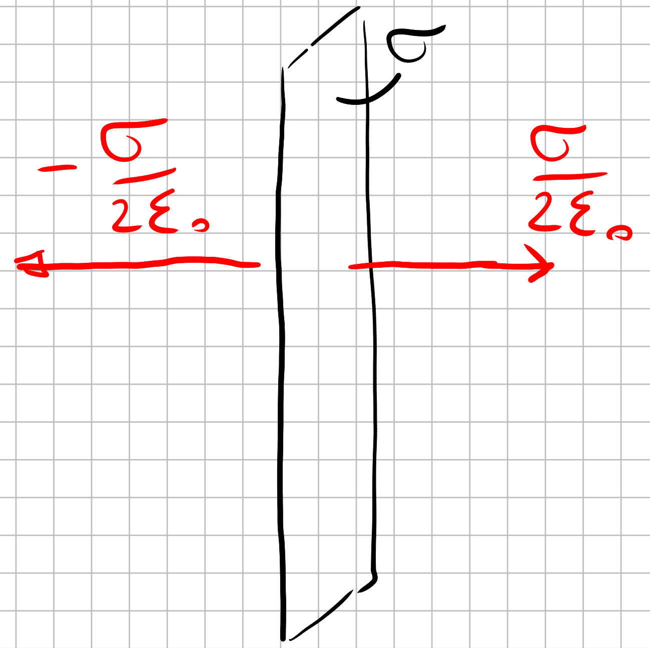
$$E_x = \int_{\text{DISCO}} dE_x = \int_0^R \frac{\sigma r x\, dr}{2\varepsilon_0 (x^2+r^2)^{3/2}} = \frac{\sigma x}{2\varepsilon_0} \int_0^R \frac{r\, dr}{(x^2+r^2)^{3/2}} \Rightarrow$$

$$E_x = \frac{\sigma x}{2\varepsilon_0}\left(-\frac{1}{\sqrt{r^2+x^2}}\right)_0^R = \frac{\sigma x}{2\varepsilon_0}\left(-\frac{1}{\sqrt{R^2+x^2}}+\frac{1}{|x|}\right) =$$

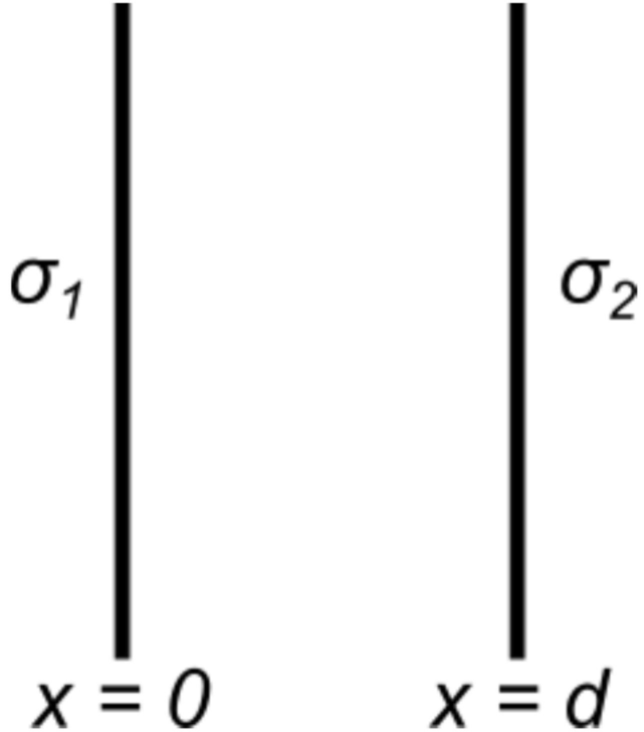
$$= \frac{\sigma}{2\varepsilon_0}\left(\frac{x}{|x|} - \frac{x}{\sqrt{R^2+x^2}}\right)$$

$$E_x \xrightarrow[R\to\infty]{} \frac{\sigma}{2\varepsilon_0}\frac{x}{|x|} = \boxed{\frac{\sigma}{2\varepsilon_0}\,\mathrm{sgn}(x)}$$

il campo generato da un piano carico con σ

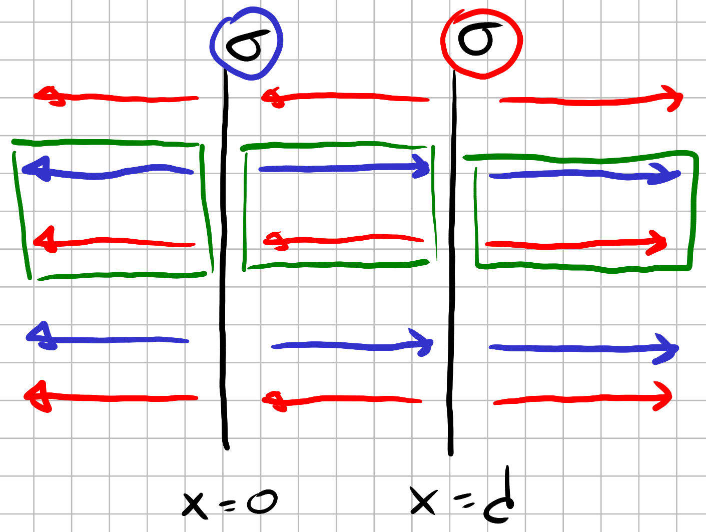


ESERCIZIO 7

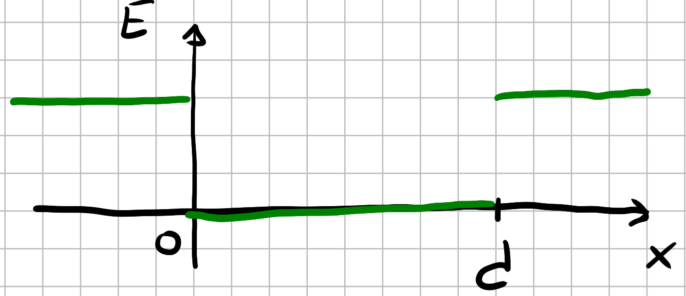


Calcolare e disegnare il potenziale e il modulo del campo elettrostatico generati in tutto lo spazio da due piani indefiniti paralleli uniformemente carichi con densità superficiale σ_1 e σ_2 e posti a $x_1 = 0$ e $x_2 = d$ lungo l'asse x,

1. Nel caso in cui $\sigma_1 = -\sigma_2$.
2. Nel caso in cui $\sigma_1 = \sigma_2 = \sigma$.



SVOLGIMENTO



$$|\vec{E}_1| = \frac{|\sigma|}{2\varepsilon_0}$$

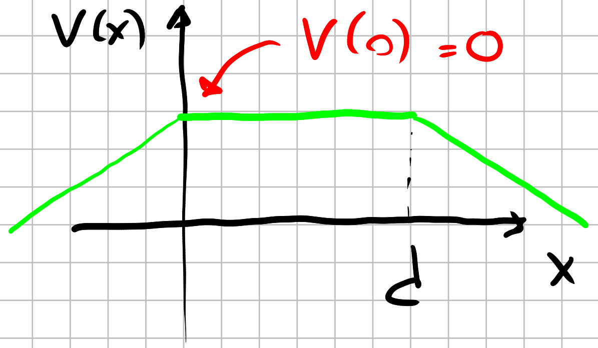
$$|\vec{E}_2| = \frac{|\sigma|}{2\varepsilon_0}$$

$$|\vec{E}_{TOT}| = \begin{cases} 0 & \text{se } 0 \leq x \leq d \\ \dfrac{|\sigma|}{\varepsilon_0} & \text{altrimenti} \end{cases}$$

$$\int_A^B \vec{E} \cdot d\vec{s} = V(A) - V(B) = -\left(V(B) - V(A)\right) = -\Delta V_{AB}$$

$$\int_0^d \vec{E} \cdot d\vec{s} = \int_0^d \vec{E} \cdot d\vec{x} = 0 = -\Delta V_{0d} = -\left[V(d) - V(0)\right] \Rightarrow$$

$$V(0) = V(d) \Rightarrow V(0) = V(x) \quad \text{per} \quad 0 \leq x \leq d$$



$$\boxed{V(0) = V(d) = V(x)} \quad \text{se } 0 \le x \le d$$

$$\int_d^{x>d} \vec{E}\cdot d\vec{s} = \int_d^{x>d} \vec{E}\cdot d\vec{x}' = \int_d^{x>d} E\, dx = \int_d^{x>d} \frac{\sigma}{\varepsilon_0}\, dx = \frac{\sigma}{\varepsilon_0}\int_d^{x>d} dx = \frac{\sigma}{\varepsilon_0}(x-d)$$

$$= -\big(V(x) - V(d)\big) \implies V(x) = \boxed{V(d) - \frac{\sigma}{\varepsilon_0}(x-d)} \quad x>d$$

$$\int_0^{|x|} \vec{E}\cdot d\vec{s} = \int_0^{|x|} E(-\hat{x})(-d\vec{x}) = \int_0^{|x|} E\, dx = \int_0^{|x|} \frac{\sigma}{\varepsilon_0}\, dx = \frac{\sigma}{\varepsilon_0}|x| = -\frac{\sigma}{\varepsilon_0}x =$$

$$= -\big(V(x) - V(0)\big) \implies \boxed{V(x) = \frac{\sigma}{\varepsilon_0}x + V(0)} \quad x<0$$