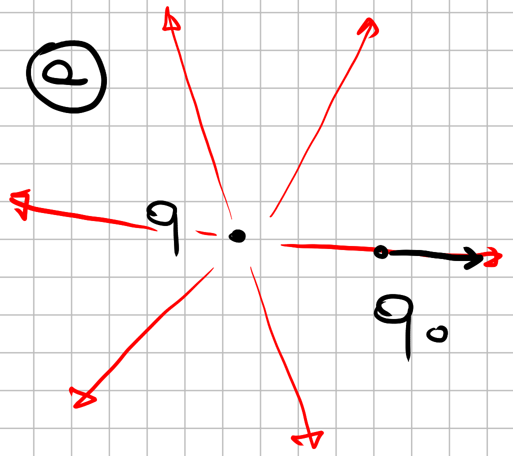
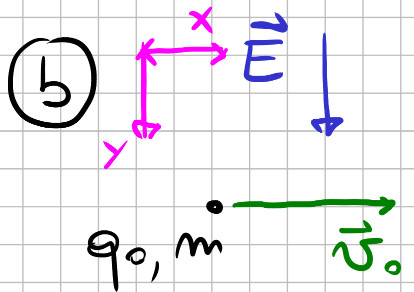
MOTO DI UNA CARICA IN UN CAMPO





al tempo 0 q_0 si trova (x_0, y_0, z_0)

con $\vec{v}_0 = (v_{0x}, v_{0y}, v_{0z})$

prendiamo un campo costante e uniforme

$$\vec{F} = q_0 \vec{E} = m\vec{a} \Rightarrow \vec{a} = \frac{q_0 \vec{E}}{m} = \left(0, \frac{q_0 E}{m}, 0\right) \Rightarrow$$

$$\begin{cases} v_x(t) = v_{0x} \\ v_y(t) = v_{0y} + a\,t = v_{0y} + \frac{q_0 E}{m} t \\ v_z(t) = v_{0z} \end{cases}$$

$$\begin{cases} x(t) = x_0 + v_{0x} t \\ y(t) = y_0 + v_{0y} t + \frac{1}{2} a t^2 \\ z(t) = z_0 + v_{0z} t \end{cases}$$

ENERGIA CINETICA

$$U_k(t) = \frac{1}{2} m v^2(t) ,$$

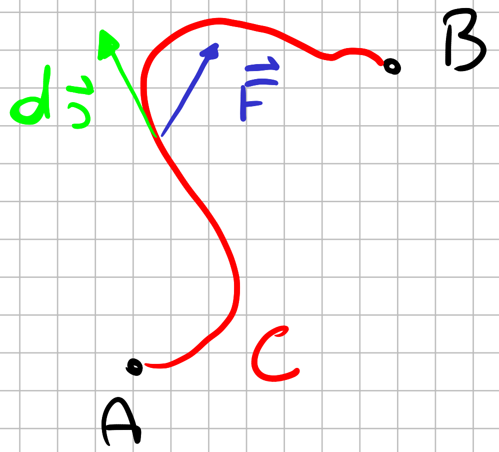
$$\Delta U_k(t) = U_k(t) - U_k(0) = \frac{1}{2} m \Big[\overbrace{\cancel{v_{0x}^2} + (v_{0y} + at)^2 + \cancel{v_{0z}^2}}^{U_k(t)} -$$

$$- \underbrace{\left(\cancel{v_{0x}^2} + v_{0y}^2 + \cancel{v_{0z}^2}\right)}_{U_k(0)} \Big] = \frac{1}{2} m \left(\cancel{v_{0y}^2} + a^2 t^2 + 2 v_{0y} a t - \cancel{v_{0y}^2} \right) =$$

$$= \frac{1}{2} m \left(a^2 t^2 + 2 v_{0y} a t \right) = \underbrace{ma}_{F} \Big(\underbrace{\frac{1}{2} a t^2 + v_{0y} t}_{x(t) - x_0 = \Delta x(t)} \Big) =$$

$$= F \Delta x = W$$ LAVORO

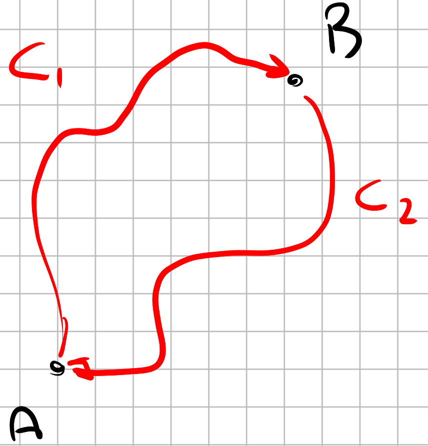
$$W = \Delta U_k$$ TEOREMA DELL'ENERGIA CINETICA



IL LAVORO

$$W_c = \int_C \vec{F}\cdot d\vec{s} = \int_A^B \vec{F}\cdot d\vec{s}$$

SE LA FORZA È CONSERVATIVA



$$W_{C_1+C_2} = W_{C_1} + W_{C_2} = \int_{C_1} \vec{F}\cdot d\vec{s} + \int_{C_2} \vec{F}\cdot d\vec{s} =$$

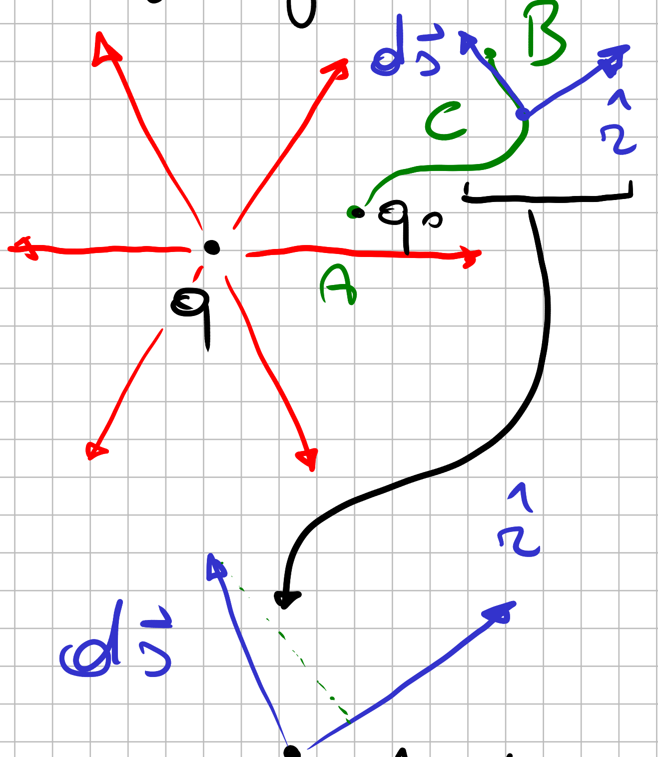
$$= \int_A^B \vec{F}\cdot d\vec{s} + \int_B^A \vec{F}\cdot d\vec{s} = \int_A^B \vec{F}\cdot d\vec{s} - \int_A^B \vec{F}\cdot d\vec{s} =$$

$$= 0$$

LAVORO E FORZA ELETTRICA

In generale le forze elettriche NON sono conservative

La forza elettrostatica È conservativa



$d\vec{s} \cdot \hat{r} = dr$

$$W_{AB} = \int_C \vec{F}_e \cdot d\vec{s} \quad , \quad \vec{F}_e = \frac{q q_0}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2} \, ,$$

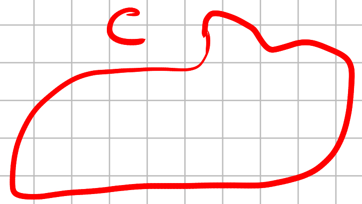
$$\vec{F}_e \cdot d\vec{s} = \frac{q q_0}{4\pi\varepsilon_0 r^2} \, \hat{r} \cdot d\vec{s} = \frac{q q_0}{4\pi\varepsilon_0} \frac{dr}{r^2} \implies$$

$$W_{AB} = \int_C \frac{q q_0}{4\pi\varepsilon_0} \frac{dr}{r^2} = \int_A^B \frac{q q_0}{4\pi\varepsilon_0} \frac{dr}{r^2} =$$

$$= \frac{q q_0}{4\pi\varepsilon_0} \int_A^B \frac{dr}{r^2} = - \frac{q q_0}{4\pi\varepsilon_0} \left(\frac{1}{r_B} - \frac{1}{r_A} \right)$$

$$\underbrace{\oint_c \vec{F}_e \cdot d\vec{s}}_{\text{CIRCUITAZIONE}} = \oint_c q_0 \vec{E} \cdot d\vec{s} = 0$$

$\oint_c$ INTEGRALE SU UN PERCORSO CHIUSO



$$\boxed{\oint_c \vec{E} \cdot d\vec{s} = 0}$$

CAMPO <u>ELETTROSTATICO</u>

È CONSERVATIVO

POTENZIALE ELETTRICO

$$\int_c \vec{E} \cdot d\vec{s} = \int_A^B \vec{E} \cdot d\vec{s} \equiv V(A) - V(B) = -\big(V(B) - V(A)\big)$$

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a) = G(a) - G(b), \quad G(x) = -F(x)}$$

$V = V(x,y,z)$ è il potenziale elettrico

$$W_{AB} = -q_0\big(V(B) - V(A)\big) \equiv -q_0 \boxed{\Delta V_{AB}}$$ differenza di potenziale

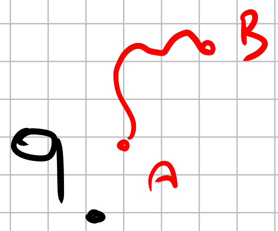
$$W_{AB} = \Delta U_K \overset{\text{CONS.}}{=} - \Delta U_e = -q_0 \Delta V_{AB} \Rightarrow$$

$$U_e(A) = q_0 V(A) \longleftrightarrow \vec{F}_e = q_0 \vec{E}$$

UNITÀ DI MISURA

$$[V] = \frac{J}{C} = \frac{N}{C}\,m = \text{Volt}\ \ (V)$$

POTENZIALE GENERATO DA q



$$\int_A^B \vec{E}\cdot d\vec{s} = \frac{\int \vec{F}\cdot d\vec{s}}{q_0} = -\overbrace{\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_B}-\frac{1}{r_A}\right)} = -\big(V(B)-V(A)\big)$$

$$V(A) = \frac{q}{4\pi\varepsilon_0}\frac{1}{r_A} + C$$

FUNZIONE POTENZIALE

$\neq$

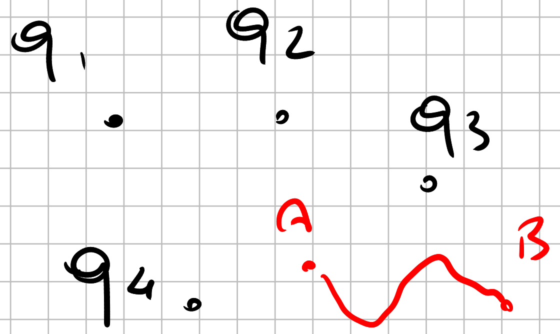
ΔV DIFFERENZA DI POTENZIALE

$$V(R) \xrightarrow[R\to\infty]{} 0 \implies C = 0$$

SCELTA PIÙ COMUNE

MA NON OBBLIGATORIA

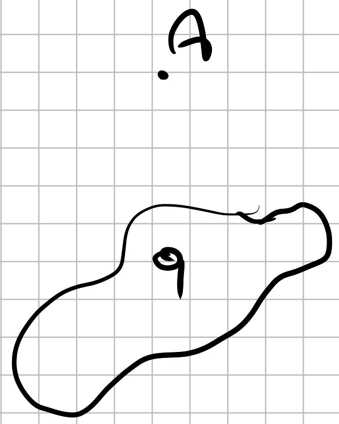
PRINCIPIO DI SOVRAPPOSIZIONE



$$\Delta V_{AB} = -\int_A^B \vec{E}\cdot d\vec{s} = -\int_A^B \sum_i \vec{E}_i \cdot d\vec{s} = -\sum_i \int_A^B \vec{E}_i \cdot d\vec{s} =$$

$$= \sum_i \Delta V_{AB,i} = \sum_i \left(V_i(B) - V_i(A) \right) \Rightarrow$$

$$\boxed{V(A) = \sum_i V_i(A)} \longleftrightarrow \vec{E}(A) = \sum_i \vec{E}_i(A)$$

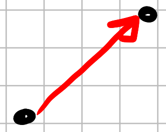


$$V(A) = \int_V dV = \frac{1}{4\pi\varepsilon_0}\int_V \frac{dq}{r}$$ ← UN INTEGRALE

RELAZIONE INVERSA TRA $\vec{E}$ E V

$$\Delta V_{AB} = -\int_A^B \vec{E}\cdot d\vec{s} = \int_A^B dV \Rightarrow dV = -\vec{E}\cdot d\vec{s},$$

$B(x+dx, y+dy, z+dz)$



$d\vec{s} = (dx, dy, dz)$

$A(x, y, z)$

$$-\vec{E}\cdot d\vec{s} = -E_x dx - E_y dy - E_z dz = dV, \text{ ma}$$

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz \Rightarrow$$

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z} \Rightarrow$$

$$\vec{E} = -\left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right) = -\underbrace{\vec{\nabla} V}_{\text{GRADIENTE DI } V}, \qquad \vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

NABLA

IL CAMPO È IRROTAZIONALE

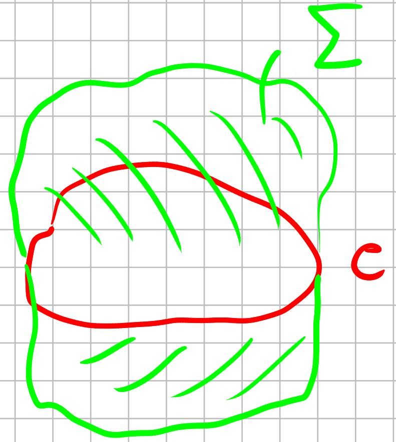
$$\oint \vec{E} \cdot d\vec{s} = \int_{\Sigma} \vec{\nabla} \times \vec{E} \cdot \hat{n} \, d\Sigma = 0$$

TEOREMA DI STOKES

$$\Downarrow$$

$$\boxed{\vec{\nabla} \times \vec{E} = 0}$$

IL CAMPO È IRROTAZIONALE



IL ROTORE DI UN CAMPO

$$|\vec{\nabla} \times \vec{E}|$$