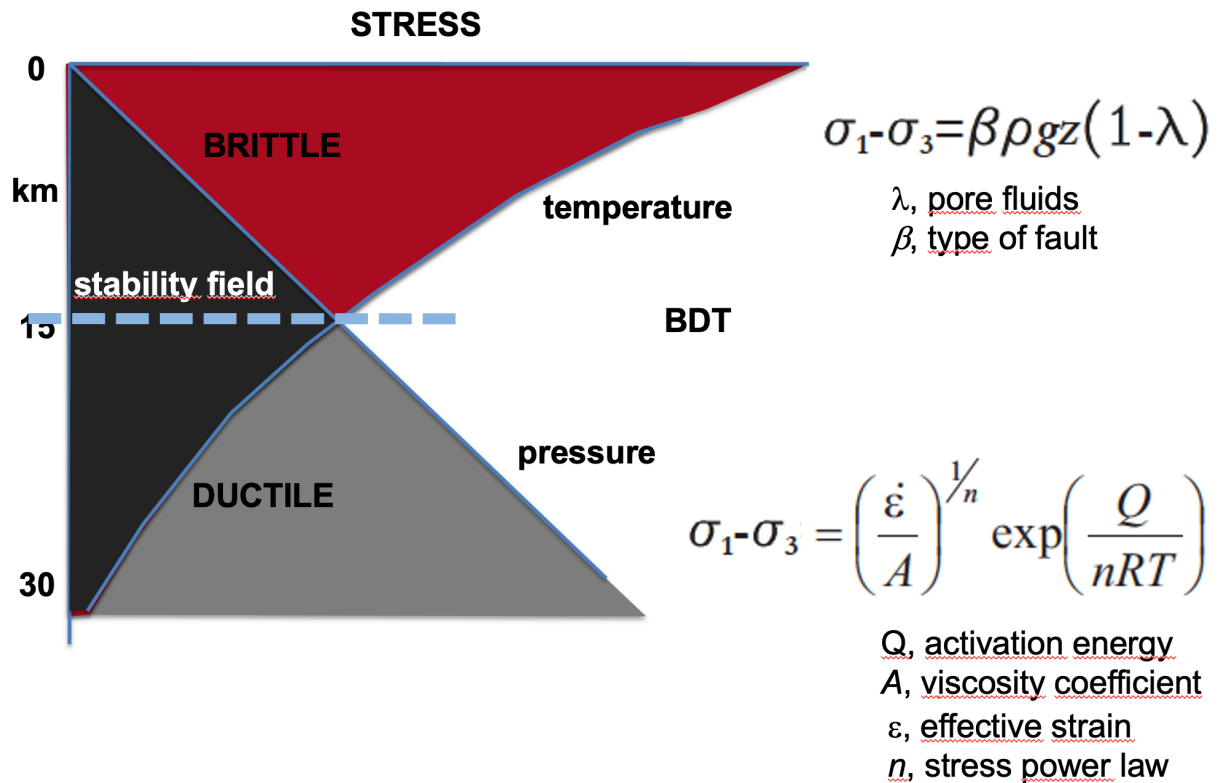


Profilo reologico



$\beta=3$ thrust, 1.2 strike-slip, 0.75 normal fault

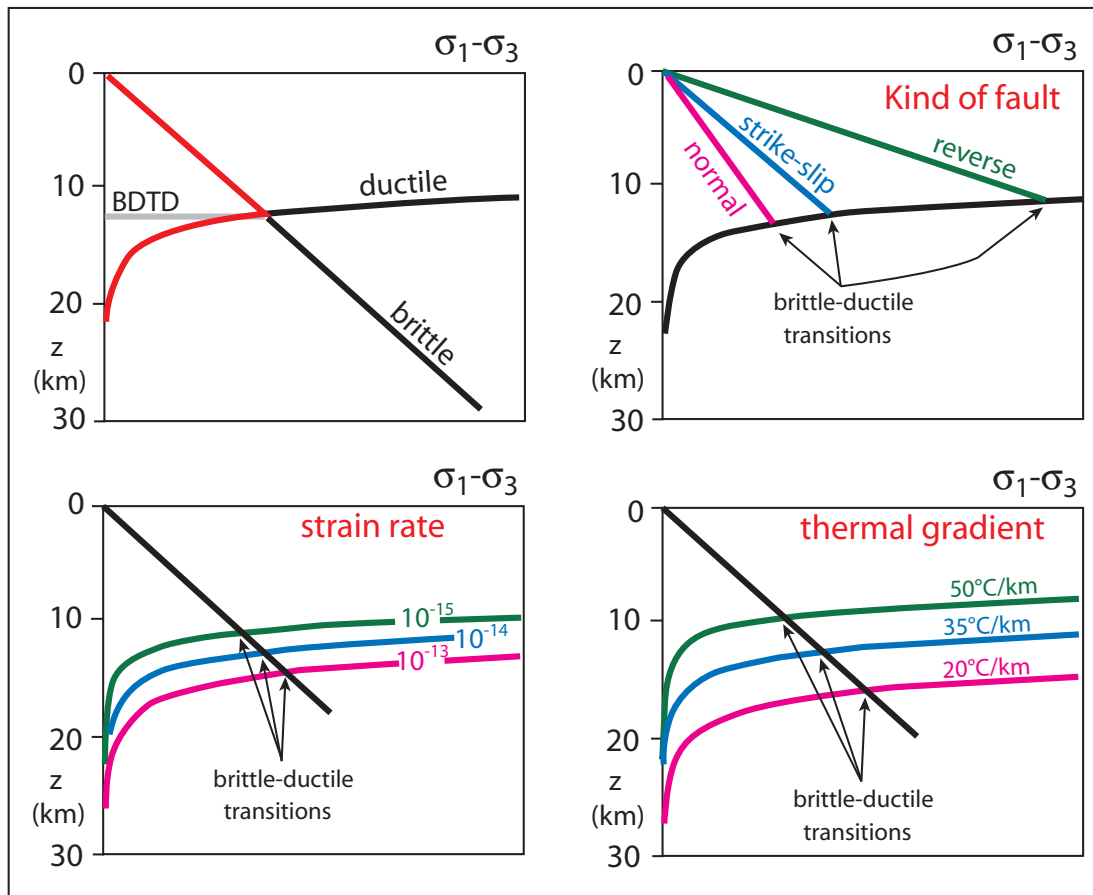


Fig. 3 - The brittle-ductile transition depth (BDTD) is strongly dependent on the kind of structural regime active in a given region, on the strain rate and on the thermal gradient. In this figure, the sensitivity of the BDTD on these parameters is schematically shown. In all the panels the lithology is kept constant. Areas characterised by thrust faulting are characterised by larger brittle strength with respect to areas characterised by strike-slip and normal faulting. As a consequence, keeping constant the other controlling parameters, the brittle-ductile transition depth in contractional areas will be shallower than in strike-slip (intermediate depth) and extensional (deeper depth) areas. Strain rate strongly controls the ductile strength. Typical strain rates in deforming areas vary between 10^{-15} s^{-1} (slow deformation) and 10^{-13} s^{-1} (fast deformation). Increasing strain rates induce higher ductile rock strengths. This is mainly due to the fact that, for fast deformations, migration of lattice defects, recrystallization and other plastic deformation mechanisms are not capable to keep the pace with deformation. Keeping constant the other controlling parameters, areas characterised by faster deformations will be characterised by deeper brittle-ductile transition depths. The thermal gradient of an area is controlled by the regional mantle heat flow, by the crustal radiogenic heat production (that is in turn controlled by crustal lithologies) and by local effects (e.g., magmatic intrusions, heat production generated along major regional faults). An area characterised by larger thermal gradient will be characterised, at a fixed depth, by higher temperatures with respect to areas characterised by low thermal gradients. Since ductile strength is inversely proportional to temperature, at the same depth areas characterised by high gradients will also be characterised by low ductile strength. Therefore, keeping constant the other controlling parameters, larger thermal gradients will be associated to shallower brittle-ductile transition depths.

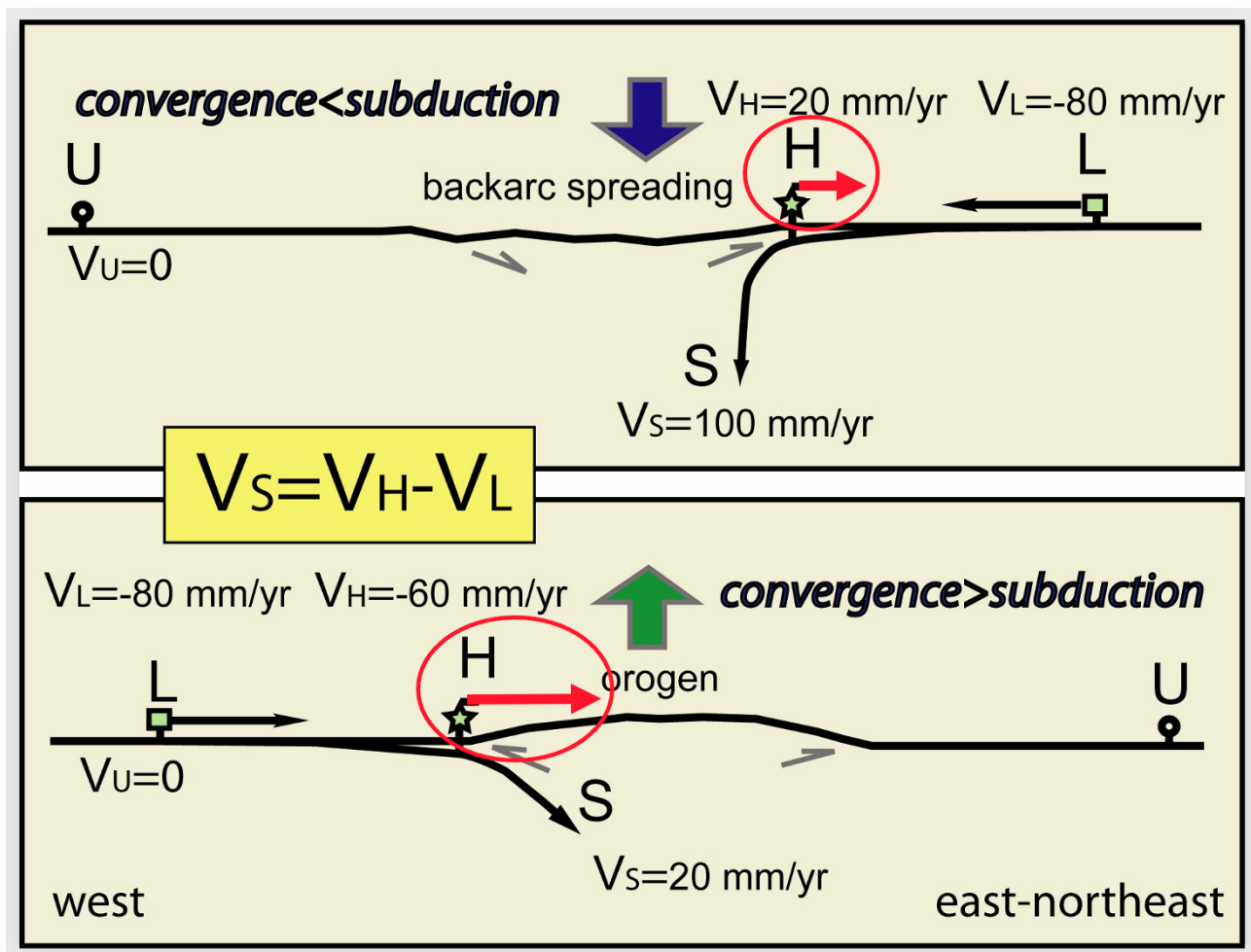
Velocità subduzione placche

$$V_s = V_h - V_L$$

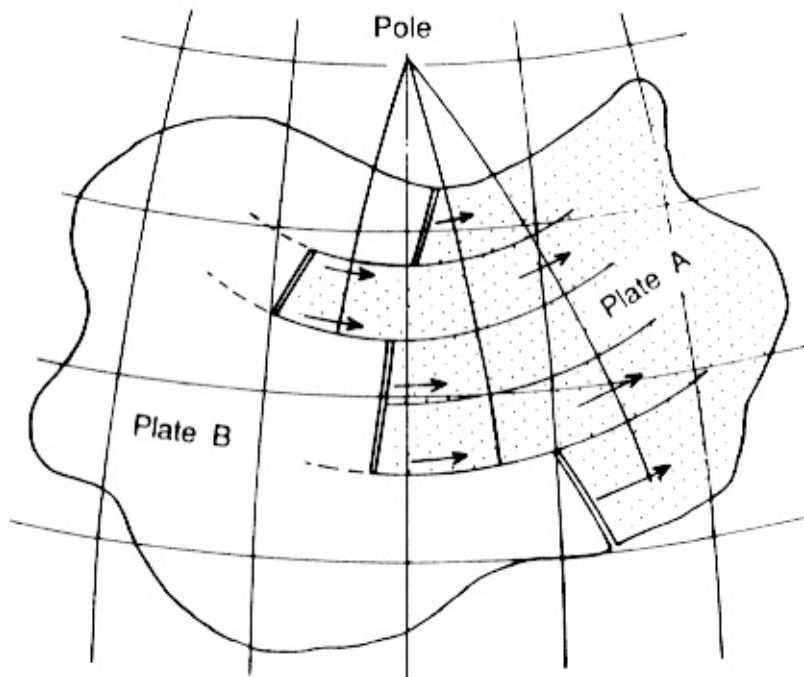
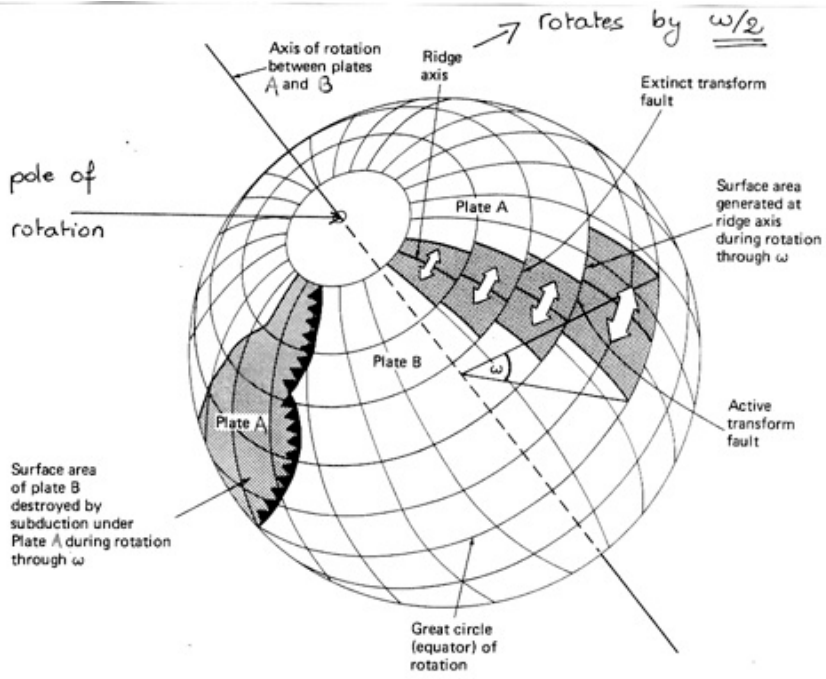
V_s velocità subduzione

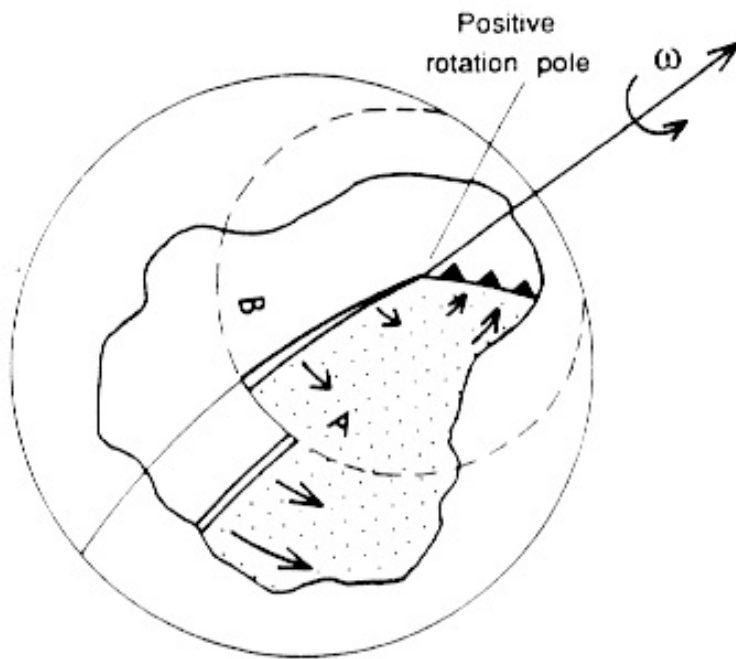
V_h velocità cerniera rispetto placca a tetto

V_L velocità placca a letto rispetto placca a tetto



Eulero





$$v = \omega R \sin \theta$$

$$v = \omega R \sin \phi \quad (1.1)$$

where v is the linear velocity, ω is the angular velocity rate, R is the Earth's radius, and ϕ is the angular distance between the Euler pole and a single plate point.

The factor of $\sin \phi$ implies that, for example, along a boundary the linear velocity v change with the positions from a minimum velocity $v = 0$, when $\phi = 0, \pi$, to a maximum value $v = \omega R$, when $\phi = \frac{\pi}{2}$ from the rotation pole.

Gutenberg-Richter

$$N=10^{a-bM}$$

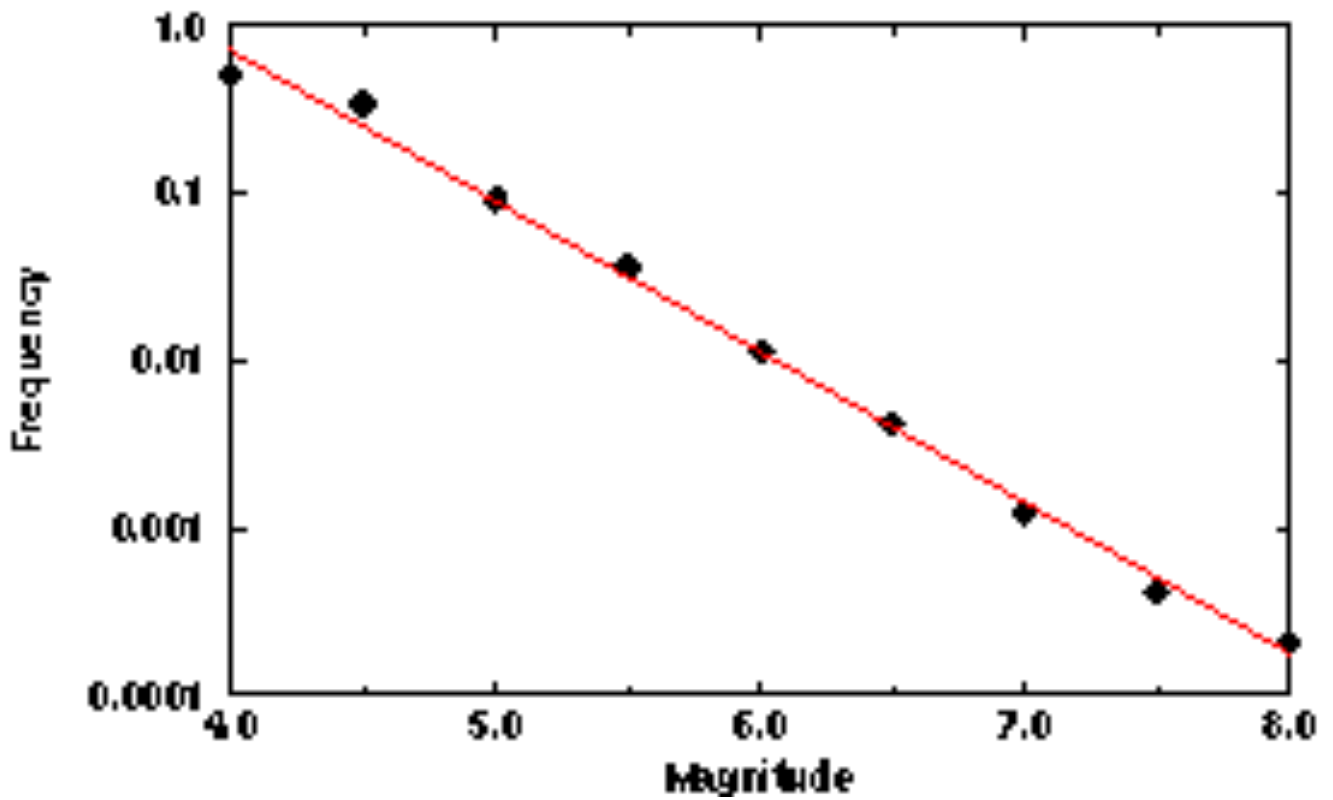
o

$$\log_{10}N=a-bM$$

N numero di terremoti con magnitudo uguale o >M

a numero totale di eventi in una data area

b circa uguale a 1 (maggiore per faglie normali 1.1 e minore per sovrascorrimenti 0.9)



Sclater Parson

Nei primi 10 milioni d'anni (Ma) il fondo marino allontanandosi dalla dorsale ha una subsidenza di circa 1000 m; nei successivi 26 Ma ha una subsidenza di altri 1000 m. Questa variazione è descritta dalla semplice formula

$$z = k\sqrt{Myears}$$

dove z è la profondità del fondo marino al di sotto della dorsale, k è una costante di circa 320, e anni è l'età della crosta oceanica in milioni di anni. Questa importante relazione è anche detta curva di Sclater, e permette di calcolare la profondità del mare al di sotto di una dorsale fino all'età di circa 60-80 milioni d'anni. Oltre questa età i fondali marini non sembrano più approfondirsi per effetto termico.

$$3 \times 10^7 \text{ s} = 1 \text{ anno}$$

diffusion creep migrazione difetti puntuali processi lineari
dislocation creep dislocazioni nel reticolo cristallino processi
non lineari

$$\text{viscosità} = 1/2 \times \text{stress/strain rate Pa s}$$

Modelli post glacial rebound basati su comportamento lineare

Shear Heating
Simple shear
Pure shear

Momento sismico

$$M_0 = \mu A u$$

Where μ is the **shear modulus** of the rocks involved in the earthquake, typically 30 **gigapascals**

A is the area of the rupture along the **geologic fault** where the earthquake occurred,

and u is the average displacement along the fault

The moment magnitude M_w is a **dimensionless number** defined by

$$M_w = \frac{2}{3} \left(\log_{10} \frac{M_0}{\text{N} \cdot \text{m}} - 9.1 \right) = \frac{2}{3} \left(\log_{10} \frac{M_0}{\text{dyn} \cdot \text{cm}} - 16.1 \right),$$

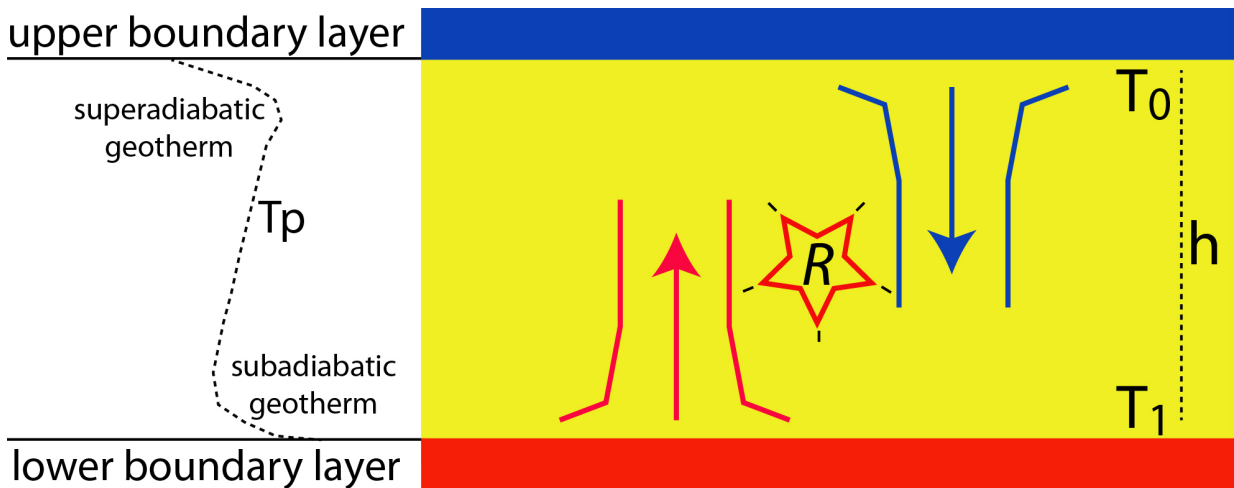
Rayleigh - Convection

Un livello di spessore h con temperature costanti T_0 e T_1 sopra e sotto il livello ha un numero Ra

$$\rho^2 g c_p \alpha (T_1 - T_0) h^3$$

$$Ra = \frac{\rho^2 g c_p \alpha (T_1 - T_0) h^3}{\eta \kappa}$$

$\mu \kappa$



$$Ra = \frac{\rho^2 g \alpha (T_1 - T_0) h^3}{\eta \kappa} \cdot \frac{Q h + R h^2}{\kappa}$$

basal heat

internal heat

- Ra Rayleigh number
- ρ density
- g gravity
- α thermal expansion
- T temperature
- h thickness
- η viscosity
- κ conductivity
- Q heat flow at the base
- R internal radiogenic heat

Al numeratore vi sono elementi che favoriscono il galleggiamento, mentre al denominatore sono i fattori viscosi che lo rallentano, cioè la crescita dei parametri al numeratore favorisce la convezione, mentre la crescita dei parametri al denominatore favorisce la conduzione. Quindi un alto numero di Rayleigh favorisce la convezione, un basso valore indica che la conduzione è dominante. La transizione tra i due regimi è detta numero critico di Rayleigh.

The notation we will use is as follows:

Property Symbol SI Units

stress Pascals, Pa

strain dimensionless

strain rate @

@t s⁻¹

viscosity Pas

rigidity μ Pa

Young's modulus E Pa

1

Property	Symbol	SI Units
stress	σ	Pascals, Pa
strain	ϵ	dimensionless
strain rate	$\frac{\partial \epsilon}{\partial t}$	s ⁻¹
viscosity	η	Pa-s
rigidity	μ	Pa
Young's modulus	E	Pa

The *Young's modulus*, E and *rigidity*, μ , both describe how an elastic material deforms under the application of stress. You may sometimes see μ written as G and referred to as the *modulus of rigidity*. As we discussed in class, for a linear elastic material (one for which Hooke's Law holds) stress is directly proportional to strain so $\sigma \propto \epsilon$, and so we can write $\sigma = \text{constant} * \epsilon$.

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The strain in the y-direction is smaller than the strain in the x-direction by a factor ν , where ν is a quantity known as Poisson's ratio. The same is in fact true in the z-direction but this is harder to see because the elastic band is thin to start with in this direction. So in the x and z directions strain is related to stress by $\epsilon = -\nu * \sigma / E$, and the constant of proportionality in Hooke's Law is now $-E/\nu$. The minus sign is there because the deformation in the y and z directions has the opposite sense from that in the x-direction. For the elastic band, the band extends in the x-direction and contracts in the y- and z-directions.

Physically, Poisson's ratio tells us (indirectly) about the *compressibility* of the material. For an incompressible material – one which cannot undergo a net change in volume $\nu = 0.5$ (see T&S, p. 106-107). This is the case for fluids, and so in the Earth's outer core, $\nu = 0.5$. Poisson's ratio of most rocks varies between 0.1 and 0.4, and in geophysics and planetary science we often take a value of 0.25 as representative of crustal or mantle rocks.

Rigidity, μ is the *resistance of an elastic material to shear stress*. μ , ν and E are not all independent quantities – if we know two of them we can calculate the third. The shear modulus is related to Young's modulus and Poisson's ratio by $\mu = \frac{E}{2(1+\nu)}$.

One can see that for our nominal mantle material ($\nu = 0.25$), the rigidity and Young's modulus are related by $\mu = 0.4 * E$. Typical values for E are 10-100 GPa.

The *compressibility*, β , of a material describes the fractional change in volume, Δ , in response to an isotropic stress or pressure, p through $\Delta = \beta p$. Clearly, an incompressible material has $\beta = 0$. The *bulk modulus*, K is defined as the reciprocal of the compressibility, $K = \frac{1}{\beta}$. See T&S section 3-8. Thus, as a material becomes more incompressible, its bulk modulus tends to infinity.

Seismic waves

P-wave velocity is given by: $v_p = \sqrt{\frac{(K + \frac{4\mu}{3})}{\rho}}$

S-wave velocity is given by: $v_s = \sqrt{\frac{\mu}{\rho}}$

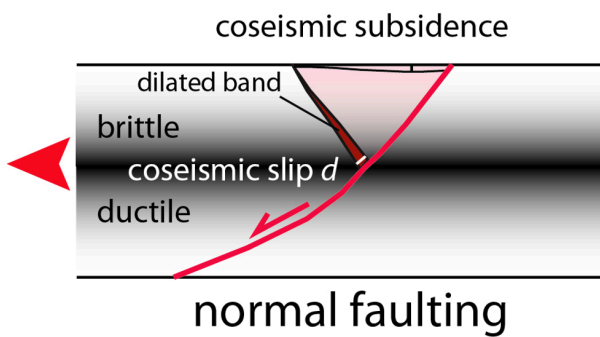
where K is the bulk modulus and μ is the shear modulus as defined above. SO: you can see that shear waves do not propagate through liquids because μ , the shear modulus is zero. However the bulk modulus is non-zero and so P-waves can propagate through liquids. You can also see that for a given region of the crust or mantle (solid) the P-wave speed is always greater than the S-wave speed, explaining why P-waves are the first arrivals on seismograms, followed by S-waves. **See also** page 14 of the pdf on line for Lecture

3:2 Viscosity, η , μ , and ν !

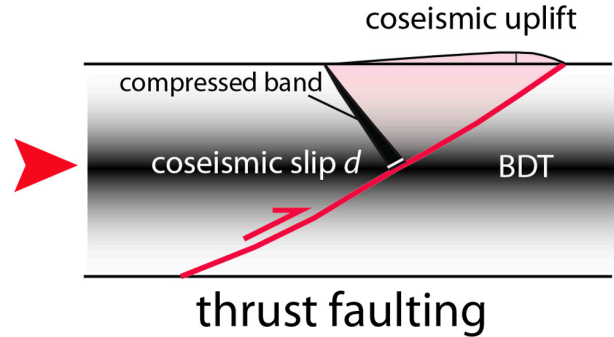
Viscosity η describes the *resistance of a fluid to shear stress*. It is sometimes referred to as the *dynamic viscosity*. Confusingly, sometimes in fluid dynamics a quantity known as the *kinematic viscosity* is used. To add to confusion, the dynamic viscosity is sometimes given the symbol μ , (NOT rigidity in this context!), and the kinematic viscosity is given the symbol ν (NOT Poisson's ratio in this context!). The kinematic viscosity is related to the viscosity, η simply by dividing η by the density of the material, so $\nu = \frac{\eta}{\rho}$. ν behaves like a diffusivity - it describes how momentum diffuses in a moving liquid (just as the thermal diffusivity describes how heat diffuses).

EARTHQUAKE ENERGY

GRAVIQUAKE



ELASTOQUAKE



$$E = mgh(\mu_s, \Theta)$$

$$E = \frac{1}{2} K \left(\frac{\Delta V (\mu_s, \Theta)}{V} \right)^2 \frac{m}{\rho}$$

Ridge push

L'elevazione di una dorsale oceanica causa un aumento dell'energia potenziale gravitazionale, che viene chiamata ridge push o spinta della dorsale. Non è legata all'inserimento del magma lungo la dorsale, ma viene considerato solo il peso in più determinato dalla maggiore elevazione della dorsale. Una possibile quantificazione del ridge push può essere la seguente:

$$F_{ridge\ push} = g\rho_m \int h dx - g\rho_w \int w dx$$

Dove g è l'accelerazione di gravità, ρ_m è la densità del mantello, h è l'elevazione della dorsale rispetto al fondo marino, x è la larghezza in orizzontale dell'area in eccesso data dalla dorsale, e ρ_w è la densità dell'acqua. Il ridge push ottenuto considerando anche l'effetto del raffreddamento della litosfera e il peso dell'acqua è di circa $3,9 \times 10^{12} \text{ Nm}^{-1}$ (Turcotte e Schubert, 2002).

Slab pull

Lo slab pull, o tiro della subduzione è la forza che è stata interpretata per la minore temperatura dello slab in subduzione rispetto al mantello circostante più caldo. I basalti oceanici, andando in subduzione si possono trasformare in eclogiti, rocce di altissima pressione e alta densità, generando un gradiente negativo di densità del piano di subduzione rispetto al mantello superiore che lo contiene. L'espressione più semplice per il tiro dello slab è la seguente:

$$F_{slab\ pull} = g dz (\rho_l - \rho_m)$$

dove $F_{slab\ pull}$ è il tiro verso il basso supposto per il maggior peso della litosfera più fredda, assumendo che litosfera e mantello abbiano la stessa

composizione, e vi sia solo una stratificazione termica; g è l'accelerazione di gravità; d è lo spessore della litosfera in subduzione; z è la profondità del piano di subduzione; ρ_l è la densità della litosfera e ρ_m è la densità del mantello. Assumendo valori come 10 per g , 100 km per d , 660 km la profondità dello slab z , e 3300 kg/m^3 e 3220 kg/m^3 rispettivamente le densità della litosfera e del mantello si ottiene uno slab pull di circa $5,2 \times 10^{13} \text{ Nm}^{-1}$. Tuttavia lo spessore della litosfera in subduzione è sovente più sottile, e soprattutto il mantello superiore ha densità ben superiori ai 3220 kg/m^3 , anche perché molto probabilmente il mantello superiore ha stratificazioni chimico-mineralogiche con un graduale aumento della densità dall'alto verso il basso. Turcotte e Schubert (2002) calcolano uno slab pull di circa $3,3 \times 10^{13} \text{ Nm}^{-1}$. Il PREM (Preliminary Reference Earth Model, Anderson, 1989) propone per esempio una densità di 3970 kg/m^3 per il mantello a 600 km di profondità.

Post Glacial Rebound:

Displacement is described by:

$$w = w_m e^{-t/tr}$$

Where:

w_m = initial displacement of surface

w = current displacement

t = time

tr = relaxation time

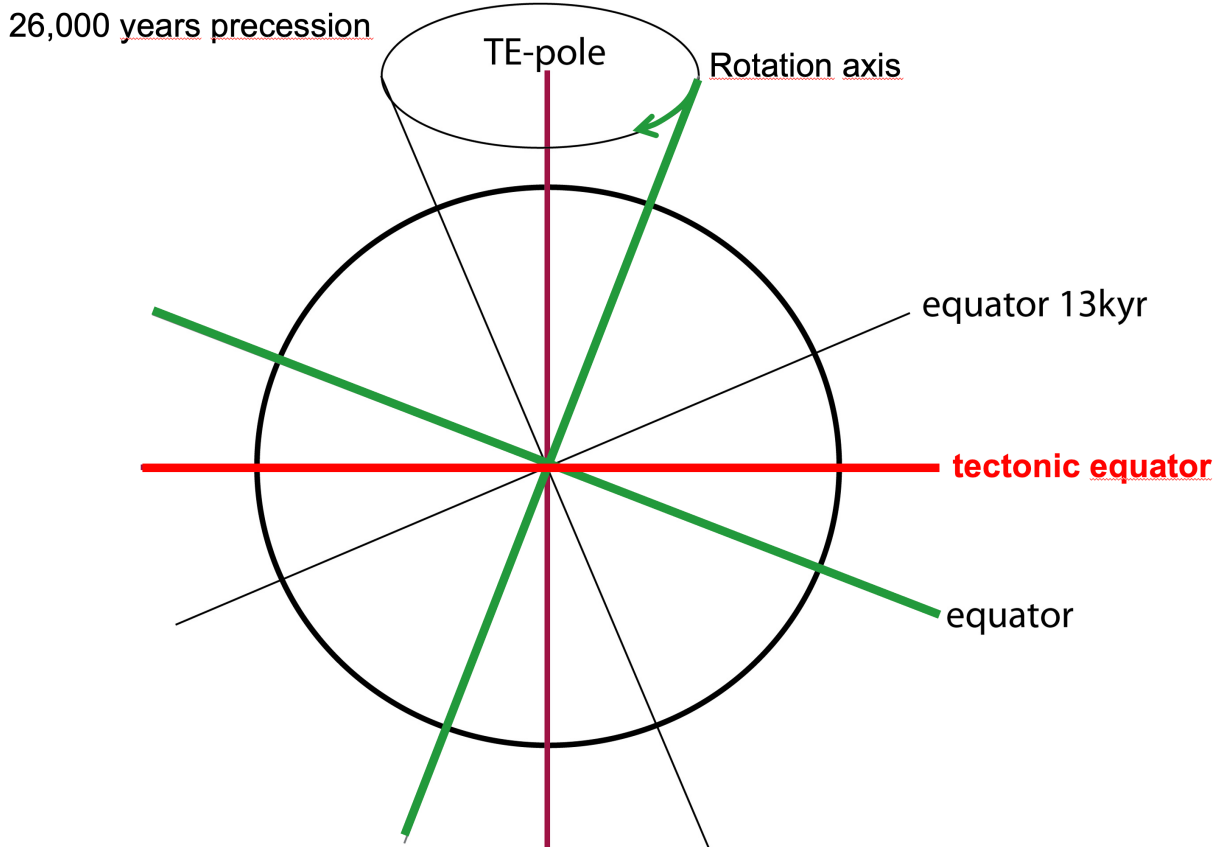
$$tr = 4\pi\mu/\rho g\lambda$$

μ = viscosity

λ = wavelength

ρ = density

Tectonic equator – Maxwell Time



$$\text{Maxwell relaxation time} = \frac{\text{viscosity}}{\text{rigidity}} = \frac{10^{22} \text{ Pa s}}{10^{11} \text{ Pa}} = 10^{11} \text{ s}$$