Estremo superiore ed estremo inferiore - 2022/09/30

Dire quali dei seguenti sottoinsiemi di $\mathbb{R}$ sono limitati (superiormente, inferiormente) e determinare, se esistono, i loro massimi, minimi, estremi superiori e inferiori (verificando con la caratterizzazione di inf/sup)

$$A = \{x \in \mathbb{R} : 4 < x^2 \leq 9\} \qquad \left[\begin{array}{l} \sup A = \max A = 3 \\ \inf A = \min A = -3 \end{array}\right]$$

$$B = \{x \in \mathbb{R} : 4 \leq x^2 < 9\} \qquad \left[\begin{array}{l} \sup B = 3 \\ \inf B = -3 \end{array}\right]$$

$$C = \left\{\frac{1}{n+3}, \; n \in \mathbb{N}\right\} \qquad \left[\begin{array}{l} \sup C = \max C = \frac{1}{3} \\ \inf C = 0 \end{array}\right]$$

$$D = \{p^3 : p \in \mathbb{Z}\} \qquad \left[\begin{array}{l} \sup D = +\infty \\ \inf D = -\infty \end{array}\right]$$

$$E = \left\{\frac{4n-2}{n+1} : n \in \mathbb{N}\right\} \qquad \left[\begin{array}{l} \sup E = 4 \\ \inf E = \min E = -2 \end{array}\right]$$

$$F = \left\{\frac{5}{2n-1} : n \in \mathbb{Z}\right\} \qquad \left[\begin{array}{l} \sup F = \max F = 5 \\ \inf F = \min F = -5 \end{array}\right]$$

$$G = \{\sqrt{n^2+3} : n \in \mathbb{N}\} \qquad \left[\begin{array}{l} \sup G = +\infty \\ \inf G = \min G = \sqrt{3} \end{array}\right]$$

$$H = \left\{(-1)^n - \frac{1}{n+1} : n \in \mathbb{N}\right\} \qquad \left[\begin{array}{l} \sup H = 1 \\ \inf H = \min H = -\frac{3}{2} \end{array}\right]$$

$$I = \left\{\frac{n + (-1)^n (n+1)}{n} : n = 1, 2, 3, \ldots\right\}$$

$$\left[\begin{array}{l} \sup I = \max I = \frac{5}{2} \\ \inf I = \min I = -1 \end{array}\right]$$

$$J = \left\{ (-1)^n \frac{2n+1}{n} : n = 1, 2, \dots \right\} \quad \left[\begin{array}{l} \sup J = \max J = \frac{5}{2} \\ \inf J = \min J = -3 \end{array} \right]$$

$$K = \left\{ \frac{n}{n^2+20} : n = 1, 2, \dots \right\} \quad \left[\begin{array}{l} \sup K = \max K = \frac{1}{9} \\ \inf K = 0 \end{array} \right]$$

$$L = \left\{ \frac{n^2+5n+1}{n^2} : n = 1, 2, \dots \right\} \quad \left[\begin{array}{l} \sup L = \max L = 7 \\ \inf L = 1 \end{array} \right]$$

$$M = \left\{ \sqrt{1 - \frac{1}{2n+5}} : n \in \mathbb{N} \right\} \quad \left[\begin{array}{l} \sup M = 1 \\ \inf M = \min M = \frac{2}{\sqrt{5}} \end{array} \right]$$

$$N = \left\{ n - \frac{1}{n} : n = 1, 2, \dots \right\} \quad \left[\begin{array}{l} \sup N = +\infty \\ \inf N = \min N = 0 \end{array} \right]$$

$$O = \left\{ x \in \mathbb{R} : \sqrt{x^2+2} \leq x+3 \right\} \quad \left[\begin{array}{l} \sup O = +\infty \\ \inf O = \min O = -\frac{7}{6} \end{array} \right]$$

$$(*) \quad P = \left\{ \frac{1}{m^2} - \frac{3}{n} : m, n = 1, 2, 3, \dots \right\} \quad \left[\begin{array}{l} \sup P = 1 \\ \inf P = -3 \end{array} \right]$$