SAPIENZA
UNIVERSITÀ DI ROMA

Environmental geophysics

Giorgio De Donno

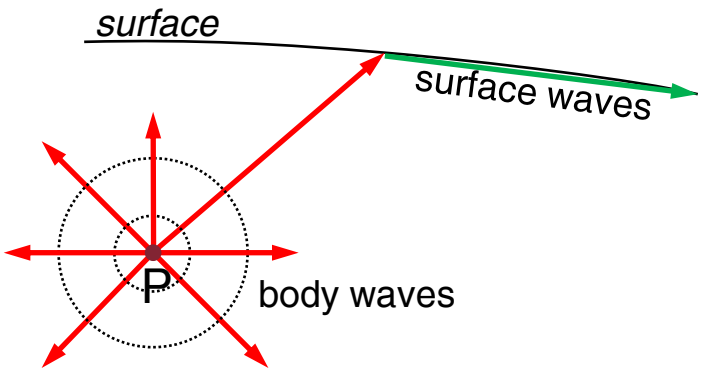
1. Seismic methods

Basic principles

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Seismic waves



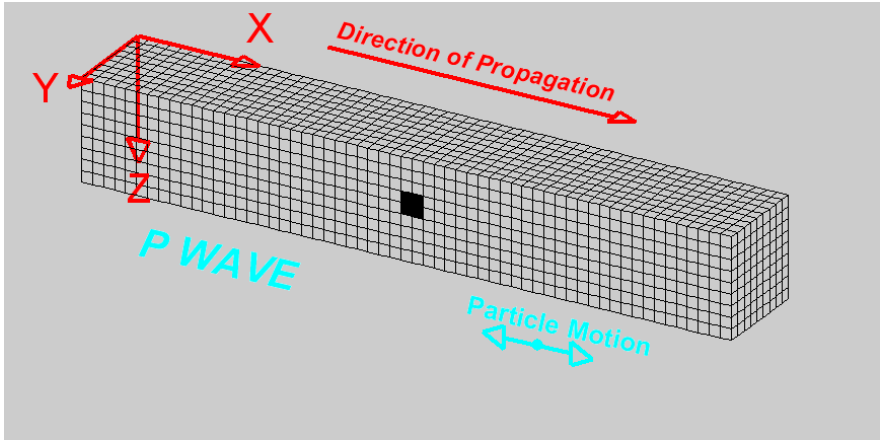
When a stress is applied (or released) on a point source P the corresponding strain propagates out of the source both within the body (**body waves**) and on surface (**surface waves**).

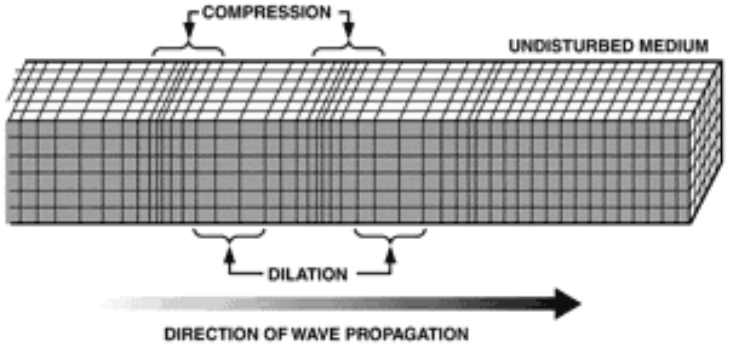
The body waves can be divided into **primary** and **secondary** waves, while the most studied surface waves are the **Rayleigh** and **Love** waves.



Body waves - P-wave

- The **wave** propagates **parallel** to the direction of **particle motion**.
- Particle motion consists of alternating **compression and dilation** (extension).
- Material returns to its original shape after the wave passes.

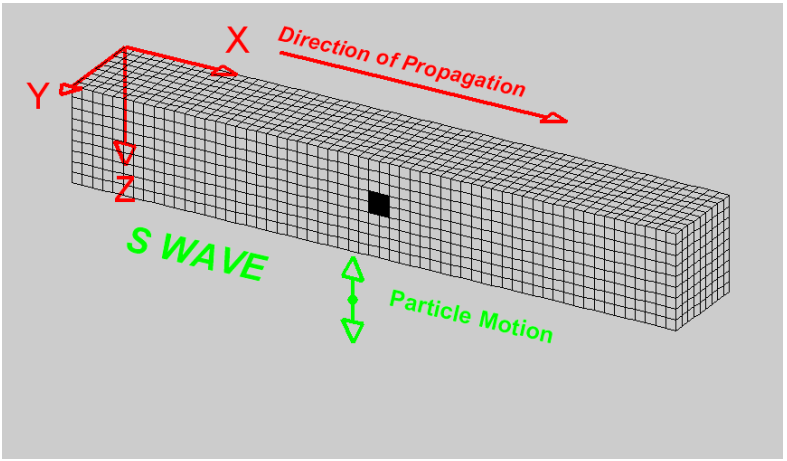




This wave is called **compressional, longitudinal** or **primary** (**P**), because it represents the first arrival

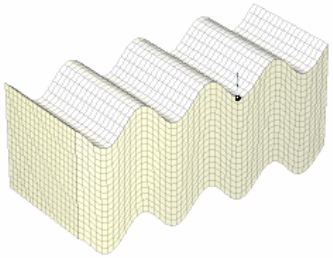
Body waves - S-wave

- The **wave** propagates **normal** to the direction of **particle motion**.
- Particle motion consists of alternating **transverse** motion.
- Material returns to its original shape after the wave passes.

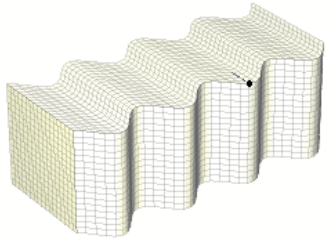


Transverse motion can be in the vertical (z) direction (**SV wave**) or in the horizontal (y) direction (**SH wave**)

SV wave



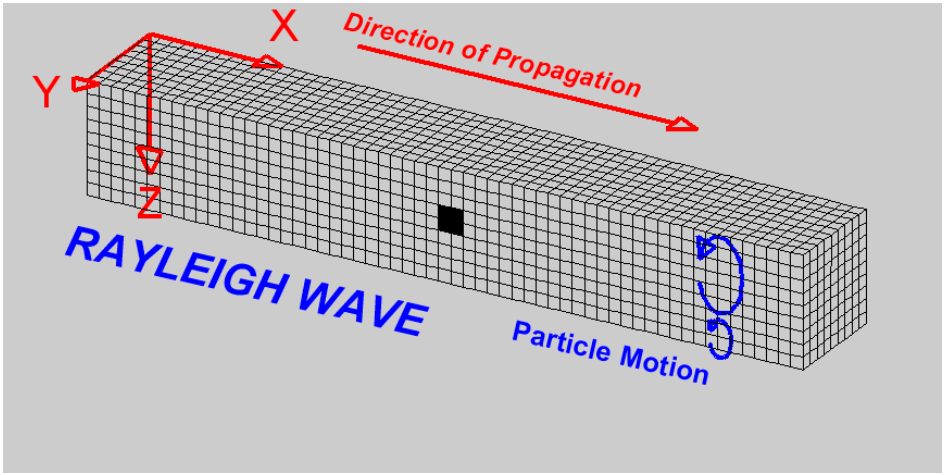
SH wave

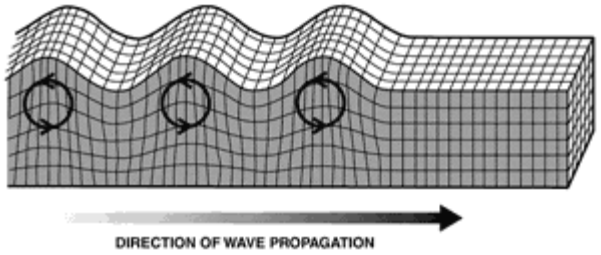


This wave is called **shear, transverse** or **secondary (S)**, because it represents the second arrival.

Surface waves – Rayleigh wave

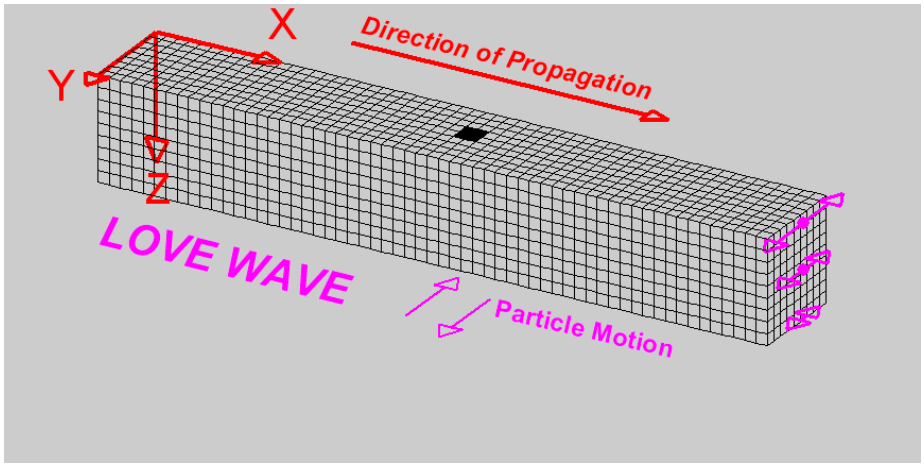
- Particle motion consists of **retrograde elliptical motions** in the **vertical plane** and **parallel to the direction of propagation**.
- Amplitude decreases with depth
- Material returns to its original shape after the wave passes.

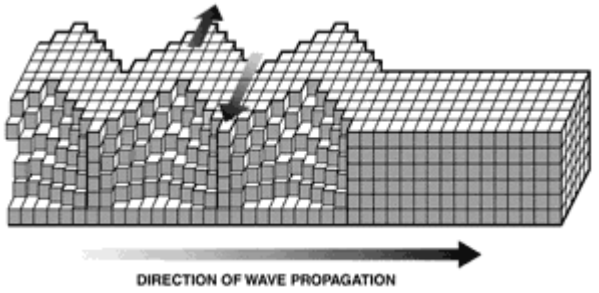




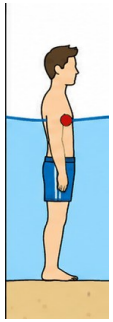
Surface waves - Love wave

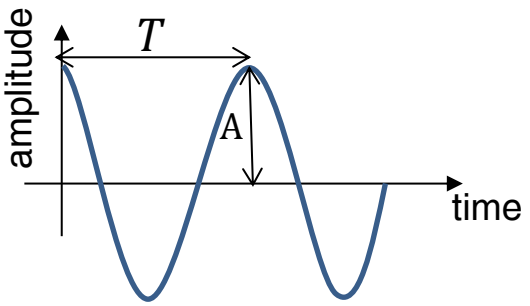
- Particle motion consists of **alternating transverse motions**, horizontal and **perpendicular to the direction of propagation** (transverse).
- Amplitude decreases with depth.
- Material returns to its original shape after the wave passes.

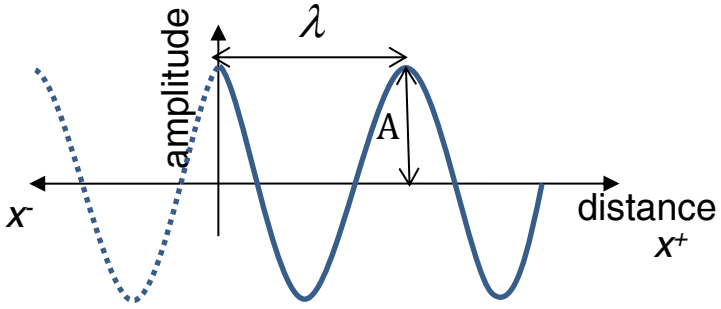


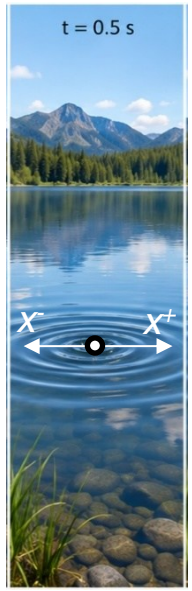


Wave parameters







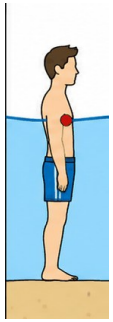


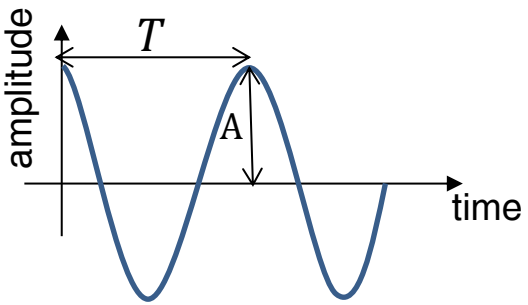
- **period** (*T*): duration of an oscillation cycle [s]
- **wavelength** (λ): distance of an oscillation cycle [m]
- **peak amplitude** (*A*): maximum absolute value of signal

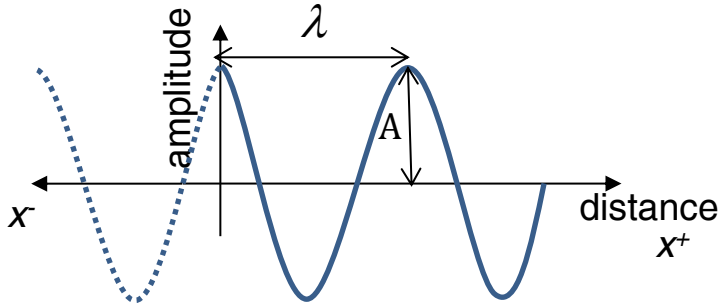
From these, we can derive:

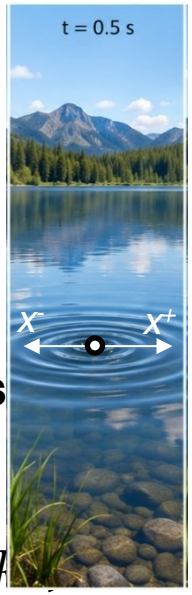
- **frequency** (*f*): $f = \frac{1}{T}$ [Hz]
- **angular frequency** (ω): $\omega = \frac{2\pi}{T} = 2\pi f$ [rad/s]
- **velocity** (*v*): $v = \frac{\lambda}{T} = \lambda f$ [m/s]
- **wavenumber** (*k*): $k = \frac{2\pi}{\lambda} = \frac{\omega}{v}$ [rad/m]

Wave parameters









Monochromatic signal = signal with only one frequency = sinusoidal s

In this case the signal is:

$$u(t) = A\cos\left(\frac{2\pi}{T}t\right) = A\cos(\omega t) \qquad u(x) = A\cos\left(\pm\frac{2\pi}{\lambda}x\right) = A\cos(\pm k$$

That is, using complex notation:

$$u(t) = \Re e\{A\mathrm{e}^{i\omega t}\} \qquad u(t) = \Re e\{A\mathrm{e}^{\pm ikx}\}$$

Since the observed signal are real-valued (e.g. displacements), in the following we will use a complex notation neglecting the imaginary part.

For a wave propagating through space and time, the resulting signal is:

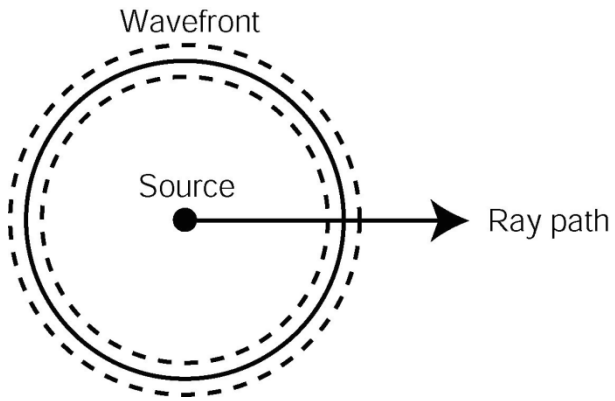
$$u(x,t) = A\mathrm{e}^{i(\omega t \pm kx)}$$

Propagation

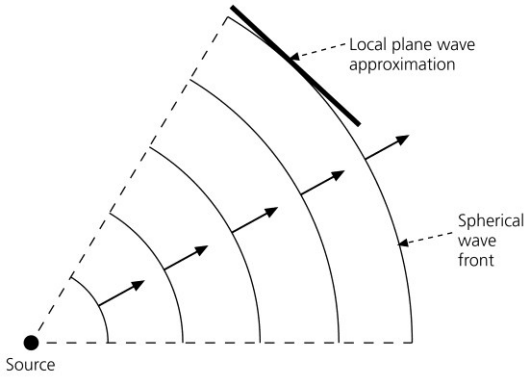
The **wavefront** is the surface where the elastic wave has a constant phase

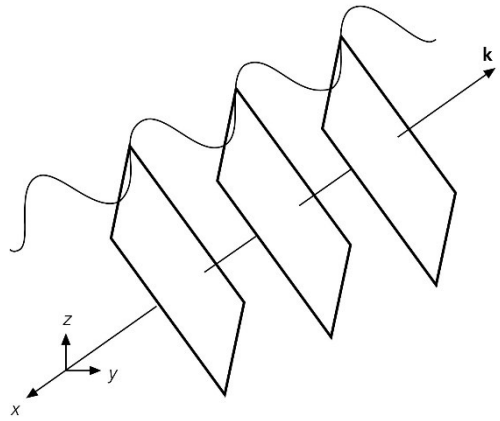
The **ray** is the path that one parcel of the wavefront travels along.

Spherical wavefront



Plane wavefront





These hypothesis hold for:

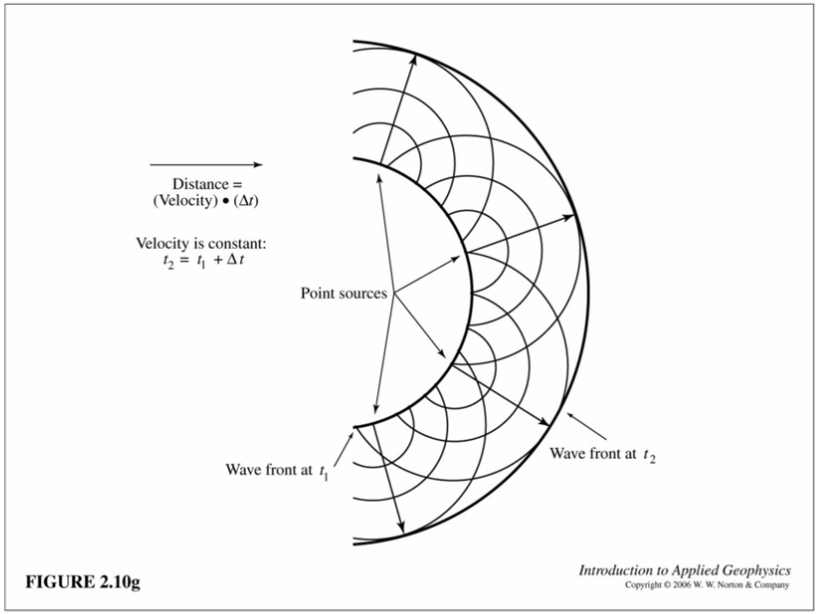
- Point source

- Point source, far enough from the source
- Plane source with stress distributed over a surface

Propagation

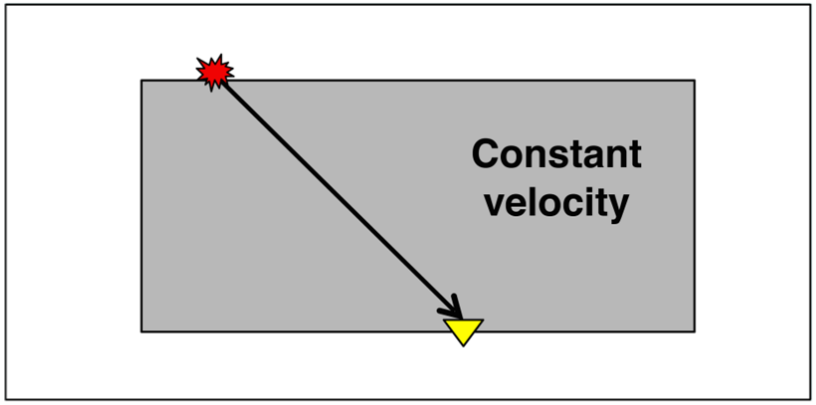
Huygens principle:

- each point on wavefront acts as a point source
- spherical waves radiate outward from each point source
- envelope of waves is the overall wavefront



Fermat's Principle:

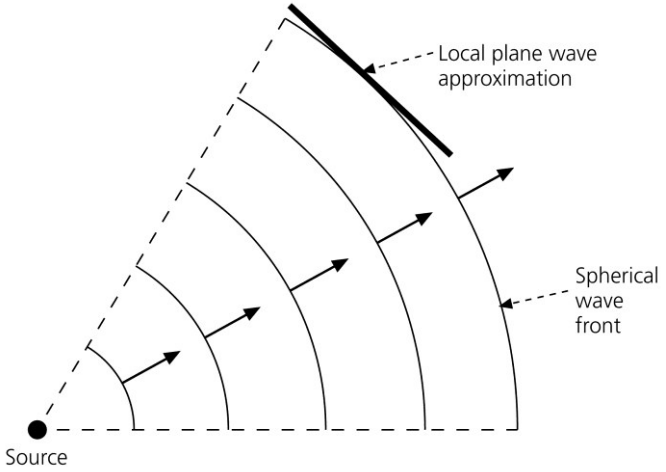
- rays propagate along the path which yields the smallest travel time (principle of least time)



Attenuation

Elastic wave fields lose energy through Earth due to:

• **geometrical spreading**: the wavefront must always contain a constant amount of energy E_0



The energy of a unit area of the growing wave front is equal to:

$$E(r) = \frac{E_0}{4\pi r^2}$$

surface of a sphere

Therefore, the energy decreases proportionally to r^2

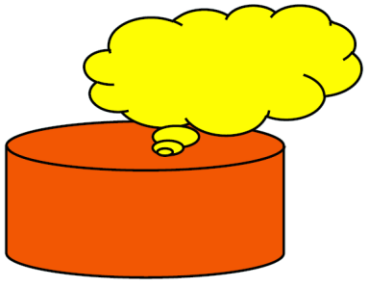
Q. What about the wave amplitude? It decrease proportionally to…

A. Proportionally to r! Remember that $E \propto a^2$

Attenuation

Elastic wave fields lose energy through Earth due to:

- **intrinsic attenuation:** energy loss due to shear heating at grain boundaries, mineral dislocations etc.

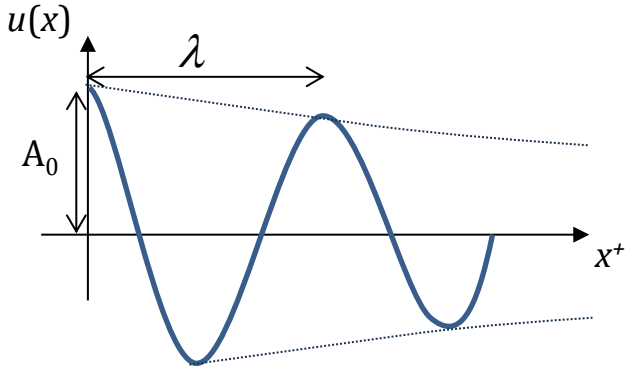


$$u(x,t) = A_0\, e^{-\alpha x}\, e^{i(\omega t - kx)} = \frac{A_0}{e^{\alpha x}}\, e^{i(\omega t - kx)}$$

For small dissipation (low losses):

$$\alpha \propto \frac{f}{Qv}$$

Attenuation coefficient

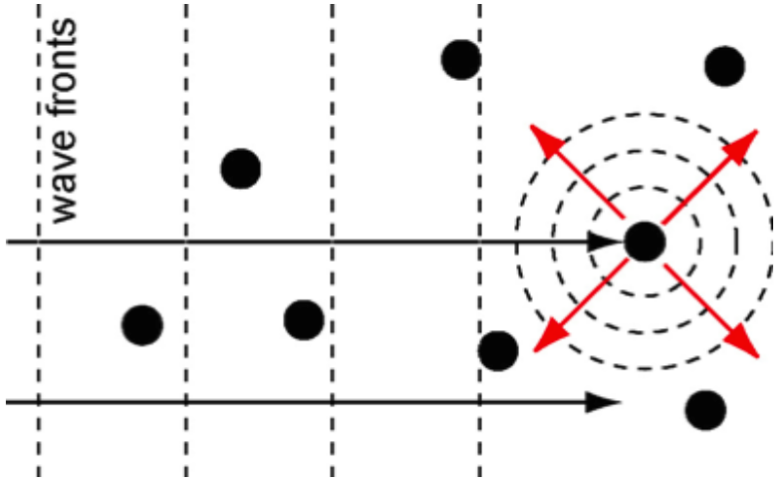


with Q (**quality factor**):

$$Q = Q(T, P, lithology, porosity, saturation, cracks)$$

Elastic wave fields lose energy through Earth due to:

• **scattering:** most materials contain small heterogeneities (grains, mineral boundaries, pore edges, cracks, etc.)



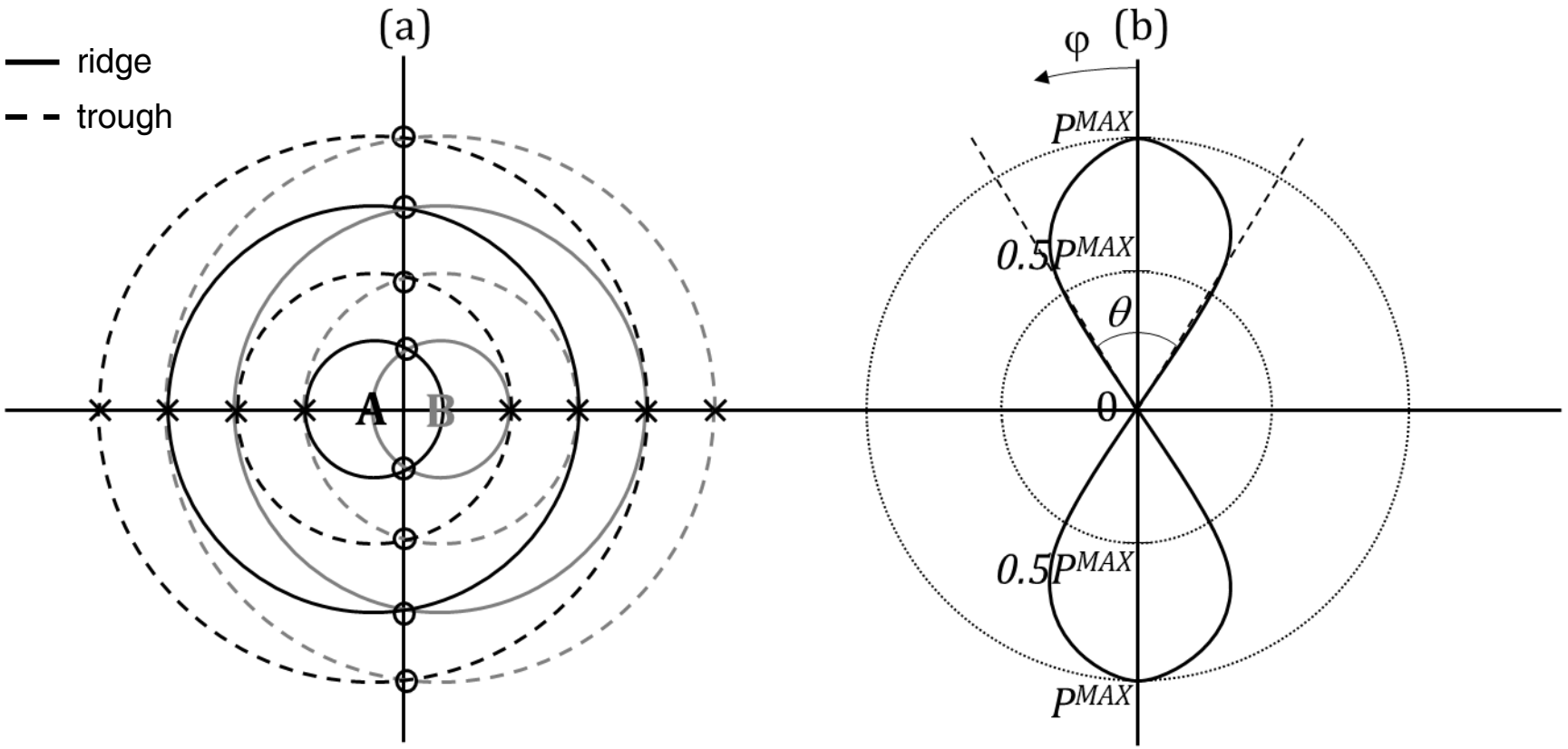
Some seismic energy is scattered when it encounters these features: according to Huygens' principles the small heterogeneities are new sources of waves.
This phenomenon depends on the ratio of the heterogeneity size d to the wavelength λ (it must be comparable)

$$\lambda \sim d$$

Interference

Waves can interact and the resulting wave is the effect of the superposition of two or more wave fronts

Interference
2 sources at $d=\lambda/2$



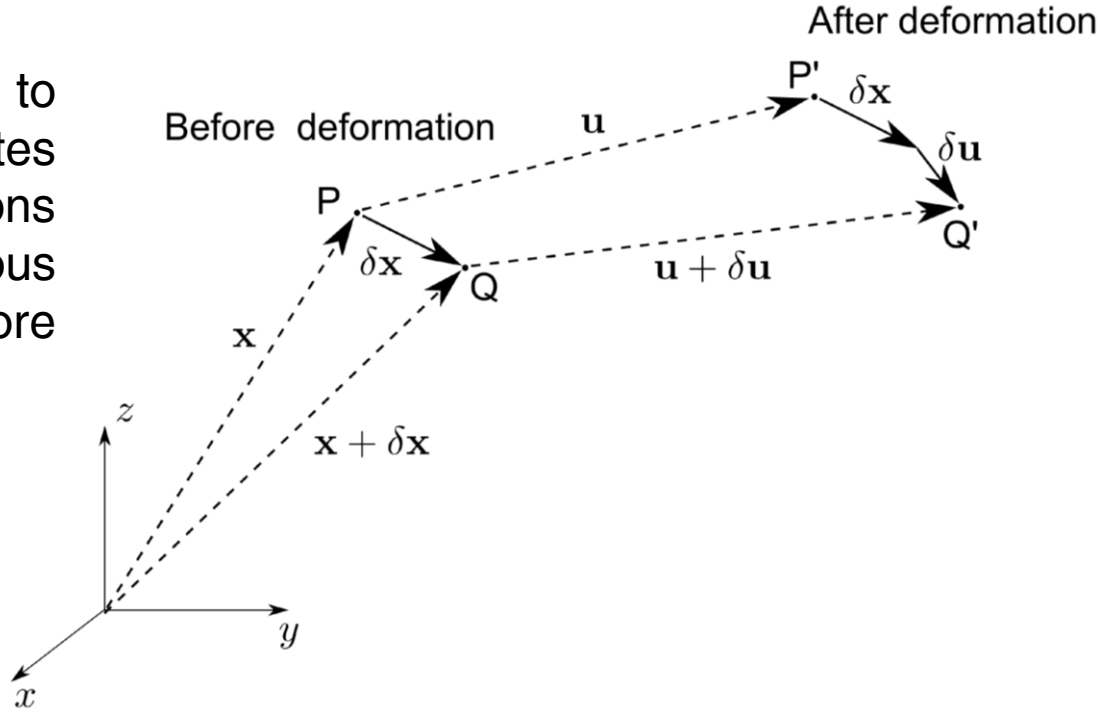
Radiation pattern

Deformation

Let us consider two material points P and Q belonging to a non-rigid continuum, which are located at a short distance from each other. Following a deformation, they are brought into positions P' and Q'.
Having defined the displacement vector from P to P' as $\mathbf{u} \equiv (u, v, w)$,

if the derivatives of $\mathbf{u}$ with respect to at least one of the three coordinates are non-zero, there will be variations in the displacement of the various points of the body, and therefore deformations $\boldsymbol{\delta u}$.

$$[\boldsymbol{\delta u}] = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix}$$



Deformation

We can decompose $[\boldsymbol{\delta u}]$ into a symmetric part $[\boldsymbol{\varepsilon}]$ and an antisymmetric part $[\boldsymbol{\Omega}]$:

$$[\boldsymbol{\delta u}] = [\boldsymbol{\varepsilon}] + [\boldsymbol{\Omega}],$$

where:

2D case

$$[\boldsymbol{\varepsilon}] = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) & \frac{\partial w}{\partial z} \end{bmatrix} \qquad [\boldsymbol{\Omega}] = \begin{bmatrix} 0 & \frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \\ -\frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) & 0 & \frac{1}{2}\left(\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\right) \\ -\frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) & -\frac{1}{2}\left(\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\right) & 0 \end{bmatrix}$$

The matrix $[\boldsymbol{\varepsilon}]$ represents the angular strains of the system, while $[\boldsymbol{\Omega}]$ represents the rigid rotations about the coordinate axes. Using a compact notation, the strain components of the matrix $[\boldsymbol{\varepsilon}]$ are written as:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right),$$

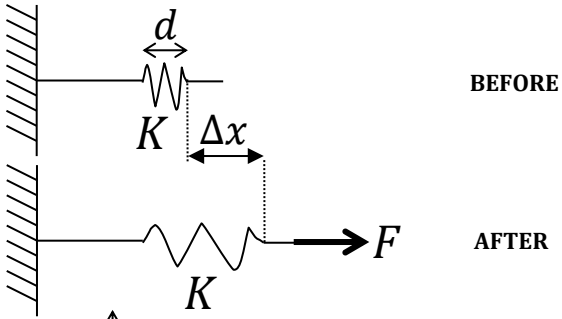
where the coordinates x, y, and z are associated respectively with 1, 2, and 3 in the permutation of the indices i and j from 1 to 3.

Linear elasticity



Seismic sources release energy as waves that causes a certain level of **stress** and a consequent **strain**, though not permanent for our low-energy applications (**elastic behaviour**)

$$\underset{\sigma}{\text{Stress}} \quad \begin{matrix} \varepsilon = f\,(\sigma) \\ \longleftrightarrow \\ \sigma = f^{-1}\,(\varepsilon) \end{matrix} \quad \underset{\varepsilon}{\text{Strain}}$$

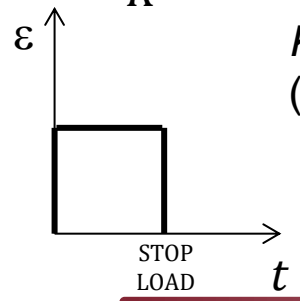


Hooke's Law: Strain is directly proportional to the stress (**linear elasticity**)

$$F = K\Delta x$$

$$F_x/A = K\,\Delta x/d$$

stress $\sigma_x = K\varepsilon_x$ strain

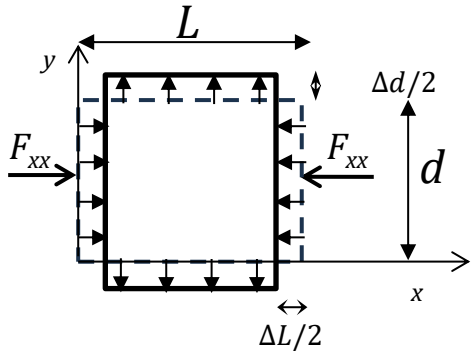


K is the spring constant (**material property**)

Elastic parameters

2D case

1. Longitudinal stress



E: Young's modulus [Pa]

$$E = \frac{F_{xx}/A}{\Delta L/L} = \frac{\sigma_{xx}}{\varepsilon_{xx}} = \frac{\sigma_{xx}}{\frac{\partial u}{\partial x}}$$

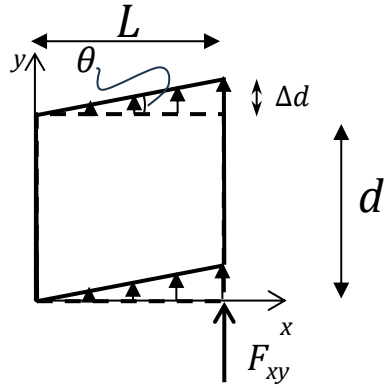
$$\varepsilon_{xx} = \frac{1}{E}\,\sigma_{xx}$$

ν: Poisson's ratio [dim-less]

$$\nu = -\frac{\Delta d/d}{\Delta L/L} = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}} = -\frac{\frac{\partial v}{\partial y}}{\frac{\partial u}{\partial x}}$$

$$\varepsilon_{yy} = -\frac{\nu}{E}\,\sigma_{xx}$$

2. Shear stress



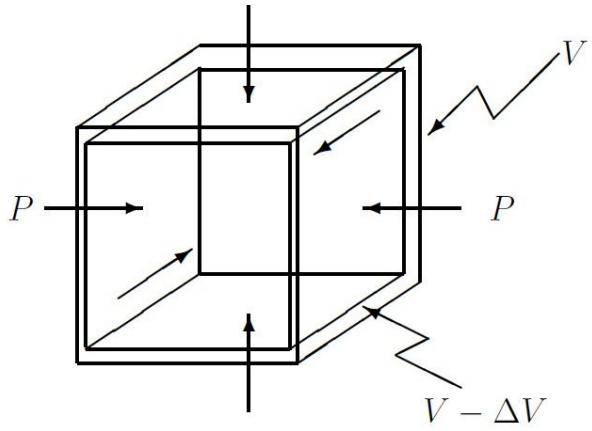
μ: Shear modulus [Pa]

$$\mu = G = \frac{F_{xy}/A}{\tan(\theta)} = \frac{F_{xy}/A}{\Delta d/L} = \frac{\sigma_{xy}}{\gamma_{xy}} = \frac{\sigma_{xy}}{\frac{\partial v}{\partial x}} = \frac{\sigma_{xy}}{2\varepsilon_{xy}}$$

$$\sigma_{xy} = 2G\,\varepsilon_{xy} = 2\mu\,\varepsilon_{xy}$$
$$\varepsilon_{xy} = \frac{1}{2G}\,\sigma_{xy} = \frac{1}{2\mu}\,\sigma_{xy}$$

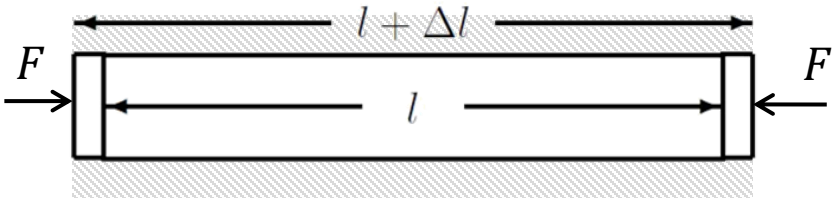
Elastic parameters

3. Volumetric (bulk stress)



$$K = \frac{vol.stress}{vol.strain} = \frac{P}{\Delta V/V}$$

4. Longitudinal stress with no lateral strain



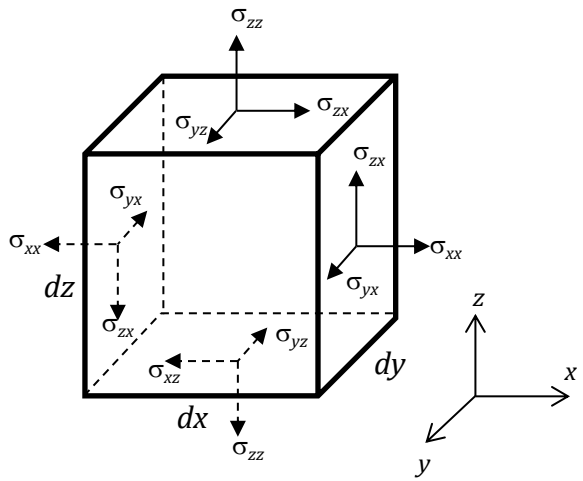
$$M = \frac{long.stress}{long.strain} = \frac{F/A}{\Delta l/l}$$

Bulk modulus [Pa]

$$M = K + \frac{4}{3}\mu$$

Longitudinal modulus [Pa]

Linear elastic homogeneous isotropic stress-strain relationship



$$\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \varepsilon_{xy}\\ \varepsilon_{xz}\\ \varepsilon_{yz}\end{bmatrix} = \begin{bmatrix} 1/E & -\nu/E & -\nu/E & & & \\ -\nu/E & 1/E & -\nu/E & & 0 & \\ -\nu/E & -\nu/E & 1/E & & & \\ & & & 1/2\mu & 0 & 0 \\ & 0 & & 0 & 1/2\mu & 0 \\ & & & 0 & 0 & 1/2\mu \end{bmatrix} \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{xy}\\ \sigma_{xz}\\ \sigma_{yz}\end{bmatrix}$$

inverting

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{xy}\\ \sigma_{xz}\\ \sigma_{yz}\end{bmatrix} = \begin{bmatrix} \lambda+2\mu & \lambda & \lambda & & & \\ \lambda & \lambda+2\mu & \lambda & & 0 & \\ \lambda & \lambda & \lambda+2\mu & & & \\ & & & 2\mu & 0 & 0 \\ & 0 & & 0 & 2\mu & 0 \\ & & & 0 & 0 & 2\mu \end{bmatrix} \begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \varepsilon_{xy}\\ \varepsilon_{xz}\\ \varepsilon_{yz}\end{bmatrix}$$

λ, μ: Lamé parameters

For the 2D case:

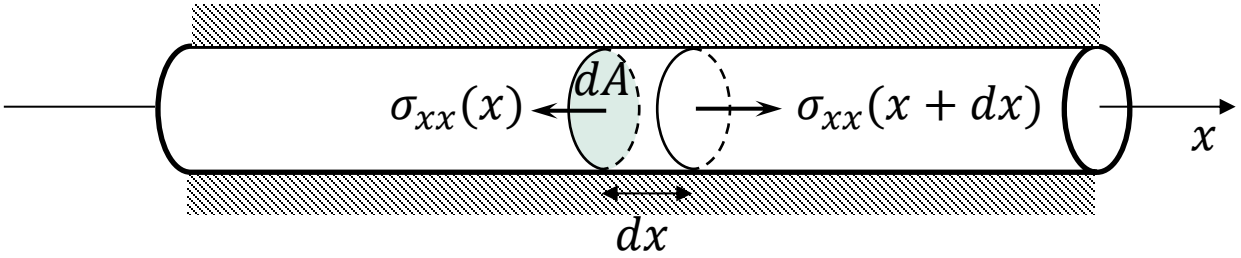
$$\sigma_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x} + \lambda\frac{\partial v}{\partial y}$$

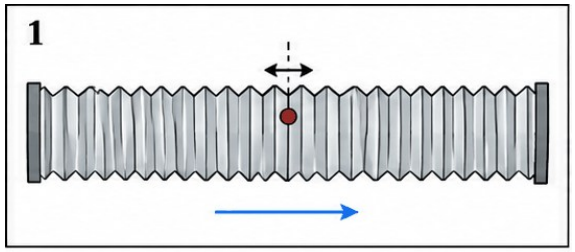
$$\sigma_{xy} = 2\mu\frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \mu\frac{\partial v}{\partial x}$$

The **linear elastic homogeneous isotropic** stress-strain relationship is defined by only **two constants** λ, μ (**Lamé parameters**)

Waves in an elastic bar

Consider an elastic bar subjected to stress at its ends in the x direction—for example, **an accordion** - *hp. the bar cannot deform in the y direction (lateral strain is constrained)*





If $\sigma_{xx}(x)$ and $\sigma_{xx}(x+dx)$ are the stress applied to both ends of the element, respectively, considering only the longitudinal displacement, from force equilibrium (Newton's second law):

$$\sum_i F_{x,i} = ma$$

$$m = \rho V = \rho dx dA$$

$$a = \frac{\partial^2 u}{\partial t^2} = \ddot{u}$$

m: mass
ρ: density
V: volume
a: acceleration

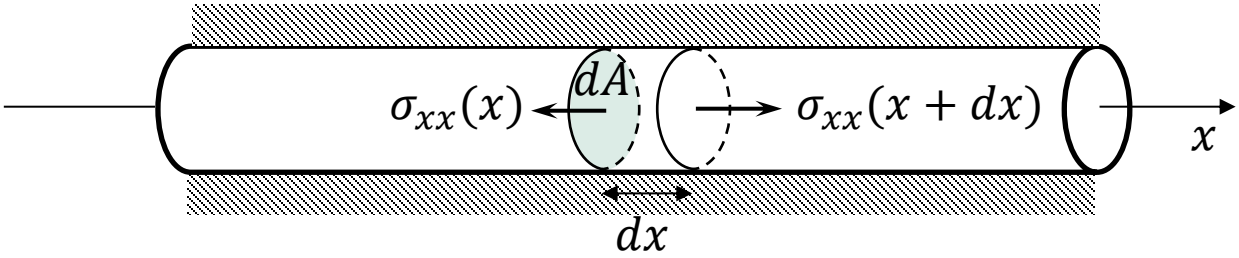
$$\sigma_{xx}(x+dx)dA - \sigma_{xx}(x)dA = m\ddot{u} = (\rho dx dA)\ddot{u}$$

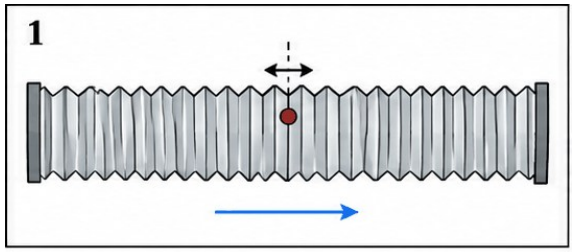
$$\sigma_{xx}(x+dx) = \sigma_{xx}(x) + \frac{\partial \sigma_{xx}}{\partial x} dx$$

$$\rho\ddot{u} = \frac{\partial \sigma_{xx}}{\partial x}$$

Waves in an elastic bar

Consider an elastic bar subjected to stress at its ends in the x direction—for example, **an accordion** - *hp. the bar cannot deform in the y direction (lateral strain is constrained)*





Assuming strain occurs only along *x* we can write:

$$\sigma_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x}$$

Therefore:

$$\rho\ddot{u} = \rho\frac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2}$$

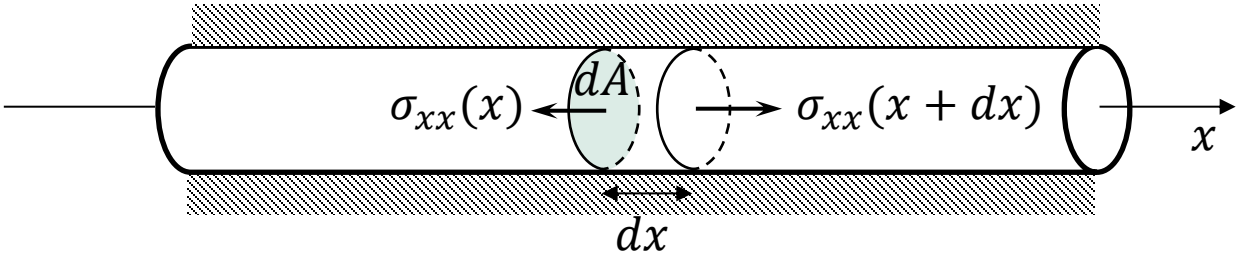
or:

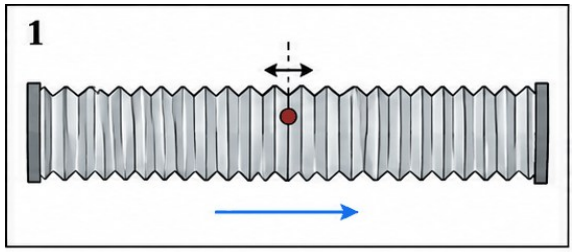
$$\frac{\partial^2 u}{\partial t^2} = \frac{(\lambda + 2\mu)}{\rho}\frac{\partial^2 u}{\partial x^2}$$

which is the **equation of a compressional wave** propagating along the x-direction with velocity $\alpha = \sqrt{(\lambda+2\mu)/\rho}$ and strain in the same direction.

Waves in an elastic bar

Consider an elastic bar subjected to stress at its ends in the x direction—for example, **an accordion** - *hp. the bar cannot deform in the y direction (lateral strain is constrained)*





$$\frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$$

A separable harmonic solution can be assumed for the wave equation:

$$u(x,t) = Ae^{i(\omega t - k_\alpha x)}$$

where the wavenumber k_α is:

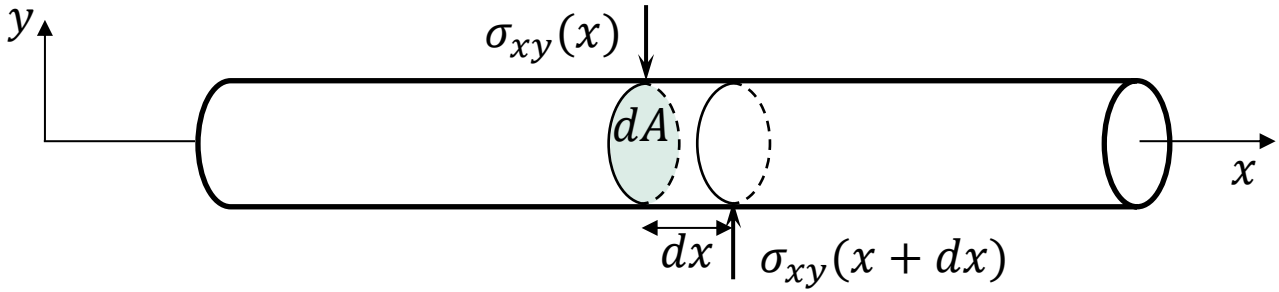
$$k_\alpha = \frac{\omega}{\alpha}$$

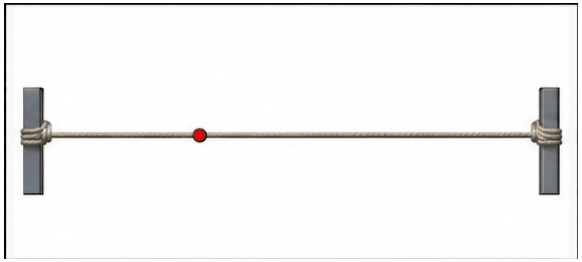
being ω the angular frequency.

Thus, this is a plane wave propagating in the positive *x+.*

Waves in an elastic bar

Now consider an elastic bar subjected to stress in the y direction—for example, **a string** plucked at one end.





If $\sigma_{yx}(x)$ and $\sigma_{yx}(x+dx)$ are the shear stress applied to both ends of the element, respectively, considering only the transversal displacement $v(x)$ we get, following the same procedure as in the previous case:

$$\sigma_{xy}(x+dx)dA - \sigma_{xy}(x)dA = m\ddot{v} = (\rho dx dA)\ddot{v}$$

$$\sigma_{xy}(x+dx) = \sigma_{xy}(x) + \frac{\partial \sigma_{xy}}{\partial x}dx$$

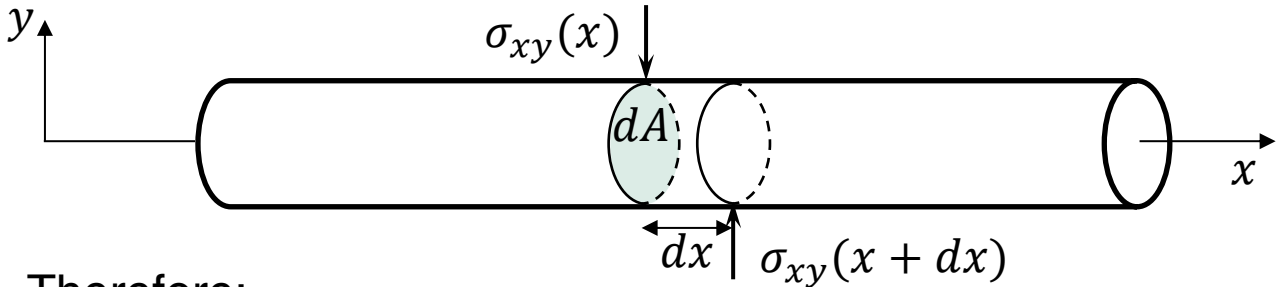
$$\rho\ddot{v} = \frac{\partial \sigma_{xy}}{\partial x}$$

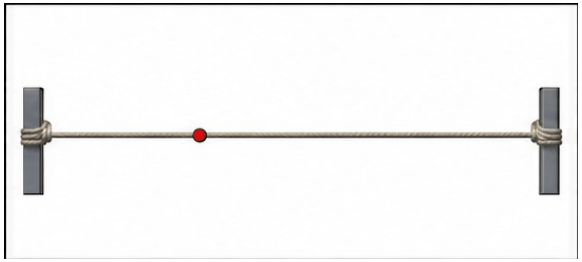
Assuming strain occurs only along *y* we can write:

$$\sigma_{xy} = \mu \frac{\partial v}{\partial x}$$

Waves in an elastic bar

Now consider an elastic bar subjected to stress in the y direction—for example, **a string** plucked at one end.





Therefore:

$$\rho\ddot{v} = \rho\frac{\partial^2 v}{\partial t^2} = \mu\frac{\partial^2 v}{\partial x^2}$$

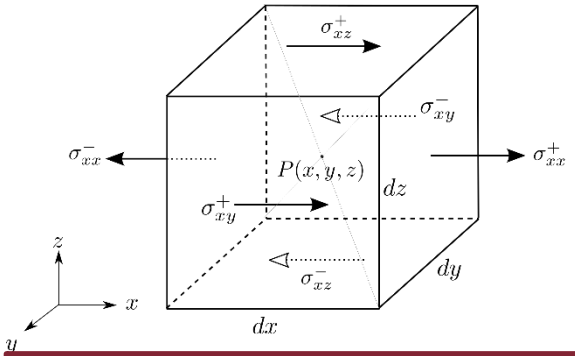
$$\frac{\partial^2 v}{\partial t^2} = \frac{\mu}{\rho}\frac{\partial^2 v}{\partial x^2}$$

which is the **equation of a shear wave** propagating along the x-direction with velocity $\beta = \sqrt{\mu/\rho}$ and strain in the normal y direction, whose solution is, similarly to the previous case:

$$\frac{\partial^2 v}{\partial t^2} = \beta^2\frac{\partial^2 v}{\partial x^2}$$

$$v(x,t) = Be^{i(\omega t - k_\beta x)} \text{ with } k_\beta = \omega/\beta$$

Waves in a 3D elastic medium



For a 3D elastic medium, a procedure similar to that used for the elastic bar is adopted, but all stresses acting on the infinitesimal element are considered and the equilibrium equation is applied along the three axes. The figure, for example, shows only the stresses acting in the x direction, for which: $\sum_i F_{x,i} = m\ddot{u}$

We get two distinct equations:

1 scalar **&** **1 vector**

$$\frac{\partial^2 f}{\partial t^2} = \alpha^2 \frac{\partial^2 f}{\partial x^2}$$

$$v_P = \alpha = \sqrt{\frac{\lambda+2\mu}{\rho}}$$

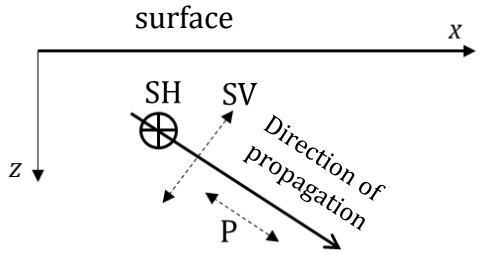
Compressional-wave velocity (also called primary or P-wave velocity)

$$\frac{\partial^2 \mathbf{g}}{\partial t^2} = \beta^2 \frac{\partial^2 \mathbf{g}}{\partial x^2}$$

$$v_S = \beta = \sqrt{\frac{\mu}{\rho}}$$

Shear-wave velocity (also called secondary or S-wave velocity)

Transverse waves therefore have displacement in the two directions (y and z) perpendicular to the propagation direction. Two independent transverse-wave polarizations can thus occur, as in light polarization. The S wave polarized in the horizontal (xy) plane is called the **SH wave**, whereas the wave polarized in the vertical (xz) plane is called the **SV wave.**



For a wave propagating in the positive *x* direction x^+, the solutions are:

$$f(x,t) = Ae^{i(\omega t - k_\alpha x)}$$

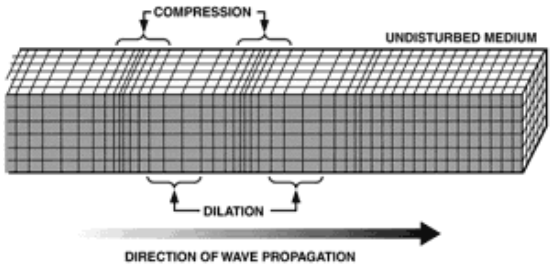
$$\mathbf{g}(x,t) = \mathbf{B}e^{i(\omega t - k_\beta x)}$$

P-wave velocity

$$v_P = \sqrt{\frac{\lambda + 2\mu}{\rho}} = \sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}}$$

density [kg/m^3]

P-wave velocity [m/s]



Dimensional analysis

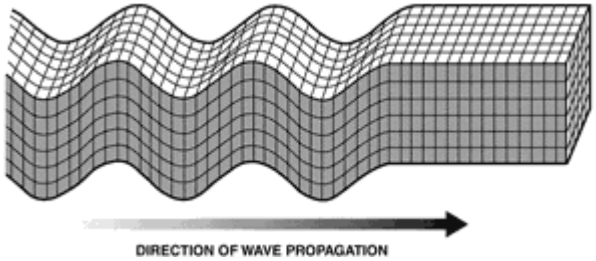
$$v_P = \left(\frac{[\mathrm{Pa}]}{[\mathrm{kg/m^3}]}\right)^{1/2} = \left(\frac{[\mathrm{N/m^2}]}{[\mathrm{kg/m^3}]}\right)^{1/2} = \left(\frac{[\frac{\mathrm{kg}}{\mathrm{m^2}}\frac{\mathrm{m}}{\mathrm{s^2}}]}{[\mathrm{kg/m^3}]}\right)^{1/2} = \left(\frac{[\mathrm{kg/ms^2}]}{[\mathrm{kg/m^3}]}\right)^{1/2}$$

$$v_P = \left(\left[\frac{\cancel{\mathrm{kg}}}{\cancel{\mathrm{m}}\mathrm{s}^2}\right]\left[\frac{\mathrm{m}^{\cancel{3}\,2}}{\cancel{\mathrm{kg}}}\right]\right)^{1/2} = \left(\left[\frac{\mathrm{m}^2}{\mathrm{s}^2}\right]\right)^{1/2} = [\mathrm{m/s}]$$

S-wave velocity

$$v_S = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{E}{\rho}\frac{1}{2(1+\nu)}}$$

S-wave velocity [m/s]

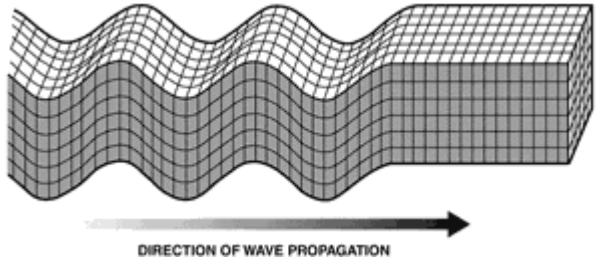


Q. Which is the S-wave velocity in water?

S-wave velocity

$$v_S = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{E}{\rho}\frac{1}{2(1+\nu)}}$$

S-wave velocity [m/s]



Q. Which is the S-wave velocity in water?

A. The S-wave velocity in water is null!

v_s^{water}=0 water does not carry shear waves

Q. Why?

A. Because its shear modulus is null!

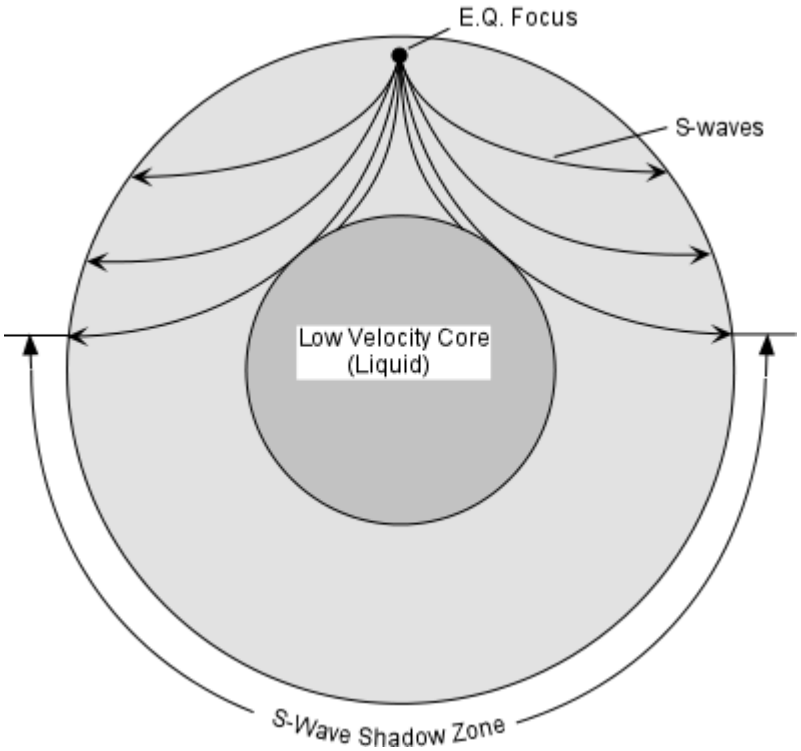
$$\mu = \frac{sh.stress}{sh.strain} = \frac{arbitrary}{\infty} = 0$$

In water, the shear modulus is zero because if you apply an arbitrary force *F*, the liquid simply flows and the strain becomes arbitrarily large: strain $\rightarrow \infty$

$$v_S = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{0}{1000\text{kg/m}^3}} = 0$$

Elastic moduli from seismic velocities

Fluids do not carry shear waves. This knowledge, combined with earthquake observations, is what lead to the discovery that the earth's outer core is a liquid rather than a solid – "shear wave shadow".



Elastic moduli from seismic velocities

Given the seismic velocities v_P and v_S, i.e. from geophysical measurements, we can derive the elastic moduli:

$$M = \rho {v_P}^2$$

$$\nu = \frac{{v_P}^2 - 2{v_S}^2}{2({v_P}^2 - {v_S}^2)}$$

$$K = \rho({v_P}^2 - \frac{4}{3}{v_S}^2)$$

$$\mu = \rho {v_s}^2$$

Generally

$$v_S \approx 0.6 v_P$$

$${v_P}^{freshwater} \approx 1450 \text{ m/s}$$

$${v_P}^{saltwater} \approx 1530 \text{ m/s}$$

$${v_P}^{air} \approx 330 \text{ m/s (speed of sound)}$$

Velocity of rocks and soils

Type of formation	P wave velocity (m/s)	S wave velocity (m/s)	Density (g/cm^3)	Density of constituent crystal (g/cm^3)
Scree, vegetal soil	330-700	100-300	1.7-2.4	-
Dry sands	400-1200	100-500	1.5-1.7	2.65 quartz
Wet sands	1500-2000	100-500	1.9-2.1	2.65 quartz
Saturated shales and clays	1100-2500	200-800	2.0-2.4	-
Marls	2000-3000	750-1500	2.1-2.6	-
Saturated shale and sand sections	1500-2200	500-750	2.1-2.4	-
Porous and saturated sandstones	2000-3500	800-1800	2.1-2.4	2.65 quartz
Limestones	3500-6000	2000-3300	2.4-2.7	2.71 calcite
Chalk	2300-2600	1100-1300	1.8-3.1	2.71 calcite
Salt	4500-5500	2500-3100	2.1-2.3	2.1 halite
Anhydrite	4000-5500	2200-3100	2.9-3.0	-
Dolomite	3500-6500	1900-3600	2.5-2.9	(Ca, Mg) $CO_3$2.8-2.9
Granite	4500-6000	2500-3300	2.5-2.7	-
Basalt	5000-6000	2800-3400	2.7-3.1	-
Gneiss	4400-5200	2700-3200	2.5-2.7	-
Coal	2200-2700	1000-1400	1.3-1.8	-
Water	1450-1500	-	1.0	-
Ice	3400-3800	1700-1900	0.9	-
Oil	1200-1250	-	0.6-0.9	-

Velocity of a multi-phase medium

Soils and rocks are multiphase media

The effective velocity is the combination of the velocity of the single component

Solid
Minerals
(**often high velocity**)

Liquid
Mainly water
(**P-wave velocity=1450-1530 m/s**)

Gas
Mainly air
(**P-wave velocity=330-340 m/s**)

Velocity of the multiphase medium
Combination of the three contributions

Petrophysical parameters

Porosity

$$\phi = \frac{V_v}{V_t}$$

V_V pore volume
V_T bulk volume (including both solid and pores components)
ϕ is dimensionless and theoretically can vary between 0 (solid material) and 1 (only pores). Practically ϕ varies from 0.01 (solid granite) to 0.5 (clay)

Water saturation

$$S_w = \frac{V_w}{V_v} = \frac{V_w}{\phi V_t}$$

V_w water volume
S_w is dimensionless and can vary between 0 (dry material) and 1 (saturated material)

Water content

$$\theta = \phi S_w$$

θ is dimensionless and can theoretically vary between 0 (dry material) and 1 (saturated material with open pores). Practically it only reaches values lower than 1(see above limitations for the porosity)

Bulk density

$$\rho = (1-\phi)\rho_r + \theta\rho_w + \phi(1-S_w)\rho_a$$

ρ_r rock density
ρ_w water density (1 g/cm^3 for freshwater)
ρ_a air density → 0

$$\rho \cong (1-\phi)\rho_r + \theta$$

for a medium saturated by freshwater and air

Velocity vs. porosity (two-phase medium)

The overall rock properties are the average of the matrix (V_M) and pore fluid (V_F) properties, weighted by the porosity ϕ:

Time-average (Wyllie) equation (empirical) TWO-PHASE EQUATION

$$\frac{1}{v_P} = \frac{\phi}{v_F} + \frac{1-\phi}{v_M}$$

Ex. 1: travertine having a matrix velocity of 3 km/s, a porosity of 30% (0.3) filled by fresh water (V_F = 1.45 km/s)



$$\frac{1}{v_P} = \frac{0.3}{1.45} + \frac{0.7}{3} = 0.207 + 0.233 = 0.44$$

$$v_P = \frac{1}{0.44} = 2.27 \text{ km/s} = 2270 \text{ m/s}$$

Velocity vs. porosity vs. saturation (three-phase medium)

Defining the three fractions:

$$\begin{cases} f_r = 1 - \phi \\ f_w = \phi\, S_w \\ f_a = \phi(1 - S_w) \end{cases}$$

THREE-PHASE EQUATION (Timur 1968 for frozen soils)

$$\frac{1}{v_P} = \frac{f_r}{v_r} + \frac{f_w}{v_w} + \frac{f_a}{v_a}$$

Ex. 2: travertine having a matrix velocity of 3 km/s, a porosity of 30% (0.3) only partially (S_w =50%) filled by fresh water (v_f = 1.45 km/s).

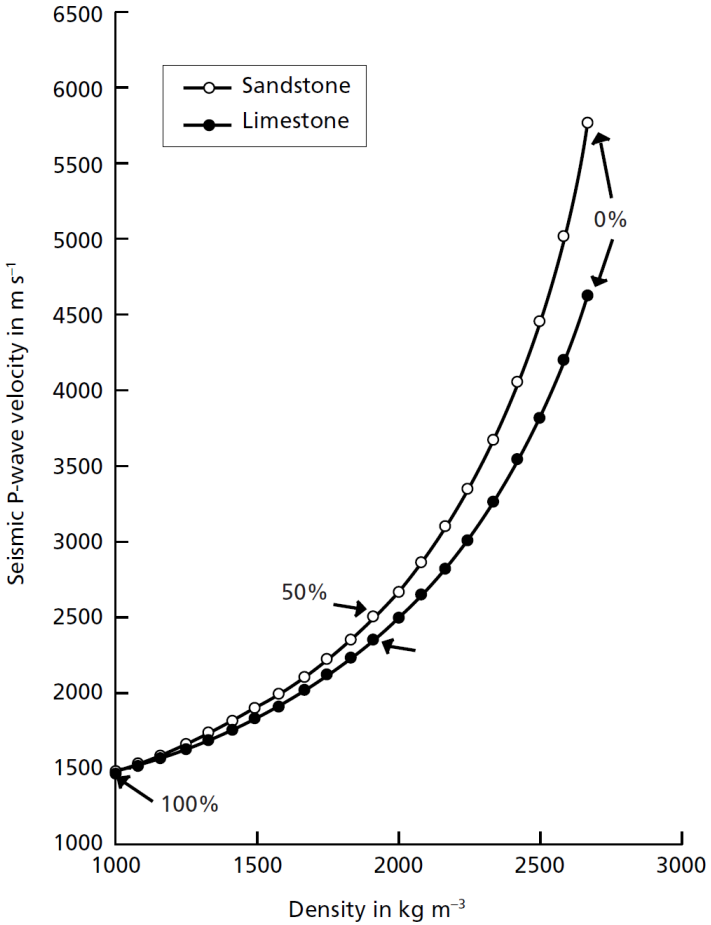


$$\frac{1}{v_P} = \frac{0.7}{3} + \frac{0.15}{1.45} + \frac{0.15}{0.33} = 0.79$$

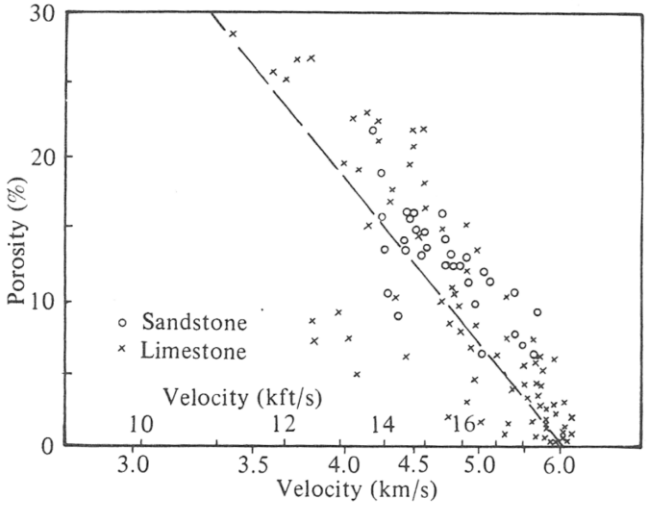
$$v_P = \frac{1}{0.79} = 1.26\ \text{km/s} = 1260\ \text{m/s}$$

Velocity vs. porosity, density, pressure and saturation

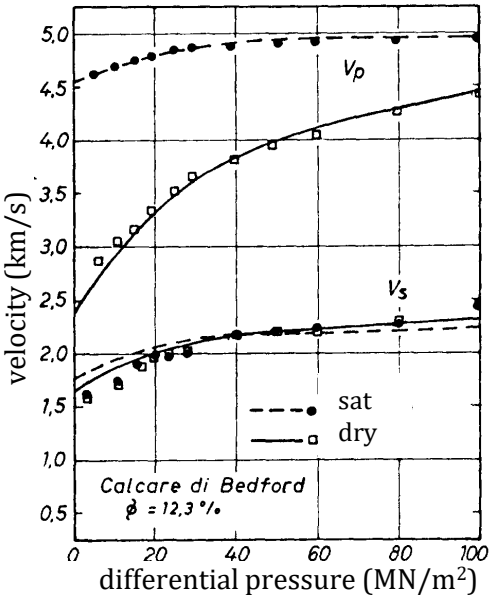
Velocity vs. density



Porosity vs. velocity



Velocity vs. pressure and saturation



Velocity vs. depth

Seismic velocities generally increase with depth due to:

$$v = C(zT)^{1/6}$$

z – depth
T – age of the rock
C – experimental constant

- increased compaction and consequent reduced pore space
- elastic moduli increase with pressure

However, near surface or within anthropogenic elements, we can have some velocity decrease **(velocity inversion)**

