the tubes emit radiation $\sigma T^4_4$, which equals $\sigma T^4_2 F_2$. To find $F_2$, use Fig. 10-51, curve 5, from which $F_2 = 0.81$. Then from Eq. (10-200)

$$F_2 = \frac{1}{(1/0.81) + (1/0.5 - 1/0.5) \times 0.81} = 0.702$$

This amounts to saying that the system of refractory-backed tubes is equal in radiating power to a continuous plane $A_0$, replacing the tubes and refractory back of them, having a temperature equal to that of the tubes and an equivalent or effective emissivity of 0.702.

The new simplified furnace now consists of an enclosure formed by two 1.85 by 3.05-m (6 by 10-ft) radiating sidewalls (area $A_0$, emissivity 0.702), a 1.52 by 3.05-m (5 by 10-ft) receiving plane on the floor $A_4$, and refractory surfaces in the ceiling and sides of the enclosure to complete the enclosure (ends, roof, and floor side strips). The device is a black box.

$$\varphi_0 = \frac{(T - T_0)A_4 F_2}{T_0}$$

where $T$ is the temperature of the enclosure $A_4$ and $T_0$ is the temperature of the black box. To evaluate $F_2$, start with the direct interchange factor $F_2$ and $F_0$ from $A_0$ to $A_3$ (a strip of $A_0$, along $A_3$, which has a common edge with $A_0$) minus $F$ from $A_0$ to the strip only. These two F's may be evaluated from Fig. 10-54. For the first $F$, $Y = 1.83/3.05$, $Z = 1.83/3.05$, and $F = 0.236$; for the second $F$, $Y = 1.83/3.05$, $Z = 0.45/3.05$, and $F = 0.100$. Then $F_2 = 0.236 - 0.100 = 0.139$. Now $F$ may be evaluated from Eq. (10-201) et seq.

$$A_4 F_2 = \frac{2}{1 + \frac{1}{1 + \sqrt{2} \times 2}} + \frac{1}{1 + \frac{1}{1 + \sqrt{2} \times 2}} + \frac{1}{1 + \frac{1}{1 + \sqrt{2} \times 2}}$$

Since the variables are $A_4$, $A_3$, and some of itself (the plane opposite), $F_2 = 1 - F = F_0 - F_2$. The direct interchange factor between parallel 1.85 by 3.05-m (6 by 10-f t) rectangles separated by 2.44 m (8 ft) may be taken as the geometric mean of the factors for 1.85-m (6-ft) squares separated by 2.44 m (8 ft) and for 3.05-m (10-ft) squares separated by 2.44 m (8 ft). These come from Fig. 10-48, curve 8, according to which $F_0 = 0.100 \times 0.255 = 0.182$. Then $F_0 = 1 - 0.182 - 0.139 = 0.679$. The other required direct factor is $F_0 = 1 - F = F_2 F_0 A_3 A_4 = 0.339 (11.14/4.65) = 0.696$. Then $F_2 = 0.139 + 0.696 = 0.835$. Having $F_2$, we may now evaluate the factor $F_0$.

$$F_0 = \frac{1}{1 + \frac{1}{1 + \sqrt{2} \times 2}} + \frac{1}{1 + \frac{1}{1 + \sqrt{2} \times 2}} + \frac{1}{1 + \frac{1}{1 + \sqrt{2} \times 2}}$$

Enclosures of Surfaces That Are Not Diffuse Reflectors In no restriction that the surfaces be diffuse emitters and reflectors is imposed, Eq. (10-202) becomes much more complex. The $W$'s are replaced by $\sigma T^4$ and $\epsilon L$, and $\epsilon_L$ is the product of the angle of the leaving beam, and $\rho_L$ goes inside the integral and becomes a function of angles of incidence and reflection. Seldom are such details of reflectance known. When they are and a solution is needed, the Monte Carlo method of tracing the history of a large number of beams emitted from random positions and in random initial directions is probably the best method of obtaining a solution. Another approach is possible, however, because of the tendency of most surfaces to fit a simpler reflection model. The total reflectance $\rho_0 = 1 - \epsilon$ can be represented by the sum of a diffuse component $\rho_0$ and a specular component $\rho_s$. For applications see Hotell and Sarofim (op. cit., chap. 5). The method yields the following relation for exchange between concentric spheres or infinite cylinders:

$$\epsilon_L^2 = \frac{1}{A_0 \epsilon_L^2 + (1/A_0 \epsilon_L^2 - 1) + (\rho_0 \epsilon_L^2 (1 - \rho_0))/A_0}$$

When there is no specular reflectance, the third term in the denominator drops out, in agreement with Eq. (10-199). When the reflectance is exclusively specular, the denominator becomes $1/A_0(1 + \rho_0 A_0)$, easily derivable from first principles.

EMISSIVITIES OF COMBUSTION PRODUCTS

The radiation from a flame is due to radiation from burning solid particles of microscopic and submicroscopic dimensions, from suspended larger particles of coal, coke, or ash, and from the water vapor and carbon dioxide in the hot gaseous combustion products. The contribution of radiation emitted by the combustion process itself, so-called chemiluminescence, is relatively negligible. Common to these problems is the effect of the shape of the emitting volume on the radiative flux; this is considered first.

Mean Beam Lengths Evaluation of radiation from a nonisothermal volume is beyond the scope of this section (see Hotell and Sarofim, Radiative Transfer, McGraw-Hill, New York, 1967, chap. 11). Consider an isothermal gas confined within the volume bounded by the solid angle $d\Omega$ with vertex at $dA$ and making the angle $\theta$ with the normal to $dA$. The ratio of the emission to $dA$ from the gas to that from a blackbody at the gas temperature and filling the field of view $d\Omega$ is known as the gas emissivity $\epsilon$. Clearly, $\epsilon$ depends on the path length $L$ through the volume to $dA$. A hemispherical volume radiating to a spot on the center of its base represents the only case in which $L$ is independent of direction. Flux at that spot relative to hemispherical blackbody flux is thus an alternative way to visualize emissivity. The flux density on an area of interest on the envelope of an emitting volume of any shape can be matched by that at the base of a hemispherical volume of some radius $L$, which is called the mean beam length. It is found that although the relation of $L$ to a characteristic dimension $D$ of the shape varies with opacity, the variation is small enough for most engineering purposes to permit use of a constant ratio $L_0/D$, where $L_0$ is the mean average beam length. $L_0$ can be defined to apply either to a spot on the envelope or to any finite portion of its area. An important limiting case is that of opacity approaching zero ($PD \to 0$, where $P$ is partial pressure of the emitting constituent). For this case, $L_0$ equals $4V/A$ ($V$ is gas volume, $A$ is bounding surface). When interest is in radiation to the entire envelope. For the range of $PD$ encountered in practice, $L_0$ (now $L_{\text{bd}}$) is always less. For various shapes, 0.8 to 0.95 times $L_0$ has been found optimum (see Table 10-18); for shapes not reported in Table 10-18, a factor of 0.88 (or $L_0 = 0.88L_{\text{bd}} = 3.57V/A$) is recommended.

Instead of using the average-mean-beam-length concept to approximate $A_0 \epsilon_L^2$ (the flux per unit black emissive power from a gas volume to a surface of area $A_0$), one may calculate the flux rigorously by integration, over the gas volume and over $A_0$. Since the gas volume is a diffuse reflector, the total flux emitted $F_0$ is equal to the total surface area $A_0$ times the emissivity $\epsilon$ of the gas volume, as represented by $A_0 \epsilon_0$. The total radiative flux reaching the surface $A_0$ is equal to $A_0 \epsilon_0 F_0$.
TABLE 10-18 Mean Beam Lengths for Volume Radiation

<table>
<thead>
<tr>
<th>Characteristic dimension, ( \beta )</th>
<th>( L_\beta/D )</th>
<th>( L_{\beta L}/D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>0.67</td>
<td>0.60</td>
</tr>
<tr>
<td>Diameter</td>
<td>1.0</td>
<td>0.90</td>
</tr>
<tr>
<td>Diameter</td>
<td>0.51</td>
<td>0.63</td>
</tr>
<tr>
<td>Diameter</td>
<td>0.76</td>
<td>0.71</td>
</tr>
<tr>
<td>Diameter</td>
<td>0.67</td>
<td>0.80</td>
</tr>
<tr>
<td>Diameter</td>
<td>0.47</td>
<td>0.45</td>
</tr>
<tr>
<td>Diameter</td>
<td>0.52</td>
<td>0.48</td>
</tr>
<tr>
<td>Diameter</td>
<td>0.90</td>
<td>0.45</td>
</tr>
<tr>
<td>Diameter</td>
<td>0.73</td>
<td>0.60</td>
</tr>
<tr>
<td>Diameter</td>
<td>0.82</td>
<td>0.76</td>
</tr>
<tr>
<td>Diameter</td>
<td>0.80</td>
<td>0.73</td>
</tr>
<tr>
<td>Radius</td>
<td>1.36</td>
<td></td>
</tr>
<tr>
<td>Edge</td>
<td>0.67</td>
<td>0.60</td>
</tr>
<tr>
<td>Shortest edge</td>
<td>0.90</td>
<td>0.82</td>
</tr>
<tr>
<td>Shortest edge</td>
<td>0.86</td>
<td>0.71</td>
</tr>
<tr>
<td>Shortest edge</td>
<td>0.89</td>
<td>0.81</td>
</tr>
<tr>
<td>Shortest edge</td>
<td>1.18</td>
<td>1.84</td>
</tr>
<tr>
<td>Shortest edge</td>
<td>1.24</td>
<td>1.84</td>
</tr>
<tr>
<td>Shortest edge</td>
<td>1.27</td>
<td>1.84</td>
</tr>
<tr>
<td>Clearance</td>
<td>2.00</td>
<td>1.76</td>
</tr>
<tr>
<td>Clearance</td>
<td>3.4</td>
<td>2.8</td>
</tr>
<tr>
<td>Clearance</td>
<td>4.65</td>
<td>3.8</td>
</tr>
<tr>
<td>Clearance</td>
<td>4.4</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Expression 4k \( \text{d} \text{r} \\text{d} \cos \theta / \pi r^2 \). Here \( k \) is the emission coefficient of the gas, and \( \tau \) is the transmission through the distance \( r \) between \( \text{d} \text{r} \) and \( \text{d} \text{a} \). The result has the dimensions of area and, by analogy to \( \sigma \), is called \( \text{g} \). The direct-exchange area between the gas and the surface zone (Gottel and Sarofim, op. cit., chap. 7). The use of \( A_g(L_a) \) instead of \( \text{g} \) is adequate when the problem is such that all the gas can be treated as a single zone having a mean radiating temperature, but the \( \text{g} \) concept is clearly useful if allowance is to be made for temperature variations within the gas.

Gaseous Combustion Products - Radiation from water vapor and carbon dioxide occurs in spectral bands in the infrared. In magnitude it overshadows convection at furnace temperatures.

Carbon Dioxide - The emissivity \( \epsilon_c \) of a CO₂-containing gas volume depends on gas temperature \( T_g \) on the CO₂ partial-pressure-beam-length product \( P_{\beta L} \) and, to a much lesser extent, on the total pressure. Figure 10-55 gives \( \epsilon_c \) for carbon dioxide at a total pressure of 10.3 kPa (1 atm). The gas absorptivity \( \alpha_c \) equals the emissivity when the absorbing gas and the emitter are at the same temperature. When the emitter surface temperature is \( T_e \), \( \alpha_c \) is \( (T_e/T_g)^{10/2} \) times the \( \epsilon_g \) read from Fig. 10-55 at \( T_g \) instead of \( T_e \) and at \( T_e \). Line broadening due to increases either in total pressure or in partial pressure of CO₂ makes necessary a correction to the emissivity read from Fig. 10-55. However, at a total pressure of 10.3 kPa (1 atm) the correction factor may be ignored, since it decreases with increase in temperature and is never more than 4 percent at temperatures above 1111 K (2000°F). Estimations of the correction in systems up to 10.3 kPa (10 atm) are given by Gottel and Sarofim (op. cit., p. 239), and by Edwards (J. Opt. Soc. Am., 50, 617 (1960)), who in addition presents data on CO₂-band emission for use in calculations involving spectrally selective surfaces. The principal emission bands of CO₂ are at about 2.64 to 2.84, 4.13 to 4.5, and 13 to 17.

Water Vapor - The gas emissivity depends on \( T_g \) and \( P_{\beta L} \) and on total pressure \( P_T \) and on the partial pressure of water vapor \( P_{w} \). Emissivity due to water vapor is given in Fig. 10-56 as a function of \( T_g \) and \( P_{\beta L} \), for the special case of \( P_w = 0 \) and \( P_T = 1 \). Allowance for departure from these special conditions is made by multiplying \( \epsilon_c \) from Fig. 10-56 by a factor \( C_n \), read from Fig. 10-57 as a function of \( P_w + P_T \) and \( P_{\beta L} \). The absorptivity \( \alpha_c \) of water vapor for blackbody radiation is \( \epsilon_c \) from Fig. 10-56, read at \( T_g \) and at \( P_{\beta L}/(T_g/T_g)^{10/2} \) instead of \( P_{\beta L} \), then multiplied by \( (T_e/T_g)^{10/2} \). The correction factor \( C_n \) still applies. Spectral data for water vapor, tabulated for 371 wavelength intervals from 1 to 40 µm, are also available [Ferrie, Ludwig, and Thompson, J. Quant. Spectrosc. Radiat. Transfer, 4, 241-273 (1969)]. The principal emisssion line is in bands at about 2.55 to 2.84, 5.6 to 7.6, and 12 to 25 µm.

Carbon Dioxide-Water-Vapor Mixtures - When these gases are present together, the total radiation due to both is somewhat less than the sum of the separately calculated effects, because each gas is somewhat opaque to radiation from the other in the wavelength regions 2.7 and 15 µm. The spectral-overlap correction factor \( C_{\text{overlap}} \) by which to reduce the sum of \( \epsilon_c \) for CO₂ and \( \epsilon_c \) for H₂O (each evaluated as if the other were absent) to obtain the \( \epsilon_c \) due to the two together is read from Fig. 10-58. The same type of correction applies in calculating \( \alpha_c \).

To summarize, the gas emissivity and absorptivity due to carbon dioxide and water vapor are formulated from the following:

\[
\epsilon_c = (C_{\text{overlap}} \epsilon_{c_{\text{CO}_2}} + \epsilon_{c_{\text{H}_2\text{O}}})/(1 - C_{\text{overlap}}) \\
\alpha_c = (C_{\text{overlap}} \alpha_{c_{\text{CO}_2}} + \alpha_{c_{\text{H}_2\text{O}}})/(1 - C_{\text{overlap}}) \\
\epsilon_{c_{\text{CO}_2}} = (C_{\text{overlap}} \epsilon_{c_{\text{CO}_2}} + \epsilon_{c_{\text{H}_2\text{O}}})/(1 - C_{\text{overlap}}) \\
\alpha_{c_{\text{H}_2\text{O}}} = (C_{\text{overlap}} \alpha_{c_{\text{CO}_2}} + \alpha_{c_{\text{H}_2\text{O}}})/(1 - C_{\text{overlap}})
\]

The long subscripts indicate the values of \( T \) and \( P \), at which Figs. 10-55 and 10-56 are read.

Effective use can sometimes be made of the fact that, at furnace temperatures and for gases containing H₂O and CO₂ in a fixed ratio, \( \epsilon_c \) decreases with rising temperature in such a way that the product \( \epsilon_c T_g \) depends almost exclusively on \( P_T + P_{\beta L} \). Figure 10-59, constructed on this basis, gives the total emissivity (temperature) prod-
uct versus \((P_u + P_d)L\) for combustion products with \(P_u/P_d = 1\) (dashed line) and \(2\) (solid line); it shows as well the temperature range (shaded zone) within which the error in total emissivity is less than 4 percent. The \(P_u/P_d\) range of 1 to 2 almost brackets industrial hydrocarbon fuels. Figure 10-59 has value, however, only for problems in which the gas temperature is so much higher than the surface temperature as to make the value of gas absorptivity unimportant.

**Example 6** Flue gas containing 6 percent CO\(_2\) and 11 percent H\(_2\)O vapor, wet basis, flows through a bank of tubes of 0.105-m (4-in) outside diameter on equilateral 0.203-m (8-in) triangular centers. In a section in which the gas and tube surface temperatures are 691°C (664 K, 1275°F) and 415°C (696 K, 775°F), what are the emissivity and absorptivity of the gas? From Table 10-18, \(L_u = \frac{2.85(0.102)}{0.0986 \times 0.94 \text{ ft}} = 3.20 \text{ ft atm}\) and \(P_d = 3.18 \text{ kPa atm} (0.102 \text{ ft atm}).\) From Fig. 10-55, \(c_{\text{T},\text{CO}_2} = 0.065, c_{\text{T},\text{H}_2\text{O}} = 0.065, c_{\text{CO}_2} = 0.054 (664/666) = 0.067,\) From Fig. 10-56, \(c_{\text{T},\text{CO}_2} = 0.065,\) and \(c_{\text{T},\text{H}_2\text{O}} = 0.065\) at \(P_u = 0.\) From Fig. 10-57, \(C_u = 1.06.\) From Fig. 10-58, overlap correction = 0.02.

\[
\begin{align*}
\epsilon_{\text{CO}_2} & = 0.065 + (0.065)(1.065)1 - 0.02 = 0.130 \\
\alpha_{\text{CO}_2} & = 0.065 + (0.065)(1.065)1 - 0.02 = 0.149
\end{align*}
\]

If the approximation represented by Fig. 10-59 is accepted, then for \(P_u/P_d = 11.6\) and \(P_u + P_d L = 4.91,\) the temperature range for 4 percent accuracy is found to be 1500 to 3600 K (2700 to 6000°F), and at \(T = 964 \text{ K (1735°F)},\) \(c_{\text{T},\text{CO}_2} = 153.3, c_{\text{T},\text{H}_2\text{O}} = 0.138\) (versus 0.130).

**Other Gases** Because of their practical importance, the emissivities of CO\(_2\) and H\(_2\)O have been studied much more extensively than those of other gases, and the values summarized in the preceding paragraphs are based on extensive measurement of both total and integrated spectral values. A summary of the less adequate information on other gases appears in Table 10-19.

**Flames and Particle Clouds**

**Luminous Flames** Luminosity conventionally refers to soot radiation; it is important when combustion occurs under such conditions that the hydrocarbons in the flame are subject to heat in the absence of sufficient air well mixed on a molecular scale. Because soot particles are small relative to the wavelength of the radiation of interest [diameters \(2 \times 10^{-6}\) to \(1 \times 10^{-5}\) m (200 to 1400 Å)], the monochromatic emissivity \(\epsilon_\lambda\) depends on the total particle volume.
per unit volume of space $f$, regardless of particle size. It is given by

$$\varepsilon = 1 - e^{-K/c_0}$$  \hspace{1cm} (10-207)

where $K$ is the path length. Integration of this over the energy spectrum gives the total emissivity $\varepsilon$ as

$$\varepsilon = 1 - \frac{1}{(1 + KTf/L/c_0)}$$  \hspace{1cm} (10-208)

where $c_0$ is the second Planck constant. $K/c_0$ can be obtained from the complex refractive index of soot, in turn dependent on its hydrogen-carbon ratio $H/C$. According to a study of coals, $K/c_0$ varies from 480 m$^{-1}$ K$^{-1}$ at $H/C = 0$ to 2.4 at $H/C = 0.4$; some experimental work at 11muiden (International Flame Foundation) on two oil-flame types leads to 4.4 and 9.8. A tentative value of 500 m$^{-1}$ K$^{-1}$ (95 ft$^{-1}$ R$^{-1}$) is recommended.

There is at present we method of predicting soot concentration of a luminous flame analytically; reliance must be placed on experimental measurement on flames similar to that of interest. Visual observation is misleading; a flame so bright as to hide the wall behind it may be far from a "black" radiator.

The International Flame Foundation has recorded data on many luminous flames from gas, oil, and coal (see J. Inst. Fuel, 1956 to present). Addition of 0.1 to nonluminous-gas emissivity to allow for soot luminosity is often sufficient. If calculations of total flame emission are to be based on a mean flame temperature; this is true because emission from the flame comes more from its cool envelope than from its hot core, especially as its emissivity goes up.

Clouds of Large Black Particles

The emissivity $\varepsilon_0$ of a cloud of particles with a perimeter large compared with wavelength $\lambda$ is

$$\varepsilon_0 = 1 - e^{-\alpha/\alpha_0}$$  \hspace{1cm} (10-209)

where $\alpha_0$ is the projected area of the particles in unit volume of space. If the particles have no negative curvature (a particle can see none of itself) and are randomly oriented, $\alpha$ is $\alpha/A'$, where $\alpha'$ is the actual surface area; and if the particles are uniform, $\alpha/\alpha_0 = cA = cA'/4$ where $A$ and $A'$ are the projected and total areas of each particle and $c$ is the number concentration of particles. For spherical particles this gives

$$\varepsilon_0 = 1 - e^{-\alpha/\alpha_0} = 1 - e^{-M_{A'/A}}$$  \hspace{1cm} (10-209a)
HEAT TRANSMISSION

FIG. 10-37 Correction factor for converting emissivity of water vapor to values of \( P_w \) and \( P_r \), other than 0 to 1 atm, respectively. To convert atmosphere-feet to kilopascal-meters, multiply by 50.86; to convert atmosphere to kilopascals, multiply by (1.01325)/10.

FIG. 10-38 Spectral-overlap correction \( C_p \) for mixtures of carbon dioxide and water vapor. To convert atmosphere-feet to kilopascal-meters, multiply by 50.86.

As an example, consider heavy fuel oil (C\(_{17}\), specific gravity, 0.95) atomized to a surface mean particle diameter of \( d \), burned with 20 percent excess air to produce coke residue particles having the original drop diameter and suspended in combustion products at 1204°C (2200°F). The flame emissivity due to the particles along a path of \( L \) m will be, in micrometers,

\[ \varepsilon = 1 - e^{-4.6 M / d} \]

(10-206b)

With 200-μm particles and an \( L \) of 3.05 m (10 ft), the particle contribution to emissivity will be 0.31. Soot luminosity will increase this; particle burnout will decrease it.

Clouds of Nonblack Particles The correction for nonblackness of the particles is complicated by multiple scatter of the radiation reflected by each particle. The emissivity \( \varepsilon_p \) of a cloud of gray particles of individual surface emissivity \( \varepsilon_s \) can be estimated by the use of Eq. (10-209), with its exponent multiplied by \( \varepsilon_s \), if the optical thickness (\( \varepsilon / \varepsilon_s \)) does not exceed about 2. Modified Eq. (10-209) would predict an approach of \( \varepsilon_p \) to 1 as \( L \to \infty \), an impossibility in a scattering system; the asymptotic value of \( \varepsilon_p \) can be read from Fig. 10-44 as \( \varepsilon_p \), with \( \alpha \) (albedo) given by particle-surface reflectance \( 1 - \varepsilon_s \). Particles with a perimeter lying between 0.3 and 5 times the wavelength of interest can be handled with difficulty by use of the Mie equations (see Hotell and Sarofim, op. cit., chaps. 12 and 13).

Summation of Separate Contributions to Gas or Flame Emissivity The combined emissivity due to several kinds of emitters can be calculated from the separately calculated emissivities, provided only one of these (gaseous combustion products) is a selective emitter. If \( \varepsilon_g = \varepsilon_w = \varepsilon_l \) are the separate emissivities due to gas, soot, and massive particles, each calculated as though no other emitter were present, the combined emissivity is

\[ 1 - (1 - \varepsilon_g)(1 - \varepsilon_w)(1 - \varepsilon_l) = \varepsilon \]

This assumes soot radiation to be gray; rigorous allowance for its nongrayness would necessitate knowledge of the detailed spectral distribution of emission from all emitters present.

RADIATIVE EXCHANGE BETWEEN GASES OR SUSPENDED MATTER AND A BOUNDARY

Local Radiative Exchange The interchange rate \( Q \) between an isothermal gas mass at \( T_G \) and its isothermal black bounding surface of area \( A_t \) is given by

\[ Q = A_t \varepsilon(T_G - T_{\text{iso}}) \]

(10-210)

Evaluation of \( \varepsilon_G \) is unnecessary when \( T_G \) is less than one-half \( T_{\text{iso}} \). \( \varepsilon_G \) may then be assumed equal to \( \varepsilon_L \). A better approximation is to evaluate \( \varepsilon_G \) and \( \varepsilon_L \) at \( T_L \) and \( P_L \).

If the bounding surface is gray rather than black, multiplication of Eq. (10-210) by surface emissivity \( \varepsilon_s \) allows properly for reduction of the primary beams, gas-to-surface or surface-to-gas, but secondary reflections are ignored. The correction then lies between \( \varepsilon_L \) and 1, and for most industrially important surfaces with \( \varepsilon_L > 0.8 \) a value of \( (1 + \varepsilon_L)/2 \) is adequate. Rigorous allowance for this and other factors is presented later.

If the bounding walls are mostly sink-type surfaces of area \( A_t \) and temperature \( T_L \), but in small part refractory surfaces of area \( A_r \), in radiative equilibrium at unknown temperature \( T_R \), an energy balance on \( A_r \) is in principle necessary to determine \( T_R \) and the effect on energy flux. However, the total heat transfer to the sink may be visualized as corresponding to its having an effective area equal to its own plus a fraction \( x \) of that of the refractory, with the only temperatures involved being those of the gas and the heat sink. The fraction \( x \) varies from zero when the ratio of refractory to heat-sink surface is very high to unity when the ratio is very low and the value of \( \varepsilon_G \) is low. If \( A_r \) is small compared with \( A_t \), a value for \( x \) of 0.7 may be used in the approximate method.

Long Exchanger This case, in which axial radiative flux is ignored, includes most radiatively modified heat exchangers of interest to chemical engineers. When the gas temperature transverse to the flow direction is reasonably uniform and the chamber is long compared with its mean hydraulic radius, the opposed upstream and downstream flows through the flow cross section will substantially cancel (but combustion products through tubes or across tube banks, tunnel kilns, billet-reheating furnaces, Example 7). Under these conditions, the radiative contribution to local flux density \( q \) may be formulated in terms of local temperatures and beam lengths or exchange areas evaluated for a two-dimensional system infinite in the flow direction. The local flux density at the sink \( A_t \) is then

\[ q(T_G, T_L) = q(T_G, T_L) + h(T_L - T_G) \]

(10-211)

where \( h \) is the local convective heat-transfer coefficient and \( q(T_G, T_L) \) the radiation contribution calculated from \( T_G, T_L, \varepsilon_L, \) and \( \varepsilon_L \) by using the approximate treatment in the preceding subsection or the more rigorous treatment in the following subsection. If \( mC_p \) is the hourly heat capacity of the gas stream, the temperature of which changes by \( dT_G \) over the sink-area increment \( dA_t \), then

\[ (q(T_G, T_L)) \ dA_t = -mC_p \ dT_G \]

(10-212)

from which

\[ A_t = m \int_{T_G, T_L, \varepsilon_L} C_p \ dT_G \]

(10-213)

The area under a curve of \( C_p/q \) versus \( T_G \) or \( 1/q \) versus the specific enthalpy \( i \) may be used to solve for the area \( A_t \) required to obtain a...
given outlet temperature to or obtain the outlet temperature given $A_t$. Three points generally suffice to determine the area under the curve within 10 percent.

Instead of using graphical integration, which can handle any complexity of variation of flux density $q$ with $T_G$ and $T_t$ along an interchanger flow path, one may evaluate a mean flux density based on mean gas and sink temperatures, based in turn on terminal temperatures. It has been found empirically that fair results are obtained by the use of a mean surface temperature equal to the arithmetic mean of the terminal surface temperatures and by the use of a mean gas temperature equal to the mean surface temperature plus the logarithmic mean of the temperature difference, gas to surface, at the two ends of the exchanger. When radiation dominates the transfer process, however, graphical integration is safer.

**Example 7**

Fuel gas containing 6 percent carbon dioxide and 11 percent water vapor by volume (wet basis) flows through the convection bank of an oil tube, still consisting of rows of 0.105 by 4.5-m (4-in.) tubes on 0.385 by 8-in. centers, since 7.62 by 24-ft) tubes in a row, the rows staggered to put the tubes on equilateral triangular centers. The fuel gas enters at $871.1^\circ C$ (1144 K, 1600°F) and leaves at 538.7°C (811 K, 1000°F). The oil flows in a countercurrent direction to the gas and rises from 316 to 427°C (600 to 800°F). Tube-surface emissivity is 0.8. What is the average heat-input rate, due to gas radiation alone, per square meter of external tube area?

With each row of tubes there is associated $0.203(\sqrt{4}/3) = 0.176$ m (5.77 ft) of wall height, of area: $0.203(\sqrt{4}/3) = 0.176$ m (5.77 ft). One row of tubes has an area of $0.002177$, giving $22.0$ m² (236 ft²). If the recommended factor of 0.7 on the refractory area is used, the effective area of the tubes is $22.0(0.7)/0.333 = 12.1$ m² (130 ft²) of actual area. The exact evaluation of the outside tube area from the known oil temperature would involve a knowledge of the oil film coefficient, tube-wall resistance, and rate of heat flow into the tube, the evaluation usually involving trial and error. However, for the present purpose the temperature drop through the tube wall and oil film will be assumed to be 47°C (75°F), making the tube surface temperature 350°C (675°F) and 469°C (875°F); the average is 412°C (757°F). The radiating gas temperature is $t_f = 412 + 2.5 \log \left( \frac{871 - 469}{538 - 357} \right) = 412 + 278 = 690^\circ F (174^\circ C)$. These temperatures, partial pressures, and dimensions were used in Example 6 to determine gas emissivity and absorptivity $\epsilon = 0.150$; $\alpha = 0.149$. The approximate effective emissivity of the boundary is $(0.8 + 1/2)(0.12) = 0.9$. Then

<table>
<thead>
<tr>
<th>Carbon Dioxide</th>
<th>6%</th>
<th>11%</th>
<th>16%</th>
<th>20%</th>
<th>25%</th>
<th>30%</th>
<th>35%</th>
<th>40%</th>
<th>45%</th>
<th>50%</th>
<th>55%</th>
<th>60%</th>
<th>65%</th>
<th>70%</th>
<th>75%</th>
<th>80%</th>
<th>85%</th>
<th>90%</th>
<th>95%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water Vapor</td>
<td>0.047</td>
<td>0.067</td>
<td>0.087</td>
<td>0.107</td>
<td>0.127</td>
<td>0.147</td>
<td>0.167</td>
<td>0.187</td>
<td>0.207</td>
<td>0.227</td>
<td>0.247</td>
<td>0.267</td>
<td>0.287</td>
<td>0.307</td>
<td>0.327</td>
<td>0.347</td>
<td>0.367</td>
<td>0.387</td>
<td>0.407</td>
<td></td>
</tr>
<tr>
<td>NH₃</td>
<td>0.020</td>
<td>0.030</td>
<td>0.040</td>
<td>0.050</td>
<td>0.060</td>
<td>0.070</td>
<td>0.080</td>
<td>0.090</td>
<td>0.100</td>
<td>0.110</td>
<td>0.120</td>
<td>0.130</td>
<td>0.140</td>
<td>0.150</td>
<td>0.160</td>
<td>0.170</td>
<td>0.180</td>
<td>0.190</td>
<td>0.200</td>
<td></td>
</tr>
<tr>
<td>SO₂</td>
<td>0.001</td>
<td>0.002</td>
<td>0.003</td>
<td>0.004</td>
<td>0.005</td>
<td>0.006</td>
<td>0.007</td>
<td>0.008</td>
<td>0.009</td>
<td>0.010</td>
<td>0.011</td>
<td>0.012</td>
<td>0.013</td>
<td>0.014</td>
<td>0.015</td>
<td>0.016</td>
<td>0.017</td>
<td>0.018</td>
<td>0.019</td>
<td></td>
</tr>
<tr>
<td>CH₄</td>
<td>0.020</td>
<td>0.030</td>
<td>0.040</td>
<td>0.050</td>
<td>0.060</td>
<td>0.070</td>
<td>0.080</td>
<td>0.090</td>
<td>0.100</td>
<td>0.110</td>
<td>0.120</td>
<td>0.130</td>
<td>0.140</td>
<td>0.150</td>
<td>0.160</td>
<td>0.170</td>
<td>0.180</td>
<td>0.190</td>
<td>0.200</td>
<td></td>
</tr>
<tr>
<td>CO₂</td>
<td>0.020</td>
<td>0.030</td>
<td>0.040</td>
<td>0.050</td>
<td>0.060</td>
<td>0.070</td>
<td>0.080</td>
<td>0.090</td>
<td>0.100</td>
<td>0.110</td>
<td>0.120</td>
<td>0.130</td>
<td>0.140</td>
<td>0.150</td>
<td>0.160</td>
<td>0.170</td>
<td>0.180</td>
<td>0.190</td>
<td>0.200</td>
<td></td>
</tr>
<tr>
<td>NO</td>
<td>0.0096</td>
<td>0.0192</td>
<td>0.0288</td>
<td>0.0384</td>
<td>0.0480</td>
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<td>0.1248</td>
<td>0.1344</td>
<td>0.1440</td>
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<td>0.1632</td>
<td>0.1728</td>
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</tr>
<tr>
<td>HCl</td>
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<td>0.00044</td>
<td>0.00066</td>
<td>0.00088</td>
<td>0.00110</td>
<td>0.00132</td>
<td>0.00154</td>
<td>0.00176</td>
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<td>0.00330</td>
<td>0.00352</td>
<td>0.00374</td>
<td>0.00396</td>
<td>0.00418</td>
<td></td>
</tr>
</tbody>
</table>

Note: Figures in this table are taken from plots in Hostel and Saradin, Radiative Transfer, McGraw-Hill, New York, 1967, chap. 6. Values in parentheses are extrapolated. To convert degrees Rankine to Kelvin, multiply by $5.556 \times 10^{-3}$. To convert atmosphere-feet to kilopascal-meters, multiply by 0.909.

*Calculations of Guerriot (in Chemical Engineering, MIT, 1940) at 1-atm total pressure, $L = 1.68$ ft, $T = 2000^\circ R$.


*Calculations of Malkmus and Thompson [J. Quant. Spectrosc. Radiat. Transfer, 1, 16 (1962)], to $T = 5400^\circ R$ and $PL = 30$ atm-ft.