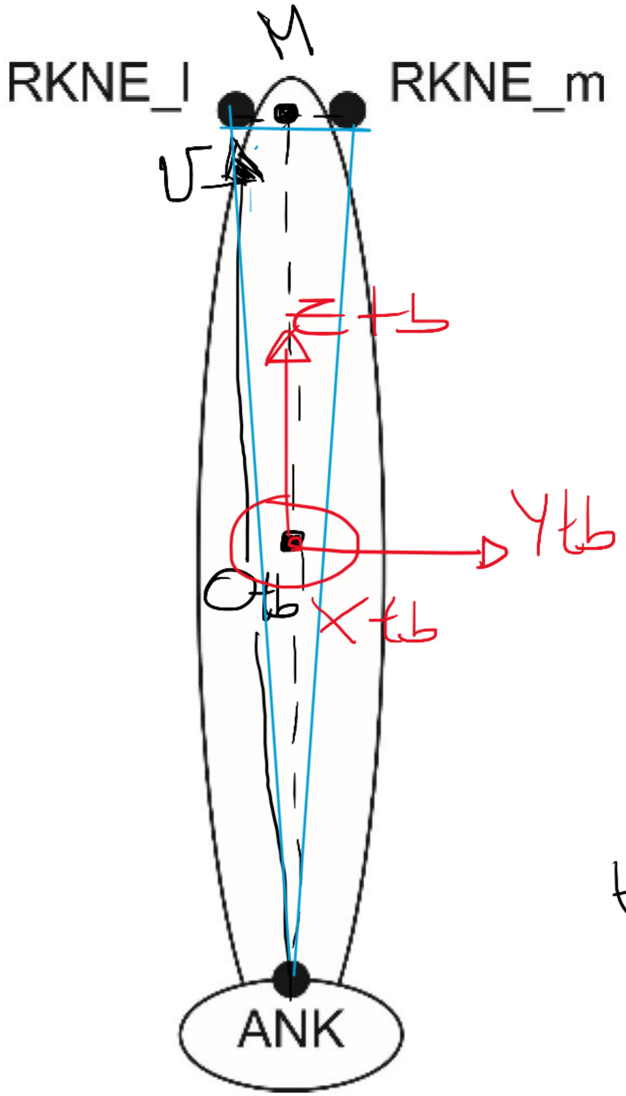
ESAME 15/02/2021

1a)



$$M = \frac{RKNE\_l + RKNE\_m}{2}$$

$$O_{tb} = \frac{ANK + M}{2}$$

$$\hat{z}_{tb} = \text{vers}(M - O_{tb})$$

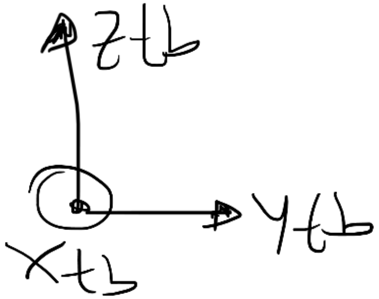
$$v = RKNE\_l - ANK$$

$$\hat{x}_{tb} = \text{vers}(\hat{z}_{tb} \times v)$$

$$\hat{y}_{tb} = \hat{z}_{tb} \times \hat{x}_{tb}$$

$$^{tb}T_0 = \begin{bmatrix} | & | & | & | \\ \hat{x} & \hat{y} & \hat{z} & O_{tb} \\ | & | & | & | \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1b)



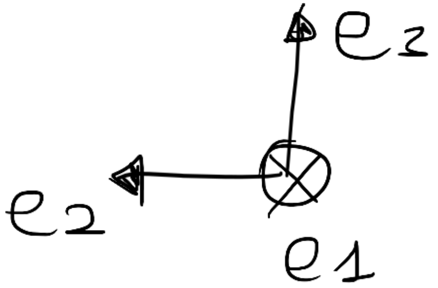
FRONTALE

$e_1 \parallel y_{tb}$

$e_3 \parallel y_{ft}$

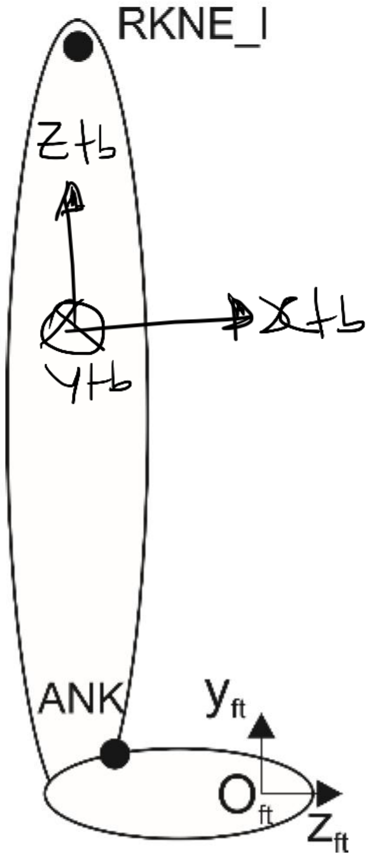
$e_2 = e_3 \times e_1$

$\Rightarrow$



SEQ. EULERO

Y X Y



ROTAZIONI POSITIVE:

- FLESSIONE PLANTARE
- EVERSIONE
- ROTAZIONE INTERNA

1c) SEQUENZA YXY

$$\omega = H(\alpha)\dot{\alpha} = H(\alpha)\begin{pmatrix}\dot{\varphi}\\ \dot{\theta}\\ \dot{\psi}\end{pmatrix} = \overbrace{\begin{pmatrix}0 & C\varphi & S\varphi C\theta\\ 1 & 0 & C\theta\\ 0 & -S\varphi & C\varphi S\theta\end{pmatrix}}^{H(\alpha)}\begin{pmatrix}\dot{\varphi}\\ \dot{\theta}\\ \dot{\psi}\end{pmatrix}$$

$$\omega_\varphi = \begin{pmatrix}0\\ \dot{\varphi}\\ 0\end{pmatrix}$$

$$\omega_\theta = R_y(\varphi)\begin{pmatrix}\dot{\theta}\\ 0\\ 0\end{pmatrix} = \begin{pmatrix}C\varphi & 0 & S\varphi\\ 0 & 1 & 0\\ -S\varphi & 0 & C\varphi\end{pmatrix}\begin{pmatrix}\dot{\theta}\\ 0\\ 0\end{pmatrix} = \begin{pmatrix}C\varphi\,\dot{\theta}\\ 0\\ -S\varphi\,\dot{\theta}\end{pmatrix}$$

$$\omega_\psi = R_y(\varphi)R_x(\theta)\begin{pmatrix}0\\ \dot{\psi}\\ 0\end{pmatrix} = \begin{pmatrix}C\varphi & 0 & S\varphi\\ 0 & 1 & 0\\ -S\varphi & 0 & C\varphi\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & C\theta & -S\theta\\ 0 & S\theta & C\theta\end{pmatrix}\begin{pmatrix}0\\ \dot{\psi}\\ 0\end{pmatrix} =$$

$$= \begin{pmatrix}C\varphi & 0 & S\varphi\\ 0 & 1 & 0\\ -S\varphi & 0 & C\varphi\end{pmatrix}\begin{pmatrix}0\\ C\theta\,\dot{\psi}\\ S\theta\,\dot{\psi}\end{pmatrix} = \begin{pmatrix}S\varphi\, S\theta\, \dot{\psi}\\ C\theta\,\dot{\psi}\\ C\varphi\, S\theta\,\dot{\psi}\end{pmatrix}$$

$$\Rightarrow \omega_\varphi + \omega_\theta + \omega_\psi = \begin{pmatrix}0 + C\varphi\,\dot{\theta} + S\varphi\, C\theta\,\dot{\psi}\\ \dot{\varphi} + 0 + C\theta\,\dot{\psi}\\ 0 - S\varphi\,\dot{\theta} + C\varphi\, S\theta\,\dot{\psi}\end{pmatrix} = H(\alpha)\dot{\alpha}$$

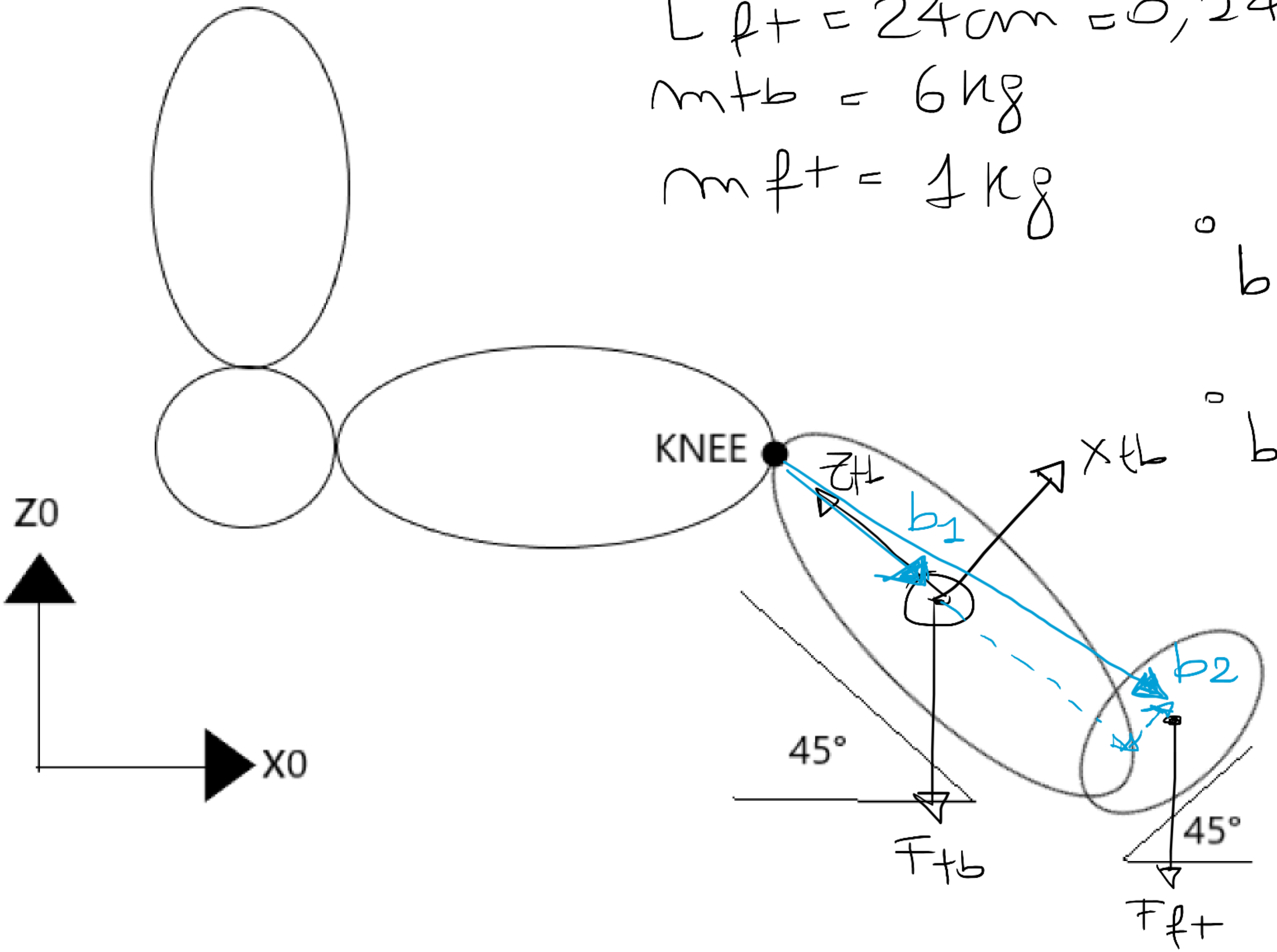
1d)

$L_{tb} = 40\,cm = 0{,}4\,m$

$L_{ft} = 24\,cm = 0{,}24\,m$

$m_{tb} = 6\,kg$

$m_{ft} = 1\,kg$



IN CS0

$$^{0}b_{tb} = \begin{pmatrix} \frac{L_{tb}}{2}\cos(45°) \\ 0 \\ \frac{L_{tb}}{2}(-\sin(45°)) \end{pmatrix} = \begin{pmatrix} 0{,}14 \\ 0 \\ -0{,}14 \end{pmatrix} m$$

$$^{0}b_{ft} = \begin{pmatrix} L_{tb}\cos(45°) + \frac{L_{ft}}{2}\cos(45) \\ 0 \\ L_{tb}(-\sin(45)) + \frac{L_{ft}}{2}\sin(45) \end{pmatrix}$$

$$= \begin{pmatrix} 0{,}37 \\ 0 \\ -0{,}2 \end{pmatrix} m$$

$$^{0}F_{tb} = \begin{pmatrix} 0 \\ 0 \\ -58{,}9 \end{pmatrix} N$$

$$^{0}F_{ft} = \begin{pmatrix} 0 \\ 0 \\ -9{,}8 \end{pmatrix} N$$

$${}^{0}m_{tb} = {}^{0}b_{tb} \times {}^{0}F_{tb} = \begin{pmatrix} i & j & k \\ 0{,}14 & 0 & -0{,}14 \\ 0 & 0 & -58{,}9 \end{pmatrix} = 0 \cdot \hat{i} - (-8{,}4)\hat{j} + 0 \cdot \hat{k} = \begin{pmatrix} 0 \\ 8{,}4 \\ 0 \end{pmatrix} Nm$$

$${}^{0}m_{ft} = {}^{0}b_{ft} \times {}^{0}F_{ft} = \begin{pmatrix} i & j & k \\ 0{,}37 & 0 & -0{,}2 \\ 0 & 0 & -9{,}8 \end{pmatrix} = 0\,\hat{i} - (-3{,}7)\hat{j} + 0 \cdot \hat{k} = \begin{pmatrix} 0 \\ 3{,}7 \\ 0 \end{pmatrix} Nm$$

$$\Rightarrow \quad {}^{0}m_{TOT} = {}^{0}m_{tb} + {}^{0}m_{ft} = \begin{pmatrix} 0 \\ 8{,}4 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 3{,}7 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 12{,}1 \\ 0 \end{pmatrix} Nm$$

In CS_{tb}

$${}^{tb}R_0(45°) = \begin{pmatrix} \cos(45) & 0 & \sin(45) \\ 0 & 1 & 0 \\ -\sin(45) & 0 & \cos(45) \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$$

$$\Rightarrow {}^{tb}m_{TOT} = {}^{tb}R_0 \; {}^{0}m_{TOT} = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} 0 \\ 12{,}1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 12{,}1 \\ 0 \end{pmatrix} Nm$$