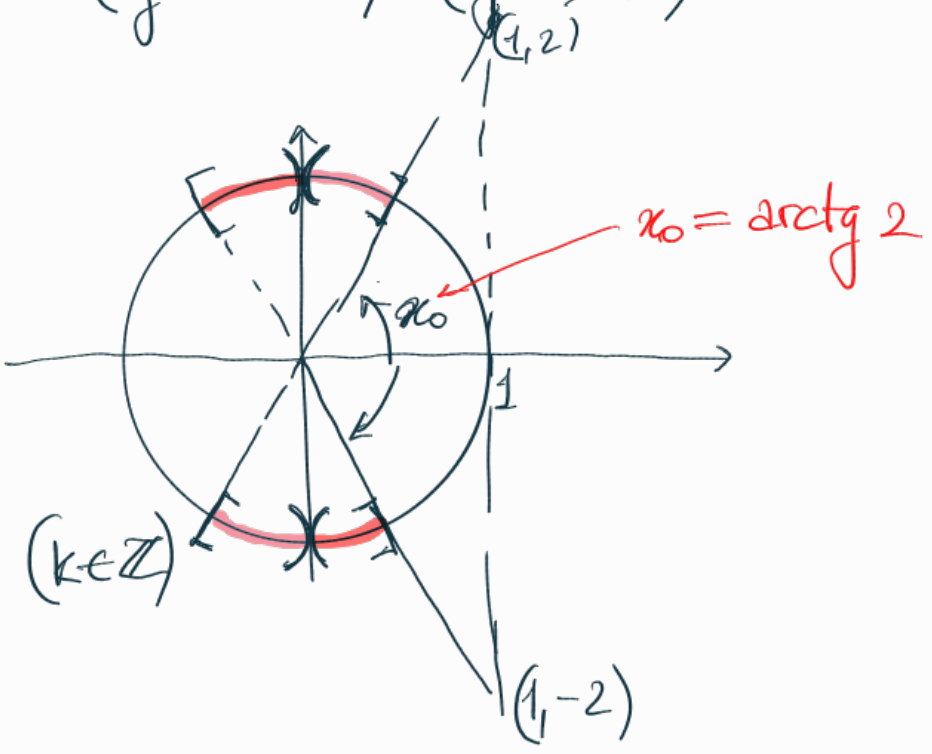
Uso delle funz. trigonometriche inverse per risolvere equaz/diseq.

$$\underbrace{tg^2 x}_{\parallel (tg\,x)^2} \geq 4 \iff (tg\,x \leq -2) \vee (tg\,x \geq 2)$$



$tg\,x \geq 2$

$\Downarrow$

$\operatorname{arctg} 2 + k\pi \leq x < \frac{\pi}{2} + k\pi \quad (k \in \mathbb{Z})$

$tg\,x \leq -2$

$\Downarrow$

$-\frac{\pi}{2} + k\pi < x \leq \underbrace{\operatorname{arctg}(-2)}_{\parallel -\operatorname{arctg} 2} + k\pi$

Soluzioni:

$$\left(-\frac{\pi}{2} + k\pi < x \leq -\operatorname{arctg} 2 + k\pi\right) \vee \left(\operatorname{arctg} 2 + k\pi \leq x < \frac{\pi}{2} + k\pi\right)$$

oppure, scritto diversamente:

$$\operatorname{arctg} 2 + k\pi \leq x \leq \pi - \operatorname{arctg} 2 + k\pi \qquad x \neq \frac{\pi}{2} + k\pi$$

Due lezioni fa abbiamo risolto (in tre modi diversi) la disequazione

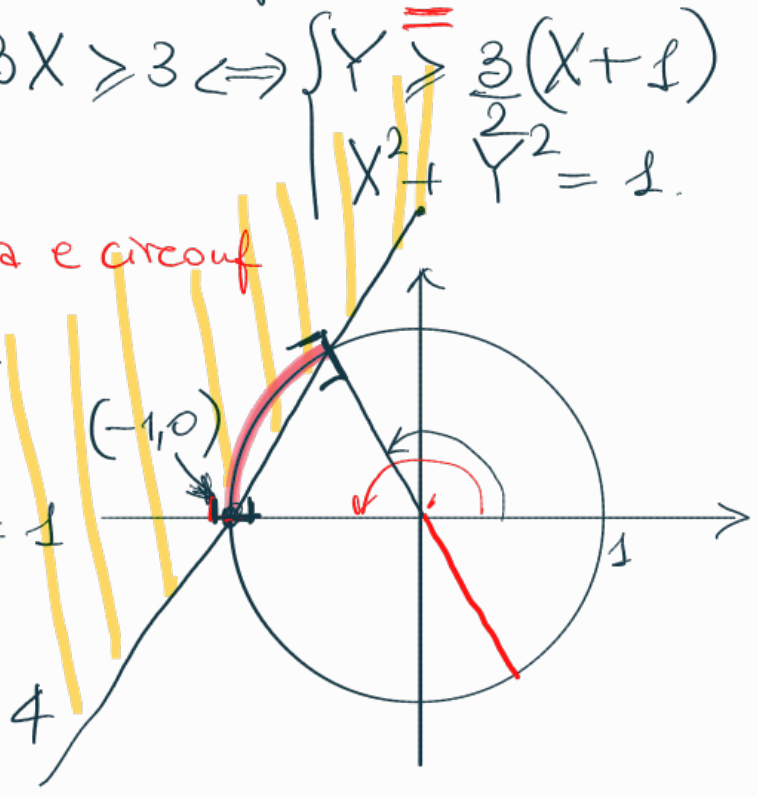
$$\sin x - \sqrt{3}\cos x \geq \sqrt{3}$$

Cosa succede se al posto di $\sqrt{3} \approx 1{,}7$ metto $\frac{3}{2} = 1{,}5$?

$$\sin x - \frac{3}{2}\cos x \geq \frac{3}{2} \iff 2\sin x - 3\cos x \geq 3$$

1° modo (Intersezione con la circ. goniometrica)

$$\begin{cases} \cos x = X \\ \sin x = Y \end{cases} \Rightarrow \quad 2Y - 3X \geq 3 \iff \begin{cases} Y \geq \frac{3}{2}(X+1) \\ X^2 + Y^2 = 1 \end{cases}$$



Studio le intersezioni tra retta e circonf

$$X^2 + \left[\frac{3}{2}(X+1)\right]^2 = 1$$

$$X^2 + \frac{9}{4}(X^2+2X+1) = 1$$

$$4X^2 + 9X^2 + 18X + 9 = 4$$

$$13X^2 + 18X + 5 = 0$$

$$X_{1,2} = \frac{-9 \pm \sqrt{81-65}}{13} = \frac{-9 \pm 4}{13} = \begin{cases} -1 \\ -\frac{5}{13} \end{cases}$$

$$X = -1 \Rightarrow Y = 0 \iff x = \pi + 2k\pi$$

$$X = -\frac{5}{13} \Rightarrow Y = \frac{3}{2}(X+1) = \frac{3}{2} \cdot \frac{8}{13} = \frac{12}{13}$$

come trovo il secondo valore di x ?

$$\begin{cases} \cos x = -\frac{5}{13} \\ \sin x = \frac{12}{13} \end{cases} \Downarrow \quad \text{tg } x = -\frac{12}{5}$$

$x = \arccos\left(-\frac{5}{13}\right)$ Ok!

~~$x = \arcsin\left(\frac{12}{13}\right)$~~ $\pi - \arcsin\left(\frac{12}{13}\right)$

~~$x = \text{arctg}\left(-\frac{12}{5}\right)$~~ $\text{arctg}\left(-\frac{12}{5}\right) + \pi$

$=$

$\pi - \text{arctg}\left(\frac{12}{5}\right)$

Sol^mi della diseq:

$$x_0 + 2k\pi \leq x \leq \pi + 2k\pi$$

dove $x_0 = \arccos\left(\frac{-5}{13}\right) = \pi - \arcsin\left(\frac{12}{13}\right) = \pi - \text{arctg}\left(\frac{12}{5}\right)$

2° modo (metodo dell'angolo aggiunto)

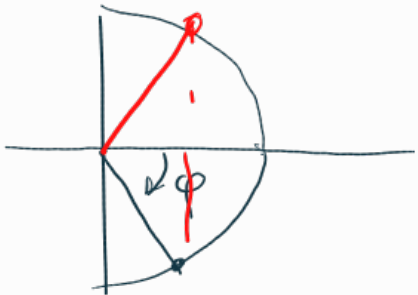
$$2\sin x - 3\cos x \geq 3$$

$$A\sin x + B\cos x = \sqrt{A^2+B^2}\ \sin(x+\varphi)$$

dove φ verifica $\begin{cases} \cos\varphi = \frac{A}{\sqrt{A^2+B^2}} \\ \sin\varphi = \frac{B}{\sqrt{A^2+B^2}} \end{cases}$

Nel nostro caso $(A=2,\ B=-3)$

$$\begin{cases} \cos\varphi = \frac{2}{\sqrt{13}} \\ \sin\varphi = \frac{-3}{\sqrt{13}} \end{cases}$$



$\varphi = \arccos \frac{2}{\sqrt{13}}$ ~~$\arccos \frac{2}{\sqrt{13}}$~~ $- \arccos \frac{2}{\sqrt{13}}$

$\varphi = \arcsin\left(\frac{-3}{\sqrt{13}}\right) = \boxed{-\arcsin\left(\frac{3}{\sqrt{13}}\right)}$

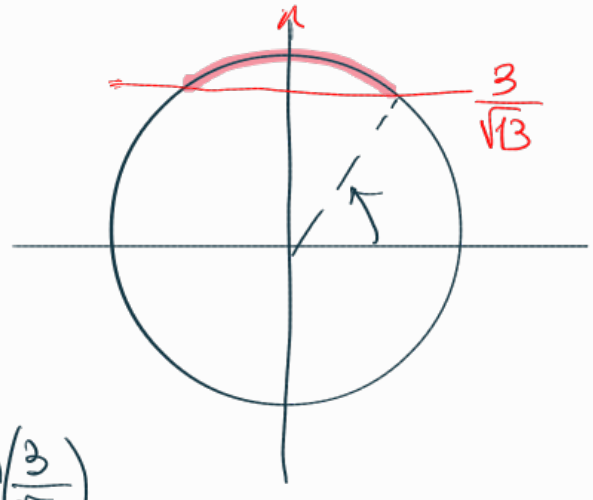
$\varphi = \operatorname{arctg}\left(-\frac{3}{2}\right) = -\operatorname{arctg}\left(\frac{3}{2}\right)$

$2\sin x - 3\cos x \geq 3$ diventa

$\sqrt{13}\ \sin(x+\varphi) \geq 3$

$\sin(x+\varphi) \geq \frac{3}{\sqrt{13}}$

$\| \quad t$



$\arcsin\left(\frac{3}{\sqrt{13}}\right) \leq t \leq \pi - \arcsin\left(\frac{3}{\sqrt{13}}\right)$

$\| \quad x+\varphi$

$2\arcsin\left(\frac{3}{\sqrt{13}}\right) + 2k\pi \leq x \leq \pi + 2k\pi$.

È lo stesso risultato di prima? sì se

$\arccos\left(-\frac{5}{13}\right) = 2\arcsin\left(\frac{3}{\sqrt{13}}\right)$ è vero?

$[0,\pi]$ $[0,\pi]$.

Sono uguali se e solo se i coseni sono uguali

$-\frac{5}{13} = \cos\left(2\arcsin\left(\frac{3}{\sqrt{13}}\right)\right)$

$$\cos(2t) = 1 - 2\sin^2 t$$

$$\cos\left(2\arcsin\left(\frac{3}{\sqrt{13}}\right)\right) = 1 - 2\left(\sin \arcsin \frac{3}{\sqrt{13}}\right)^2 =$$

$$= 1 - 2\left(\frac{3}{\sqrt{13}}\right)^2 = 1 - 2 \cdot \frac{9}{13} =$$

$$= 1 - \frac{18}{13} = -\frac{5}{13}$$