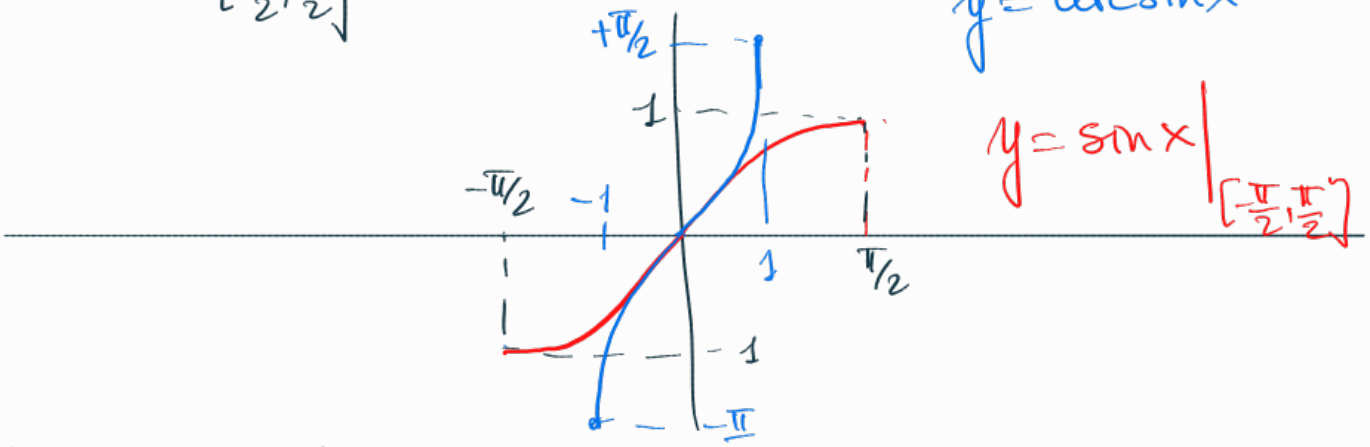
Funzioni trigonometriche inverse

$f(x) = \sin x \Big|_{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]} : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$ biiettiva

$y = \arcsin x$



$\Rightarrow$ Resta definita la funzione inversa

$f^{-1} = \arcsin : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$y \mapsto$ $\arcsin y =$ l'unico $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ t.c. $\sin x = y$

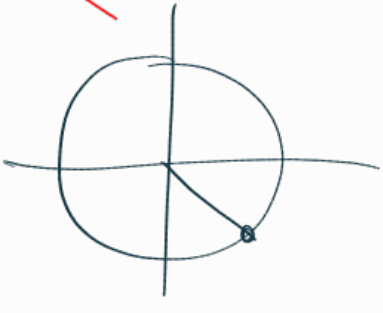
$\arcsin \frac{1}{2} =$ l'unico $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ t.c. $\sin x = \frac{1}{2} =$

$= \frac{\pi}{6}$

~~$\arcsin \frac{1}{2} = \frac{5}{6}\pi$~~

~~$\arcsin \frac{1}{2} = -\frac{11}{6}\pi$~~

~~$\arcsin \frac{1}{2} = \frac{\pi}{6} + 2k\pi$~~



$\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

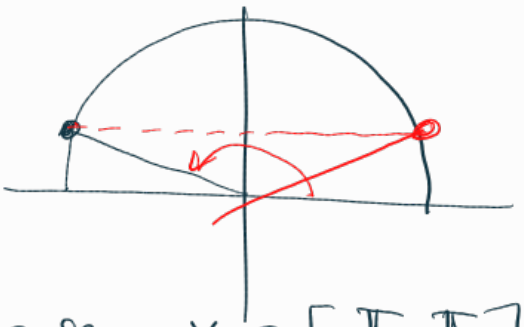
Se $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $y \in [-1, 1]$, allora

$\sin x = y \iff x = \arcsin y.$

$\arcsin\left(\sin\frac{1}{4}\right) = \frac{1}{4}$

$\arcsin(\sin 3) = \pi - 3$

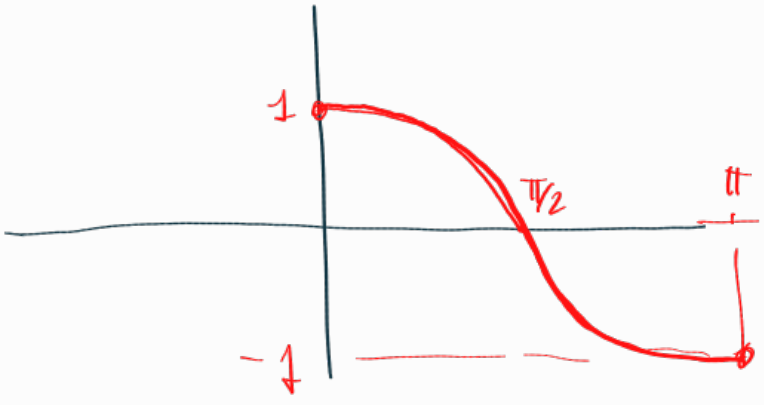
$\arcsin(\sin x) = x$ solo se $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



$\sin(\arcsin x) = x \quad \forall x \in [-1,1]$

$\cos : \mathbb{R} \to \mathbb{R}$ non è invertibile

$\cos|_{[0,\pi]} : [0,\pi] \to [-1,1]$ è biiettiva.



$y = \cos x|_{[0,\pi]}$

Iniettiva perché strett. decrescente.
Suriettiva perché continua in un intervallo $\Rightarrow$ assume tutti i valori compresi tra -1 e 1.

Resta definita f^{-1}

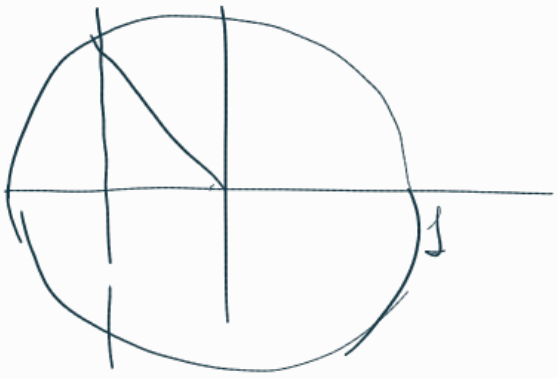
$f^{-1} = \arccos : [-1,1] \to [0,\pi]$

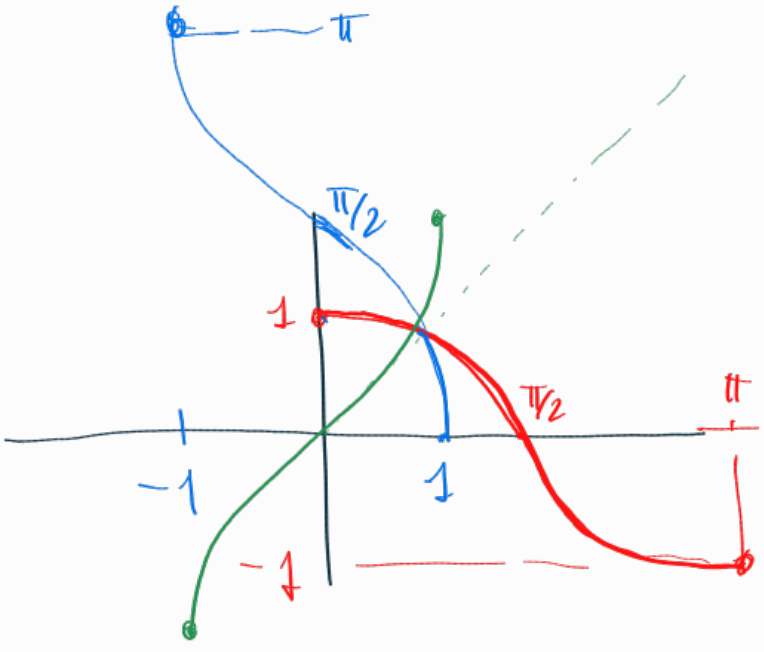
$y \mapsto \arccos y = $ l'unico $x \in [0,\pi]$ t.c. $\cos x = y$

$\arccos 1 = 0$

$\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3}{4}\pi$

$\arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$





$y = \arccos x$

$y = \arcsin x$

$y = \cos x\big|_{[0,\pi]}$

$\arccos\left(\cos\frac{7}{4}\pi\right) = \frac{\pi}{4}$

$\arccos\left(\cos\frac{5}{4}\pi\right) = \frac{3}{4}\pi$



$\arccos(\cos x) = x$ solo se $x \in [0, \pi]$

$\cos(\arccos x) = x$ $\forall x \in [-1, 1]$.

Dal grafici sembra che

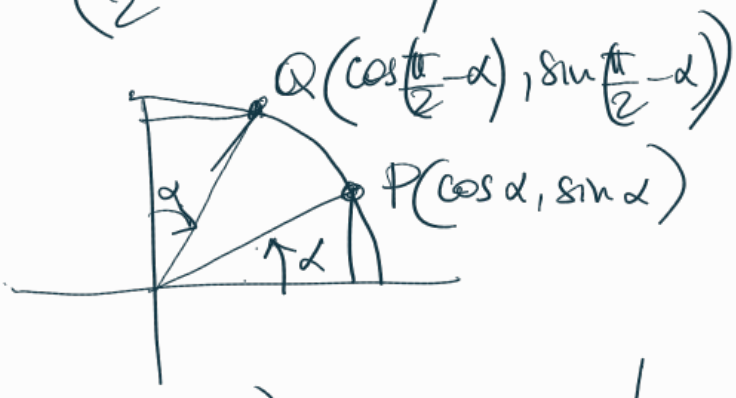
$$\arccos x = \frac{\pi}{2} - \arcsin x \qquad \forall x \in [-1, 1]$$

Sia $x \in [-1, 1]$, mi chiedo se $\frac{\pi}{2} - \arcsin x = $ l'unico $t \in [0, \pi]$ t.c $\cos t = x$.

$t = \frac{\pi}{2} - \underbrace{\arcsin x}_{\in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]} \overset{?}{\in} [0, \pi]$ sì

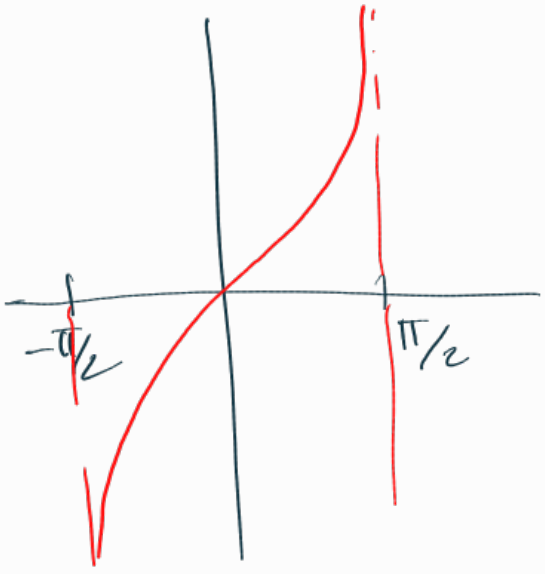
$\cos t \overset{?}{=} x$ $\qquad \cos\left(\frac{\pi}{2} - \arcsin x\right) \overset{?}{=} x$

$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$



$\cos\left(\frac{\pi}{2} - \arcsin x\right) = \sin(\arcsin x) = x$!

$\operatorname{tg} x\big|_{\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)} : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$ biiettiva



$f^{-1} = \operatorname{arctg} : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

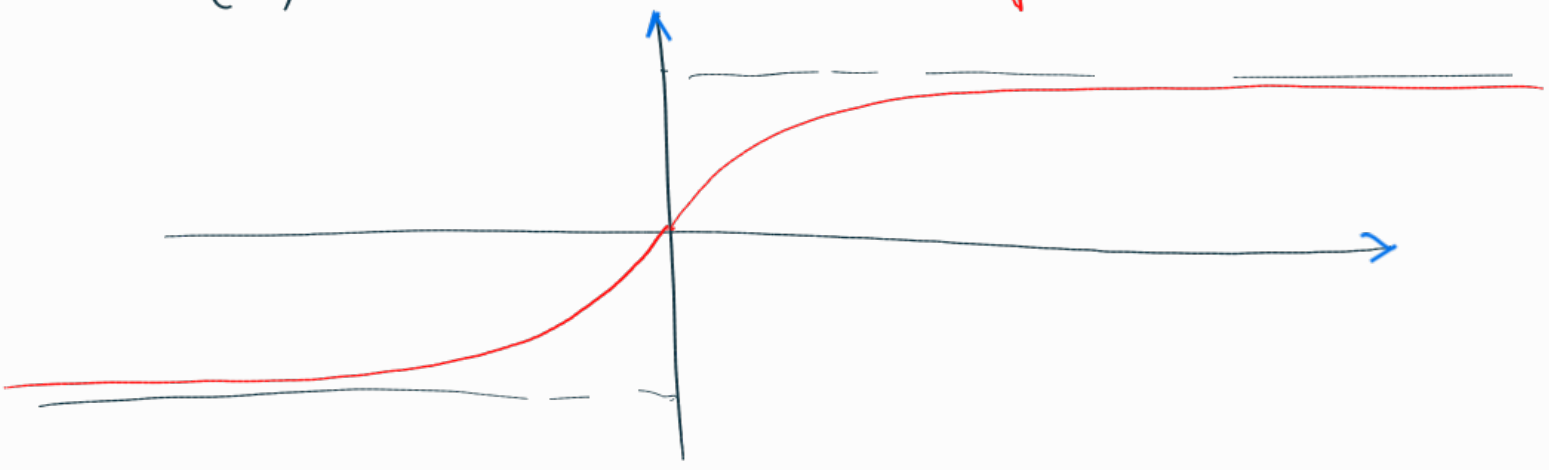
$y \mapsto \operatorname{arctg} y =$ l'unico $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ t.c. $\operatorname{tg} x = y$.

$\operatorname{arctg}\sqrt{3} = \frac{\pi}{3}$

$\arctg 1 = \frac{\pi}{4}$

$\arctg\left(\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$

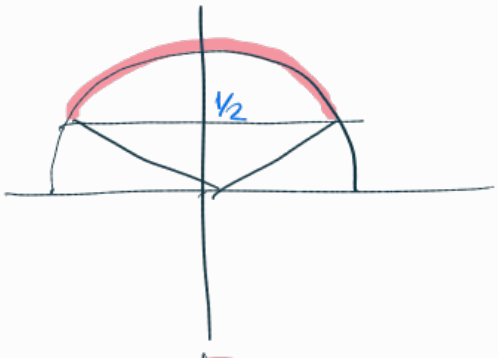
$y = \arctg x$



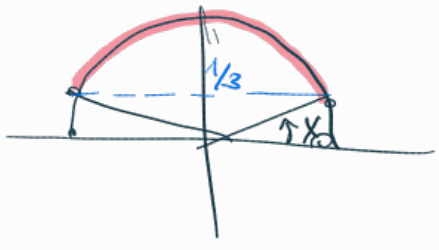
Uso delle f. inverse per risolvere diseq.

$3 \sin x > 1 \iff \sin x > \frac{1}{3}$

$\sin x > \frac{1}{2} \iff \frac{\pi}{6} + 2k\pi < x < \frac{5}{6}\pi + 2k\pi$



$\text{sen } x > \frac{1}{3} \iff$



$x_0 + 2k\pi < x < \pi - x_0 + 2k\pi$

$x_0 = \arcsin \frac{1}{3}$

soluzioni:

$\arcsin \frac{1}{3} + 2k\pi < x < \pi - \arcsin \frac{1}{3} + 2k\pi \qquad (k \in \mathbb{Z})$