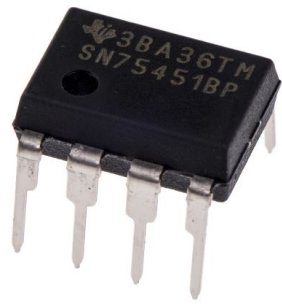


Measurements for industrial sustainability and environmental safety

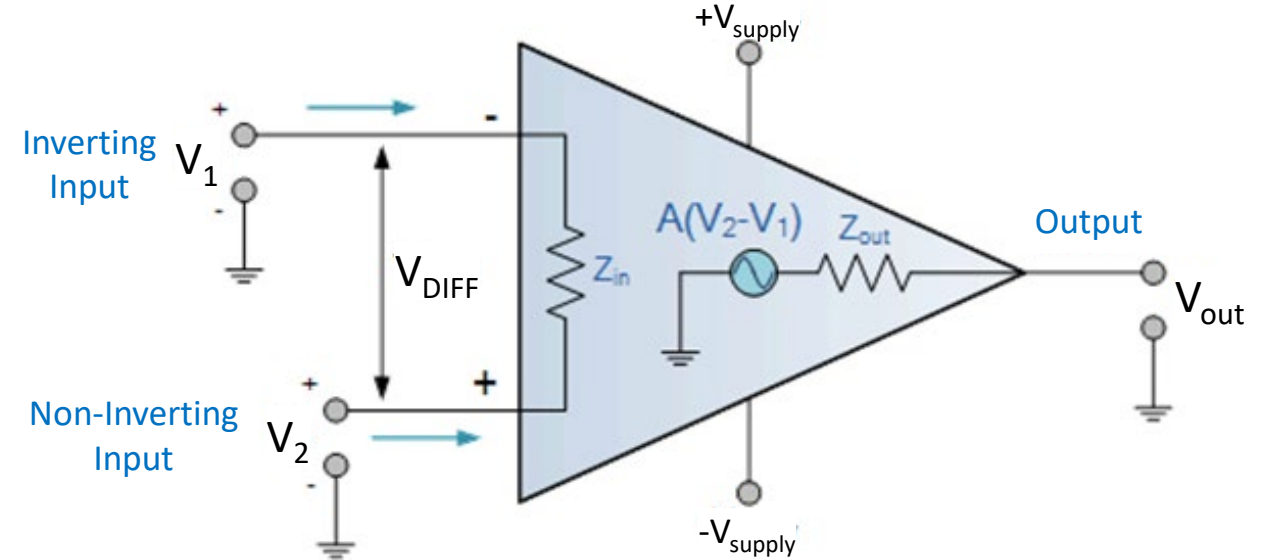
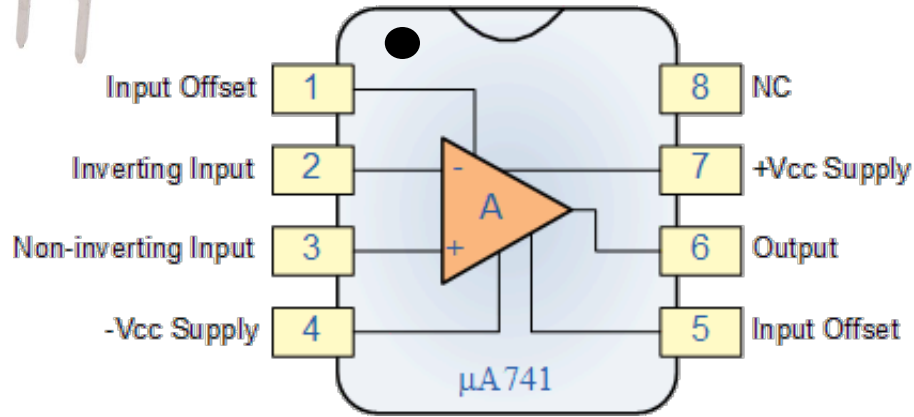
PART II
Lesson 6

A.Y. 2026-27
Ludovica Apa





The operational amplifier (OA)



Ideal OA characteristics :

Amplification

$$A \rightarrow \infty \quad (10^7)$$

Input impedance

$$Z_i \rightarrow \infty \quad (10^{10} \Omega)$$

Output impedance

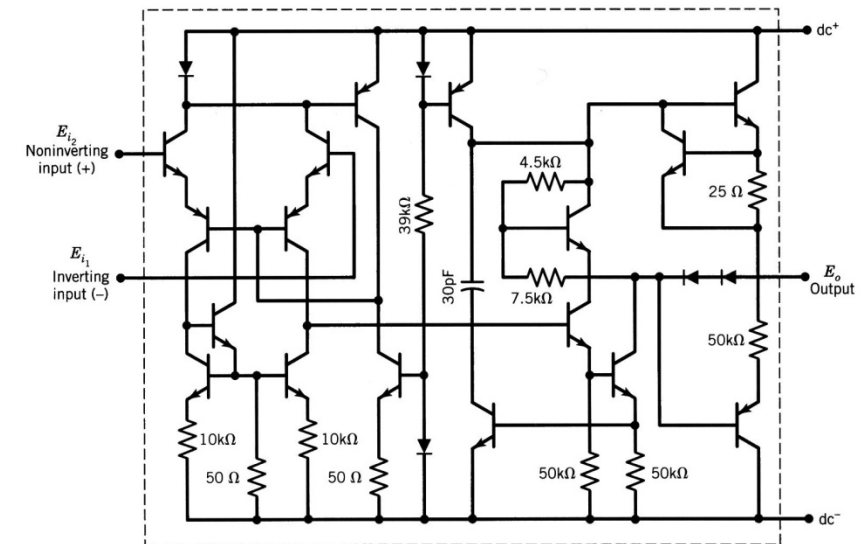
$$Z_o \rightarrow 0 \quad (100 \Omega)$$

Band width

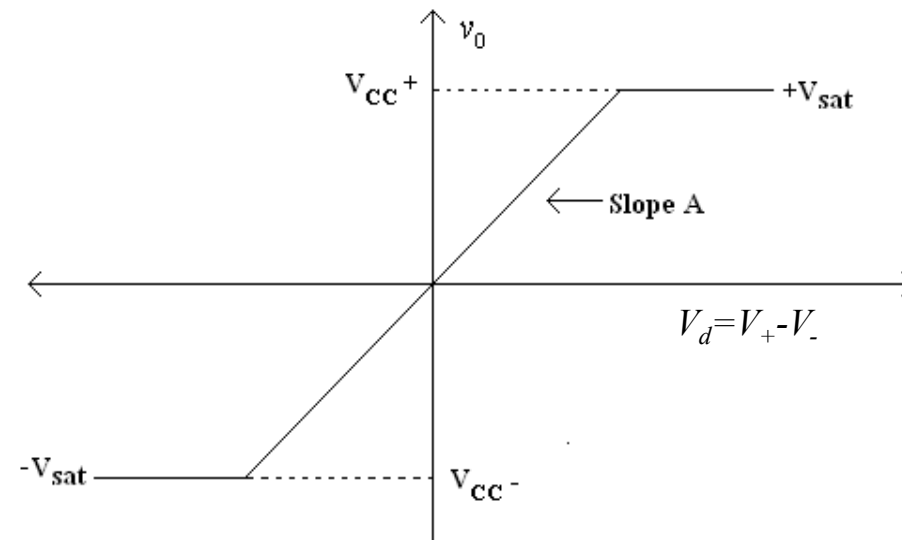
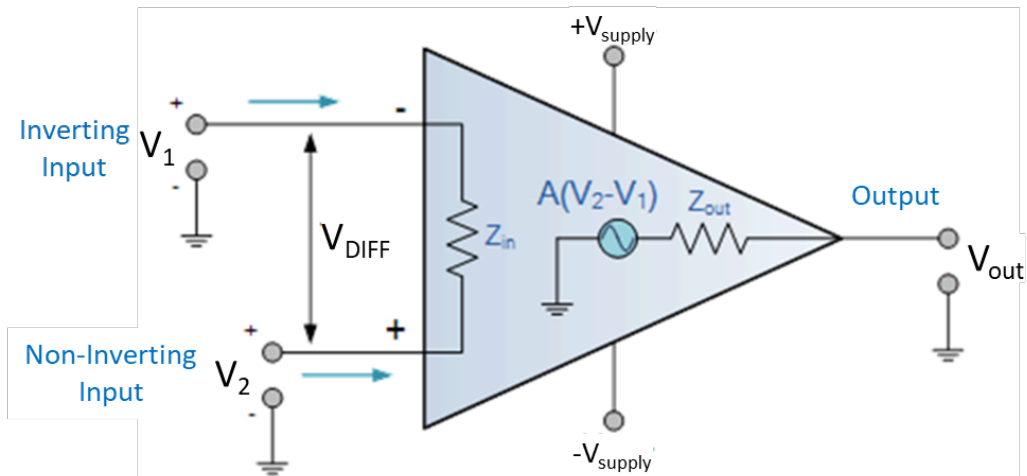
$$BW \rightarrow \infty$$

Offset voltage

$$V_{io} \rightarrow 0$$



Type 741 circuit diagram



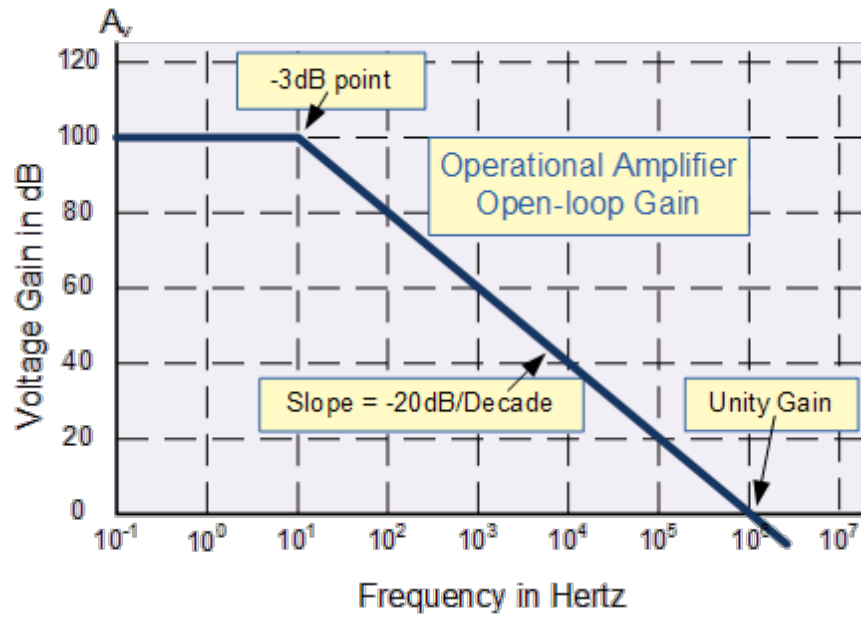
The open loop OA can amplify a voltage signal $V_o = A \cdot (V_+ - V_-)$ but NOT over its supply voltages !

Example:

If we have $V_{cc} = \pm 10\text{ V}$ and $A = 10^7$, Then $\pm 10\text{ V} = 10^7 \cdot (V_+ - V_-)_{MAX}$

and $(V_+ - V_-)_{MAX} = \frac{20\text{ V}}{10^7} = 2\mu\text{ V}$, which is the maximum input signal the OA can handle!

The OA goes into saturation for: $|V_i| = |V_+ - V_-| > 2\mu\text{ V}$



The real *open-loop frequency response* curve is determined by the **Gain Bandwidth Product (GBP)** ...

$$f_T = A_0 * f_H \quad \text{or frequency at the unitary gain}$$

The BW of the AO in the open loop configuration is very narrow

Nonetheless, the Operational Amplifier CAN NOT be used as an amplifier in the *open loop configuration* !

Inverting operational Amplifier configuration

We connect two resistors at the terminals as shown in the figure

At the virtual earth summing junction we have: $V_- \cong V_+ = 0$

- input signal: $V_i = R_i \cdot i_i$
- output signal: $V_o = R_f \cdot i_f$

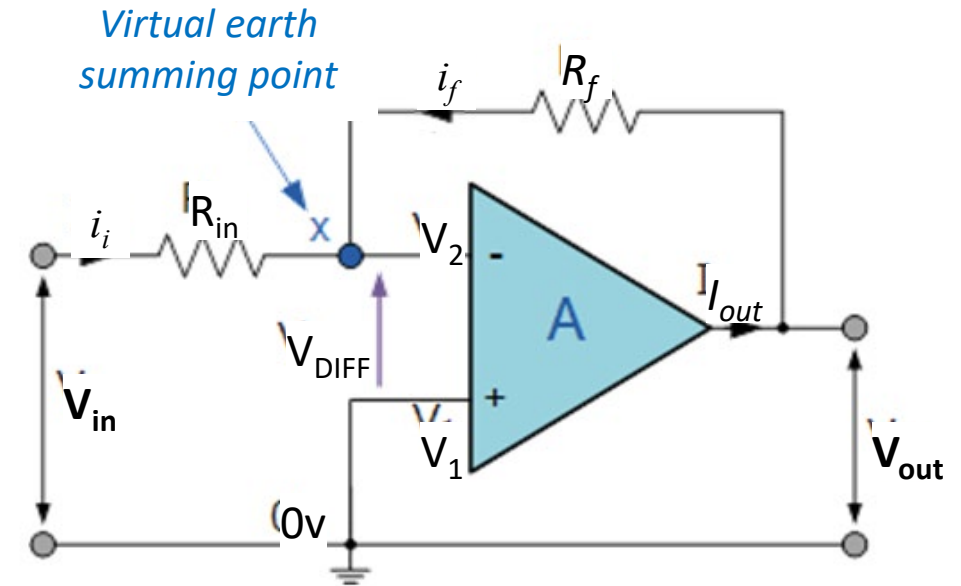
And the current at the virtual earth point is: $i_i + i_f = 0$. Now, since $i_f = -i_i$ we have:

$$V_o = -R_f i_i = -R_f \frac{V_i}{R_i}$$

$$G = \frac{V_o}{V_i} = -\frac{R_f}{R_i}$$

Static characteristic curve of the device

Gain (negative!)



Non-Inverting operational Amplifier configuration

To get a “positive” signal amplification...

Because of the “high input impedance” of the OA device, there is *virtually no current* entering the “- terminal” of the OA and all the current flows away through R_i :

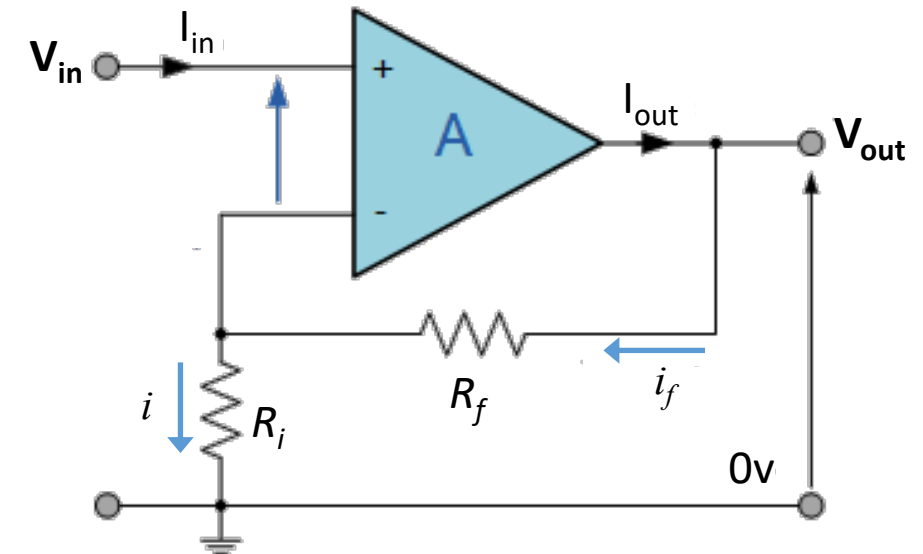
$$i_f = i \quad \text{and} \quad V_- \cong V_+ = V_i$$

→ The output signal can be written as $V_o = V_f + V_i$

where $V_f = R_f \cdot i_f$ and $V_i \cong V_- = R_i \cdot i$

In the end we have: $V_o = R_f i_f + V_i = R_f i + V_i = R_f \frac{V_i}{R_i} + V_i$

$$V_o = V_i \left(\frac{R_f}{R_i} + 1 \right)$$



The **static characteristic curve**

Positive GAIN !

Both configurations have *much lower amplification* than in the open loop configuration ($10^6 - 10^7$). However the amplification “can be designed” by choosing the two resistance values !

Which value for the resistors?

$$V_o = V_i \left(\frac{R_f}{R_i} + 1 \right)$$

$$R_f = 8 \text{ M}\Omega$$

$$R_i = 2 \text{ M}\Omega$$

$$R_f = 8 \text{ k}\Omega$$

$$R_i = 2 \text{ k}\Omega$$

$$R_f = 8 \text{ }\Omega$$

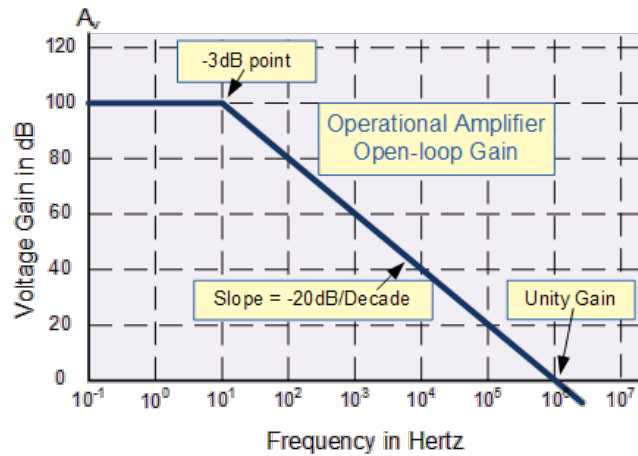
$$R_i = 2 \text{ }\Omega$$

The gain is the same, but the resistors are not the same!

For low values of R, the current values are high, so that there is a high dissipation of power.

- To keep low the power dissipation it is good to use high values of R.
- However, if R values are too high, the circulating current value could be very low, and become comparable with the current entering the AO... hypothesis?
- Even small termic or offset current might affect AO

...k Ω ...



$$f_T = A_0 * f_H$$

or frequency at the unitary gain

The BW of the AO in the open loop configuration is very narrow

GBW can be used to calculate the closed-loop gain bandwidth:

If GBW is 10 MHz and the closed loop gain is 10 : closed loop BW = 10 MHz/10 = 1 MHz

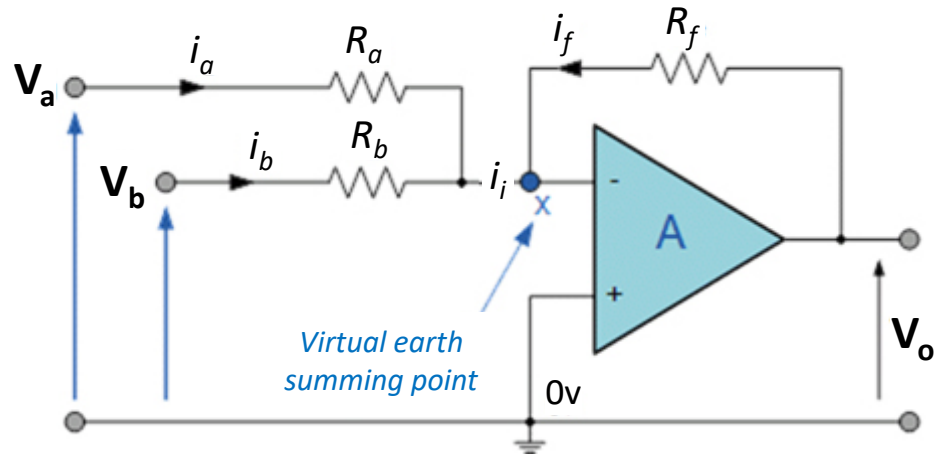
If GBW is 1 MHz and the closed loop gain is 70 db, which is the closed loop BW ?

$$\text{gain : } 20\log A = 70\text{db} \rightarrow \log A = 70/20 = 3.5 \rightarrow A = 10^{3.5} = 3162$$

$$\text{Closed-loop BW} = 1 \text{ MHz} / 3162 = 316 \text{ Hz}$$

Even if the non-inverting configuration may seem preferable, the inverting configuration has important applications ...

The voltage summing OA :



Here we wish to sum two voltages V_a and V_b

For the inverting OA we have: $V_o = -\frac{R_f}{R_i} V_i = -R_f i_i$

with $i_i = \frac{V_i}{R_i} = i_a + i_b = \frac{V_a}{R_a} + \frac{V_b}{R_b}$

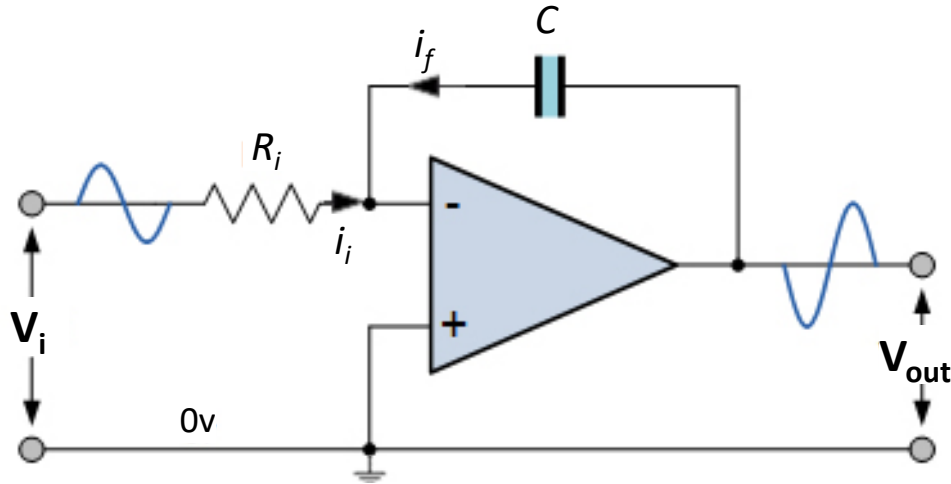
If we design the circuit inputs with the two resistance values being equal: $R_a = R_b = R$

We can simplify the above equation and we obtain :
which is the ***sum of the two input voltages*** !

$$V_o = -\frac{R_f}{R} (V_a + V_b)$$

Note that summing two voltages means physically to sum the two input currents !

The active low-pass filter (integrator) :

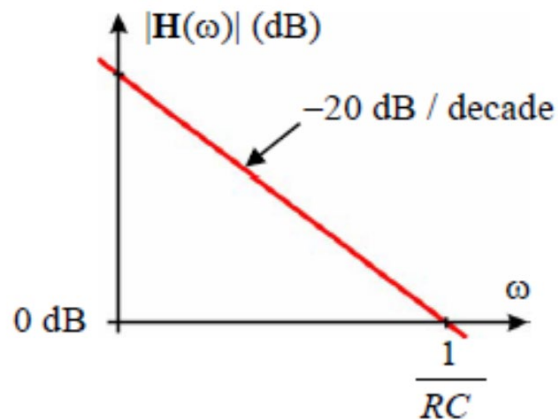


The gain of the inverting OA is: $v_o = -\frac{Z_f}{Z_i} v_i$

with $Z_i = R_i$ and $Z_f = X_c = \frac{1}{j\omega C}$

Therefore $v_o = -\frac{1/j\omega C}{R_i} v_i = -\frac{1}{j\omega C R_i} v_i = -\frac{1}{j\omega} \omega_c v_i$

the *output signal* is the electric integral of the *input signal* !



For $\omega \rightarrow 0$, $G \rightarrow \infty$

- C becomes charged, and behaves as an open circuit
- The integrator behaves as an open-loop AO
- The continuous component *yields to an infinite* V_{out}

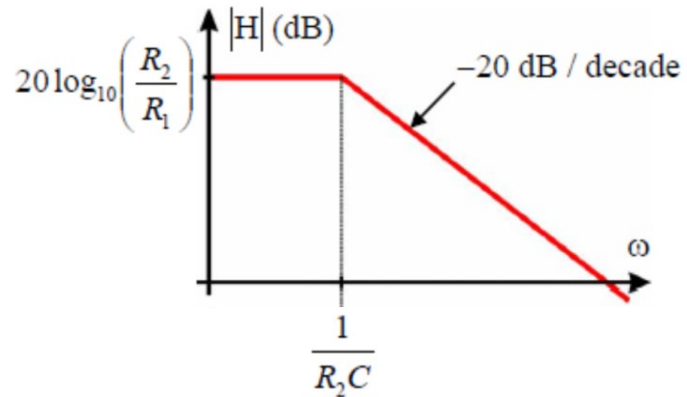
The active low-pass filter (integrator) :

These issues may be solved adding a resistor R_2 in parallel to the capacitor C

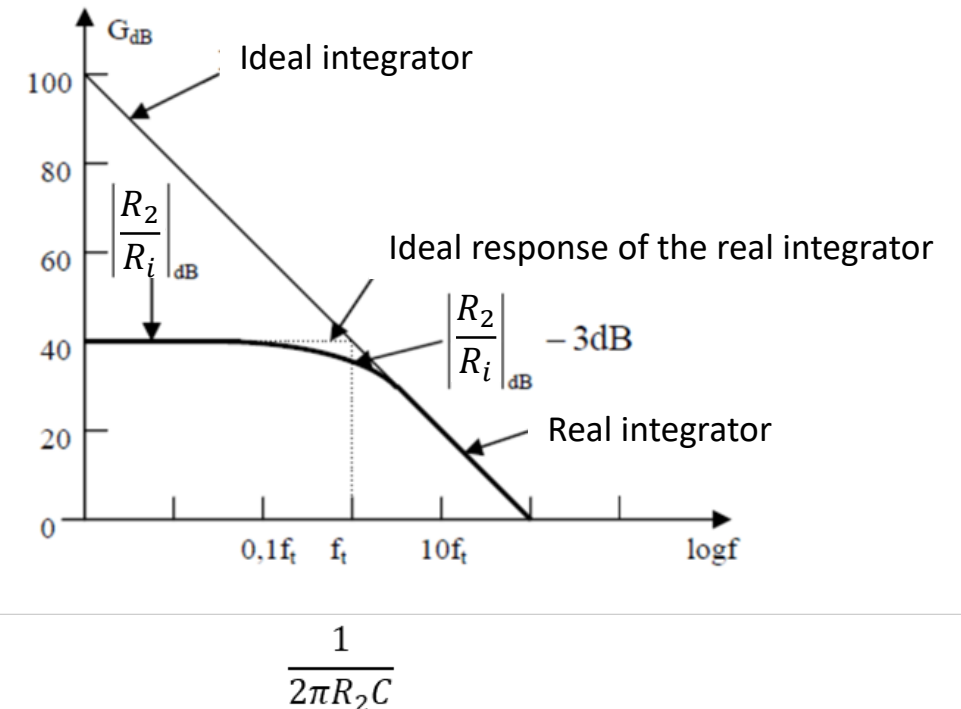
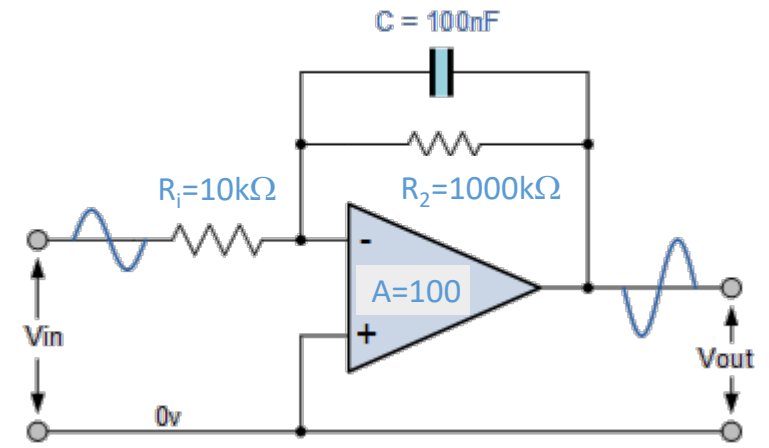
- It behaves as different from the ideal integrator as R_2 decreases

$$v_0 = -\frac{R_2}{R_i} \frac{1}{1 + j\omega R_2 C}$$

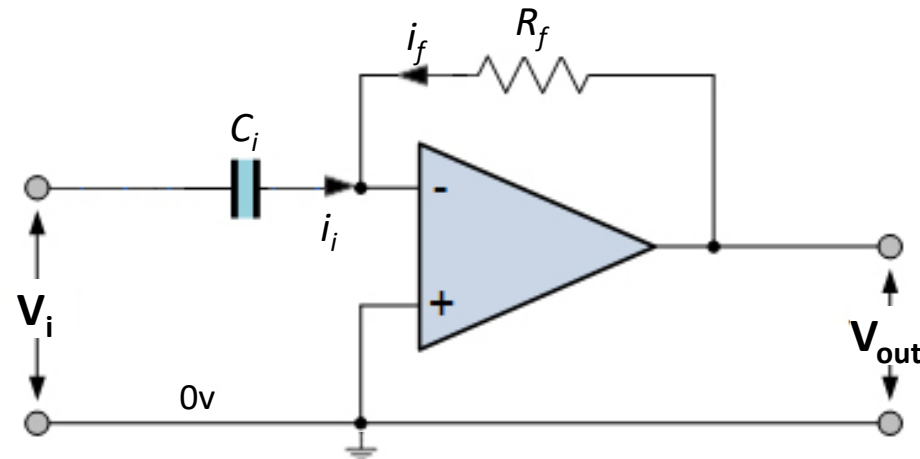
$$\omega_t = \frac{1}{R_2 C} \rightarrow f_t = \frac{1}{2\pi R_2 C}$$



Amplifier / Integrator



The active high-pass filter (derivative) :

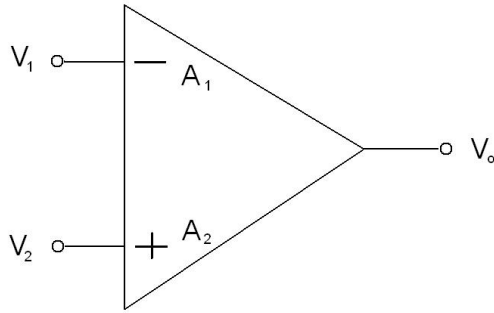


The gain is still:
$$v_o = -\frac{Z_f}{Z_i} v_i$$

with $Z_i = X_{C_i} = \frac{1}{j\omega C_i}$ and $Z_f = R_f$

so
$$v_o = -\frac{R_f}{1/j\omega C_i} v_i = -j\omega \cdot C_i R_f \cdot v_i = -j\omega \cdot \frac{1}{\omega_c} \cdot v_i$$

the electric derivative !



Consider now that the OA accepts TWO voltages at the input terminals $V_+ = V_2$ and $V_- = V_1$ both referred to earth, and amplifies its difference:
 $V_o = A(V_2 - V_1)$... But this is true in the IDEAL case !

In the reality, there are TWO slightly different amplifications at the inputs:
 $V_o = A_2V_2 - A_1V_1$... (due to imperfections..)

This situation can be effectively described by the **differential input** : $V_d = V_2 - V_1$ and the **common mode input** : $V_c = \frac{V_1 + V_2}{2}$ which is the mean distance from the earth reference of the two input voltages.

And by the **differential amplification** : $A_d = \frac{A_1 + A_2}{2}$ which operates on V_d and by the **common mode amplification** : $A_c = A_2 - A_1$ which operates on V_c

$$V_o = A_2V_2 - A_1V_1 = A_dV_d + A_cV_c$$

For an ideal OA we would have $A_1 = A_2 = A$ and the OA would amplify only the differential input, completely eliminating the common mode input ! $A_c = 0$ $A_d = A$

To minimize this problem the OA must be designed with a high A_d and a very small A_c which means $A_1 \approx A_2$

$$A_d = \frac{A_1 + A_2}{2}$$

$$A_c = A_2 - A_1$$

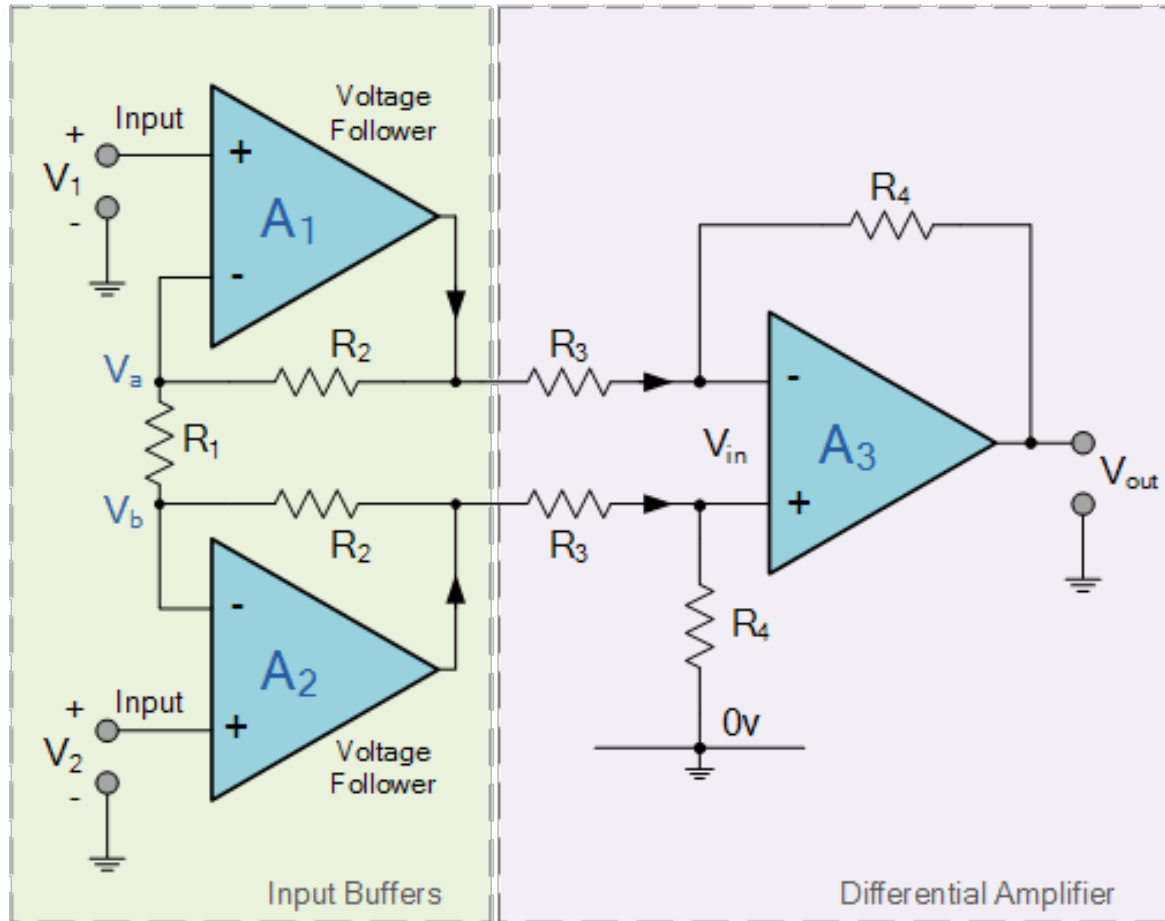
The ratio between the two amplifications is an important quality parameter of the OA, the **Common Mode**

Rejection Ratio : $CMRR = \frac{A_d}{A_c}$ often expressed in logarithmic scale

Values range from 60 dB up to 120 Db for high quality OA ...

$$CMRR = 20 \log \frac{A_d}{A_c}$$

The Instrumentation Amplifier IA :



The most employed amplifier in measurements is designed with two stages:

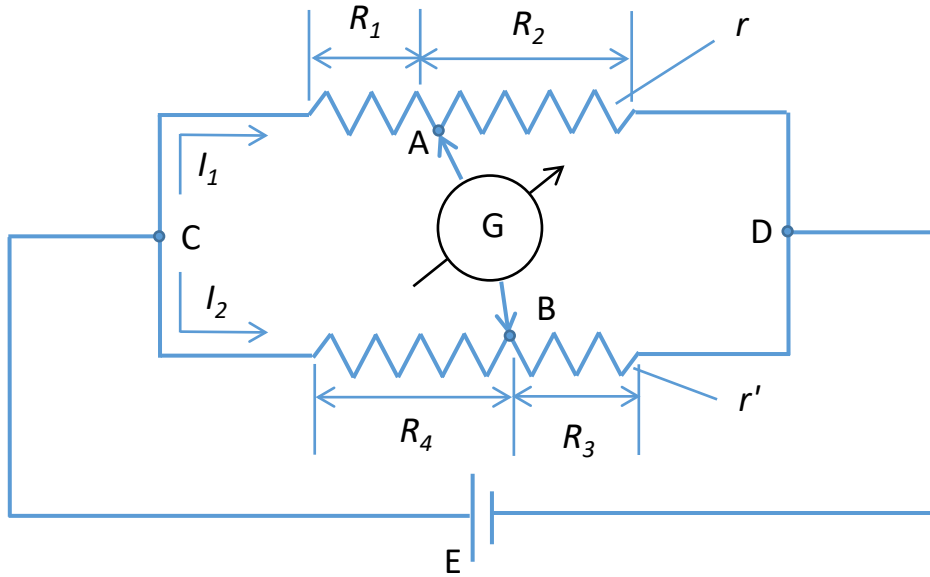
1. A very high input impedance stage ($10^{10} \Omega$)
2. A differential amplification stage

It has generally a very high **Common Mode Rejection Ratio** ($> 100 \text{ dB}$) which makes it suitable to amplify *floating signals* ($v_2 - v_1$) ...

The (differential) **GAIN** is :

$$G = \frac{v_o}{v_i} = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1} \right)$$

The Wheatstone Bridge



Two resistances r and r' are connected in parallel and a *galvanometer* G is connected in a bridge configuration across the two resistances in the points A and B ...

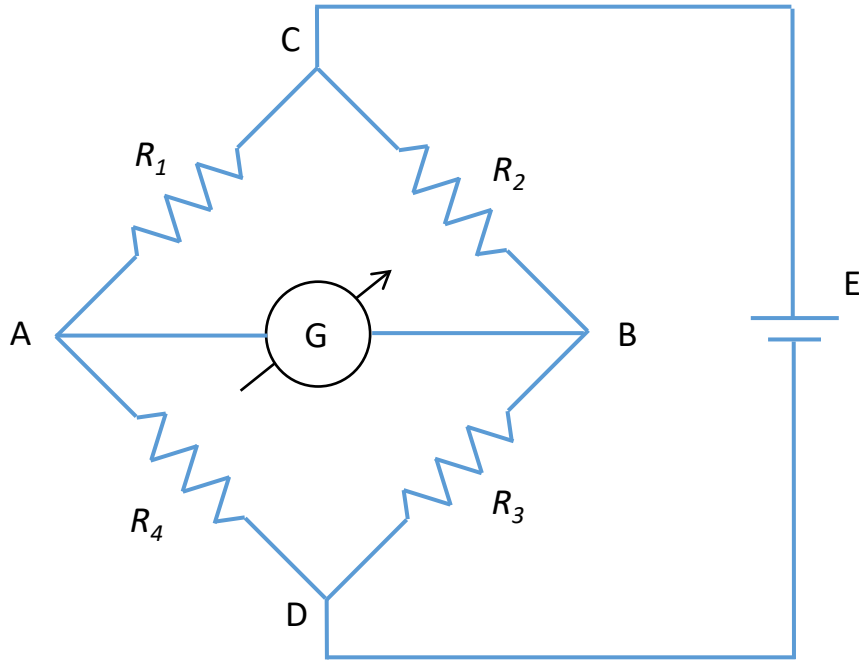
We move now the point A and B on the resistances so to have zero current though the galvanometer, which means also: $V_A = V_B$

In this situation we identified 4 resistances : R_1, R_2, R_3, R_4 for which we can write the following equations:

$$\begin{aligned} V_{CA} &= V_{CB} & \rightarrow & R_1 I_1 = R_4 I_2 \\ V_{AD} &= V_{BD} & \rightarrow & R_2 I_1 = R_3 I_2 \end{aligned}$$

Making the ratio of the two equations, we get: $\frac{R_1}{R_2} = \frac{R_4}{R_3}$ or $\boxed{R_1 R_3 = R_2 R_4}$ the bridge equilibrium equation !

No current through the bridge means also $V_A - V_B = 0$ therefore, in practical application, we can substitute the *galvanometer* with a *millivoltmeter* !



The **Wheatstone Bridge** is a “zero method” electric network used for resistance measurements:

If $R_x = R_1$ is unknown, we equilibrate the bridge by operating on R_2 and, knowing the values of R_2, R_3, R_4 , we get:

$$R_x = R_1 = \frac{R_2 R_4}{R_3}$$

However, this is NOT the main use of the Wheatstone Bridge ...

We might have a resistance R_1 that changes slightly its value ΔR_1 for physical reasons ...

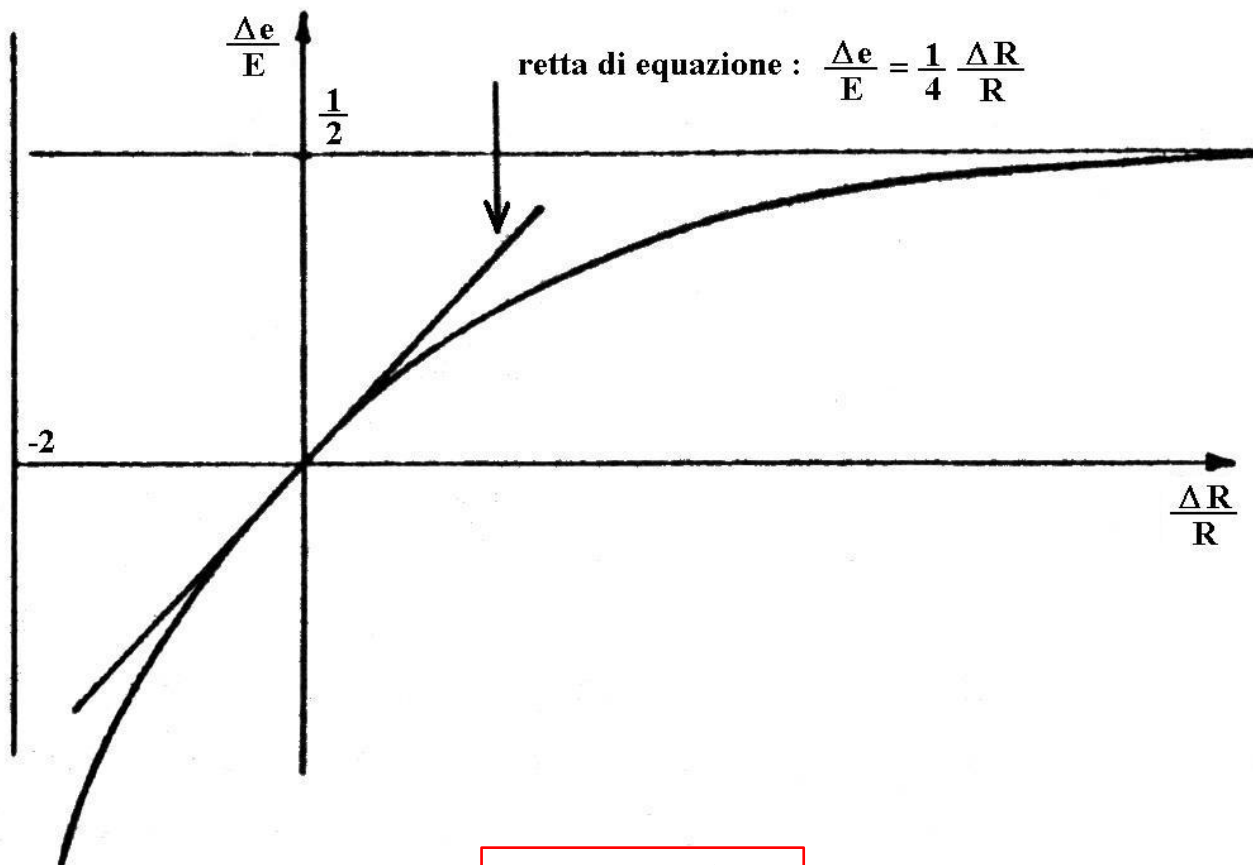
It is possible to read this small change directly on the voltmeter indicator but, to do so, we need to know the ***bridge graduation curve*** (or static characteristic equation): $V_{AB} = f(\Delta R_1)$

If we consider for simplicity a bridge with 4 equal resistances: $R_1 = R_2 = R_3 = R_4 = R$ and a variation ΔR_1 only on the resistance R_1 , we get:

The complete **Wheatstone Bridge characteristic curve** :

$$\frac{\Delta e}{E} = \frac{\frac{1}{4} \cdot \frac{\Delta R_1}{R}}{1 + \frac{1}{2} \cdot \frac{\Delta R_1}{R}}$$

which is clearly NOT linear !



However, if the variation $\Delta R_1 \ll R$ is

really very small: $\frac{\Delta R_1}{R} < 0.01$

Then, the denominator of the graduation curve can be approximated with "1"

$$\frac{1}{2} \frac{\Delta R_1}{R} \ll 1$$

and the *characteristic equation has been linearized* !

$$\frac{\Delta e}{E} \cong \frac{1}{4} \cdot \frac{\Delta R_1}{R}$$

Linearized Wheatstone Bridge characteristic curve !

Same result if the only resistance with a small variation ΔR_2 would have been R_2 on the second arm of the bridge:

$$\frac{\Delta e}{E} \cong -\frac{1}{4} \cdot \frac{\Delta R_2}{R}$$

negative because R_2 is in the negative term of the equilibrium equation: $R_1 R_3 - R_2 R_4 = 0$

In the case of simultaneous small variations of all 4 resistances of the Wheatstone Bridge , we would get:

$$\frac{\Delta e}{E} = \frac{1}{4} \left(\frac{\Delta R_1}{R} - \frac{\Delta R_2}{R} + \frac{\Delta R_3}{R} - \frac{\Delta R_4}{R} \right)$$

The ***Full Wheatstone Bridge characteristic curve*** ! (note the sign alternations ... very important property ...)