

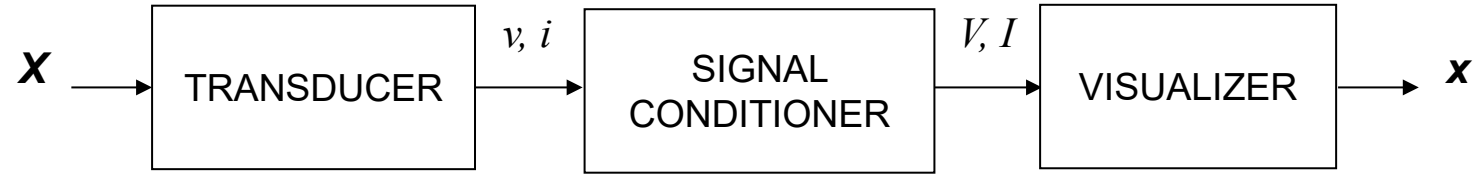
Measurements for industrial sustainability and environmental safety

PART II
Lesson 5

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We will now analyze more in depth each one of the *functional blocks* of the general **measurement chain** ...

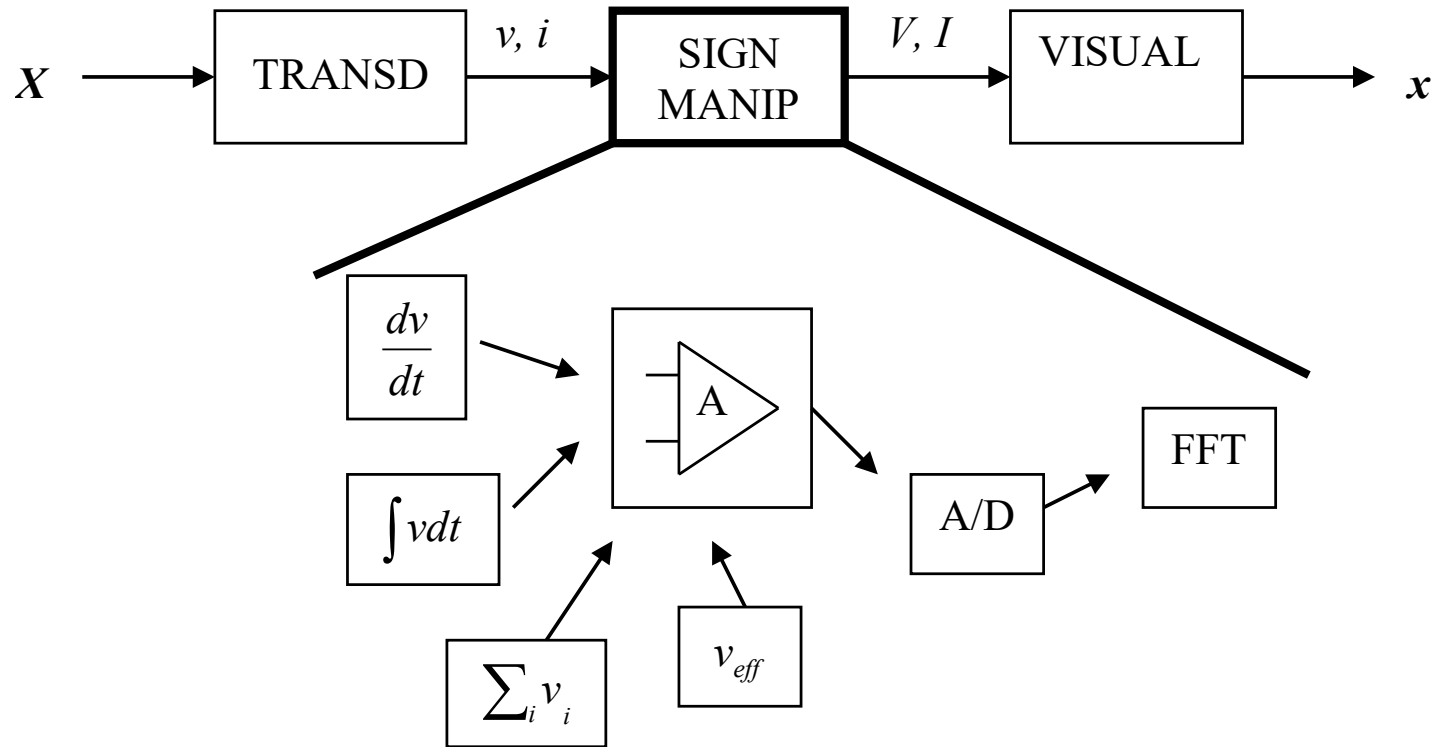


In the industry and in most modern measurement applications, the *measurement chain is electric* and the information passes through the different stages as **voltage signal** or as a **current** !

Despite the enormous technological advancements of the last 30 years, the main electrical visualizer instruments can still be included in only **three** classes:

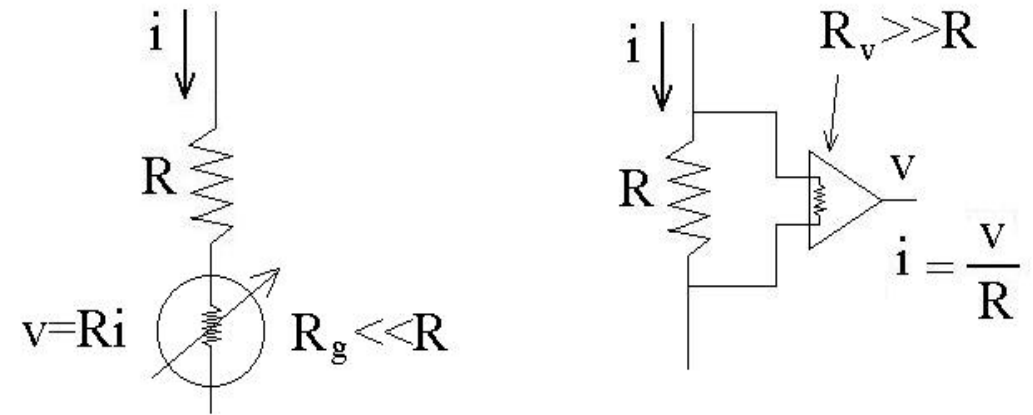
1. The **galvanometer**
2. The **oscilloscope**
3. The **digital voltmeter**

We will now study the most important **signal manipulation circuits** used in measurement instruments :



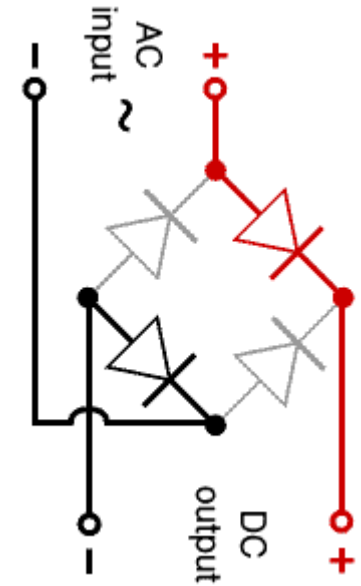
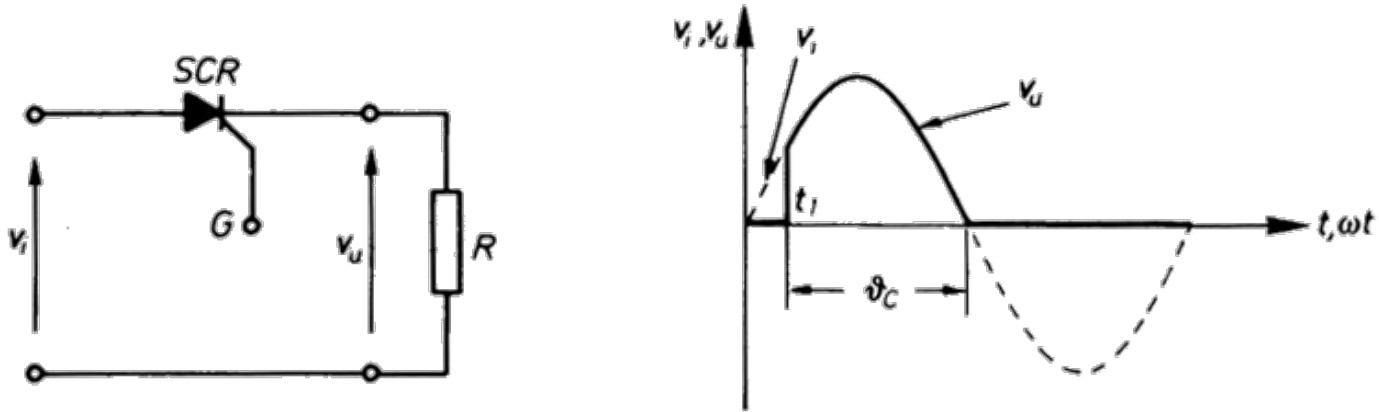
In modern and complex industry measurement system, the signal outputted by the transducer is almost always an *electrical signal v or i* for which, most of the time, we need to enhance the information content (the **intensity of the measurand**) !

It is always possible to measure a voltage with an amperometer and a current with a voltmeter, but we have to remember the different needs ...

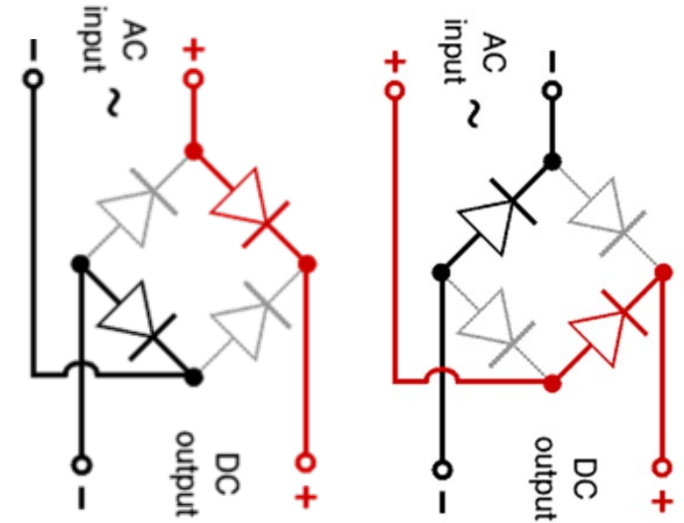
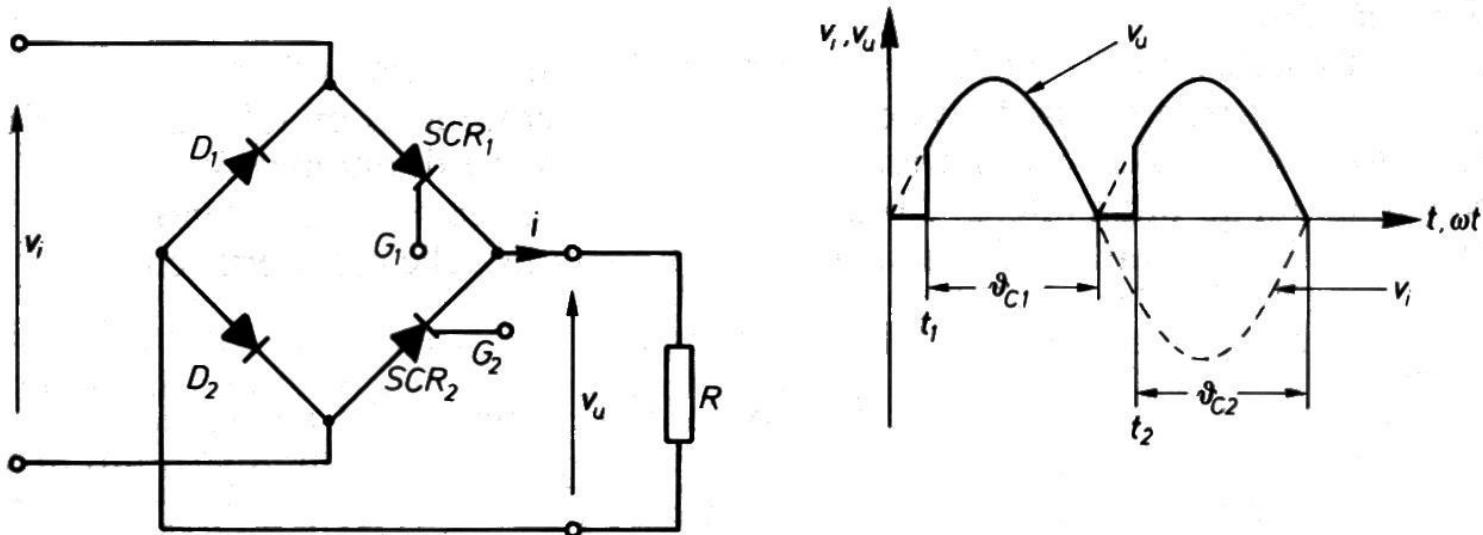


The rectifier diode :

a) The **SCR diode** works here as a half-wave rectifier !



b) The **Graetz bridge** with 4 SCR diodes works here as a full-wave rectifier !



Preliminary considerations

If we consider, for example, a sinusoidal signal $v_i(t) = V \sin \omega t = V e^{j\omega t}$
and we *integrate* and *derivate* it :

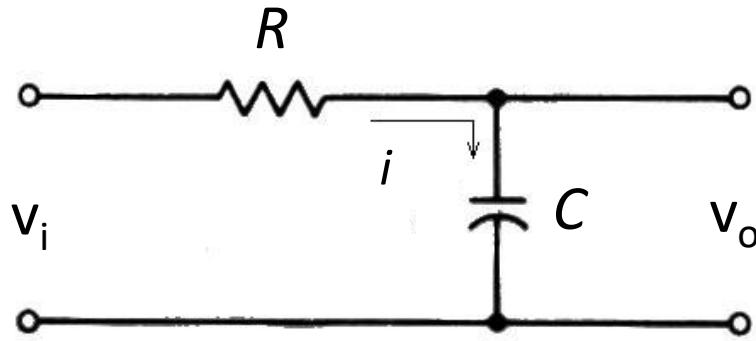
$$\int v_i(t) dt = \int V e^{j\omega t} dt = \frac{V}{j\omega} \cdot e^{j\omega t} = \left(\frac{1}{j\omega} \right) \cdot v_i(t)$$

$\left(\frac{1}{j\omega} \right) \rightarrow$ is an integration operator

$$\frac{dv_i(t)}{dt} = \frac{dV e^{j\omega t}}{dt} j\omega \cdot V e^{j\omega t} = (j\omega) \cdot v_i(t)$$

$(j\omega) \rightarrow$ is a derivative operator

The low-pass “RC” circuit



Let's study the *two electric circuits*

input (voltage) signal: $v_i = (R + X_C) \cdot i$

output (voltage) signal: $v_o = X_C \cdot i$

the current is $i = \frac{v_i}{R + X_C}$ and the output voltage: $v_o = X_C \cdot \frac{v_i}{R + X_C} = \frac{v_i}{\frac{R}{X_C} + 1}$

but $X_C = \frac{1}{j\omega C}$ is the **capacitive reactance**, therefore the output/input ratio is : $\frac{v_o}{v_i} = \frac{1}{j\omega RC + 1}$

Considering $\omega_c = \frac{1}{RC} = \frac{1}{\lambda}$ the output/input ratio is also:

$$\frac{v_o}{v_i} = \frac{1}{j\omega\lambda + 1}$$

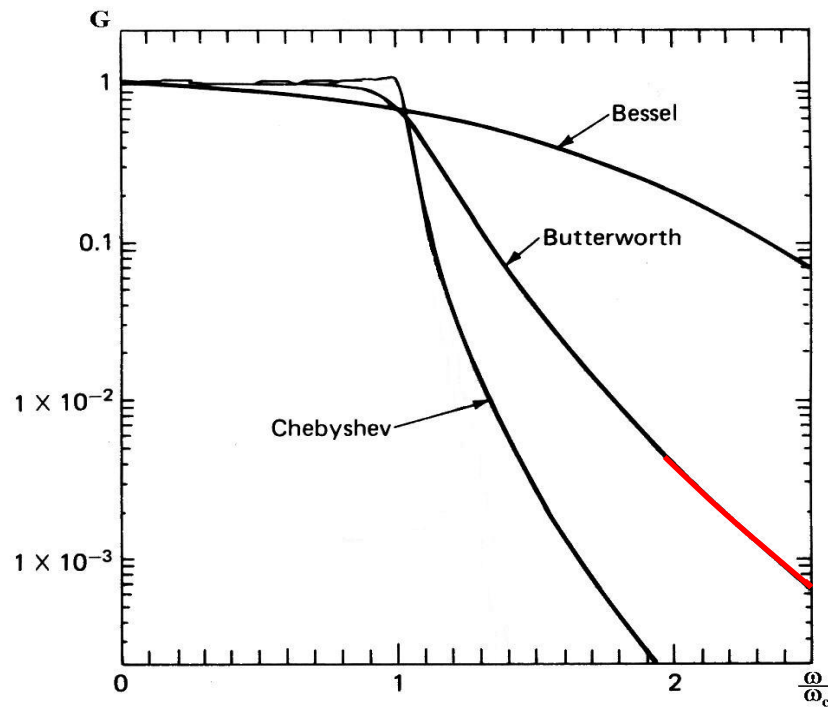
time constant

We can calculate the modulus of the previous complex function :

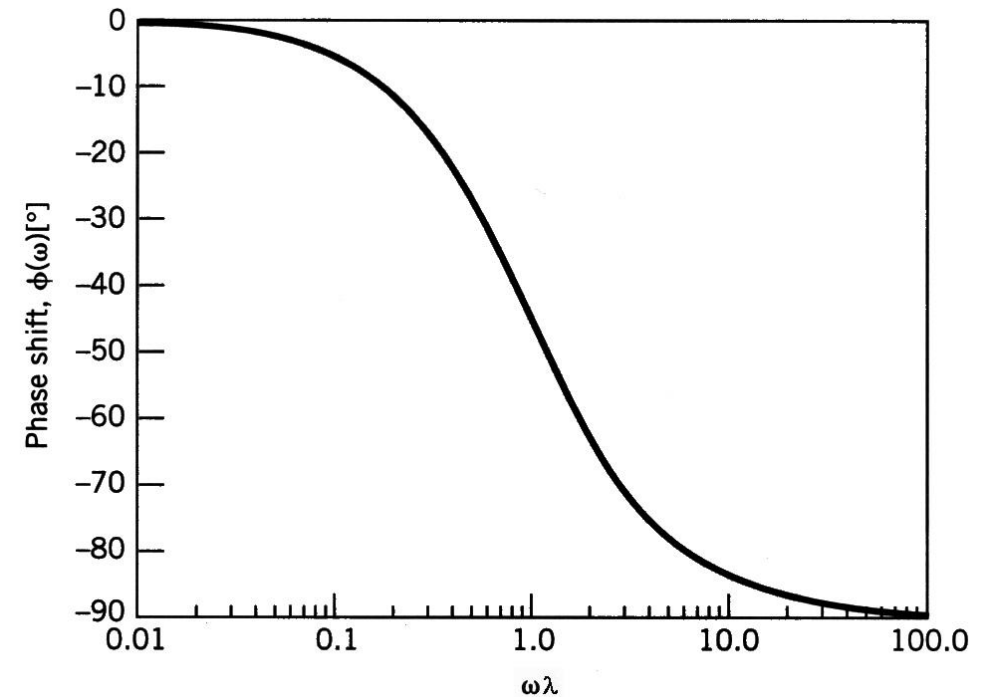
And the phase delay: $\varphi = \arctg(-\omega\lambda)$

$$G = \frac{v_o}{v_i} = \frac{1}{\sqrt{\left(\frac{\omega}{\omega_c}\right)^2 + 1}}$$

We recognize the frequency response of a simple **1st order electric signal manipulation** stage !

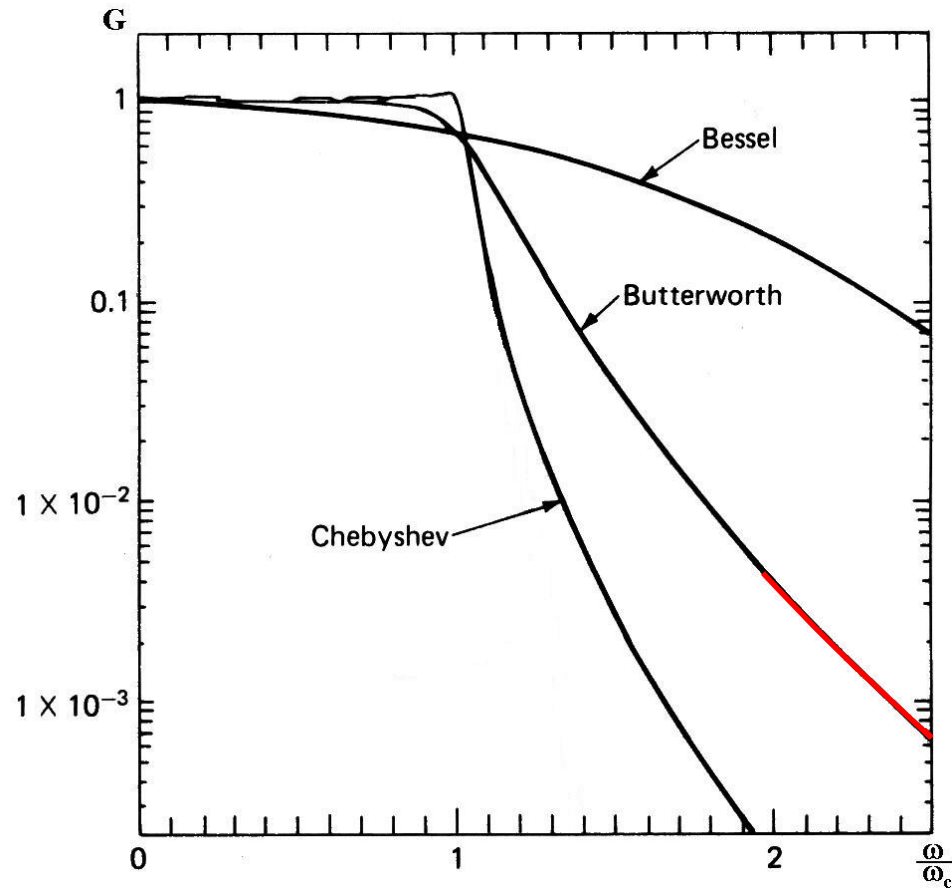


$$\begin{aligned}\omega &= 0 \\ \omega &\rightarrow \infty \\ \omega &= \omega_c\end{aligned}$$



The RC circuit is a low-pass filter (**1 pole Butterworth filter**) with a -3 db cut-off frequency:

$$f_c = \frac{1}{2\pi RC}$$



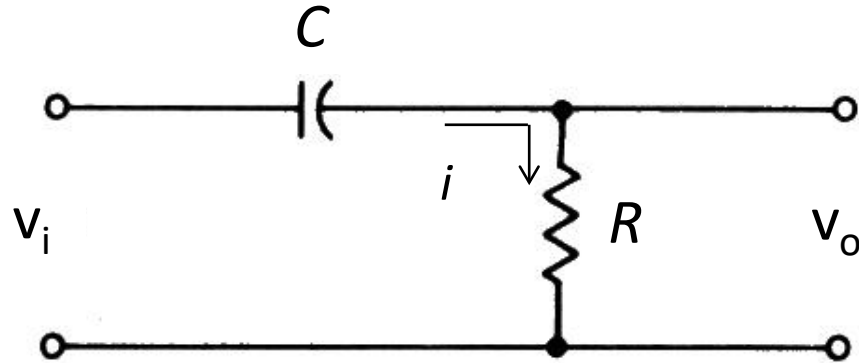
$$\frac{v_o}{v_i} = \frac{1}{j\omega RC + 1}$$

Note that for $|j\omega CR| \gg 1$ which is $\omega \gg \frac{1}{CR} = \omega_c$

the *output signal* is the “integral” of the input signal

$$v_o(t) = \frac{1}{j\omega} \cdot \frac{1}{\lambda} v_i(t)$$

The high-pass “CR” circuit



The *two electric circuits'* equations now are:

input (voltage) signal: $v_i = (X_C + R) \cdot i$

output (voltage) signal: $v_o = R \cdot i$

which now result in : $v_o = R \cdot \frac{v_i}{X_C + R}$ with the same notations: $X_C = \frac{1}{j\omega C}$ $\omega_c = \frac{1}{RC} = \frac{1}{\lambda}$

The output/input ratio is : $\frac{v_o}{v_i} = \frac{R}{X_C + R} = \frac{j\omega CR}{1 + j\omega CR} = \frac{j\omega\lambda}{1 + j\omega\lambda}$

It is also a **1st order electric signal manipulation stage** with a **gain** of :

$$G = \frac{v_o}{v_i} = \frac{\frac{\omega}{\omega_c}}{\sqrt{\left(\frac{\omega}{\omega_c}\right)^2 + 1}}$$

for

$$\omega = 0$$

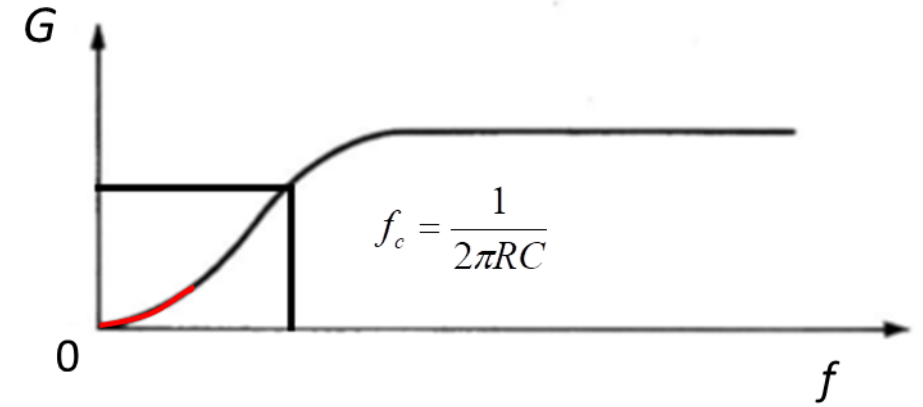
$$G = 0$$

$$\omega \rightarrow \infty$$

$$G = 1$$

$$\omega = \omega_c$$

$$G = \frac{1}{\sqrt{2}}$$



$$\varphi = \arctg\left(\frac{1}{\omega\lambda}\right)$$

for

$$\omega = 0$$

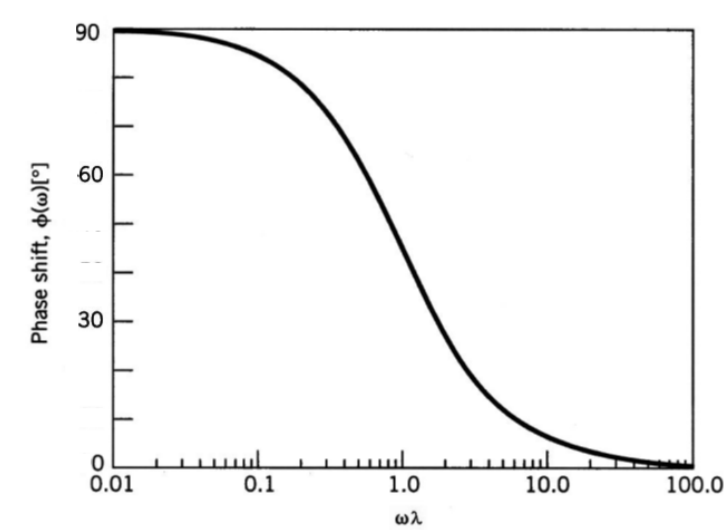
$$\varphi = 90^\circ$$

$$\omega \rightarrow \infty$$

$$\varphi = 0^\circ$$

$$\omega = \omega_c$$

$$\varphi = 45^\circ$$



$$\frac{v_o}{v_i} = \frac{R}{X_C + R} = \frac{j\omega CR}{1 + j\omega CR}$$

For $|j\omega CR| \ll 1$ or $\omega \ll \frac{1}{CR} = \omega_c$

we get $\frac{v_o}{v_i} \cong j\omega CR \ll 1$

$$\rightarrow v_o(t) = \lambda \cdot j\omega \cdot v_i(t)$$

the *output signal* is the “derivative” of the *input signal* !

