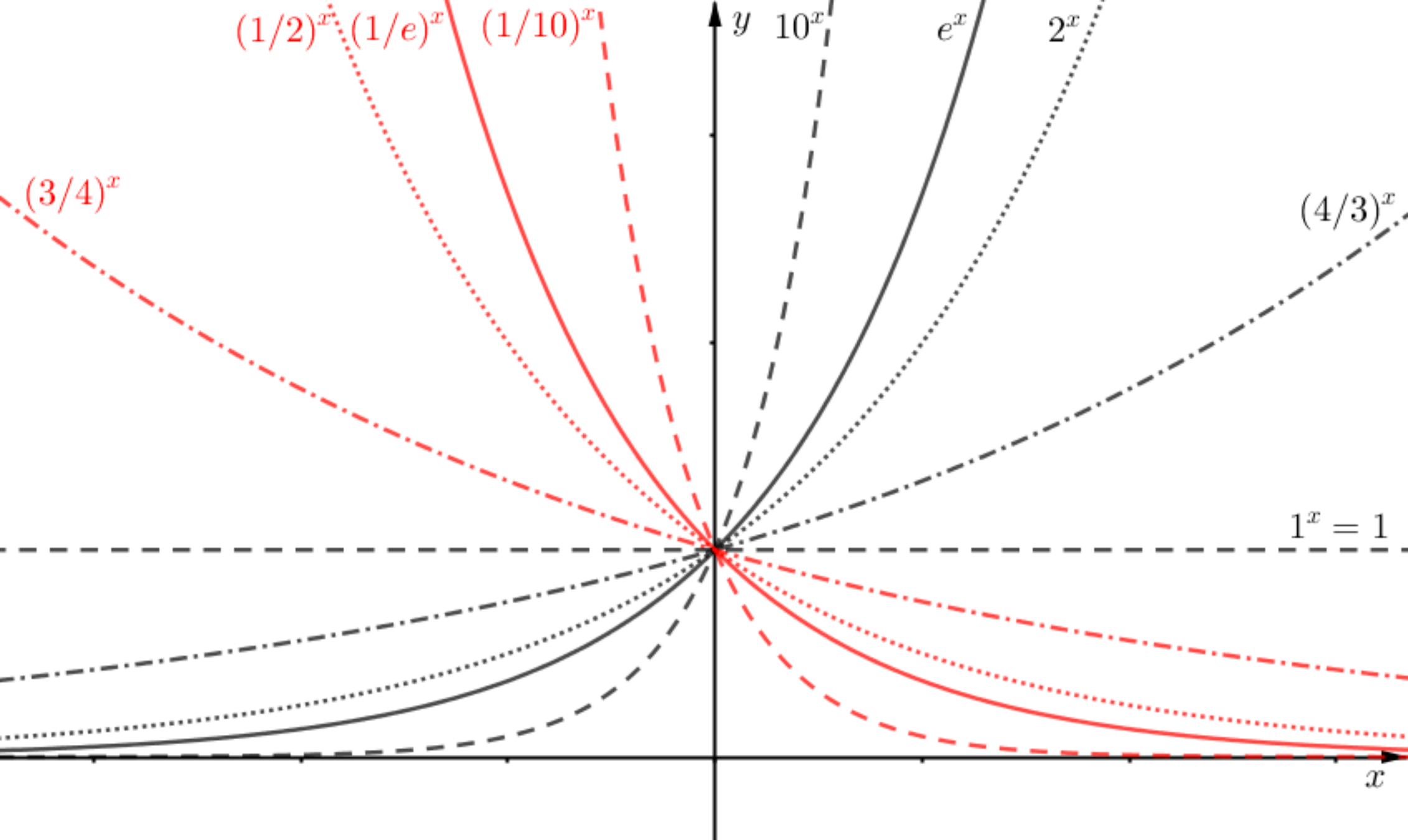




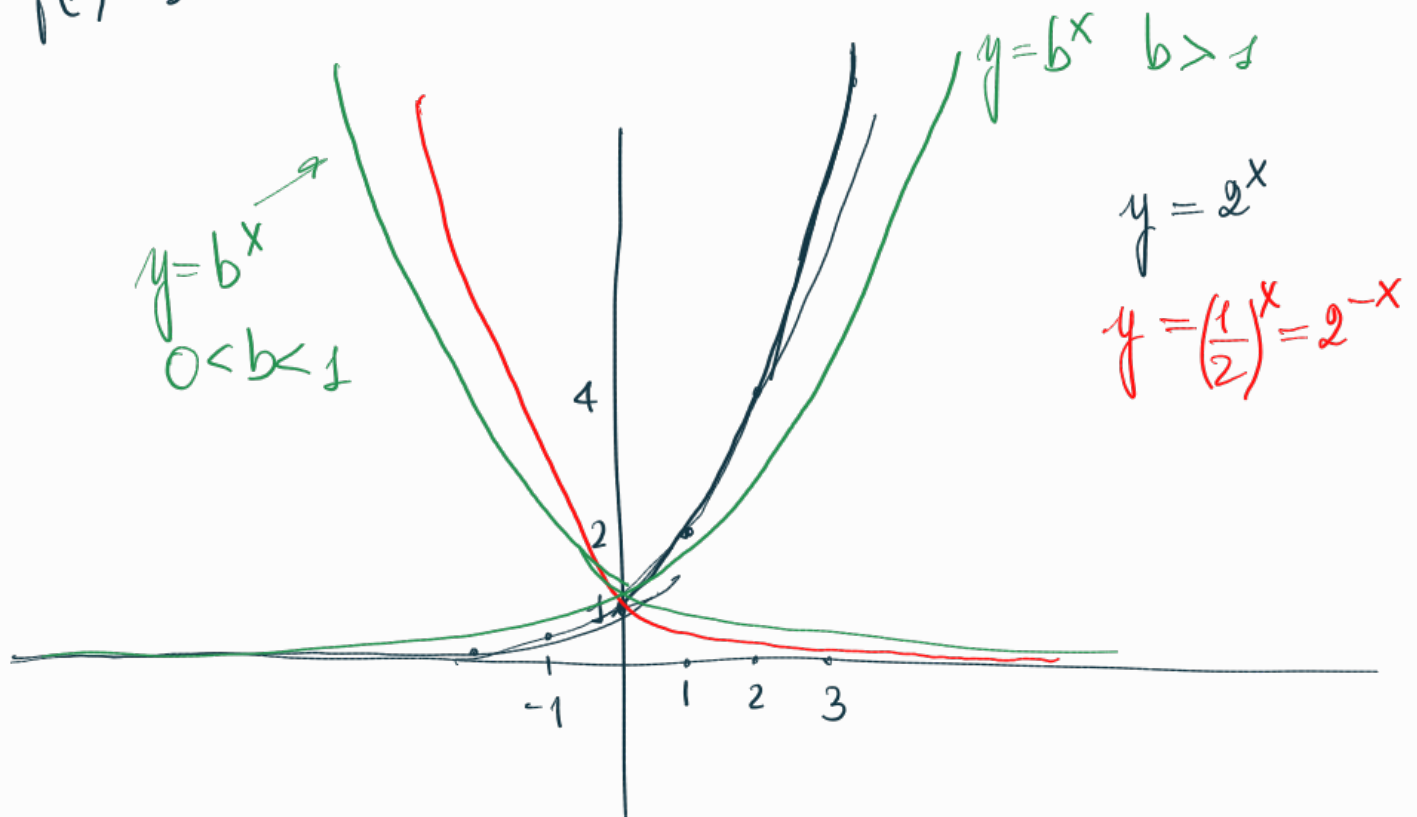
Esponenziali

$$f(x) = b^x$$

$$b > 0 \quad (x \in \mathbb{R})$$

$$f(x) = 2^x$$

$$b = 2$$



Risolvere la diseg^{ne}

$$\left(\frac{1}{9}\right)^0 = 1 < \left(\frac{1}{9}\right)^{2x-3} < 27 = \left(\frac{1}{9}\right)^{-3/2}$$

$$2x-3 < 0$$

$$2x-3 > -\frac{3}{2}$$

$$\left(\frac{1}{9}\right)^{-3/2} = 9^{3/2} = 3^3 = 27$$

oss $\left(\frac{1}{9}\right)^t$ è strett. decrescente, quindi

$$\left(\frac{1}{9}\right)^t < \left(\frac{1}{9}\right)^{-3/2} \Leftrightarrow t > -\frac{3}{2}$$

$$-\frac{3}{2} < 2x-3 < 0$$

$$\frac{3}{2} < 2x < 3$$

$$\boxed{\frac{3}{4} < x < \frac{3}{2}}$$

Logaritmi:

$$f: \mathbb{R} \rightarrow \cancel{\mathbb{R}} (0, +\infty)$$

$$x \mapsto b^x$$

$$\boxed{b > 1}$$

è biettiva.

• è iniettiva perché strett. crescente

• è suriettiva perché

una f. continua in un
intervallo assume tutti i valori
tra $\inf f = 0$ e $\sup f = +\infty$

Resta definita la f. inversa.

$$f^{-1}: (0, +\infty) \rightarrow \mathbb{R}$$

$$y \mapsto f^{-1}(y) = (\text{l'unico } x \in \mathbb{R} \text{ t.c.}$$

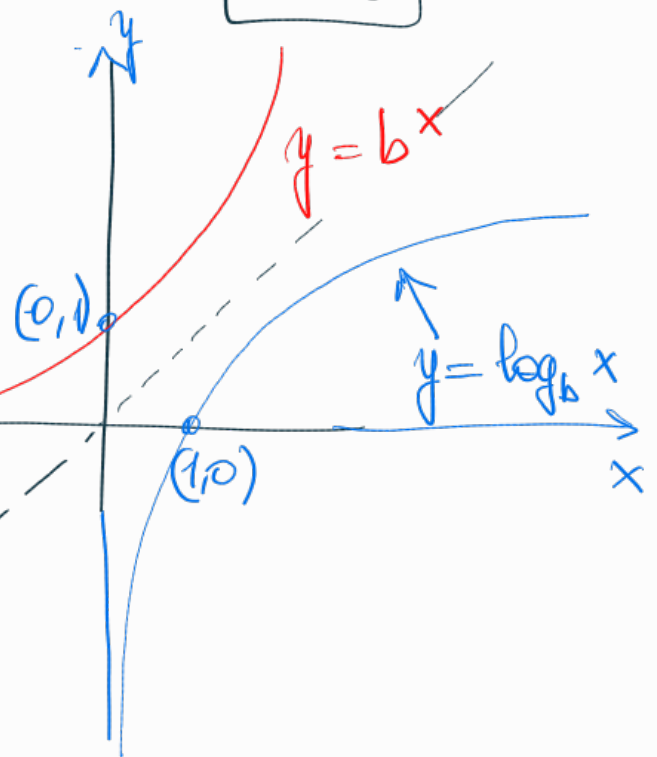
$$f(x) = y, \text{ cioè } b^x = y) =$$

$$= \log_b y$$

per $y > 0$

$\log_b y$ è l'unico $x \in \mathbb{R}$ t.c. $b^x = y$

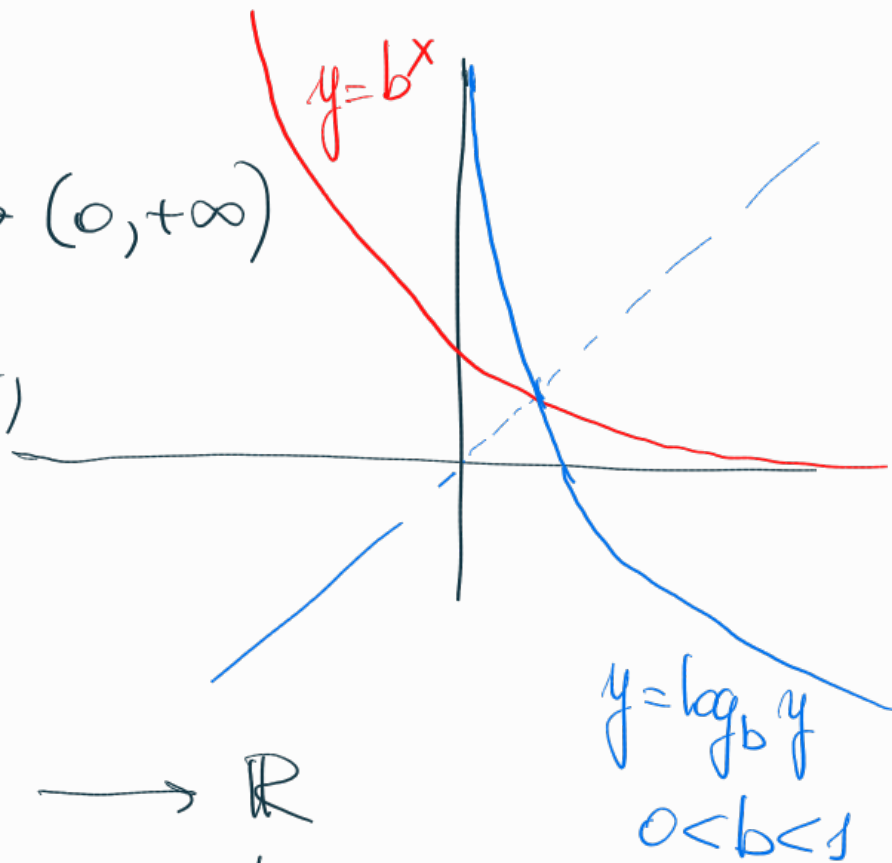
$$x = \log_b y \iff b^x = y.$$



È se $0 < b < 1$?

$$f(x) = b^x : \mathbb{R} \rightarrow (0, +\infty)$$

è biettiva (strett. decrescente)



$$f^{-1}(y) = \log_b y : (0, +\infty) \rightarrow \mathbb{R}$$

$$y \mapsto f^{-1}(y) = \log_b y =$$

$$= \text{l'unico } x \in \mathbb{R} \text{ t.c. } b^x = y.$$

Esemp:

$$\log_3 81 = 4 \iff 3^4 = 81$$

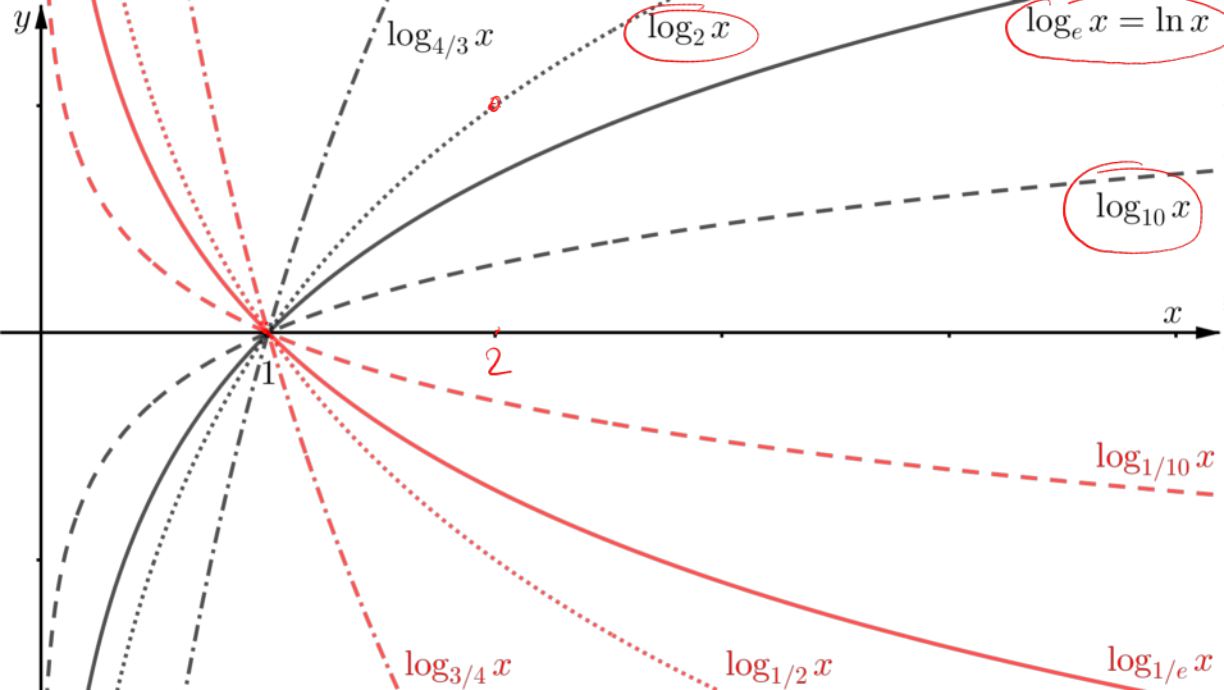
$$\log_3 \sqrt{3} = \frac{1}{2} \iff 3^{1/2} = \sqrt{3}$$

$$\log_{\sqrt{3}} 3 = 2 \iff (\sqrt{3})^2 = 3$$

$$\log_{1/2} 64 = -6 \iff \left(\frac{1}{2}\right)^{-6} \stackrel{?}{=} 64 \quad \text{OK,}$$
$$2^6 \stackrel{?}{=} 64$$

$$\log_{1/2} 1 = 0$$

$$\log_b 1 = 0 \quad \forall b > 0, b \neq 1$$



Proprietà dei logaritmi:

$$b > 0 \quad b \neq 1$$

$$1) \quad b^{\log_b x} = x \quad f(f^{-1}(x)) = x \quad \forall x > 0$$
$$\log_b(b^x) = x \quad f^{-1}(f(x)) = x \quad \forall x \in \mathbb{R}$$

$$2) \quad \log_b(xy) = \log_b x + \log_b y \quad \forall x, y > 0.$$

$$\log_b(xy) \stackrel{?}{=} t \quad \Leftrightarrow \quad b^t \stackrel{?}{=} xy$$

$$\Leftrightarrow b^{\log_b x + \log_b y} \stackrel{?}{=} xy$$

$$\Leftrightarrow \underbrace{b^{\log_b x}}_x \cdot \underbrace{b^{\log_b y}}_y \stackrel{?}{=} xy \quad \text{ok!}$$

OSS $\log(x^2) = \log(x \cdot x) = \log x + \log x = 2 \log x \quad \forall x > 0$

$$\log(x^2) = \log(|x|^2) = 2 \log |x| \quad \forall x \neq 0$$

OSS $f(x) = \log(x(x-1))$ dominio = $\{x \in \mathbb{R} : x(x-1) > 0\} =$

$$= (-\infty, 0) \cup (1, +\infty)$$

Ma se ragiono come segue

$\log(x(x-1)) = \log x + \log(x-1)$, questa funzione ha
un dominio diverso
 $(1, +\infty)$

$$\log(x(x-1)) = \begin{cases} \log x + \log(x-1) & \text{se } x > 1 \\ \log(-x) + \log(1-x) & \text{se } x < 0 \end{cases}$$

$$= \underbrace{\log |x| + \log |x-1|}_{\text{N.B. - Questo è definito anche in } (0,1)} \quad \text{se } x \in (-\infty, 0) \cup (1, +\infty)$$

N.B. - Questo è definito
anche in $(0,1)$