

$$\lim_{n \rightarrow +\infty} \frac{3n^2 - 2n + 5}{-n^3 + n + 1} = \left(\frac{+\infty}{-\infty} \right) = \lim_{n \rightarrow +\infty} \frac{\cancel{n^2} (3 + o(1))}{\cancel{n^3} (-1 + o(1))} =$$

$$= \lim_{n \rightarrow +\infty} \frac{1}{n} (-3 + o(1)) = \left(\frac{-3}{+\infty} \right) = 0$$

OSS per dire che $a_n \rightarrow l \in \mathbb{R}$, posso scrivere che $a_n = l + o(1)$

$$\lim_{n \rightarrow +\infty} \frac{3n^3 - 2n + 5}{-n^2 + n + 1} = \left(\frac{+\infty}{-\infty} \right) = \lim_{n \rightarrow +\infty} \frac{\cancel{n^3} (3 + o(1))}{\cancel{n^2} (-1 + o(1))} =$$

$$= \lim_{n \rightarrow +\infty} n (-3 + o(1)) = (+\infty \cdot (-3)) = -\infty$$

Caso generale

$$\lim_{n \rightarrow +\infty} \frac{P_k(n)}{Q_h(n)} = \left(\frac{\pm \infty}{\pm \infty} \right) =$$

$$P_k(n) = a_k n^k + \dots + a_1 n + a_0$$

$$Q_h(n) = b_h n^h + \dots + b_1 n + b_0$$

$$= \lim_{n \rightarrow +\infty} \frac{n^k (a_k + o(1))}{n^h (b_h + o(1))} = \lim_{n \rightarrow +\infty} n^{k-h} \left(\frac{a_k}{b_h} + o(1) \right) =$$

$$= \begin{cases} 0 \\ \frac{a_k}{b_h} \\ \pm \infty \end{cases}$$

se $h > k$ $\begin{pmatrix} 0^+ & \text{se } a_k, b_h \text{ concordi} \\ 0^- & \text{se } a_k, b_h \text{ discordi} \end{pmatrix}$
 se $h = k$ $\begin{pmatrix} 0^+ & \text{se } a_k, b_h \text{ concordi} \\ 0^- & \text{se } a_k, b_h \text{ discordi} \end{pmatrix}$
 se $k > h$ $\begin{pmatrix} +\infty & \text{se } a_k, b_h \text{ concordi} \\ -\infty & \text{se } a_k, b_h \text{ discordi} \end{pmatrix}$

Bisogna ricordare solo

$$\lim_{n \rightarrow +\infty} \frac{P_k(n)}{Q_h(n)} = \lim_{n \rightarrow +\infty} \frac{a_k n^k}{b_h n^h}$$

$$\lim_{n \rightarrow +\infty} \frac{n^\alpha - n}{n^\pi - n^3}$$

$$\alpha \in \mathbb{R}$$

$$1) \boxed{\alpha > 1}$$

$$n^\pi - n^3 = n^\pi (1 - n^{3-\pi}) = n^\pi (1 + o(1))$$

$$n^\alpha - n = \begin{cases} n^\alpha (1 + o(1)) \\ 0 \\ n (-1 + o(1)) \end{cases}$$

$$\alpha > 1$$

$$\alpha = 1$$

$$\alpha < 1$$

$$\lim_{n \rightarrow +\infty} \frac{n^\alpha - n}{n^\pi - n^3}$$

$$= \lim_{n \rightarrow +\infty} n^\alpha$$

$$= \begin{cases} +\infty \\ 1 \\ 0 \end{cases}$$

$$2) \alpha = 1$$

$$= \lim_{n \rightarrow +\infty} \frac{n^\alpha (1+o(1))}{n^\pi (1+o(1))}$$

$$n^{-\pi} (1+o(1)) =$$

$$\alpha > \pi$$

$$\alpha = \pi$$

$$1 < \alpha < \pi$$

$$\lim \dots = 0$$

$$3 \boxed{\alpha < 1}$$

$$\lim_{n \rightarrow +\infty} \frac{n^\alpha - n}{n^\pi + n^3} = \lim_{n \rightarrow +\infty} \frac{n(-1+o(1))}{n^\pi (1+o(1))} =$$

$$= \lim_{n \rightarrow +\infty} \frac{1}{n^{\pi-1}} \cdot (-1+o(1)) = 0^-$$

$$\lim_{n \rightarrow +\infty} \frac{2^n - 5^n}{3^n + 1} \rightarrow -\infty = \left(\frac{-\infty}{+\infty} \right) = \lim_{n \rightarrow +\infty} \frac{5^n (-1 + o(1))}{3^n (1 + o(1))} =$$

$$= \lim_{n \rightarrow +\infty} \left(\frac{5}{3} \right)^n (-1 + o(1)) = -\infty$$

$$2^n - 5^n = (+\infty - \infty) = 5^n \left(\frac{2^n}{5^n} - 1 \right) = 5^n (-1 + o(1))$$

$\left(\frac{2}{5} \right)^n \rightarrow 0$
 $\downarrow$
 $-\infty$

