

INDIVIDUAL PREFERENCES AND COLLECTIVE DECISIONS

In the welfare economics approach, the problem of social choice consists in moving from individual preferences to an evaluation of alternatives for society as a whole.

The Pareto criterion

In individual choice, consumers maximise their utility subject to their budget constraint. Their preferences normally concern the bundle of goods they consume. To study social choice, this framework can be extended by assuming that each individual has preferences over all allocations of goods among all consumers.

An allocation, or state of the world, specifies the quantity of each good held by each individual. Preferences over these states reflect the individual's view of society. If a utility function represents these preferences, the individual prefers state X to state Y when the utility associated with X is greater than that associated with Y.

Welfare economics adopts an individualistic approach: collective evaluation derives from individual evaluations. The Pareto criterion regards a change as desirable when it makes at least one individual better off without making anyone else worse off. Applying this criterion requires only ordinal utilities and does not require interpersonal comparisons of utility.

This criterion provides only a partial ordering: when some individuals gain and others lose, it cannot determine which allocation is preferable. The Pareto frontier consists of efficient allocations, but choosing a point on it requires an additional distributive criterion.

The social welfare function

A social welfare function, denoted by W , expresses collective welfare as a function of individual utilities. Social choice can then be formulated as the maximisation of W over the set of feasible utility combinations. If W satisfies the Pareto principle, the chosen allocation lies on the utility possibility frontier, or UPF.

$$W = W(u_1, \dots, u_n)$$

Ethical individualism requires the evaluation to depend on individuals' utilities, excluding paternalistic criteria unrelated to their own evaluations. The Pareto principle requires that an improvement for someone, with no losses for others, increase social welfare. The function's other properties depend on the distributive judgements adopted and on assumptions about the measurement and comparability of utilities.

In formulations that give weight to inequality, concavity expresses a preference for more egalitarian distributions, other things being equal and provided that utilities are comparable.

The main formulations

The utilitarian function sums individual utilities, assigning the same weight to each. This formulation requires a utility scale that makes summation and compensation between individuals meaningful. GDP, calculated as the sum of incomes generated by domestic production, can be interpreted as a utilitarian function under the simplifying assumption that each individual's utility coincides with their income, so that every euro contributes equally to social welfare, regardless of who receives it.

The Bergson–Samuelson approach allows for a more general function. By assigning greater weight to the utility of those who are worse off, a loss of utility for a poorer individual requires a larger increase in utility for a richer individual to compensate for it. This formulation evaluates social welfare by taking account of both the levels of individual utilities and their distribution.

The Rawlsian function evaluates social welfare on the basis of the utility of the worst-off individual. As long as this individual remains the least advantaged, no increase in the utility of a richer individual can compensate for a loss in their utility.

The social optimum is the point at which a social indifference curve is tangent to the UPF. [see graph]

Axiomatic aggregation and Arrow's theorem

An alternative approach consists in defining the desirable properties of an aggregation rule. Arrow (1951) studies how to transform complete and transitive individual orderings into a collective ordering. The properties considered are as follows.

Transitivity. If society prefers A to B and B to C, it must prefer A to C.

Unrestricted domain. The rule must work for every configuration of complete and transitive individual preferences.

Non-dictatorship. No individual must be able to impose their own ordering as the collective ordering in all cases, regardless of others' preferences.

Weak Pareto principle. If everyone prefers A to B, the collective ordering must also prefer A to B.

Independence of irrelevant alternatives. The collective comparison between A and B must depend only on individual comparisons between A and B, without being affected by preferences concerning other alternatives.

Arrow's impossibility theorem states that, with at least three alternatives to choose from, no rule satisfies all these conditions simultaneously. The result concerns the aggregation of individual orderings; it does not rule out adopting a social welfare function based on additional distributive judgements.

To overcome the impossibility result, at least one of the conditions must be modified.

Majority voting and the Condorcet paradox

In a democracy, majority voting is a common way of aggregating preferences. Suppose that N individuals must choose among K alternatives. An alternative provides a solution in pairwise majority comparisons if it defeats every other alternative: it is the Condorcet winner.

Such a winner may not exist. The voting paradox, illustrated by the Marquis de Condorcet, occurs when collective preferences are cyclical, even though individual preferences are transitive. Consider three individuals and three alternatives.

Individual	Preference ordering
1	$A > B > C$
2	$B > C > A$
3	$C > A > B$

A defeats B with the votes of individuals 1 and 3; B defeats C with those of individuals 1 and 2; C defeats A with those of individuals 2 and 3. The resulting cycle is A preferred to B, B preferred to C, and C preferred to A. Majority voting does not produce a transitive ordering, and no alternative defeats all the others.

Single-peaked preferences and the median voter (optional)

Restricting the domain of preferences can produce a well-defined outcome. If alternatives are ordered along a single dimension and each individual has unimodal, or single-peaked, preferences, each individual has a preferred alternative and regards alternatives progressively further away from it on either side as less desirable.

In this setting, with an odd number of voters, the median voter's preferred alternative is a majority-voting equilibrium. The median voter is the voter whose preferred alternative occupies the middle position in the ordering of ideal points.

For example, a committee of 11 experts must choose a speed limit. If their preferred values are 100, 110, 120, 130, 140, 150, 160, 170, 180, 190 and 200 km/h, and preferences are single-peaked, the median value is 150 km/h. Against a lower value, at least six experts prefer 150; against a higher value, at least six still prefer 150.

Preferences may, however, fail to be unimodal. Consider public spending on education: a wealthy individual who sends their children to a private school may prefer zero public spending to a positive but low level of spending, because they pay taxes without using the service. If spending exceeds a threshold that makes public education sufficiently attractive, their utility may increase as spending rises. Preferences with more than one peak may thus emerge, and the median voter result does not automatically apply.

From direct to representative democracy (optional)

The voting theory presented so far concerns direct democracy, in which voters decide on individual issues, for example through referendums. In modern representative democracies, voters instead elect representatives who make decisions once in office.

Downs (1957) shows that, in a two-party system in which parties aim to be elected (or re-elected), competition may lead them to converge on the median voter's preferred position. This conclusion applies within a framework of competition along a single dimension and under the assumptions required for the median voter result.