Nuclear Reactions

Nuclear Reactions

- First nuclear reaction was a nitrogen target bombarded with alpha particles, which emitted protons. The reaction is written as:

$$\alpha + {}^{14}_{7}\mathrm{N} \rightarrow p + {}^{17}_{8}\mathrm{O}$$

- The first particle is the projectile and the second is the nitrogen target. These two nuclei react to form proton projectiles and the residual oxygen target.
- The reaction can be rewritten in shorthand as: ${}^{14}\mathrm{N}(\alpha, p){}^{17}\mathrm{O}$. In general a reaction $\underbrace{x + X}_{\text{Entry channel}} \rightarrow \underbrace{y + Y}_{\text{Exit channel}}$ can be rewritten as

$$X(x, y)Y$$

Types of Reactions

- **Nuclear photodisintegration**: initiation of a nuclear reaction by a photon
  X(γ,*)Y.
- **Neutron or proton radioactive capture** nucleon is absorbed by the target nucleus, with energy and momentum conserved by gamma ray emission.
  X(n/p,γ)Y.
- *Elastic scattering*: entrance and exit channels are identical and particles in the exit channels are not in excited states.

  X(a,a)X
- *Inelastic scattering*: entrance and exit channels are identical but one or more of the reaction products is left in an excited state.

  X(a,a)X* → X(a,a+γ)X

Q value

Reaction $a+b \rightarrow c_1 + c_2 + \dots c_N$

- Q is the amount of mass transformed in kinetic energy

$Q=M_a+M_b-\Sigma_i M_{c,i}$

- In terms of kinetic energy (b is the target at rest)

$Q=\Sigma_i T_{c,i}-T_a$

Three cases:

Q>0 (exothermic) T_a can be as small as one wants (spontaneous reaction, e.g. fusion and fission)

Q=0 elastic

Q<0 (endothermic): T_a needs to be above a threshold

Projectile energy threshold

Reaction $a+b \rightarrow c_1 + c_2 + \ldots c_N$

Four momentum conservation:

$a+b=\Sigma_i c_i$ $\rightarrow$ $M_a^2+M_b^2+2M_b(T_a+M_a)=2\Sigma_{i>j}c_i \bullet c_j+\Sigma_i M^2_{c,i}$

$\rightarrow$ $(M_a+M_b)^2+2M_bT_a=2\Sigma_{i>j}c_i \bullet c_j+\Sigma_i M^2_{c,i}$

Calculating the right side in the CMS, at threshold all particles in the exit channel are at rest $\rightarrow$

$(M_a+M_b)^2+2M_bT_{thr}=(\Sigma_i M_{c,i})^2 \rightarrow$

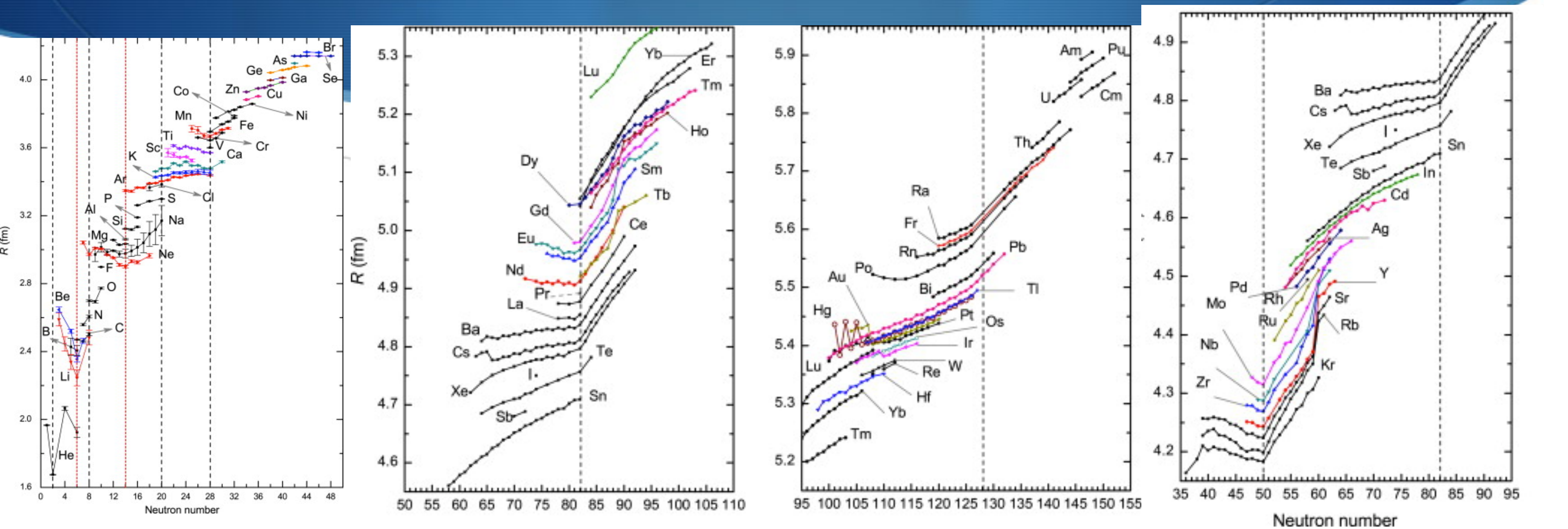
$$Ta>Tthr=[(\Sigma_i M_{c,i})^2-(M_a+M_b)^2]/2M_b$$

Coulomb Barrier

- If projectile is charged, it needs to overcome the Coulomb barrier. Assuming homogeneous charge distribution in the nucleus of radius R

$\Delta E = zZe^2/4\pi\varepsilon_0 R = zZ\alpha\hbar c/R = 1.45(\text{MeV fm})\ zZ/R$

Nuclear Radius



http://www.sciencedirect.com/science/article/pii/S0092640X12000265

- Empirically $R(fm) \sim 1.15 * A^{0.294}$
- Coulomb barrier

$\Delta E[MeV] \sim 1.3\ zZ/A^{0.294}$

Cross sections

As in the photon interactions with matter, the particle beam is attenuated by the occurrence of nuclear interactions. In unit of thickness (dx)

$$d\Phi = \sigma \, n_b \, \Phi(x) dx$$

σ : cross section

n_b: number of target nuclei per unit volume ($=\rho(g/cm3)N_A/A$)

Φ: flux (number of particles per unit time and area) or

rate (number of particles per unit time) or

fluence (number of particles per unit area)

Use of cross sections

- Estimate attenuation of beam through material
  - $\Phi(x)=\Phi(0)\exp(-x/\lambda)$
  - $\lambda=A/(\sigma\rho N_A)=1/(\mu_{mass}\rho)$
  - Note: for this application the x-section need to be inclusive of all processes

Nuclear reactions DB

https://www-nds.iaea.org/exfor/exfor.htm

Reaction codes

ABS	Absorption
EL	Elastic scattering
F	Fission
FUS	Total fusion
INL	Inelastic scattering
NON	Nonelastic (= total minus elastic)
PAI	Pair production (for photonuclear reactions)
SCT	Total scattering (elastic + inelastic)
TCC	Total charge changing
THS	Thermal neutron scattering
TOT	Total
X	Process unspecified

Particle codes

0	(no incoming/outgoing particle)
A	α
AN	Antineutrons
AP	Antiprotons
AR	Annihilation radiation
B	Decay β
B+	Decay β^+
B-	Decay β^-
D	Deuterons
DG	Decay γ
DN	Delayed neutrons
E	Electrons
EC	Electron capture
ER	Evaporation Residues
ETA	Eta mesons
FF	Fission fragments
G	γ
HCP	Heavy Charged Particle
HE2	2He
HE3	3He
HE6	6He
HF	Heavy fragment
ICE	Internal-conversion electrons
K	Kaons,unspecified
KN	Kaons,negative
KP	Kaons,positive
LCP	Light charged particle (Z<7)
LF	Light fragment
N	Neutrons
P	Protons
PI	pion,unspecified
PI0	pion,neutral
PIN	pion,negative
PIP	pion,positive
PN	Prompt neutrons
RSD	Residual nucleus
SF	Fragments from spontaneous fission
T	Tritons
XR	X-rays

Exercise 9

- Determine the energy threshold and the coulomb barrier for
  - p+18O □ n + 18F
  - n+131Xe□ p+131I
- Estimate the attenuation length of neutrons of 1meV, 1keV, 1 MeV in Pb and Paraffin (for simplicity consider it as an H target with $\rho=0.9$ g/cm^3)
- Find nuclear reactions

that allow to produce the

particles in the table

Isotope	Dec	Prod. reaction	Productio n threshold	Production max Xsection
131I	beta-			
90Y	Beta-			
188Re	Beta-			
198Au	Beta-			
11C	Beta+			
18F	Beta+	O18(n,p)F18	3MeV	0,5MeV
99mTc	Gamma			
68Ga	Beta+			

Radio-isotope Production

Isotope	Decay	Production
131I	beta-	131Xe(n,p)131I
90Y	Beta-	89Y(n,γ)90Y
188Re	Beta-	
198Au	Beta-	
11C	Beta+	
18F	Beta+	
99mTc	Gamma	
68Ga	Beta+	

Production modes in http://nucleardata.nuclear.lu.se/toi/nucSearch.asp

Radio-isotope production

- Bombard a sample of X, thickness T, with a rate R (part/s) in order to obtain Y
  - Note: R is the rate of particles on the sample, if the beam size is larger than the sample, this needs to be accounted for
  - Y is produced at a rate

$P = R\sigma Tn = R\sigma T\rho(g/cm^3)N_A/A$

  - Since Y decays with lifetime τ, the number of Y molecules is

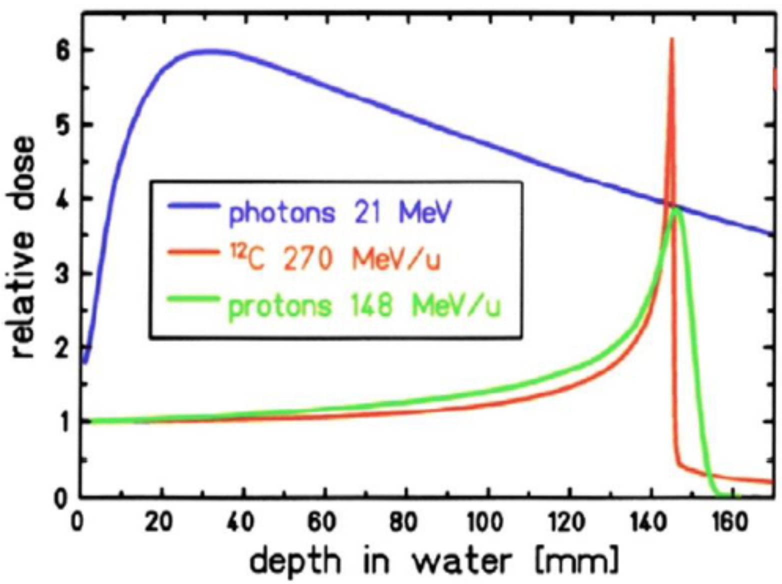
$$\frac{dN_Y}{dt} = -\frac{N_Y}{\tau} + P$$

$$N_Y = P\tau\left(1 - e^{-t/\tau}\right)$$

After a time long compared with the lifetime N_Y is stable and the activity is $N_Y/\tau = P$

Fragmentation in carbon

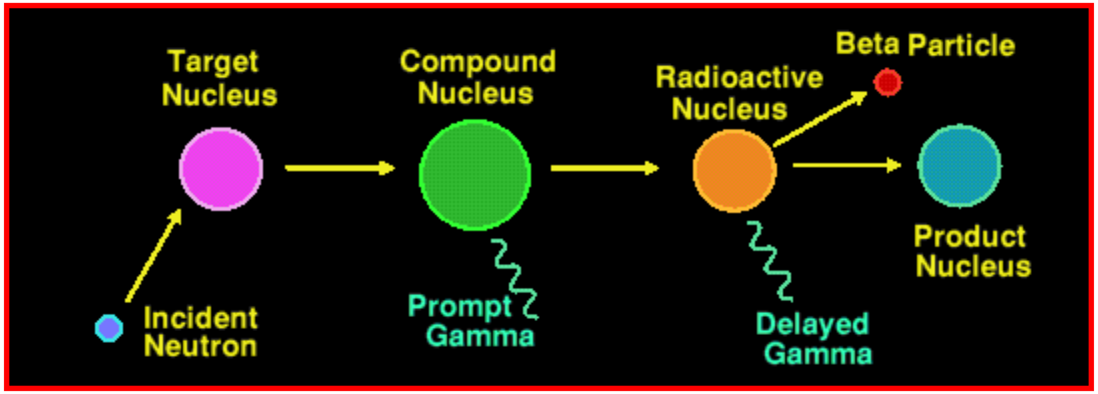
- When the projectile is a high mass nucleus it can fragment
  - X→ A + B
- The daughters have momenta comparable with the father, but
  - Smaller masses → higher beta
  - Smaller charge → lower LET



NEUTRON ACTIVATION

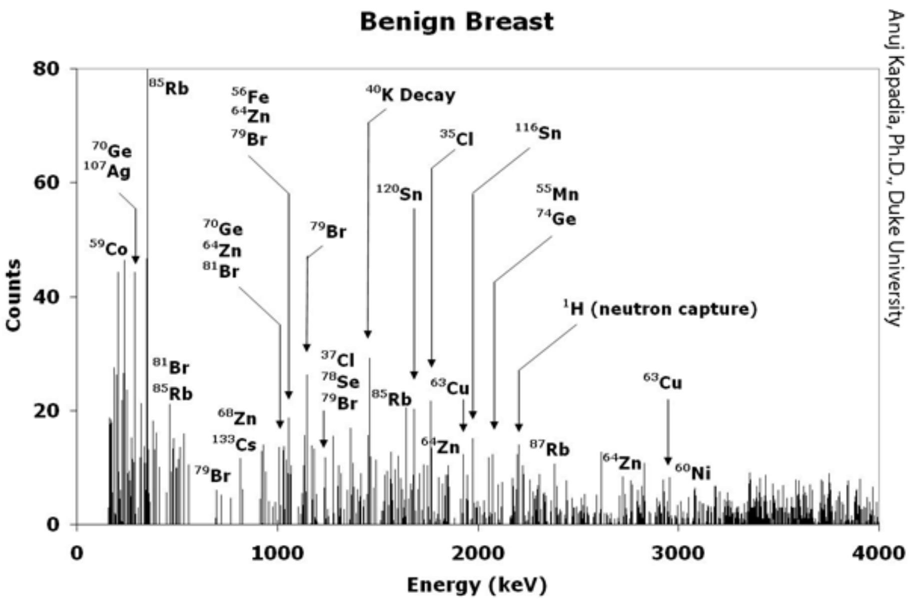
$$X(n,\gamma)Y$$

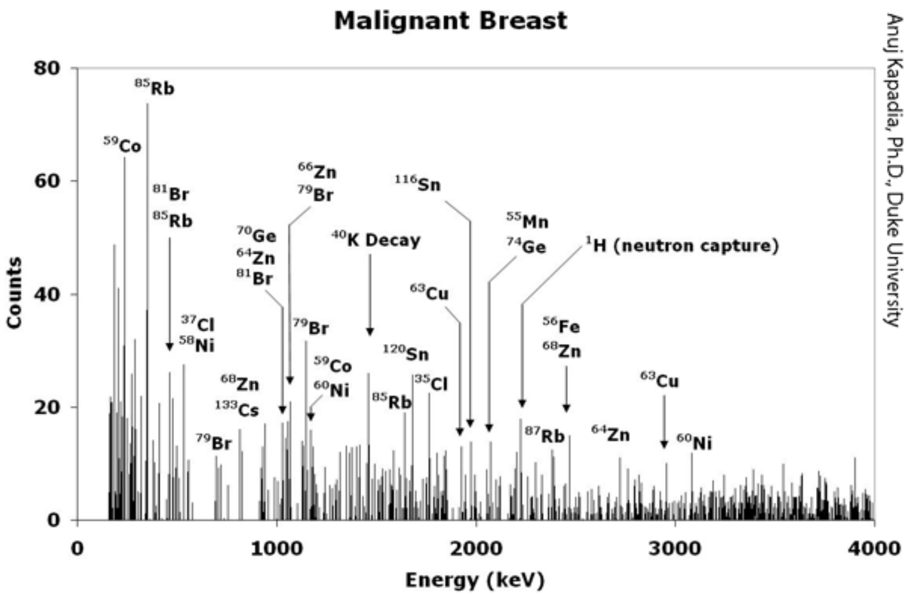
neutron is *captured* by the target, transmuting it into an unstable nucleus which then decays by fission or by the release of some particle or photon.
NAA, which uses low-energy thermal neutrons to transmute a wide range of nuclei into unstable isotopes, irradiation can take many hours while measurement of the decay energies and rates of the unstable transmuted isotopes can require days



NAAMEDICAL APPLICATIONS

Several diseases in humans → Increased element concentration

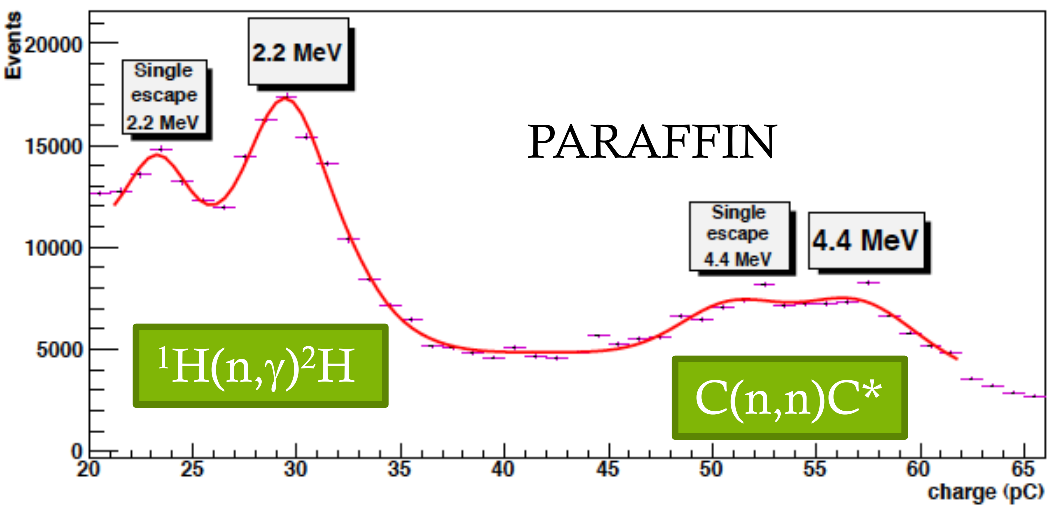


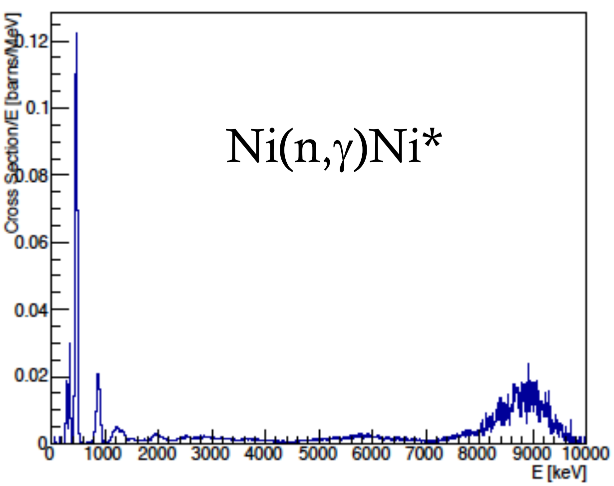


Energy keV	Element Match	Counts Benign	Counts Malignant	Diff	p-val
219	^{79}Br	6	19	13	0.01
397	^{59}Co, ^{79}Br	16	2	-14	0.01
1028	^{81}Br	13	29	16	0.05
1128	^{39}K, ^{68}Zn	0	13	13	0.001
1306	^{56}Fe	10	0	-10	0.01
2299	^{27}Al	0	13	13	0.001
2469	^{37}Cl, ^{56}Fe, ^{66}Zn	5	15	10	0.05
3635	^{35}Cl	3	14	11	0.01

Calibration Gamma Lines

- Use of paraffin and Ni to produce 2.2, 4.4 and 9.1 MeV lines





Exercise 10

- Find the processes and the energy thresholds for the production of C11, **F18**, I131 and Re188 and the achievable activities for 100mA of ~10-20MeV protons or 10^{14} thermal neutrons/s [assume 1cm of thickness and rho=water]
  - Note: production processes can be found in the Lund Nuclear Data (http://nucleardata.nuclear.lu.se/)
- Find prompt gamma neutron activation lines for Ni, C, H
  - Note: use the Prompt Gamma Neutron Activation database (http://www-nds.iaea.org/pgaa/pgaa7/index.html)

Isotope	Dec	Production reaction	Production threshold (MeV)	Production s_{max} (barn)	A_{max} (GBq)
131I	beta-				
90Y	Beta-				
188Re	Beta-				
11C	Beta+				
18F	Beta+				