

Formulario

Corso di Meccanica Quantistica, A.A. 2024-2025

Proff. R. Contino, M. Nardecchia, A. Pelissetto

Costanti Fisiche

Velocità della luce	$c = 2.99 \times 10^8 \text{ m/s}$
Costante di Planck ridotta	$\hbar = 1.05 \times 10^{-34} \text{ J s} = 6.6 \times 10^{22} \text{ MeV s}$ $\hbar c = 197 \text{ MeV fm} = 197 \text{ eV nm}, \quad \hbar c = 1.23 \cdot 10^{-6} \text{ eV m}$
Massa dell'elettrone	$m_e c^2 = 0.511 \text{ MeV}$
Massa del protone	$m_p c^2 = 938.3 \text{ MeV}$
Massa del neutrone	$m_n c^2 = 939.6 \text{ MeV}$
Raggio di Bohr	$a_0 = 0.53 \text{ Å} = 0.53 \times 10^{-10} \text{ m} \quad (\hbar^2/(m_e e^2) = \text{in unità di Gauss})$
Energia di ionizzazione idrogeno	$E_I = 13.6 \text{ eV} \quad (E_I = m_e e^4/(2\hbar^2) \text{ in unità di Gauss})$

Atomo di idrogeno

Autofunzioni radiali $R_{nl}(r)$ per il problema Coulombiano (a_0 è il raggio di Bohr):

$$\begin{aligned} R_{10}(r) &= 2 \frac{1}{a_0^{3/2}} e^{-\frac{r}{a_0}} & R_{30}(r) &= 2 \frac{1}{(3a_0)^{3/2}} \left(1 - \frac{2r}{3a_0} + \frac{2r^2}{27a_0^2} \right) e^{-\frac{r}{3a_0}} \\ R_{20}(r) &= \frac{1}{(2a_0)^{3/2}} \left(2 - \frac{r}{a_0} \right) e^{-\frac{r}{2a_0}} & R_{31}(r) &= \frac{4\sqrt{2}}{9} \frac{1}{(3a_0)^{3/2}} \frac{r}{a_0} \left(1 - \frac{r}{6a_0} \right) e^{-\frac{r}{3a_0}} \\ R_{21}(r) &= \frac{1}{\sqrt{3}} \frac{1}{(2a_0)^{3/2}} \frac{r}{a_0} e^{-\frac{r}{2a_0}} & R_{32}(r) &= \frac{2\sqrt{2}}{27\sqrt{5}} \frac{1}{(3a_0)^{3/2}} \left(\frac{r}{a_0} \right)^2 e^{-\frac{r}{3a_0}}. \end{aligned}$$

Oscillatore Armonico

Autofunzioni dell'oscillatore armonico unidimensionale:

$$\psi_n(x) = \left(\frac{m\omega}{\hbar\pi} \right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n(\rho) e^{-\rho^2/2}, \quad \rho = \frac{x}{\xi}, \quad \xi \equiv \sqrt{\frac{\hbar}{m\omega}}$$

con $H_0(\rho) = 1$, $H_1(\rho) = 2\xi$, $H_2(\rho) = 4\rho^2 - 2$.

Autofunzioni radiali $R_{n\ell}(r)$ dell'oscillatore armonico isotropo tridimensionale ($\beta = 1/\xi$):

$$\begin{aligned} R_{00}(r) &= 2 \frac{\beta^{3/2}}{\pi^{1/4}} e^{-\beta^2 r^2/2} & R_{11}(r) &= \sqrt{\frac{8}{3}} \frac{\beta^{3/2}}{\pi^{1/4}} \beta r e^{-\beta^2 r^2/2} \\ R_{20}(r) &= \sqrt{6} \frac{\beta^{3/2}}{\pi^{1/4}} \left(1 - \frac{2}{3} \beta^2 r^2 \right) e^{-\beta^2 r^2/2} & R_{22}(r) &= \sqrt{\frac{16}{15}} \frac{\beta^{3/2}}{\pi^{1/4}} \beta^2 r^2 e^{-\beta^2 r^2/2} \end{aligned}$$

Integrali

$$\Gamma(n+1) = \int_0^{+\infty} t^n e^{-t} dt = n!$$

$$\int_{-\infty}^{+\infty} \frac{dx}{(1+x^2)^n} = \frac{\pi}{((n-1)!)^2} \frac{(2n-2)!}{2^{2n-2}} \quad n \in \mathbb{N}^+$$

$$I_{n,\ell_1,m,\ell_2} = \int_0^\infty dr r^2 (r/a_0)^2 R_{n\ell_1}(r) R_{m\ell_2}(r);$$

$$I_{1,0,1,0} = 3, I_{1,0,2,0} = -512\sqrt{2}/243, I_{2,0,2,0} = 42, I_{1,0,2,1} = 1280\sqrt{2}/(243\sqrt{3}), I_{2,0,2,1} = -20\sqrt{3}/243, I_{2,1,2,1} = 30.$$

$$\int_{-\infty}^{+\infty} e^{-ax^2+bx+c} dx = \sqrt{\frac{\pi}{a}} e^{\frac{b^2}{4a}+c} \quad \text{con } \operatorname{Re} a > 0$$

$$I_n = \int_{-\pi/2}^{\pi/2} dx x^n \cos x, \quad J_n = \int_{-\pi/2}^{\pi/2} dx x^n \sin x,$$

$$I_1 = 0, I_2 = \frac{1}{2}(\pi^2 - 8), I_3 = 0, I_4 = \frac{1}{8}(\pi^4 - 48\pi^2 + 384), J_1 = 2, J_2 = 0, J_3 = \frac{3}{2}(\pi^2 - 8), J_4 = 0.$$

$$I_{m,n} = \int_{-\pi/2}^{\pi/2} dx x \cos(mx) \sin(nx)$$

$$I_{1,1} = \frac{\pi}{4}, I_{1,2} = \frac{8}{9}, I_{1,3} = \frac{\pi}{8}, I_{2,1} = -\frac{10}{9}, I_{2,2} = -\frac{\pi}{8}, I_{2,3} = \frac{26}{25}.$$

$$I_{mn} = \int_{-\pi/4}^{\pi/4} dx \cos mx \cos nx$$

$$I_{11} = (2 + \pi)/4, I_{12} = 2\sqrt{2}/3, I_{13} = 1/2, I_{22} = \pi/4, I_{23} = 2\sqrt{2}/5, I_{33} = (3\pi - 2)/12.$$