$$\boxed{\begin{cases} |z^2+1| = 1 \\ \frac{1}{2}|z|^2 - \mathrm{Re}\, z = 0 \end{cases}}$$

$z = x + iy \qquad x, y \in \mathbb{R}.$

$$|z^2+1| = 1 \iff |x^2 - y^2 + 2ixy + 1| = 1 \iff$$

$$\sqrt{(x^2-y^2+1)^2 + 4x^2y^2} = 1 \iff$$

$$\iff (x^2-y^2+1)^2 + 4x^2y^2 = 1.$$

$$\iff x^4 + y^4 + \cancel{1} + 2x^2y^2 + 2x^2 - 2y^2 + \cancel{4x^2y^2} = \cancel{1}$$

$$\iff x^4 + y^4 + 2x^2y^2 + 2x^2 - 2y^2 = 0$$

Passiamo alla 2^a eq^{ne}.

$$\frac{1}{2}(x^2+y^2) - x = 0$$

$$x^2 + y^2 - 2x = 0 \implies \boxed{y^2 = 2x - x^2}$$

$$(x^2 - 2x + 1) + y^2 = 1$$

$(x-1)^2 + y^2 = 1$. circonf. di centro $(1,0)$ e raggio 1.

$$x^4 + (2x - x^2)^2 + 2x^2(2x - x^2) + 2x^2 - 4x + 2x^2 = 0$$

$$\cancel{x^4} + 4x^2 + \cancel{x^4} - \cancel{4x^3} + \cancel{4x^3} - \cancel{2x^4} + 2x^2 - 4x + 2x^2 = 0$$

$$8x^2 - 4x = 0$$

$$x(2x-1) = 0 \implies x = 0 \quad \text{oppure} \quad x = \frac{1}{2}$$

$$y^2 = 2x - x^2$$

se $x = 0 \Rightarrow y^2 = 0 \Rightarrow y = 0$

se $x = \frac{1}{2} \Rightarrow y^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$$y = \pm \frac{\sqrt{3}}{2}$$

soluzioni

$\boxed{z = 0}$ $\quad \boxed{z = \frac{1}{2} - \frac{\sqrt{3}}{2} i}$ $\quad \boxed{z = \frac{1}{2} + \frac{\sqrt{3}}{2} i}$

$$\boxed{\operatorname{Re}\left(z - \bar{z} + i\bar{z}^2\right) = \operatorname{Im}\left(z + \bar{z} + i|z|^2\right)}$$

$z = x + iy \qquad x, y \in \mathbb{R}$.

$$\operatorname{Re}\left((\cancel{x} + iy) - (\cancel{x} - iy) + i(x - iy)^2\right) =$$

$$= \operatorname{Re}\left(+ 2iy + i\left(x^2 - y^2 - 2ixy\right)\right) =$$

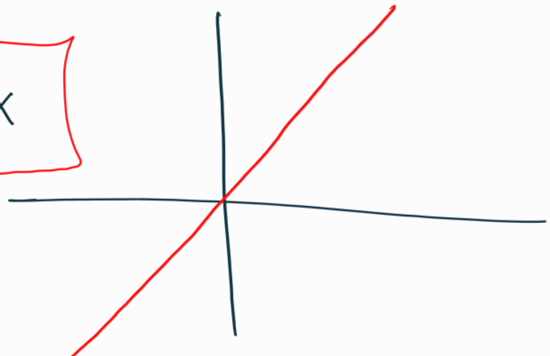
$$= \operatorname{Re}\left(2iy + i(x^2 - y^2) + 2xy \right) = 2xy$$

$$\operatorname{Im}\left(z + \bar{z} + i|z|^2\right) = \operatorname{Im}\left(x + \cancel{iy} + x - \cancel{iy} + i(x^2 + y^2)\right) = x^2 + y^2$$

L'eq^{ne} diventa $\quad 2xy = x^2 + y^2$

$$x^2 + y^2 - 2xy = 0$$

$$(x - y)^2 = 0 \qquad \boxed{y = x}$$



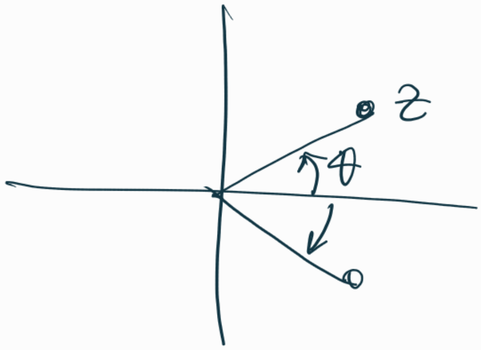
Sol^{ni} $\boxed{z = x + ix = x(1+i) \qquad x \in \mathbb{R}.}$

$\boxed{-8iz^7 = |z|^2 \bar{z}^2}$

In coord. polari $\boxed{z = |z| e^{i\theta}}$

$$z^7 = |z|^7 e^{i7\theta}$$

$$\underbrace{-8i}_{8e^{-i\frac{\pi}{2}}}\, z^7 = 8|z|^7 e^{i\left(7\theta - \frac{\pi}{2}\right)}$$



$$\bar{z} = |z| e^{-i\theta} \Rightarrow \bar{z}^2 = |z|^2 e^{-2i\theta}$$

$$|z|^2 \bar{z}^2 = |z|^4 e^{-2i\theta}$$

L'eq^{ne} diventa

$$8\,|z|^7\, e^{i\left(7\theta - \frac{\pi}{2}\right)} = |z|^4 e^{-2i\theta}$$

$$8|z|^7 = |z|^4 \quad \begin{cases} \to |z| = 0 \iff \boxed{z = 0} \\ \to 8|z|^3 = 1 \Rightarrow |z|^3 = \frac{1}{8} \Rightarrow |z| = \frac{1}{2} \end{cases}$$

$$7\theta - \frac{\pi}{2} = -2\theta + 2k\pi \qquad (k \in \mathbb{Z})$$

$$9\theta = \frac{\pi}{2} + 2k\pi.$$

$$\theta = \frac{\pi}{18} + k\underbrace{\left(\frac{2\pi}{9}\right)}$$

$$\theta_0 = \frac{\pi}{18}, \quad \theta_1 = \frac{5}{18}\pi, \quad \theta_2 = \frac{9}{18}\pi = \frac{\pi}{2}, \quad \theta_3 = \frac{13}{18}\pi$$

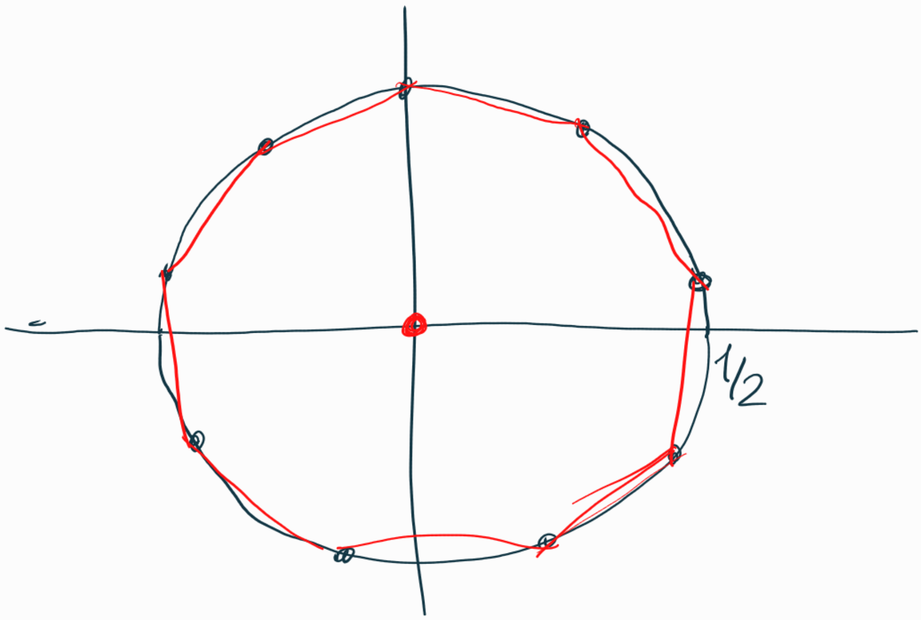
$\vartheta_4 = \frac{17}{18}\pi$, $\quad \vartheta_5 = \frac{21}{18}\pi = \frac{7\pi}{6}$ $\quad \vartheta_6 = \frac{25}{18}\pi$

$\vartheta_7 = \frac{29}{18}\pi$ $\quad \vartheta_8 = \frac{33}{18}\pi$, $\quad \vartheta_9 = \cancel{\frac{37\pi}{18} = 2\pi + \frac{\pi}{18}}$

$\cancel{k \in \mathbb{Z}}$ $\quad k = 0, 1, 2, \dots, 8$

$$z_0 = \frac{1}{2} e^{i\frac{\pi}{18}} = \frac{1}{2}\left(\cos\frac{\pi}{18} + i \sin\frac{\pi}{18}\right)$$

$$z_1 = \frac{1}{2} e^{i\frac{5\pi}{18}} = \frac{1}{2}\left(\cos\frac{5\pi}{18} + i \sin\frac{5}{18}\pi\right)$$



$$\boxed{\lim_{x \to +\infty} \left(\left(\frac{ax+2}{x}\right)^{x^2} + x^4\right) e^{-2x}} \qquad (a > 0)$$

$$\left(\frac{ax+2}{x}\right)^{x^2} = \left(a + \frac{2}{x}\right)^{x^2} \longrightarrow \begin{cases} 0 & 0 < a < 1 \\ +\infty & a = 1 \\ +\infty & a > 1 \end{cases}$$

$$\left(\frac{ax+2}{x}\right)^{x^2} \qquad \left[\left(1 + \frac{2}{x}\right)^x\right]^x \to +\infty$$

$$\left[\left(\frac{ax+2}{x}\right)^{x^2} + x^4\right] \sim \begin{cases} x^4 & \text{se} \quad a < 1 \\ \left(\frac{ax+2}{x}\right)^{x^2} & \text{se} \quad a \geq 1 \end{cases}$$

da completare ....