$$\lim_{n\to+\infty}\Big(\underbrace{(\sin x-1)}_{t\in[-2,0]}+\frac{1}{n^2+1}\Big)^{n^2} \qquad (x\in\mathbb{R}) \text{ parametro}$$

$[t=0]$

$$\lim_{n\to+\infty}\left(\frac{1}{n^2+1}\right)^{n^2}=\lim_{n\to+\infty}\frac{1}{(n^2+1)^{n^2}}=\left(\frac{1}{+\infty}\right)=0^+$$

$-2\le t<-1 \Rightarrow$ La base $\left(t+\frac{1}{n^2+1}\right)$ è definitivamente negativa.

**OSS** $\alpha<0$

$$\alpha^{n^2}=(-|\alpha|)^{n^2}=\underbrace{(-1)^{n^2}}_{\overset{\shortparallel}{(-1)^n}}|\alpha|^{n^2}=(-1)^n|\alpha|^{n^2}$$

perché $n$ pari/dispari $\Leftrightarrow$ $n^2$ pari/dispari

$-2\le t<0 \Rightarrow t+\frac{1}{n^2+1}<0$ def$^{te}$

$$\left(t+\frac{1}{n^2+1}\right)^{n^2}\underset{\text{per } n \text{ grande}}{=}(-1)^n\left(-t-\frac{1}{n^2+1}\right)^{n^2}$$

**1° caso**

se $-1<t<0 \Rightarrow 0<-t<1$

$$\left(t+\frac{1}{n^2+1}\right)^{n^2}=\underbrace{(-1)^n}_{\text{limitata}}\underbrace{\left(-t-\frac{1}{n^2+1}\right)^{n^2}}_{\downarrow \; ((-t)^{+\infty}) \; \overset{\shortparallel}{0}}\to 0$$

in quanto: limitata per infinitesima

$-2 \leq t < -1 \Rightarrow 1 < -t \leq 2$

$$\left(t + \frac{1}{n^2+1}\right)^{n^2} \underset{\text{def te}}{=} (-1)^n \underbrace{\left(-t - \frac{1}{n^2+1}\right)^{n^2}}_{\to (-t)^{+\infty} = +\infty}$$ non ha limite.

$t = -1 \Rightarrow -t = 1.$

$$\left(-1 + \frac{1}{n^2+1}\right)^{n^2} = (-1)^n \boxed{\left(1 - \frac{1}{n^2+1}\right)^{n^2}} \to \frac{1}{e} > 0$$ Il limite non esiste.

$$= \frac{\boxed{\left(1 - \frac{1}{n^2+1}\right)^{n^2+1}} \to \frac{1}{e}}{\left(1 - \frac{1}{n^2+1}\right) \to 1}$$

## Risultati

Il limite non esiste se

$t \leq -1 \Leftrightarrow \sin x - 1 \leq -1$

$\Updownarrow$

$\sin x \leq 0$

$\Updownarrow$

$$\boxed{-\pi + 2k\pi \leq x \leq 2k\pi}$$

Il limite vale zero se

$-1 < t \leq 0, \Leftrightarrow -1 < \sin x - 1 \leq 0$

$\Updownarrow$

$0 < \sin x \leq 1$ sempre

$\Updownarrow$

$$\boxed{0 + 2k\pi < x < \pi + 2k\pi}$$

Ordine di infinitesimo, per $x \to +\infty$, di $f(x) = \dfrac{\sin\left(\frac{1}{x^2}\right)\left(e^{\frac{2}{x^3}} - 1\right)}{\left(\sqrt{1+\frac{3}{x}} - 1\right)\operatorname{arctg}\left(\frac{1}{\sqrt{x}}\right)}$

$$f(x) = \frac{\overbrace{\boxed{\sin \frac{1}{x^2}}}^{\sim \frac{1}{x^2}} \overbrace{\left(e^{\frac{2}{x^3}} - 1\right)}^{\sim \frac{2}{x^3}}}{\underbrace{\left(\sqrt{1+\frac{3}{x}} - 1\right)}_{\sim \frac{3}{2x}} \underbrace{\operatorname{arctg}\left(\frac{1}{\sqrt{x}}\right)}_{\sim \frac{1}{\sqrt{x}}}} \sim \frac{1}{x^2} \frac{2}{x^3} \frac{2x}{3} \sqrt{x} = \frac{4}{3} \frac{1}{x^{7/2}}$$

$\sqrt{1+t} - 1 = (1+t)^{1/2} - 1 \sim \frac{t}{2}$ $\quad t \to 0$

Infinitesimo di ordine $\frac{7}{2}$.

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Ordine dei seguenti infinitesimi per $x \to 0^+$.

$$\boxed{f(x) = x^2 \log x - x} = x\left(\underbrace{\underbrace{x \log x}_{\to 0} - 1}_{\to -1}\right) \sim -x$$

Inf.$^{\text{mo}}$ di ordine 1.

$$\boxed{g(x) = \cos(\sqrt{2}x) - e^{x^2} + 2x^2}$$

$$\cos t = 1 - \frac{t^2}{2} + \frac{t^4}{24} + o(t^5) \qquad t \to 0$$

$$\cos(\sqrt{2}x) = 1 - x^2 + \frac{4x^4}{24} + o(x^5) \qquad x \to 0$$

$$e^t = 1 + t + \frac{t^2}{2} + o(t^2) \qquad t \to 0$$

$$e^{x^2} = 1 + x^2 + \frac{x^4}{2} + o(x^5) \qquad x \to 0$$

$$g(x) = \cancel{1} - \cancel{x^2} + \frac{x^4}{6} + o(x^5) - \cancel{1} - \cancel{x^2} - \frac{x^4}{2} + \cancel{2x^2} =$$

$$= -\frac{x^4}{3} + o(x^5) \sim -\frac{x^4}{3}$$

$\boxed{\text{infinitesimo di ordine 4.}}$

$$\boxed{h(x) = \cosh x - \cosh(x - x^2)}$$

$$\cosh x = 1 + \frac{x^2}{2} + \frac{x^4}{24} + o(x^5) \qquad x \to 0$$

$$\cosh(x-x^2) = 1 + \frac{(x-x^2)^2}{2} + \frac{(x-x^2)^4}{24} + \underbrace{o\left((x-x^2)^5\right)}_{o(x^5)}$$

$$h(x) = \cancel{1} + \frac{x^2}{2} + \frac{x^4}{24} + o(x^5) - \cancel{1} - \frac{(x-x^2)^2}{2} - \frac{(x-x^2)^4}{24} =$$

$$= \cancel{\frac{x^2}{2}} + \frac{x^4}{24} + o(x^5) - \frac{1}{2}\left(\cancel{x^2} - 2x^3 + x^4\right) - \frac{1}{24}\left(x^4 - 4x^5\right)$$

$$= x^3 + o(x^3) \sim x^3$$

$\boxed{\text{inf}^{\text{mo}} \text{ di ordine 3.}}$